

UNIT - I

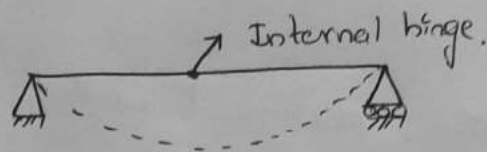
Equilibrium and Compatibility Equations:-

Equilibrium:-

Internal force equilibrates the external force. is called as equilibrium.

Mechanism:-

If no resistance against the deformation the structure is called as unstable structure.



Types of structure:-

Basically these are three types.

- (1) skeletal (1-D)
- (2) surface (2-D)
- (3) solid (3-D).

Skeletal:-

These are the structure member's are idealised to be series of lines and curves.

② Surface Structure:

These are idealised to be series of planes straight and curved.

Examples: Roof truss, Building frames.

→ These are two dimensional structure.





Ex: R.C.C slab, folded plates and shells.

③ Solid Structure:-

which can't be idealised into neither a skeleton nor a space (or) surface.

Ex: massive foundation, machine foundation.

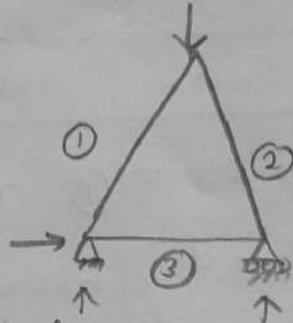
Types of Supports:-

Notation	Type of support	DOF
	Fixed	0
	Hinge	1
	Roller	2
	Free	3

DOF = degree of freedom of supports.

Statically determinate :-

If unknown reaction forces equal to no. of equation of equilibrium it is called as statically determinate structure.



3 no. of unknown reaction.

3 no. of unknown reaction member forces

6 unknown forces.

Equation of equilibrium we have in this structure is

$$\begin{aligned} \uparrow \sum F_y &= 0 \\ \rightarrow \sum F_x &= 0 \end{aligned}$$

$2 \times 3 = 6$ eq of equilibrium.

for every joint's.

$$\boxed{6 = 6}$$

Statically indeterminate :-

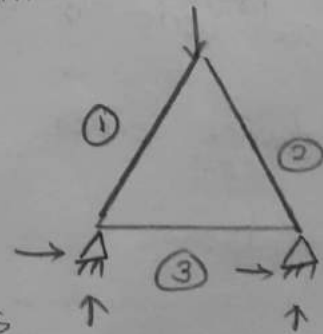
If unknown reaction forces are not equal to no. of equation of equilibrium it is called as indeterminate structure.

3 no. of unknown reaction.

4 no. of member forces

7

equation of equilibrium we have in this



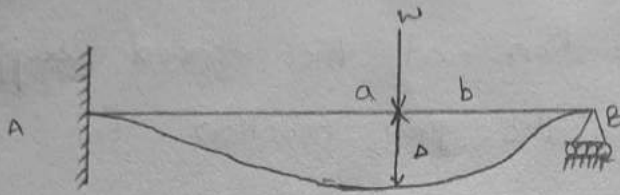
structure is '6' $6 \neq 7$ not satisfied.

Hence it is a statically indeterminate.

→ Differences between the statically determinate and statically indeterminate structure.

statically determinate structure	statically indeterminate structure
<p>① The determination of internal forces in the structures do not require the cross-sectional areas and moment of inertia of beams and frame members.</p>	<p>① The cross-sectional area and moment of inertia are required as first step of analysis. This quantities has effect on the values of internal forces and moments.</p>
<p>② The internal forces and moments are not effected by settlement of supports or lack of fit.</p>	<p>② The settlements and lack of fit are the important factors in determination of internal forces and moments.</p>
<p>③ The equilibrium equations are adequate to determine the internal forces and moments.</p>	<p>③ The compatibility conditions of displacements are required over and above the equilibrium conditions to determine the internal forces and moments.</p>

Propped Cantilever Beam:-



Consider a straight beam with the end 'A' is clamped and 'B' is supported by a prop.

→ It is a statically indeterminate to one degree as the number of unknowns are four.

→ ① vertical ② horizontal ③ fixed end moment and ④ prop reaction.

→ The available equations of equilibrium are three viz $\sum H$, $\sum V=0$, and $\sum M=0$.

→ As three equation are not sufficient to determine four unknowns, one more equation can be formulated by knowing the final position of B relative to A' is known.

→ The props are classified as.

(i) Rigid props

(ii) Elastic props.

(1) The total number of support reactions are four, i.e. three unknown reactions at the fixed support and the fourth one is the prop reaction. The available equilibrium equations are three.

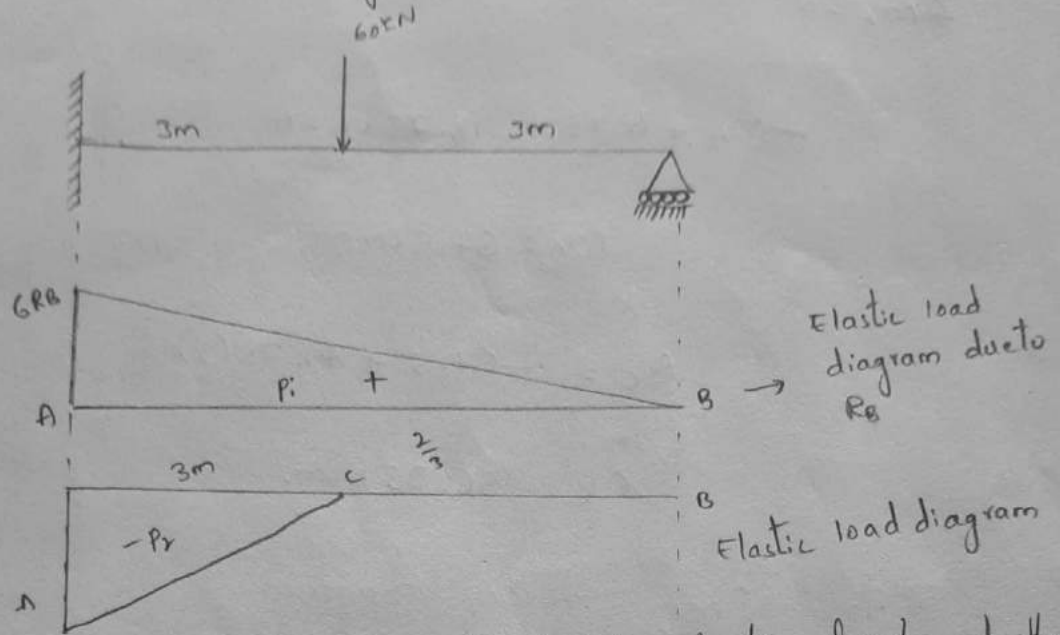
(2) As an initial step draw the elastic load diagram (M/EI). Due to the prop reaction.

The bending moment is positive if the moment is acting clockwise to the left of the section, and if it is acting anticlockwise to the right of the section.

(3) Draw the free body diagram and use the equilibrium to calculate the other reaction and moment.

(4) Draw the shear force diagram, Bending moment diagram and the elastic curve.

① Draw SFD and BMD for the propped cantilever beam loaded as shown in figure use consistent deformation method.



Thus, $\sum_{i=1}^n P_i x_i = 0$ is zero ' P_i ' is the total elastic load of the respective diagram and ' x_i ' is the centroidal distance of the elastic load diagram.

$$x_1 = \frac{2}{3} \times 6 = 4 \text{ m.}$$

$$x_2 = 3 \times \frac{2}{3} + 3 = 5 \text{ m.}$$

$$\sum M_B = 0.$$

$$P_1 x_1 + P_2 x_2 = 0.$$

$$\left(\frac{1}{2}\right)(6) \times 6R_B \times 4 - \left(\frac{1}{2} \times 3 \times 180\right) \times 5 = 0.$$

$$R_B = 18.75 \text{ kN.}$$

$$R_A = 60 - 18.75$$

$$R_A = 41.25 \text{ kN.}$$

$$\Sigma V = 0$$

$$\Sigma M_A = 0$$

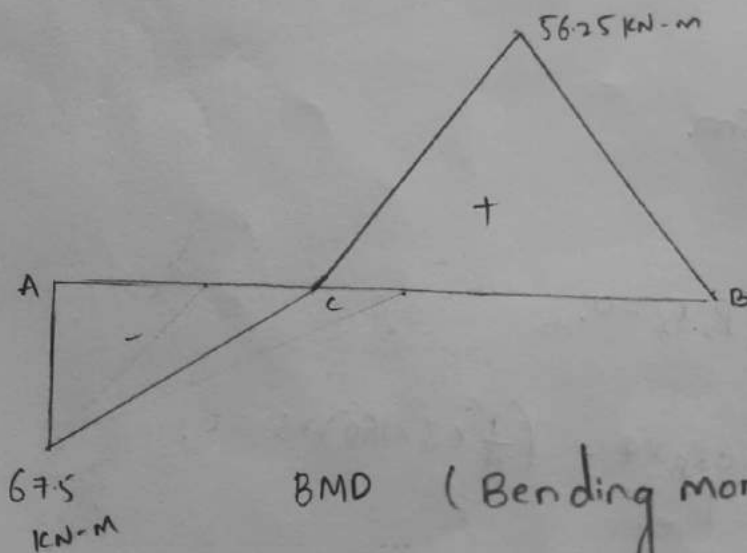
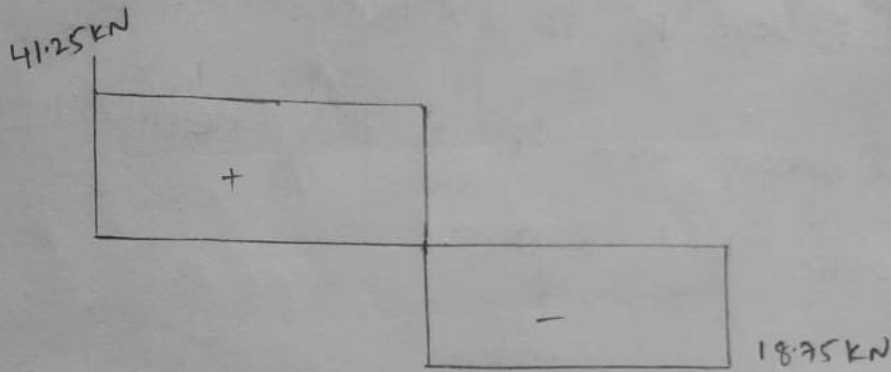
$$-M_A + 60 \times 3 - 18.75 \times 6 = 0$$

$$M_A = 67.5 \text{ kNm.}$$

$$M_C = -67.5 + 41.25(3)$$

$$M_C = 56.25 \text{ kNm.}$$

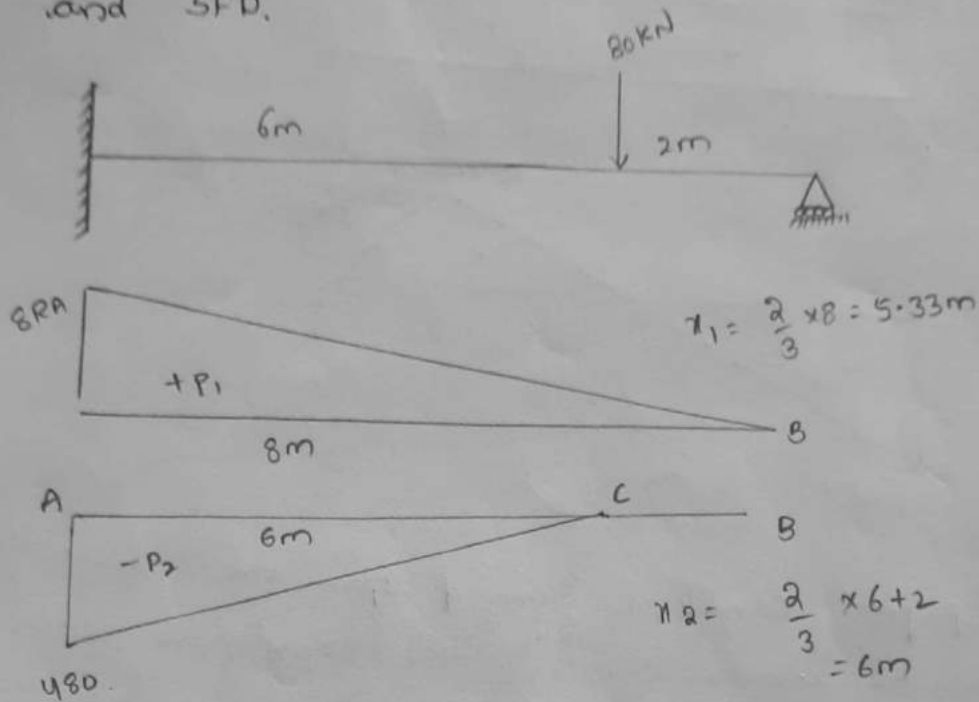
SFD: Shear force diagram



BMD (Bending moment diagram)

② Analyse the propped cantilever beam shown in sketch

BMD and SFD.



30ln Taking EI as unity the above elastic load diagrams were drawn. As the deflection at B is zero.

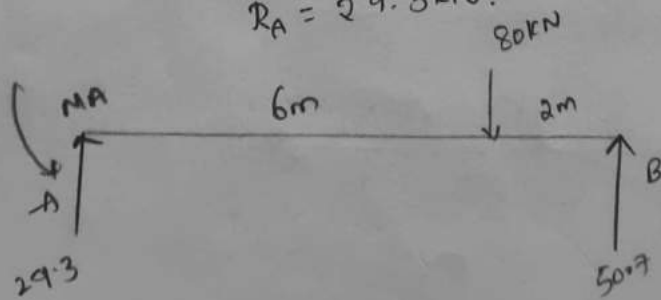
$$M_B = 0.$$

$$P_1 x_1 + P_2 x_2 = 0$$

$$\Rightarrow \left(\frac{1}{2} \times 8\right) \times 8 R_B \times 5.33 - \left(\frac{1}{2} \times 6 \times 480\right) \times 6 = 0.$$

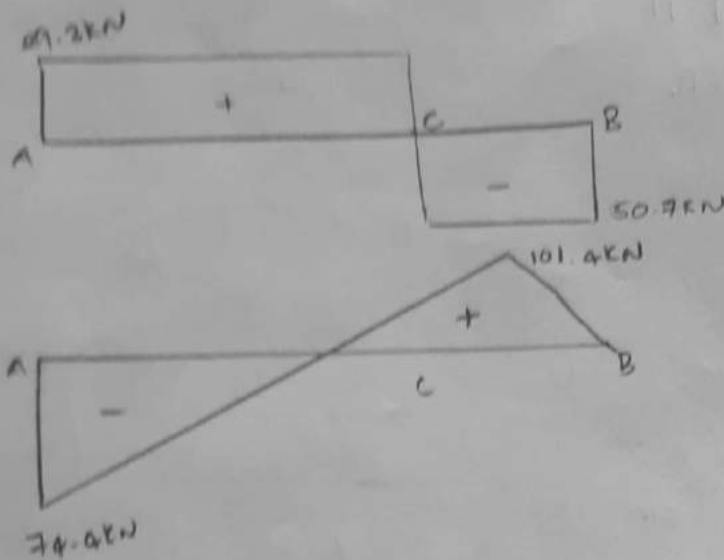
$$R_B = 50.66 \text{ kN.}$$

$$R_A = 29.3 \text{ kN.}$$

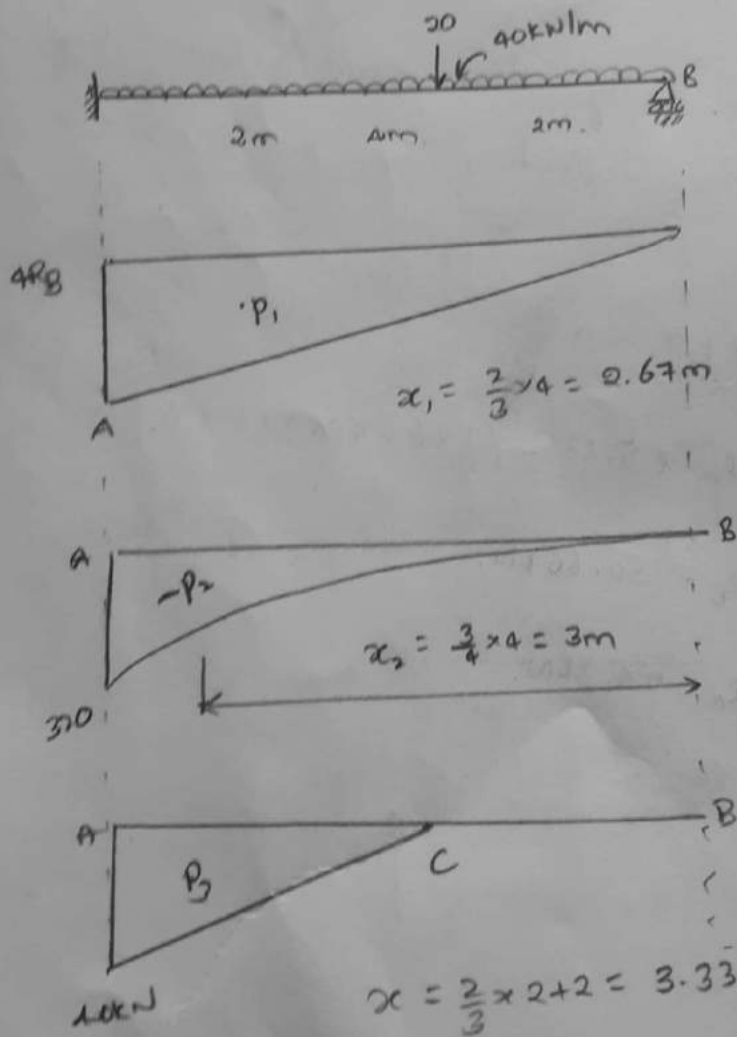


$$80 \times 6 - M_A - 50.7 \times 8 = 0 \Rightarrow M_A = 74.4 \text{ kNm.}$$

SFD:-



③ a) propped cantilever beam of span 4m is subjected to UDL of intensity 40 kN/m throughout the span draw SFD and BMD.



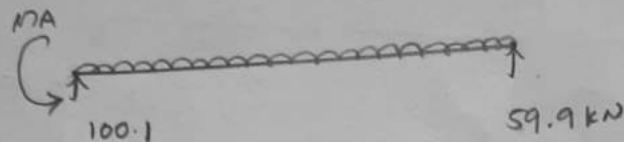
Sol. The deflection at the prop 'B' is zero, Hence

$$P_1 x_1 + P_2 x_2 = 0.$$

$$\left(\frac{1}{2}\right) 4(4R_B) \times 2.67 + \left(\frac{1}{8} \times 4 \times 320\right) \times 3 = 0.$$

$$R_B = 59.9 \text{ kN}$$

$$R_A = 160 - 59.9 = 100.1 \text{ kN} \Rightarrow R_A = 100.1 \text{ kN}.$$



Taking moment about A:

$$-M_A + (40 \times 4 \times \frac{4}{2}) - 59.9 \times 4 = 0.$$

$$M_A = 80.4 \text{ kNm}.$$

To find the maximum positive bending moment equate the shear force equation to zero

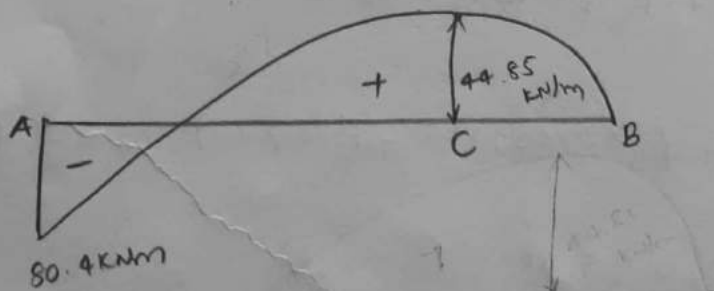
$$100.1 - 40x = 0$$

$$x = 2.5 \text{ m}.$$

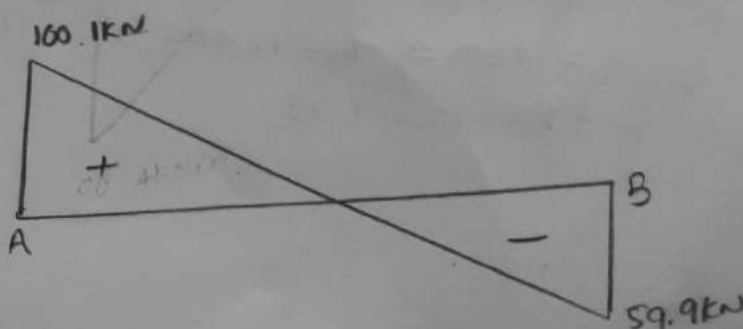
$$\text{Maximum positive BM} = -80.4 + 100.1(2.5) - 40 \times \frac{2.5^2}{2}$$

$$M_C = 44.85 \text{ kNm}.$$

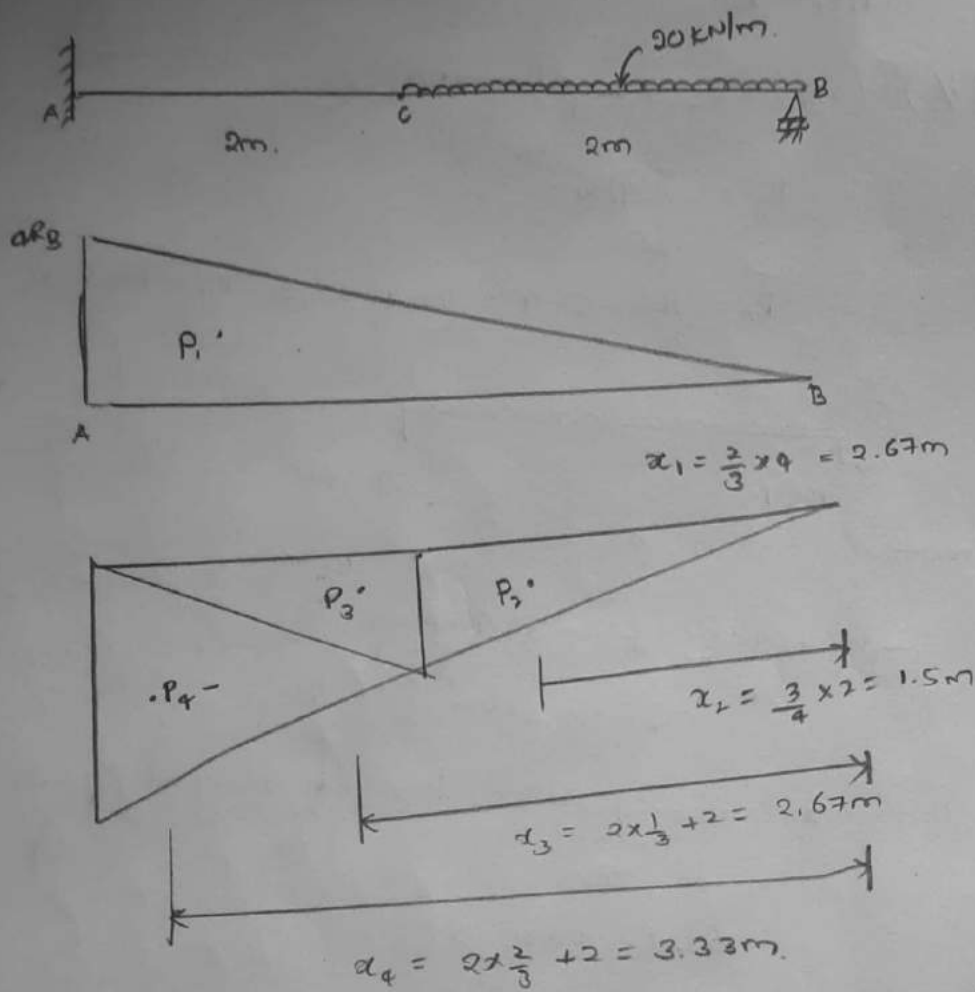
BMD



SFD



① Analyse the propped cantilevers shown in fig and draw SFD and BMD.



Elastic load diagram due to UDL.

The deflection at B is zero.

$$P_1 x_1 + P_2 x_2 + P_3 x_3 + P_4 x_4 = 0.$$

$$\left(\frac{1}{2}\right) 4 (40) 2.67 - \left(\frac{1}{3} \times 2 \times 40\right) \times 1.5 - \left(\frac{1}{2} \times 2 \times 40\right) 2.67 - \frac{1}{2} \times (2 \times 40) \times (3.33)$$

$$R_B = 25.6 \text{ kN}$$

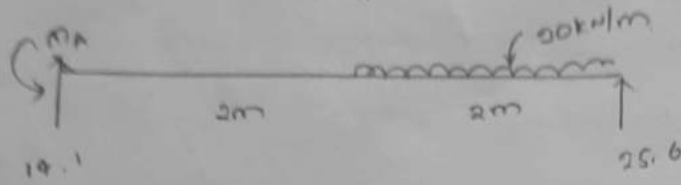
$$\sum V = 0.$$

$$R_A + R_B = 40 \Rightarrow R_A = 14.4 \text{ kN}.$$

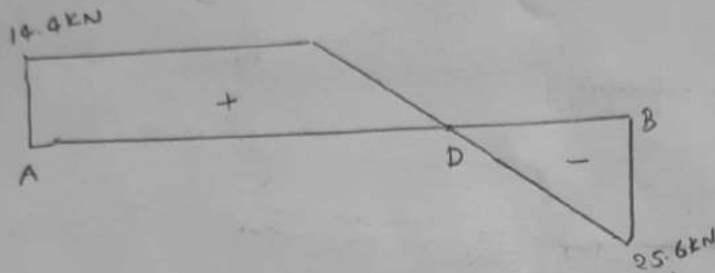
Taking moment about A;

$$-M_A + 20 \times 2 \times (2+4) - 25.6 \times 4 = 0$$

$$M_A = 17.6 \text{ kNm}$$



SFD



To obtain the maximum positive bending moment the location of zero shear is found out from 'B'.

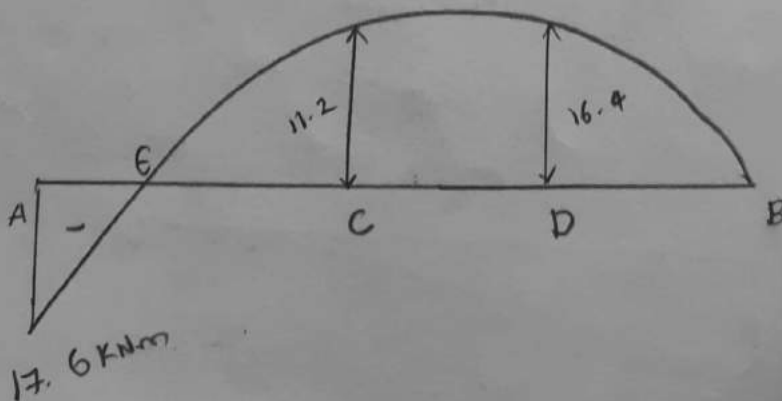
$$R_B - 20(x) = 0$$

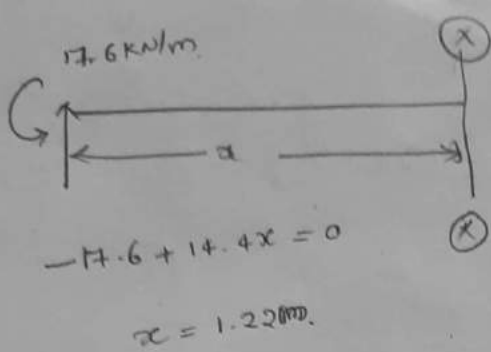
$$25.6 - 20(x) = 0$$

$$x = 1.28 \text{ m}$$

$$\text{Max +ve B.M} = 25.6(1.28) - 20 \times \frac{1.28^2}{2}$$

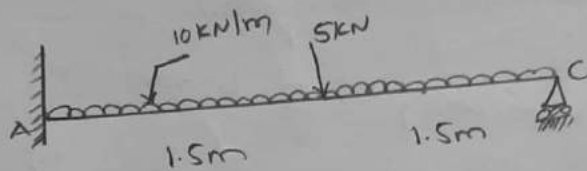
$$M_D = 16.4 \text{ kNm}$$



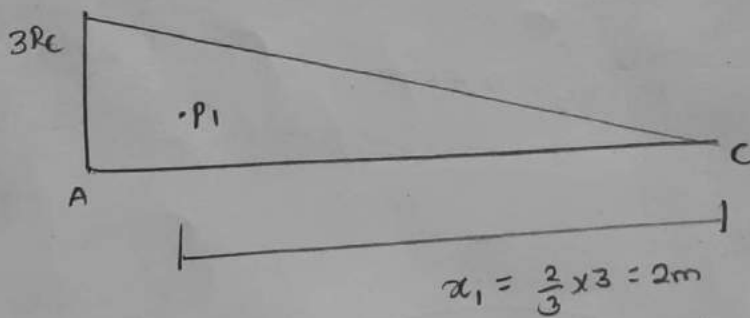


The point of contraflexure is found out by equating to bending moments is zero.

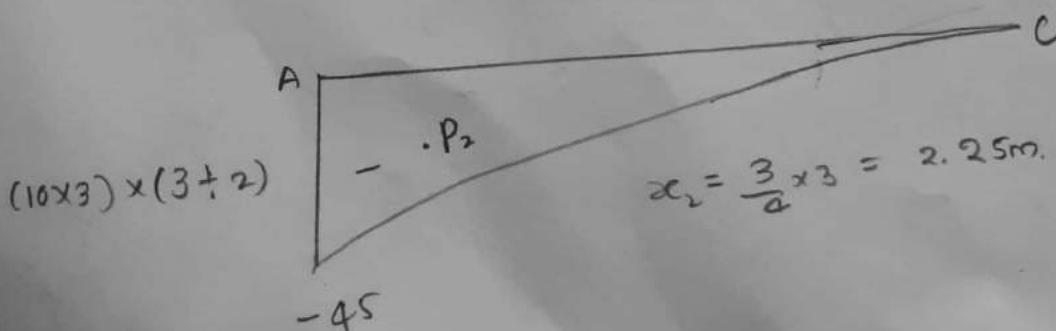
5) Analyse the propped cantilever loaded as shown in figure. Draw BMD and SFD. Assume constant EI throughout the span.

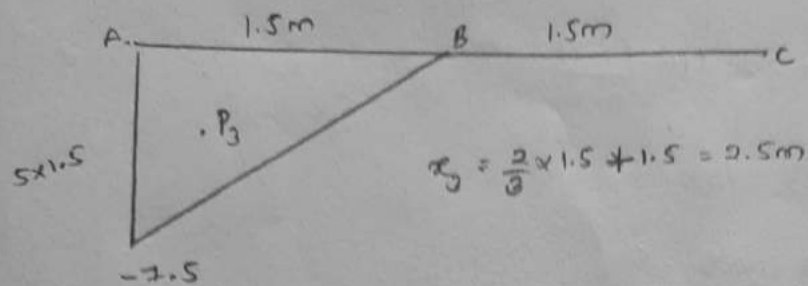


The elastic load diagram due to prop reaction applied and concentrated load are given below as EI is constant it is taken as unity.



Elastic load diagram for UDL





as the deflection at

$$P_1 x_1 + P_2 x_2 + P_3 x_3 = 0.$$

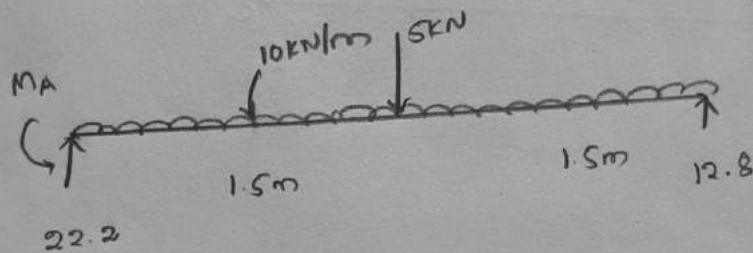
$$= \frac{1}{2}(3)(3R_c)2 - \left(\frac{1}{3}\right)3 \times 45(2.25) - \left(\frac{1}{2} \times 1.5 \times 7.5\right) \times 2.5 = 0$$

$$9R_c - 101.25 - 14.0625 = 0$$

$$R_c = 12.8 \text{ kN}$$

$$R_A = 10(3) + 5 - 12.8 = 22.2 \text{ kN}$$

$$R_A = 22.2 \text{ kN}$$



Taking moment about A:

$$-M_A + 5(1.5) + 10 \times \frac{3^2}{2} - 12.8 \times 3 = 0$$

$$M_A = 14.1 \text{ kN/m}$$

The point of contraflexure is located as.

$$-14.1 + 22.2x - 10 \cdot \frac{x^2}{2} = 0.$$

$$5x^2 - 22.2x + 14.1 = 0$$

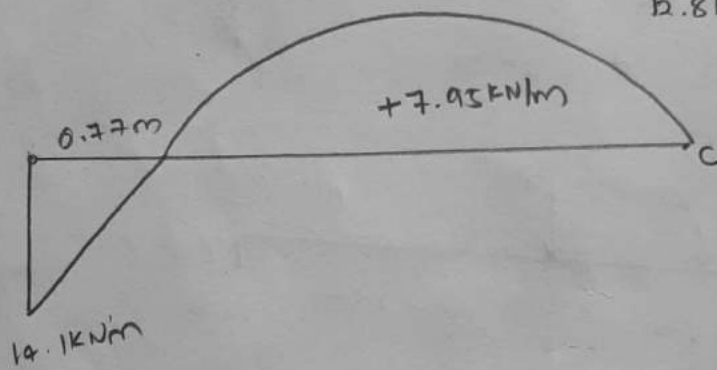
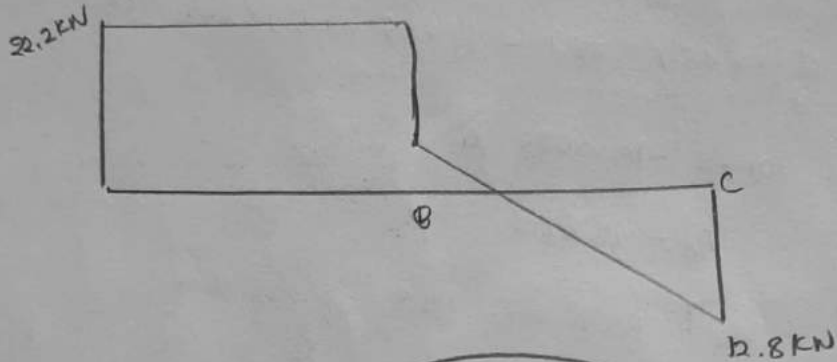
$$x = 0.77 \text{ m}$$

$$-14.1 + 22.2(2.2) - \frac{(10)(2.2)^2}{0.2}$$

$$22.2 - 5x(1.5) - 10x = 0$$

$$14.7 - 10x = 0$$

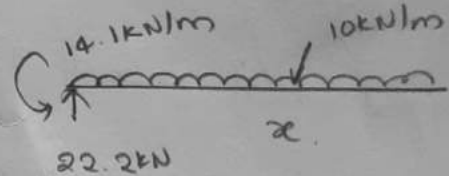
$$x = 1.47$$



$$-14.1 + 22.2x - \frac{10x^2}{2} = 0$$

$$5x^2 - 22.2x + 14.1 = 0$$

$$x = 0.77m$$



Fixed Beams :-

The fixed beam can be considered as a simply supported beam with end moments.

- The restraint moments at the ends reduce the slope of the beam and therefore they are of opposite sign to the moments due to applied load.
- The free body diagram of entire beam contains four unknowns namely R_A , R_B and M_A , M_B .
- The fixed beam is statically indeterminate to second degree as only two equilibrium equations can be used.

The analysis is done as follows :-

- (1) Remove the statically indeterminate forces and moments at one end make the beams as a cantilever beam which is statically determinate.
- (2) Draw the B.M.D due to the external loading.
- (3) Divide each ordinate of the bending moments diagram by EI and draw elastic load (M/EI) diagram.
- (4) Draw the (M/EI) diagram for the unknown forces and the moment at the end support

(5) The elastic curve of the fixed beam infers that the slope at each end is horizontal due to the built in supports.

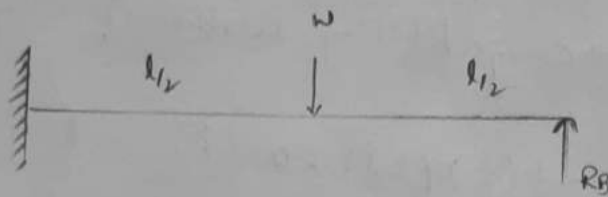
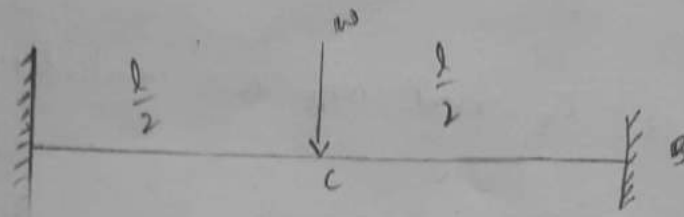
Hence the change of slope b/w the end supports is zero.

(6) A second compatibility condition is obtained by noting that tangential deviation of the support with respect to other support is zero.

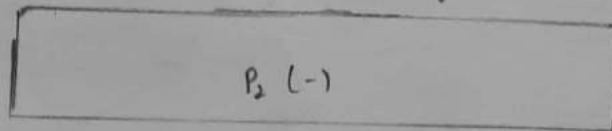
(7) Using the above two compatibility conditions the unknown reaction and moment are calculated.

(8) Draw the free body diagram and determine the other reaction and moments sketch the shear force diagram, B.M.D and the elastic curve.

① Analyse the beam calculate BM values and SF values and draw BMD and SFD for the fixed beams as shown below

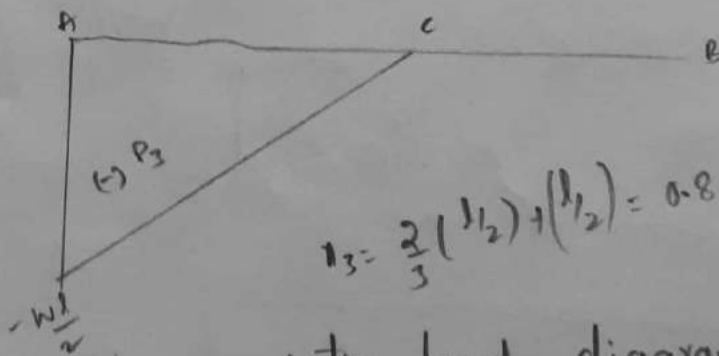


$x_1 = \frac{2l}{3}$
Elastic Load diagram due to R_B



$x_2 = l/2$

elastic load diagram due to M_B



$x_3 = \frac{2}{3} \left(\frac{l}{2} \right) + \left(\frac{l}{2} \right) = 0.833l$

elastic load diagram due to application of point load "w".

Soln The elastic load diagrams were drawn due to R_B , M_B and due to applied load. are given in the above diagram.

The values of R_B and M_B are calculated the compatibility equation.

$\sum p_i = 0$ (first compatibility condition)

$$= \left(\frac{1}{2}\right)(1)(R_B l) - M_B l - \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{wl}{2}\right) = 0$$

$$R_B \frac{l^2}{2} - M_B l = \frac{wl^2}{8} \rightarrow \textcircled{1}$$

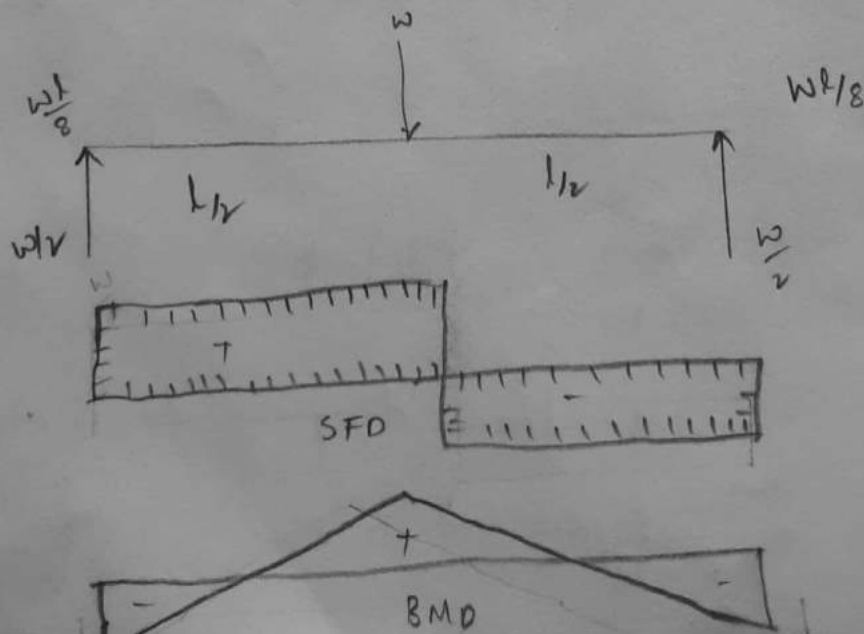
$\sum p_i \alpha_i = 0$ (second compatibility condition)

$$R_B \frac{l^2}{2} \times \left(\frac{2l}{3}\right) - M_B l \left(\frac{l}{2}\right) = \frac{wl^2}{8} \left(\frac{5l}{8}\right) (0.83)$$

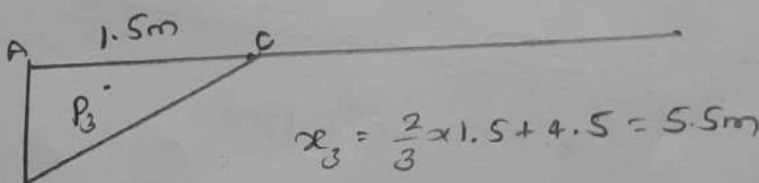
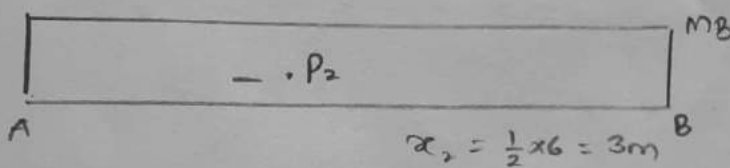
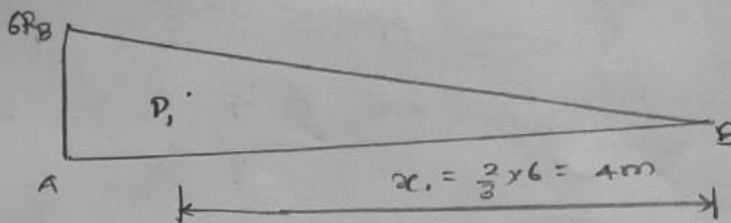
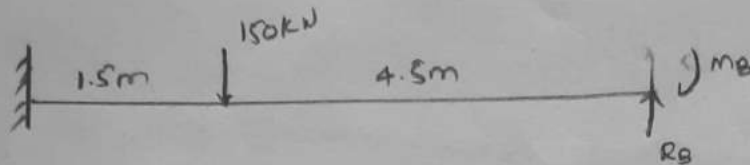
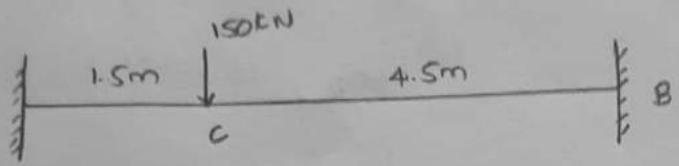
Solving $\textcircled{1}$ & $\textcircled{2}$

$$R_B = w/2$$

$$M_B = \frac{wl}{8}$$



② A fixed beam of 6m span carries a concentrated load of 150kN at a distance of 1.5m from the left support calculate the bending moment at mid span.



$\sum p_i = 0$ (first compatibility).

$$\left(\frac{1}{2} \times 6 \times 6R_B\right) - 6M_B - \frac{1}{2} \times 1.5 \times 225 = 0.$$

$$18R_B - 6M_B = 168.75 \rightarrow \textcircled{1}$$

$\sum p_i x_i = 0$ (second compatibility condition)

$$18R_B(4) - 6(M_B)(3) = 168.75(5.5)$$

$$72R_B - 18M_B = 928.13 \rightarrow \textcircled{2}$$

Solving

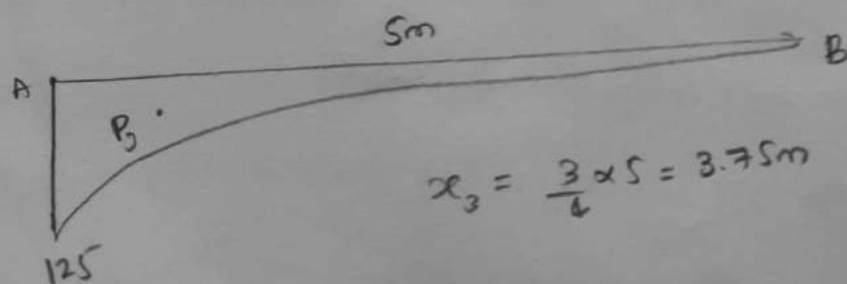
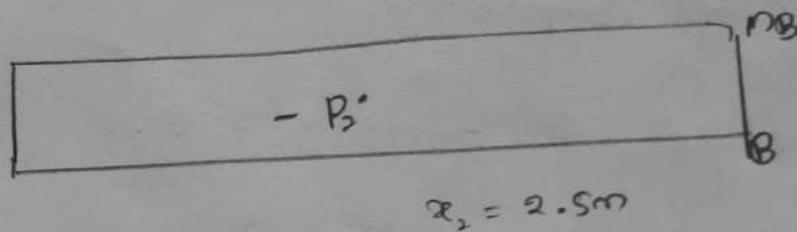
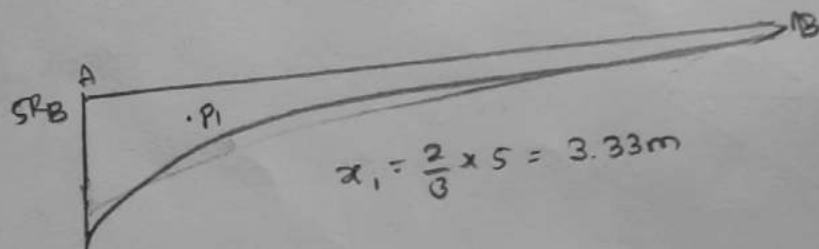
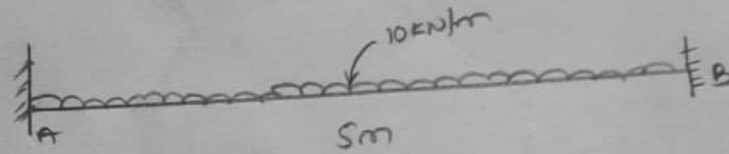
$$R_B = 23.44 \text{ kN}$$

$$M_B = 42.2 \text{ kNm}$$

$$\text{Bending moment at midspan} = 23.44(3) - 42.2$$

$$M_D = 28.12 \text{ kN}$$

- ③ For a rigidly fixed beam AB of span 5m carrying a Udl of 10kN/m over a entire span, locate the point of contraflexure and draw B.M.D and SFD.



$\sum p_i = 0$ (first compatibility equation)

$$\left(\frac{1}{2}\right)(5)(5R_B) - 5M_B - \frac{1}{3} \times 5 \times 125 = 0$$

$$12.5R_B - 5M_B = 208.33 \rightarrow \textcircled{1}$$

$\sum p_i x_i = 0$ (second compatibility equation)

$$12.5R_B(3.33) - 5M_B(2.5) - 208.33 \times 3.75 = 0$$

$$41.625R_B - 12.5M_B = 781.25 \rightarrow \textcircled{2}$$

By solving $\textcircled{1}$ & $\textcircled{2}$

$$R_B = 25 \text{ kN} \quad \text{and} \quad M_B = 21 \text{ kNm}$$

$$\sum v = 0 \quad R_A + 25 = 5 \times 10$$

$$R_A = 25 \text{ kN}$$

$$\sum m = 0 \quad -M_A + \frac{10 \times 5^2}{2} + 21 = 25 \times 5 = 0$$

$$M_A = 20.83 \text{ kNm}$$

Max +ve B.M at

$$M_c = -20.83 + 25 \times 2.5 - 10 \times \frac{2.5^2}{2}$$

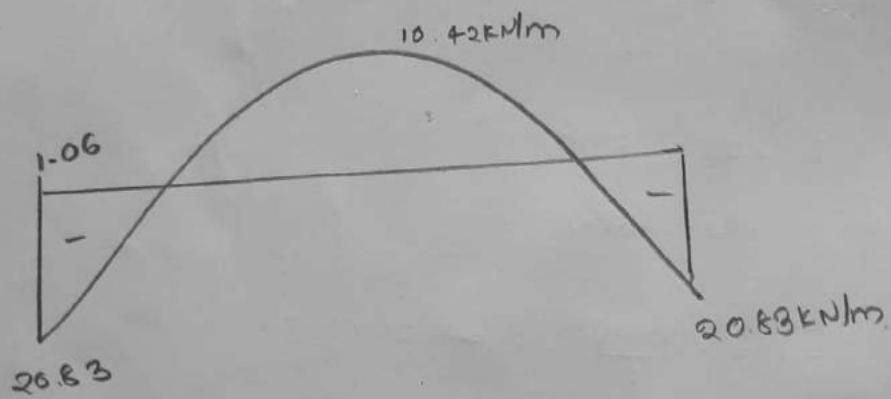
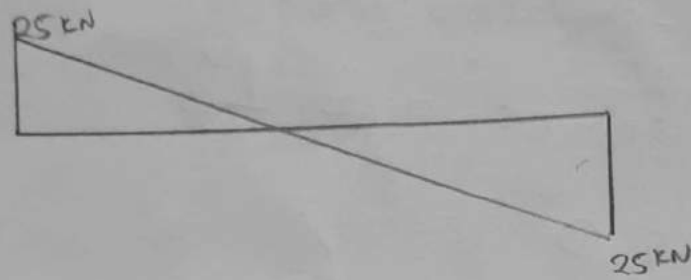
$$M_c = 10.42 \text{ kNm}$$

Contraflexure :-

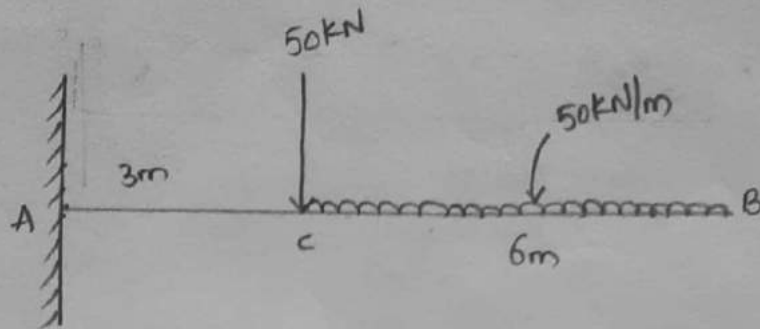
$$25x - 20.83 - 10 \frac{x^2}{2} = 0$$

$$5x^2 - 25x + 20.83 = 0$$

$$x = 3.94\text{m}, x = 1.06\text{m}$$



④ Determine the moments at the supports of a fixed beam ACB of span. AC = 3m, CB = 6m fixed at A and B it carries a concentrated load of 50kN at C and UDL of 50kN/m over the entire span.



$$\sum P_i x_i = 0 \Rightarrow \left(\frac{1}{2}\right) \times 9 \times (9R_B) - 9M_B - \frac{1}{3} \times 9 \times 2025 - \frac{1}{3} \times 3 \times 150 = 0$$

$$40.5R_B - 9M_B = 6075 + 225$$

$$40.5R_B - 9M_B = 6300 \rightarrow \textcircled{1}$$

$$\sum P_i x_i^2 = 0$$

$$40.5R_B(6) - (9M_B \times 4.5) = 6075 \times 6.75 + 225 \times 8$$

$$243R_B - 40.5M_B = 42806.425$$

By solving ① & ②

$$R_B = 237.96 \text{ kN}$$

$$M_B = 370.8 \text{ kNm}$$

$$\sum V = 0 \quad R_A + R_B = 50 + 50 \times 9 \Rightarrow R_A = 500 - 237.96$$

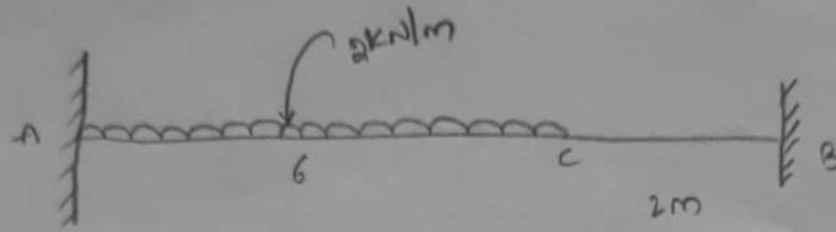
$$R_A = 262 \text{ kN}$$

$$\sum M = 0$$

$$-M_A + 50 \times 3 + 370.8 - 237.96 \times 9 + 50 \times \frac{9^2}{2} = 0$$

$$M_A = 409.2 \text{ kNm}$$

⑤ Determine the fixed end moments for the beam shown in figure.



$$\sum p_i = 0$$

$$\left(\frac{1}{2}\right)(8)(8R_B) - 8M_B - \frac{1}{3} \times 6 \times 36 = 0$$

$$32R_B - 8M_B = 72 \rightarrow \textcircled{1}$$

$$\sum p_i x_i = 0$$

$$32R_B(5.33) - 8M_B(4) = 72 \times 6.5$$

$$170.56R_B - 32M_B = 468 \rightarrow \textcircled{2}$$

By solving $\textcircled{1}$ & $\textcircled{2}$.

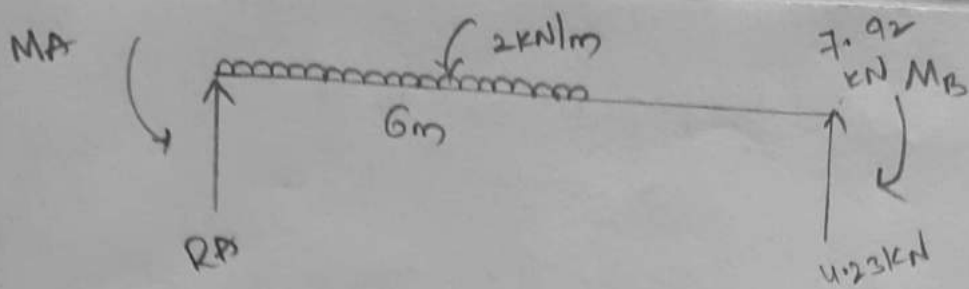
$$R_B = 4.23 \text{ kN}$$

$$M_B = 7.29 \text{ kNm}$$

$$\sum v = R_A + R_B \Rightarrow 6 \times 2$$

$$R_A + 4.23 = 12$$

$$R_A = 7.78 \text{ kN}$$



$$\sum M = 0$$

$$-M_A + 7.92 - 8 \times 4.23 + 2 \times \frac{6^2}{2} = 0$$

$$M_A = 10.1 \text{ kNm}$$

Maximum positive bending moment.

$$M_D = -10.1 + 7.78(x) - 2 \times \frac{x^2}{2}$$

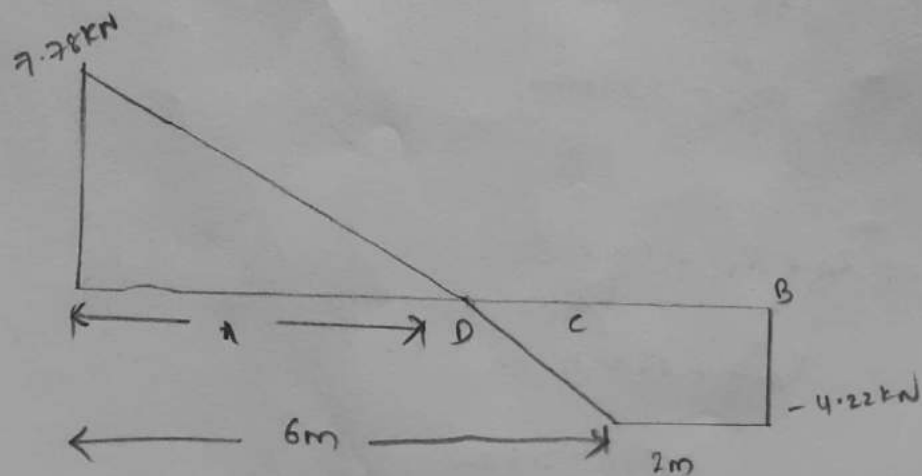
$$M_D = -10.1 + 7.78(3.89) - 2 \times \frac{3.89^2}{2}$$

$$M_D = 5$$

$$\frac{x}{6-x} = \frac{7.78}{4.22}$$

$$4.22x = 46668 - 7.78x$$

$$x = 3.89 \text{ m}$$



S.F.D

To locate x_1

$$-10.1 + 7.78(x_1) - \frac{2x_1^2}{2} = 0$$

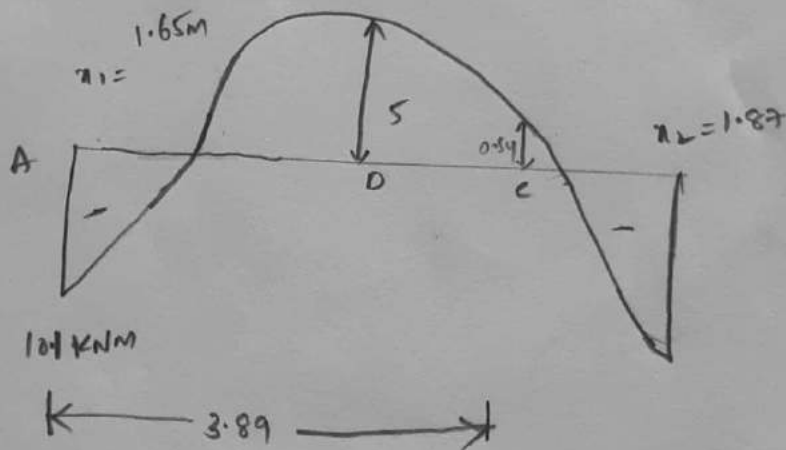
$$x_1^2 - 7.78x_1 + 10.1 = 0$$

$$x_1 = 1.65\text{m}$$

To locate $x_2 = 4.23x_2 - 7.92 = 0$.

$$x_2 = 1.87\text{m}$$

BMD :-



Analysis of Perfect Frames:

Plane frames:

→ The plane trusses are here by termed as plane frame

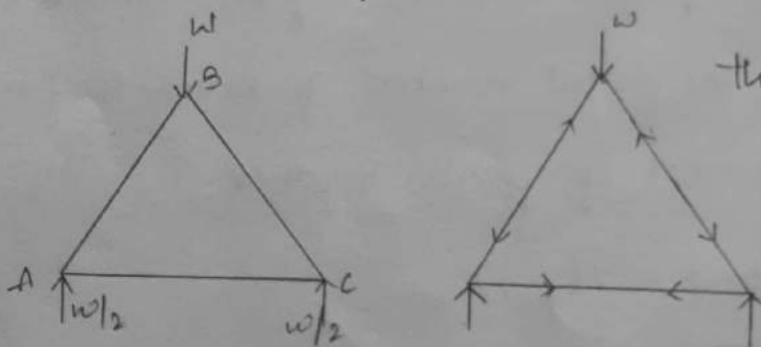
→ A statically determinate frame can be completely analysed by using statics.

→ The no. of unknown forces is the same as the no. of equation of equilibrium.

→ The main methods in analysing statically determined pin jointed plane frames.

- i) Graphical Solution
 - ii) Method of resolution at joints
 - iii) Method of section
- } 3 are used in plane frames (trusses)

iv) Tension coefficient method → For analysis of the space frame.



→ A truss is an assemblage of three (or) more members which are hinged (or) pinned.

→ A load applied on the truss is transmitted to all joints so that the members are in pure compression (or) pure tension.

Types of frames:

Basically frames are divided into 3 types:

- i) Perfect frame
- ii) Imperfect frame.
- iii) Redundant frame.

Perfect Frame: The simplest perfect frame is in the form of a triangle.

In essence the first three joints (or) pins require three bars to connect them while the remaining $(j-3)$ joints require $2(j-3)$ extra bars.

If "m" is the total number of bars (or) members required to connect "j" joints together

$$m-3 = 2(j-3)$$

$$m-3 = 2j-6$$

$$m = 2j-3$$

i.e.,

number of members to form a perfect frame

= Twice the number of joints - 3

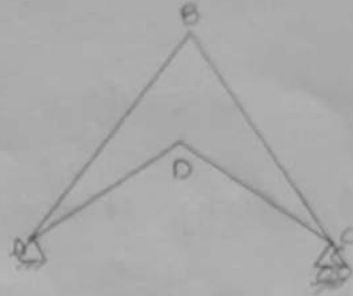
Imperfect frame:

An imperfect frame is one where the number of members is less than $(2j-3)$ then frame is called as imperfect frame.

$$m < 2j - 3$$

$$4 < (2 \times 4) - 3$$

$$4 < 5$$



Redundant Frame:

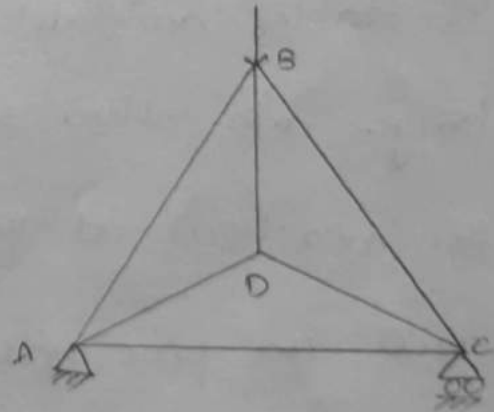
A redundant frame is one where the number of members are more than $(2j-3)$ then the frame is called as redundant frame.

$$m > (2j - 3)$$

$$6 > (2 \times 4) - 3$$

$$6 > (8) - 3$$

$$6 > 5$$



i.e., Redundant frame is having more members necessary to produce stability.

Method of Joints:

- In this method we are determined the member forces by using equilibrium conditions at particular joints.
- In this resolution of forces at the joint as well as free body diagram at the joint is considered

Procedure:

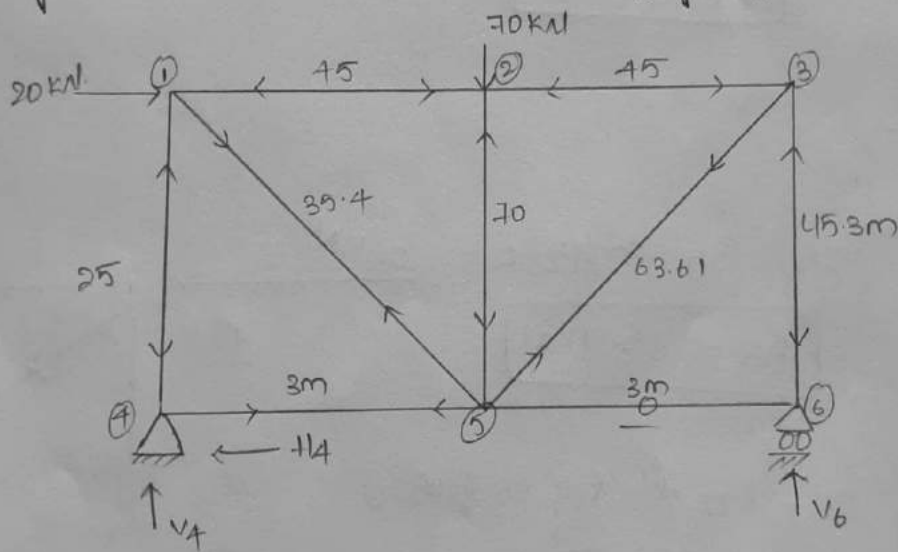
- 1) Check the stability and assess its determinacy of the truss.
- 2) If the truss is of cantilever type, the reactions need not be computed in general if the truss is stable and determinate where one support is hinge and other support is on rollers. Compute the reactions at the supports.
- 3) Draw the free body diagram at each joint and analyse the member forces at a joint where only two members meet.
- 4) Then consider the adjacent joint where only two unknown forces to be determined. This process is

repeated till the analyses of all joints are completed.

- 5) The results are tabulated along with magnitude of member forces and the nature of forces.
- 6) The forces are tensile if they are pulling (acting away) the joints.
- 7) The forces are compressive in nature if they are pushing (acting towards) the joints.

Example problems

① Analyse the truss as shown in fig by method of joints.



$$\sum H = 0$$

$$20 - H_4 = 0$$

$$H_4 = 20 \text{ kN}$$

(9) Determinacy:

$$NM + NR = 2 \times \text{Joints}$$

$$9 + 3 = 2 \times 6$$

$$12 = 12$$

$$\sum V = 0$$

$$V_4 + V_6 = 70 \text{ kN}$$

$$\sum M_4 = 0 \quad 20 \times 3 + 70 \times 3 - 6V_6 = 0$$

$$V_6 = 45 \text{ kN}$$

$$V_4 + 45 = 70 \text{ kN}$$

$$V_4 = 70 - 45 \Rightarrow V_4 = 25 \text{ kN}$$

Joint 4:

$$\sum H = 0$$

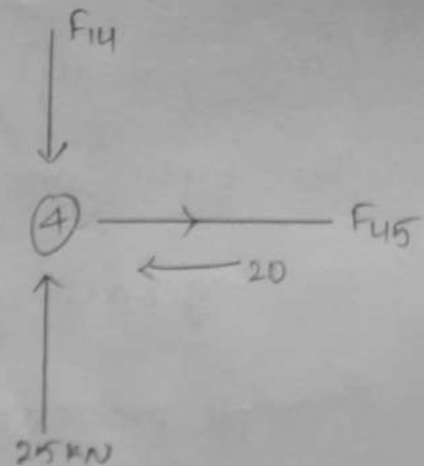
$$F_{45} - 20 = 0$$

$$F_{45} = 20 \text{ kN}$$

$$\sum V = 0$$

$$-F_{14} + 25 = 0$$

$$F_{14} = 25 \text{ kN}$$



Joint 1:

$$\sum V = 0$$

$$25 - F_{15} \sin 45 = 0$$

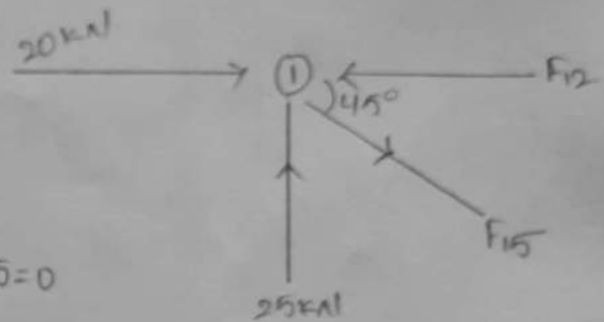
$$F_{15} = 35.4 \text{ kN}$$

$$\sum H = 0$$

$$20 - F_{12} + F_{15} \cos 45 = 0$$

$$20 - F_{12} + 35.4 \cos 45 = 0$$

$$F_{12} = 45.03 \text{ kN}$$

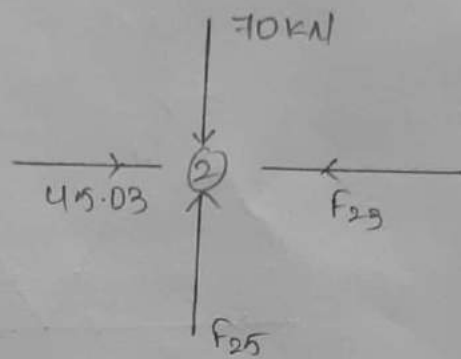


Joint-2:

$$\sum H = 0$$

$$45.03 - F_{23} = 0$$

$$F_{23} = 45.03$$



$$\sum V = 0 \quad F_{25} - 70 = 0$$

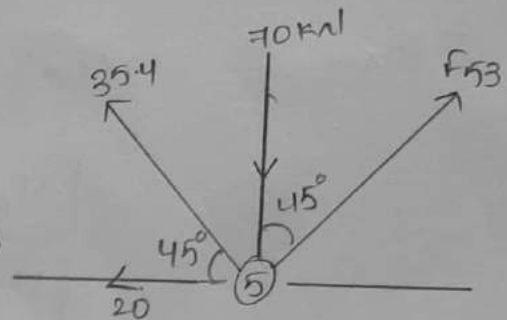
$$F_{25} = 70.0 \text{ kN}$$

Joint-5:

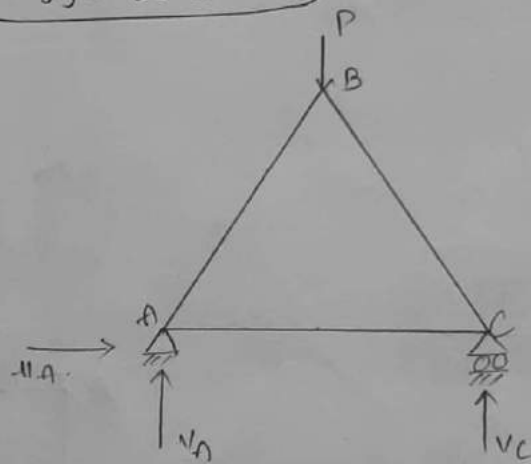
$$\sum V = 0$$

$$F_{53} \cos 45^\circ + 35.4 \sin 25^\circ - 70 = 0$$

$$F_{53} = 63.61 \text{ kN}$$

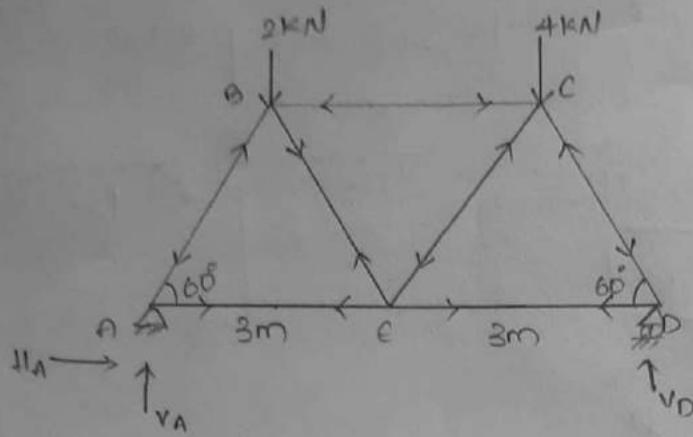


method of joints



- i) check the determinacy \rightarrow statically determinacy.
- ii) Isolate joints draw FBD
(2 unknown forces)
- iii) Apply equation of equilibrium.

Ex 2



(i) Determinacy

$$NM + NR = 2 \times J$$

$$7 + 3 = 2 \times 5$$

$$10 = 10$$

$$\sum V = 0$$

$$H_A + V_D = 2 + 4 = 6 \text{ kN}$$

$$\sum M_A = 0$$

$$2(1.5) + 4(4.5) - 6 V_D = 0$$

$$V_D = 3.5 \text{ kN}$$

$$H_A = 2.5 \text{ kN}$$

Joint A:

$$\sum V = 0$$

$$F_{AB} \sin 60 = 2.5$$

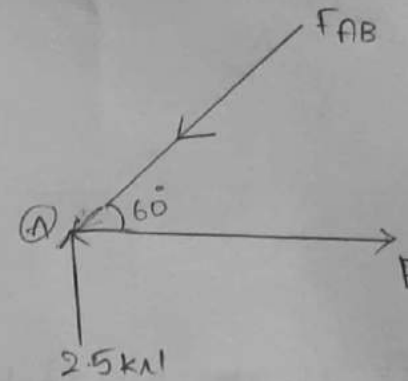
$$F_{AB} = 2.89 \text{ kN}$$

$$\sum H = 0$$

$$F_{AE} = F_{AB} \cos 60$$

$$F_{AE} = 2.89 \cos 60$$

$$F_{AE} = 1.45 \text{ kN}$$



Joint-D:

$$\sum V = 0$$

$$F_{CD} \sin 60^\circ = 3.5$$

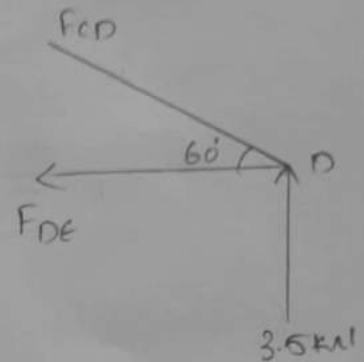
$$F_{CD} = 4.04 \text{ kN}$$

$$\sum H = 0$$

$$F_{CD} \cos 60^\circ - F_{DE} = 0$$

$$F_{DE} = 4.04 \cos 60^\circ$$

$$F_{DE} = 2.02 \text{ kN}$$



Joint-E:

$$\sum V = 0$$

$$F_{EC} \sin 60^\circ + F_{EB} \sin 60^\circ = 0$$

$$F_{EC} = -F_{EB}$$

$$\sum H = 0$$

$$2.02 + F_{EC} \cos 60^\circ - 1.45 \text{ kN} - F_{EB} \cos 60^\circ = 0$$

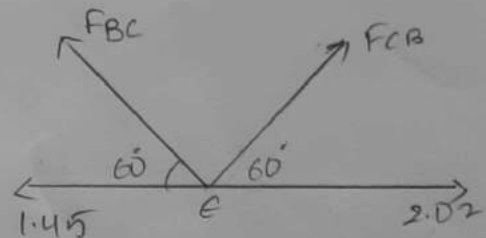
$$\therefore F_{EC} = -F_{EB}$$

$$2.02 - F_{EB} \cos 60^\circ - 1.45 - F_{EB} \cos 60^\circ = 0$$

$$0.57 = 2 F_{EB} \cos 60^\circ$$

$$F_{EB} = 0.57$$

$$F_{EC} = -0.57$$

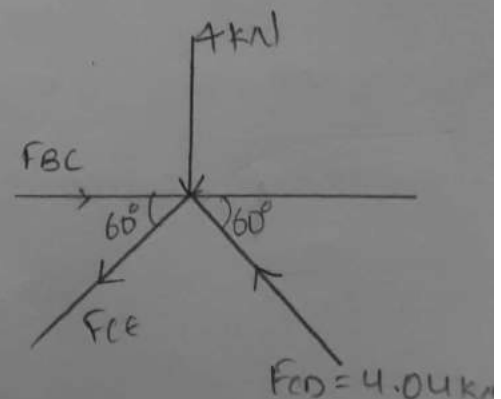


Joint C:

$$\sum H = 0$$

$$F_{BC} - F_{CE} \cos 60^\circ - F_{CD} \cos 60^\circ = 0$$

$$F_{BC} - F_{CE} \cos 60^\circ = 4.04 \cos 60^\circ$$



$$\sum V = 0$$

$$-4 - F_{CE} \sin 60^\circ + F_{CD} \sin 60^\circ = 0$$

$$-4 - F_{CE} \sin 60^\circ + 4.04 \sin 60^\circ = 0$$

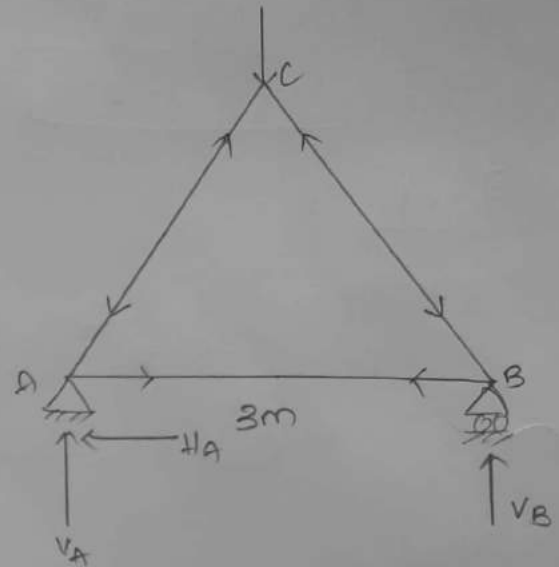
$$F_{CE} = -0.57 \text{ kN}$$

Substituting F_{CE} in the above equation

$$F_{BC} = 1.73 \text{ kN}$$

$$V_A + V_B = 10 \text{ kN}$$

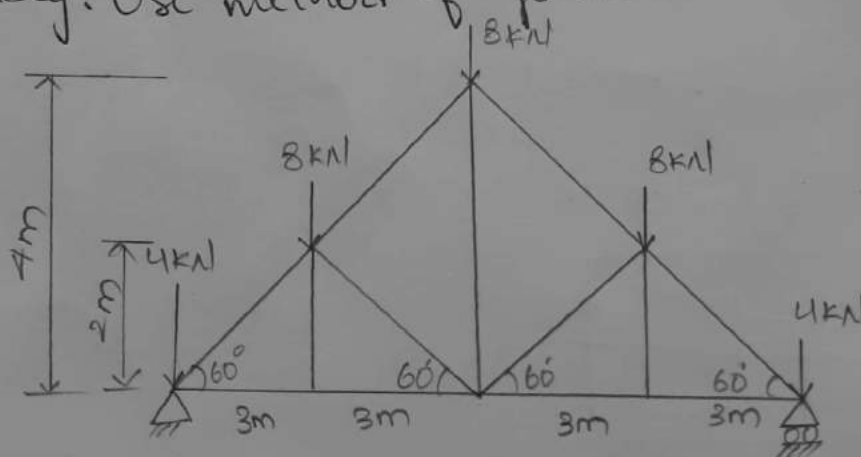
$$\sum M_A = 10(1.5) - 3V_B = 0$$



Assignment

Ex: Determine the forces in members and tabulate

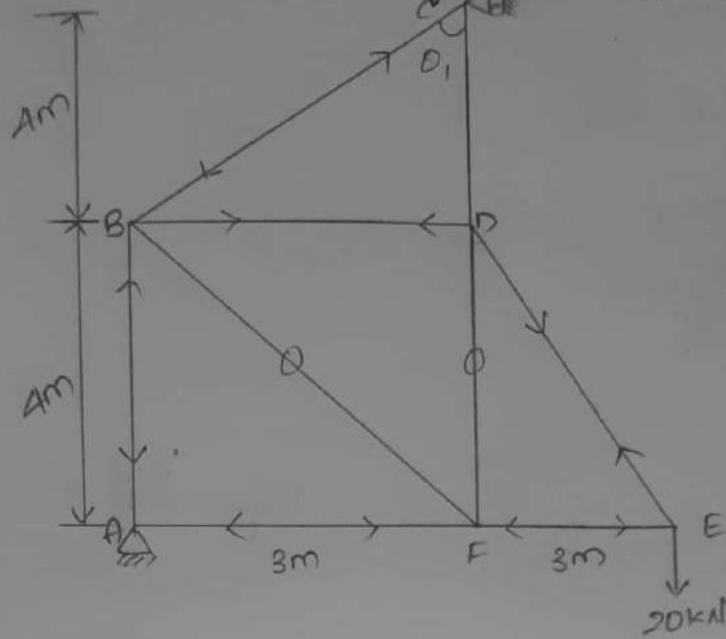
neatly. Use method of joints.



Ex: Determine the forces in members AC, BC, BD by using method of section.

Eg: Analyse the truss shown in fig., by method of joints

Indicate the members force on a neat sketch of truss.



$$\sum H = 0$$

$$H_A - H_C = 0$$

$$H_A = H_C$$

$$\therefore H_A = 15 \text{ kN}$$

$$\sum V = 0$$

$$V_A - 20 = 0$$

$$V_A = 20 \text{ kN}$$

$$\sum M_A = 0;$$

$$20(6) - 8H_C = 0$$

$$120 - 8H_C = 0$$

$$8H_C = 120$$

$$H_C = \frac{120}{8} = 15$$

$$H_C = 15 \text{ kN}$$

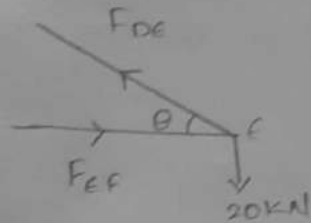
Joint - E:

$$\sum V = 0$$

$$F_{DE} \sin \theta = 20$$

$$F_{DE} = \frac{20}{\sin \theta} = \frac{20}{0.8} = 25 \text{ kN}$$

$$F_{DE} = \underline{\underline{25 \text{ kN}}}$$



$$\sum H = 0$$

$$F_{EF} - F_{DE} \cos \theta = 0$$

$$F_{EF} = F_{DE} \cos \theta$$

$$F_{EF} = 25 \times \frac{3}{5}$$

$$F_{EF} = \underline{\underline{15 \text{ kN}}}$$

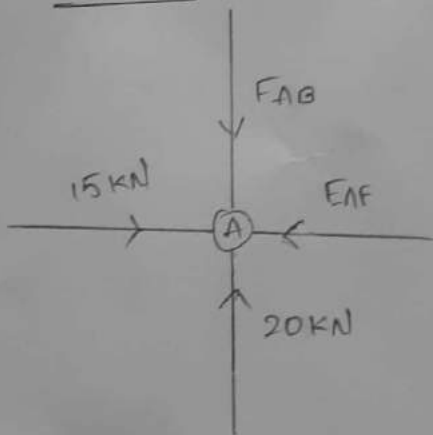
$$\begin{aligned} \text{hyp} &= \sqrt{4^2 + 3^2} \\ &= \sqrt{16 + 9} \\ &= 5 \end{aligned}$$

$$\tan \theta = \frac{4}{3} = 1.33$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5} = 0.8$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5} = 0.6$$

Joint - A:



$$\sum H = 0$$

$$F_{AF} - 15 = 0$$

$$F_{AF} = \underline{\underline{15 \text{ kN}}}$$

$$\sum V = 0$$

$$F_{AB} - 20 = 0$$

$$F_{AB} = \underline{\underline{20 \text{ kN}}}$$

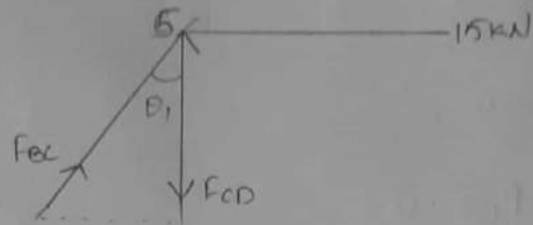
Joint-C:

$$\Sigma H = 0$$

$$F_{BC} \sin \theta_1 = 15$$

$$F_{BC} = \frac{15}{0.6}$$

$$F_{BC} = \underline{\underline{25 \text{ kN}}}$$



$$\Sigma V = 0$$

$$-F_{CD} + F_{BC} \cos \theta_1 = 0$$

$$F_{CD} = 25 \times 0.8$$

$$F_{CD} = 25 \times 0.8$$

$$F_{CD} = \underline{\underline{20 \text{ kN}}}$$

Joint-D:

$$\Sigma H = 0$$

$$25 \cos \theta - F_{DB} = 0$$

$$F_{DB}$$

$$F_{DB} = 25 \times 0.6 = \underline{\underline{15 \text{ kN}}}$$

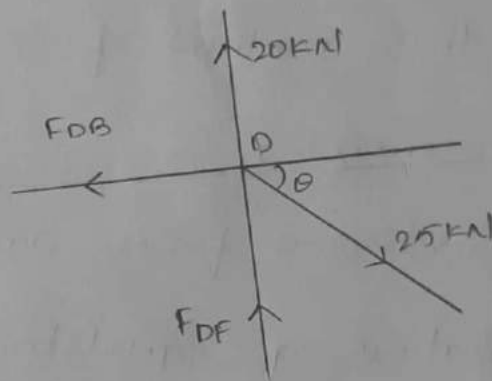
$$\Sigma V = 0$$

$$20 + F_{DF} - 25 \sin \theta = 0$$

$$20 + F_{DF} - 25 \times 0.8 = 0$$

$$20 + F_{DF} = 20$$

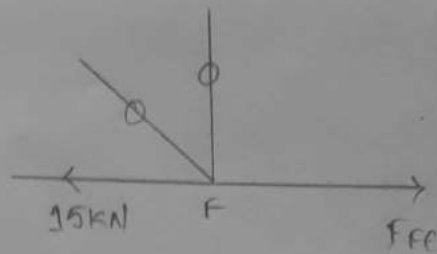
$$F_{DF} = \underline{\underline{0}}$$



Joint - F:

$$\sum V = 0$$

$$F_{DF} = 0$$

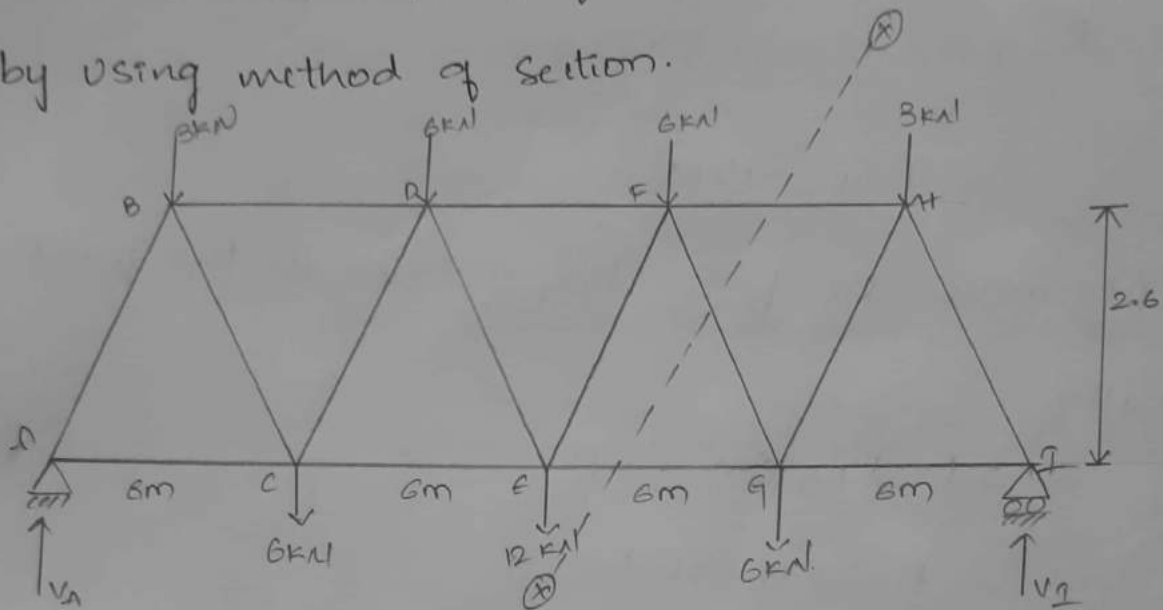


② Method of Section:

- A statically determinate frame can be completely analysed by static method.
- No. of unknowns is the same as the number of equations obtained from static equilibrium conditions.
- In this method of sections we isolate a portion of a member.
- The unknown forces are evaluated by using equations of equilibrium.
- The three equations of equilibrium are used to evaluate the unknowns.
- Thus the section should cut only three members which include the one whose force members intersect.

Example Problems:

Determine the nature and magnitude of forces in members FH, FG, EG of the truss as shown in figure by using method of section.



Sol:

$$\sum V = 0$$

$$V_A + V_I = 42$$

$$\sum M_A = 0$$

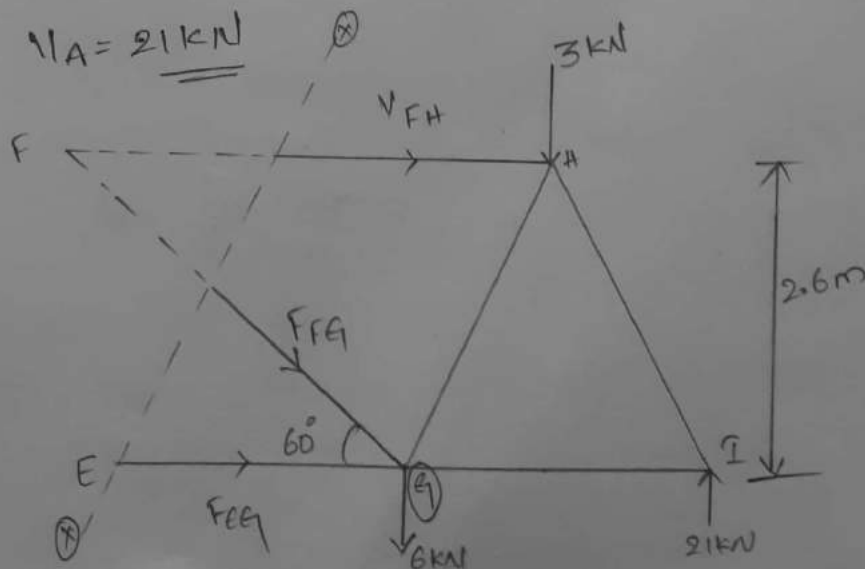
$$3(3) + 6(9) + 6(15) + 3(21) + 6(6) + 12(12)$$

$$+ 6(18) - 24 V_I = 0$$

$$V_I = \underline{\underline{21 \text{ kN}}}$$

$$V_A + V_I = 42$$

$$V_A = \underline{\underline{21 \text{ kN}}}$$



To determine the force, F_{FH} take moments about (E) where other two members of the cut section meet.

$$F_{FH} \times 2.6 + 3(3) - 21 \times 6 = 0$$

$$F_{FH} = \underline{45 \text{ kN}}$$

To determine the force F_{FE} resolve all the forces vertically.

$$21 - 3 - 6 - F_{FE} \sin 60^\circ = 0$$

$$F_{FE} = \underline{13.85 \text{ kN}}$$

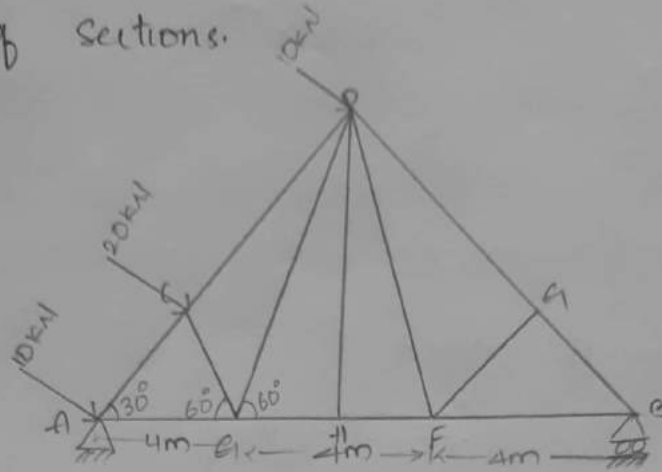
To determine the force F_{EG} take moment about where other two members of the section meet.

$$6 \times 3 + 3 \times 6 - 9 \times 21 - 2.6 F_{EG} = 0$$

$$F_{EG} = \underline{-58.85 \text{ kN}}$$

-ve sign indicates that the nature of F_{FH} is tensile.

Ex: Find the forces in the members CD, EF, FG use method of sections.



Sol: The length of panel AC, CD are found using

$$\cos 30 = \frac{6}{AD}$$

$$AD = \frac{6}{\cos(30)}$$

$$AD = \underline{6.93m}$$

$$AC = CD = \frac{AD}{2} = 3.46m.$$

The reaction V_A , H_A and V_B are determined using the equilibrium equations

$$\sum H = 0$$

$$10 \cos(60) + 20 \cos(60) + 10 \cos(60) - H_A = 0$$

$$H_A = \underline{20kN}$$

$$\sum V = 0$$

$$V_A + V_B = 10 \sin(60) + 20 \sin(60) + 10 \sin(60) + 10$$

$$V_A + V_B = \underline{44.64kN}$$

$$\sum M_A = 0$$

$$20(3.46) + 10(6.92) + 10(4) - 12V_B = 0$$

$$V_B = \underline{14.87 \text{ kN}}$$

The force in the member CD can be found by taking moment about C.

$$10 \times 3.46 - 14.87 \times 8 - F_{CD} \times CE$$

$$F_{CD} = \frac{-84.36}{CE} = \frac{-84.36}{4 \sin 30^\circ}$$

$$F_{CD} = \underline{-42.18}$$

-ve sign indicates that F_{CD} is compressive

To determine the force F_{FE} take moment about 'D' where other two members of the cut section CD and CE meet.

$$-14.87 \times 6 + F_{FE} \times 3.46 = 0$$

$$F_{FE} = \underline{25.8 \text{ kN}}$$

$$\tan 30^\circ = \frac{D_H}{6}$$

$$D_H = \underline{3.46 \text{ m}}$$

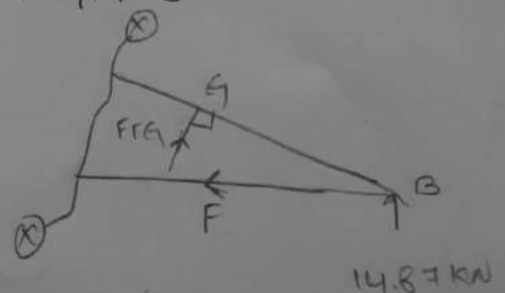
To compute the force in F_{EG} cut a

section through the member DE, EG, EB.

$$\sum M_B = 0$$

$$F_{EG} \times EB = 0$$

$$F_{EG} = 0$$



UNIT-III

Strain Energy Method

UNIT-III

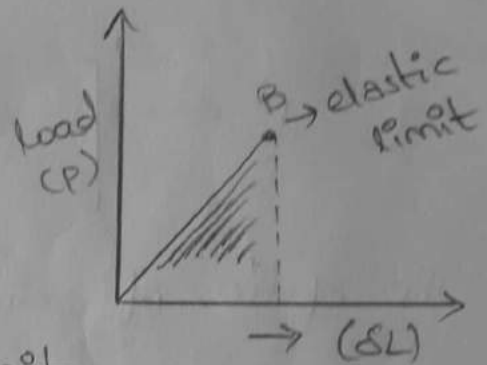
Strain energy :-

When a body is subjected to load, the points of application are displaced and the energy due to the displacements are imparted to the body

$$U = \frac{1}{2} PL \times \delta$$

$$= \frac{1}{2} \times \frac{PL}{L} \times \frac{P}{A} \times (AL)$$

$$= \frac{1}{2} \sigma \times \epsilon \times V$$



Resilience → up to elastic limit

proof Resilience → Max Energy up to elastic limit

Energy principles are extensively used for determining the displacement in structures

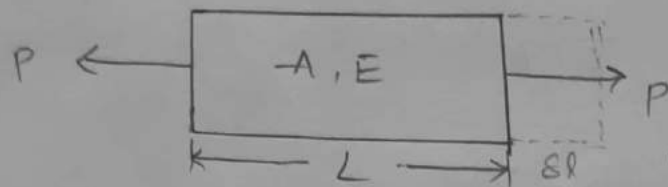
Strain energy

It is an amount of work done by an internal resistance against the deformation.

It Means when the loads are removed, the energy stored is used to restore the body to its original position.

If the imposed stresses due to the applied loads is greater than the elastic stresses then a part of the strain energy is used in permanently deforming the body which cannot be recovered.

Strain energy due to axial load :-



E : The ratio of direct stress (σ) to direct strain (ϵ) with in proportionality young's Modulus.

$$\delta L = \frac{PL}{AE} \quad \text{--- (1)}$$

$$U = \frac{1}{2} \times P \times \delta L$$

$$= \frac{1}{2} P \times \left(\frac{PL}{AE} \right)$$

$$U = \frac{P^2 L}{2AE}$$

During the application of the load no kinetic energy is created.

The work done is stored in the form of strain energy only.

③ In essence the energy stored due to strains of an elastic type is known as strain energy.

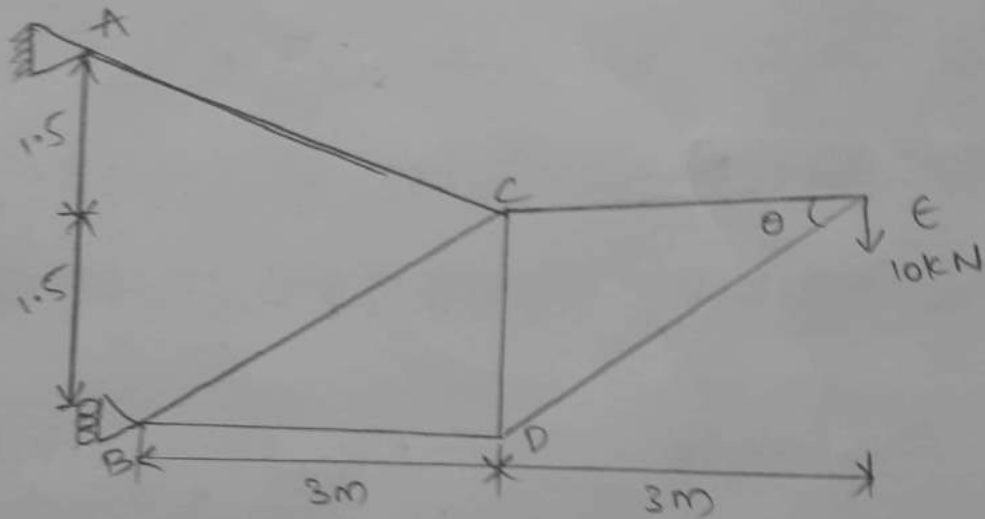
where

L = length of the bar

A = cross-section of the bar

E = Young's Modulus of the bar

Example.



Determine the vertical deflection at E of the loaded frame show in the figure cross-sectional

areas are 1000mm^2 $E = 200\text{ kN/mm}^2$

Joint E

$$F_{DE} = 22.34\text{ kN}$$

$$\sin\theta = 0.447$$

$$F_{CE} = 20\text{ kN}$$

$$\cos\theta = 0.894$$

Deflection at the point of Application of single load

$$\Delta = \frac{2U}{w} \quad \text{--- (2)}$$

$$U = \frac{w\Delta}{2} \quad \text{--- (1)}$$

$\Delta \rightarrow$ its displacement in the line of action of w .

$w \rightarrow$ is the external load

Point - D

$$F_{DB} = 20 \text{ kN}$$

$$F_{DC} = 10 \text{ kN}$$

Point - C

$$F_{AC} = 22.37$$

$$F_{BC} = 0$$

$$\delta = \frac{P^2 l}{2AE}$$

Member	length	A	Force	Area	Result
BD	3000	1000	20	1200.0	
DE	3354	1000	22.37	1678.4	
CE	3000	1000	20	1200.0	
CA	3354	1000	-22.37	1678.4	
CB	3354	1000	00	-	
CD	1500	1000	10	1500	

$\Sigma 5906.8$

In a framed structure $U = \frac{\sum P^2 L}{2AE}$

Hence deflection

$$\Delta = \frac{2U}{W}$$

$$= \frac{2}{W} \sum \frac{P^2 L}{2A}$$

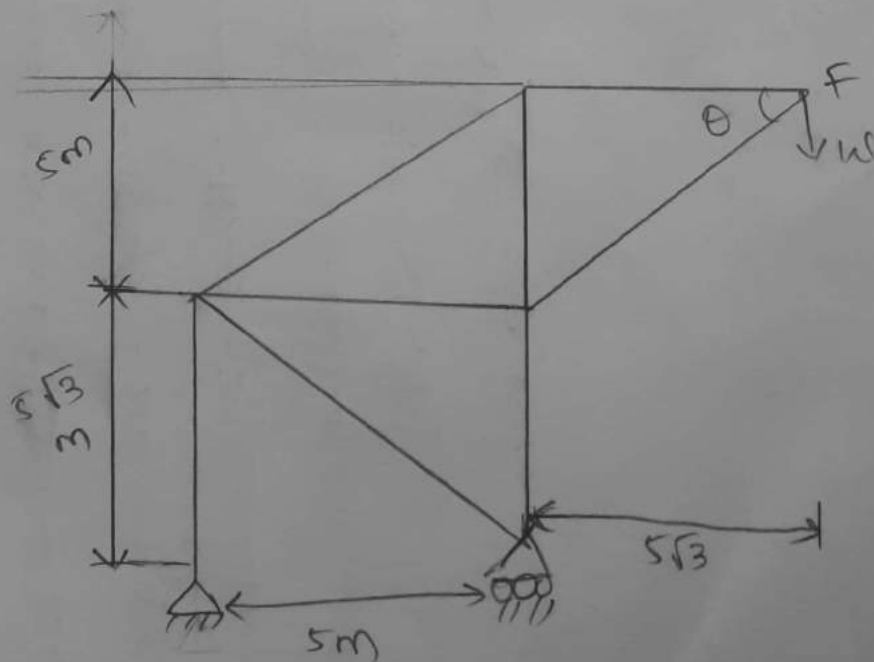
$$= \frac{\sum P^2 L/A}{WE}$$

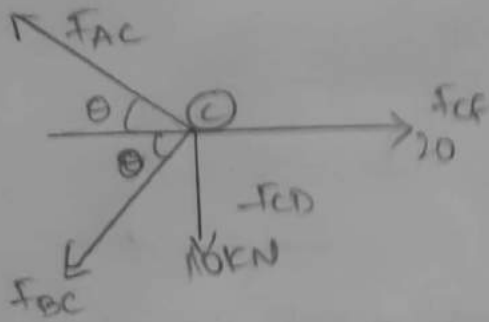
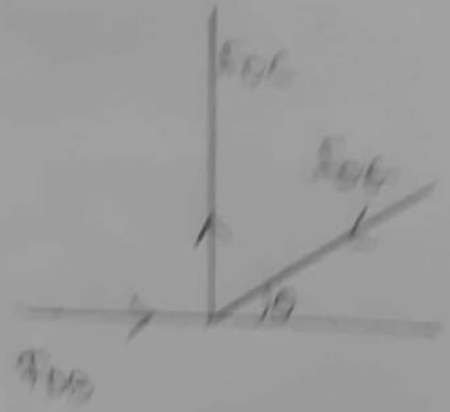
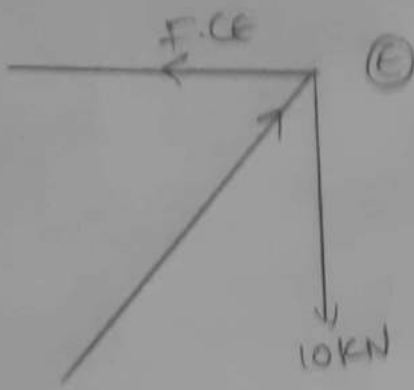
$$\Delta = \frac{5906.8}{10 \times 200} = 2.95 \text{ mm}$$

Example.

Determine the vertical deflection of the frame shown in figure. The load is such that the tension members of the frame are stressed to 150 N/mm^2 and the compression members to 75 N/mm^2 .

Take $E = 200 \text{ kN/mm}^2$





Members

Length

% (kN/m²)

(kN)

$\frac{F}{A} \times 10^3$

AC	→	8.66	→	150	→	21732w	→	22.25
BC	→	7.07	→	150	→	21150	→	21.66
CD	→	8.66	→	150	→	1732w	→	22.25
DE	→	10.00	→	-75	→	-200	→	4.50
DB	→	8.66	→	-75	→	-2732w	→	11.77
DC	→	5.00	→	-75	→	-1732w	→	90.65
DE	→	5.00	→	-75	→	-1732w	→	90.65
BC	→	10.00	→	-	→	0.00	→	0

$F_{DF} = 2w$

$F_{EF} = 1.732w$

$F_{EC} = 2.45w$

$F_{ED} = 1.732w$

$F_{DC} = 1.732w$

$F_{DB} = 2.73w$

$F_{BC} = 0$

$F_{AC} = 1.732w$

$$U = 11.67 \times 10^3 w$$

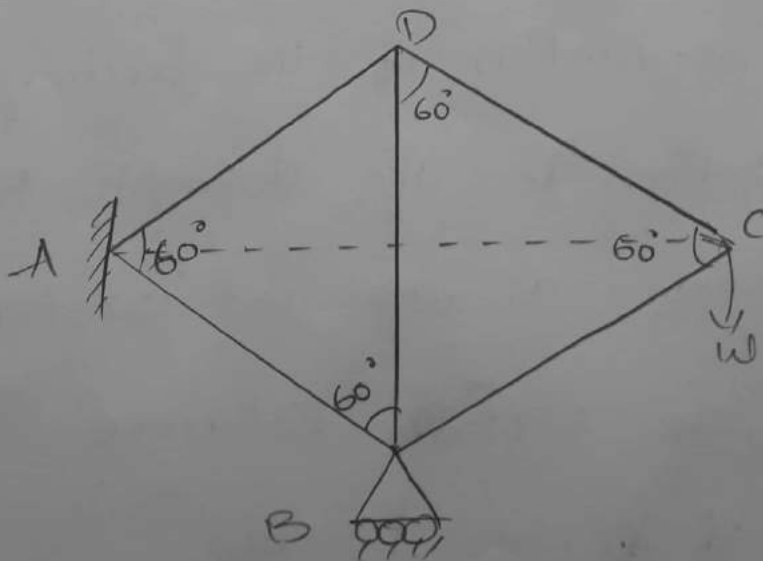
$$\begin{aligned} \Delta &= \frac{1}{wE} \sum \frac{P^2 l}{A} \\ &= \frac{1}{wE} \times \frac{P^2 l}{A} \\ &= \frac{11.67 \times 10^3 \times w}{10 \times 200} \end{aligned}$$

$$\Delta_F = 58.33 \text{ mm}$$

Problem

A frame consists of the Members as shown in fig. All the tension members are 125 mm^2 and the compression are 250 mm^2 in area find the vertical deflection of the loaded point.

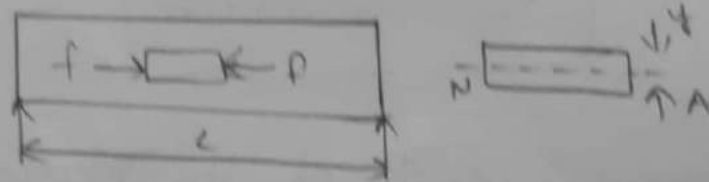
Take $E = 200 \text{ kN/mm}^2$; $W = 25 \text{ kN}$.



Strain Energy due to Bending.

* In the previous section, we have derived the expression for strain energy due to the action of direct force. either tension (or) compression.

* There are other actions, by bending, shear and torsion also produces strain energy.



considers a beam subjected to pure bending.

Let area of the element be S_a and its distance axis be 'y'. An initially straight beam is subjected to a uniform bending Moment as shown in fig.

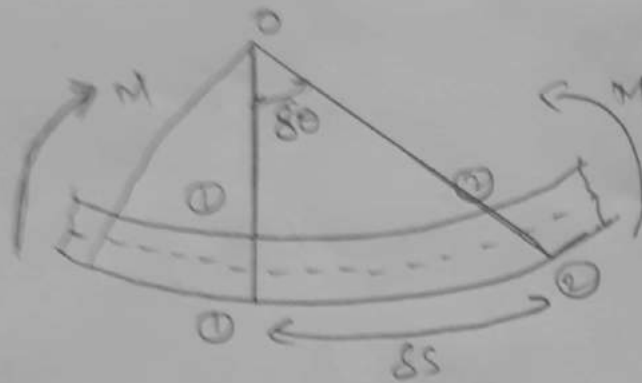
Due to the application of the bending Moment two normal sections in the straight beam.

are deformed as (1)-(1) and (2)-(2)

Let S_s be the curved distance

θ be its subtended angle.

External work is expressed in terms of the forces



Strain energy due to external loads can be expressed as

$$U = \frac{1}{2} \epsilon W \Delta$$

$$\delta U_b = \frac{1}{2} M \delta \theta \quad \text{--- (1)}$$

Where δU_b is the strain energy due to Bending

If R is the radius of curvature of the element due to the action of M

$$R \delta \theta = \delta s \quad \text{--- (2)}$$

$$\delta \theta = \frac{\delta s}{R}$$

$$\delta U_b = \frac{1}{2} M \delta \theta$$

$$\delta U_b = \frac{1}{2} M \frac{\delta s}{R} \quad \text{--- (3)}$$

from theory of bending the curvature is expressed as

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\frac{1}{R} = \frac{M}{EI} \quad \text{--- (6)}$$

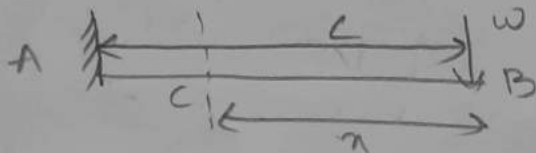
$$dU_b = \frac{1}{2} M^2 \frac{ds}{EI}$$

On Integration, we obtain the total strain energy due to the bending as

$$U_b = \int \frac{M^2 ds}{2EI}$$

case-1

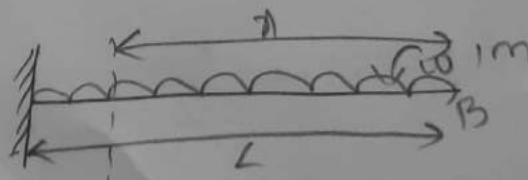
cantilever beam of span 'L' sub. to point load



$$U = \frac{W^2 L^3}{6EI}$$

case-2

cantilever beam of span 'L' subjected to uniformly distributed load



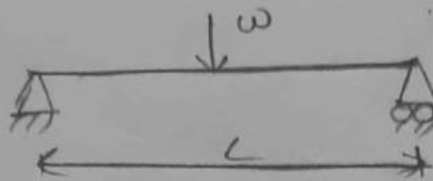
$$M_{\max} = -w \cdot \frac{3}{2} = -\frac{w \cdot 3}{2}$$

$$U = \frac{1}{2EI} \int_0^L \left(\frac{w^2 x^4}{4} \right) dx$$

$$U = \frac{w^2 L^5}{40EI}$$

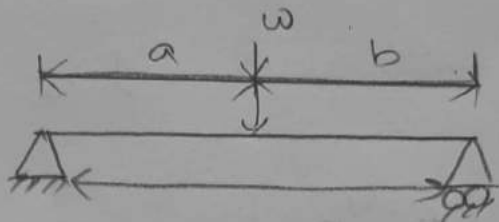
Case-3

→ A simply supported beam with concentrated load



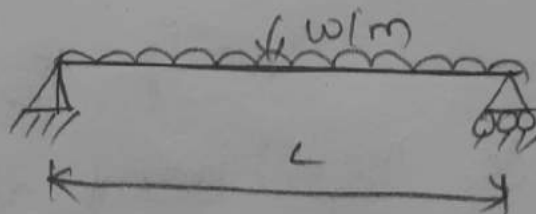
$$U = \frac{w^2 L^3}{96EI}$$

Case-4



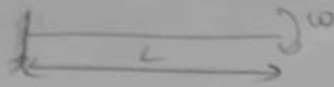
$$U = \frac{w^2 a^2 b^2}{6EIL}$$

Case-5



$$U = \frac{w^2 L^5}{240EI}$$

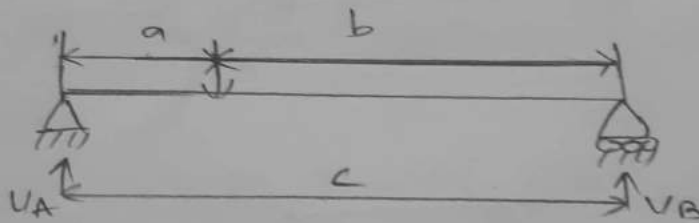
Case-6



$$U = \frac{M^2 L}{2AE}$$

Example

A simply supported beam of span l carries a concentrated load P at a distance of a & b from two ends. Find the strain energy stored in the beam and the deflection under the load by Castigliano's theorem.



Soln

$$\sum V = 0$$

$$V_A + V_B = P$$

$$\sum M = 0$$

$$Pa - V_B l = 0$$

$$V_B = \frac{Pa}{l}$$

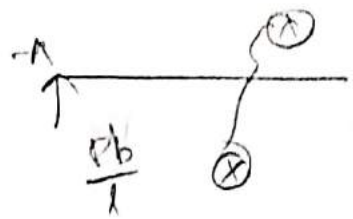
$$V_A = \frac{Pb}{l}$$

in the length of Ac the Moment at a distance

from A is $\frac{Pba}{l}$ Hence the strain energy for

this portion is

$$U_{AC} = \int_0^a \frac{M^2 dx}{2EI}$$



$$U_{AC} = \frac{1}{2EI} \int_0^a M^2 dx$$

$$= \frac{1}{2EI} \int_0^a \left(\frac{Pbx}{l} \right)^2 dx$$

$$= \frac{P^2 b^2}{2l^2 EI} \int_0^a x^2 dx$$

$$= \frac{P^2 b^2}{2l^2 EI} \left[\frac{x^3}{3} \right]_0^a$$

$$U_{AC} = \frac{P^2 a^3 b^2}{6l^2 EI}$$

Similarly for the length BC with B as the origin the strain energy for the portion BC can be obtained.

$$U_{BC} = \frac{1}{2EI} \int_0^b \left(\frac{Pax}{l} \right)^2 dx$$

$$U_{BC} = \frac{P^2 a^2 b^3}{6EI l^2}$$

Hence the total strain energy

$$U = U_{AC} + U_{BC}$$

$$= \frac{P^2 a^3 b^2}{6EI l^2} + \frac{P^2 a^2 b^3}{6EI l^2}$$

$$U = \frac{P^2 a^2 b^2}{6EI l}$$

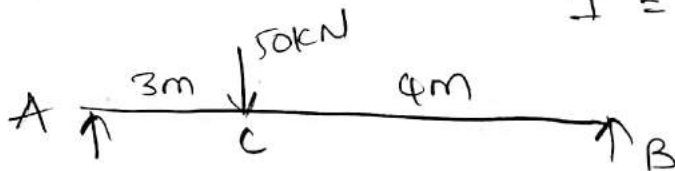
$$\Delta = \frac{2U}{P}$$

$$\Delta_c = \frac{Pa^2 b^2}{3EI l}$$

Example. Find the deflection under concentrated load for the beam shown in figure using strain energy method

$$E = 2 \times 10^8 \text{ KN/m}^2$$

$$I = 14 \times 10^5 \text{ m}^4$$



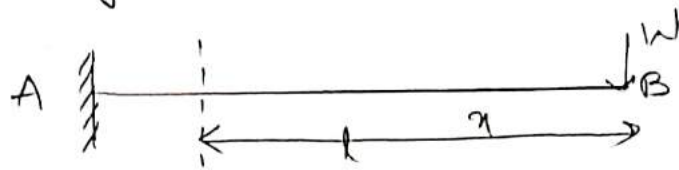
$$\Delta = \frac{Pa^2 b^2}{3EI l}$$

$$= \frac{50 \times 3^2 \times 4^2}{3 \times 2 \times 10^8 \times 14 \times 10^5 \times 7}$$

$$\Delta = 0.012 \text{ m}$$

$$= 12 \text{ mm}$$

→ calculate the deflection at the free end of a cantilever subjected to a vertical load.



The Bending moment at a distance x from

B is
$$M = -wx$$

The total strain energy can be obtained as

$$U_{AB} = \int_0^l \frac{M^2 dx}{2EI}$$

$$= \frac{w^2}{2EI} \int_0^l x^2 dx$$

$$U_{AB} = \frac{wl^3}{2EI}$$

Deflection at the free end is

$$\Delta = \frac{wl^3}{3EI}$$

Castigliano's Theorems :-

First theorem is applied to determine the linear (or) angular deflection at a point in a structure caused by the applied load.

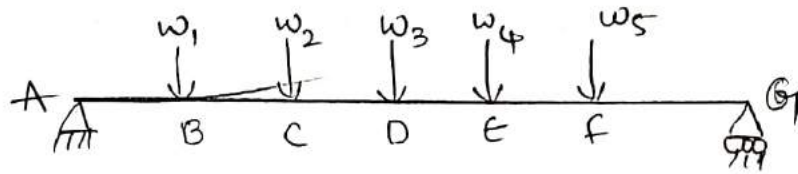
It is applicable for

- i) When the structure is stressed within its elastic limit.
- ii) The structure may obey Hooke's law but the deformation need not be linear.

Statement of Theorem - 1.

The displacement of the point of application of the load in its own line of action will be obtained by evaluating (or) calculating the first partial derivative of the total internal strain energy of the structure with respect to the applied load $\frac{\partial U}{\partial W} = \Delta$.

Proof: consider a beam AG under the action of loads w_1, w_2, w_3 etc at B, C and let displacements be $\Delta_1, \Delta_2, \Delta_3$ respectively.



Internal strain energy will be

$$U = w_1 \frac{\Delta_1}{2} + w_2 \frac{\Delta_2}{2} + w_3 \frac{\Delta_3}{2} + \dots \quad (1)$$

Let there be additional load δw_1 at B in the direction of w_1 .

Then additional displacements will be $\delta \Delta_1, \delta \Delta_2, \delta \Delta_3$, in the direction w_1, w_2, w_3

$$\delta U = w_1 \delta \Delta_1 + \frac{\delta w_1 \delta \Delta_1}{2} + w_2 \delta \Delta_2 + w_3 \delta \Delta_3 \quad (2)$$

Now the loads are replaced with another system of forces $(w_1 + \delta w_1), w_2, w_3$ at B, C, D .

Hence

Now system of displacements are

$$(\Delta_1 + \delta \Delta_1) ; -(\Delta_2 + \delta \Delta_2) ; +(\Delta_3 + \delta \Delta_3)$$

$$U_1 = \frac{(\omega_1 + \delta\omega_1) \cdot (\Delta_1 + \delta\Delta_1)}{2} + \frac{\omega_2(\Delta_2 + \delta\Delta_2)}{2} + \frac{\omega_3(\Delta_3 + \delta\Delta_3)}{2} \quad (3)$$

eq ① - ③

$$U_1 - U = \partial U = \omega_1 \frac{\delta\Delta_1}{2} + \frac{\delta\omega_1 \Delta_1}{2} + \frac{\delta\omega_1 \delta\Delta_1}{2} + \omega_2 \frac{\delta\Delta_2}{2} + \omega_3 \frac{\delta\Delta_3}{2} + \dots$$

$$2\partial U = \omega_1 \delta\Delta_1 + \delta\omega_1 \Delta_1 + \delta\omega_1 \delta\Delta_1 + \omega_2 \delta\Delta_2 + \omega_3 \delta\Delta_3 \quad (4)$$

From eq ④ - ②

$$2\partial U - \partial U = \delta\omega_1 \Delta_1 + \frac{\delta\omega_1 \delta\Delta_1}{2}$$

Hence $\partial U = \delta\omega_1 \Delta_1$

$$\Delta_1 = \frac{\partial U}{\partial \omega_1}$$

(or)

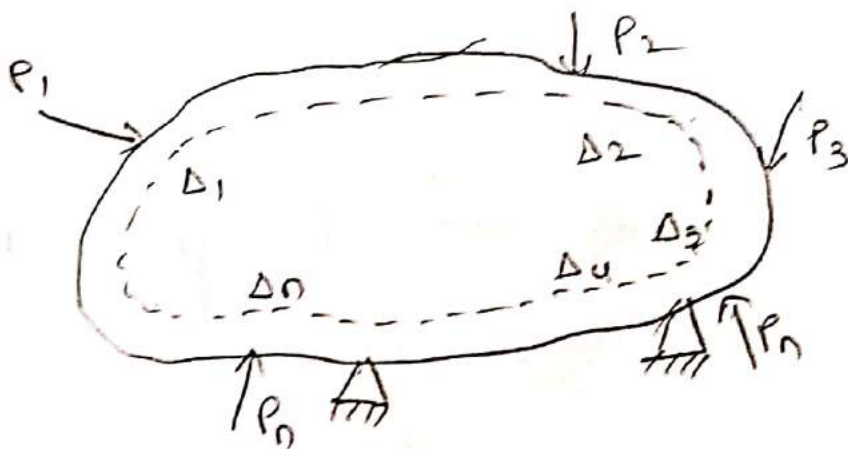
$$\Delta = \frac{\partial U}{\partial \omega}$$

unit load Method :-

consider the body shown in figure which is subjected to force $P_1, P_2, P_3, P_u, \dots, P_n$ applied gradually. Let the displacement under load at point be $\Delta_1, \Delta_2, \Delta_3, \dots, \Delta_n$ and at point c be Δ . Then

$$\text{External work done} = \frac{1}{2} \Delta_1 P_1 + \frac{1}{2} \Delta_2 P_2 + \frac{1}{2} \Delta_3 P_3 + \dots + \frac{1}{2} \Delta_n P_n$$

and strain energy stored = $\int \frac{1}{2} P e d u$.



Where P is stress

e is the strain in the element

consider

$$\frac{1}{2} \Delta_1 P_1 + \frac{1}{2} \Delta_2 P_2 + \dots + \frac{1}{2} \Delta_n P_n = \int \frac{1}{2} P e d u \quad \text{--- (1)}$$

Now consider the same body subjected to a unit load applied gradually at c when it is free of system of p forces. let the displacement

at 1, 2, 3 --- n be $\delta_1; \delta_2, \delta_3 \dots, \delta_n$ respectively and displacement at 'c' and 's'.

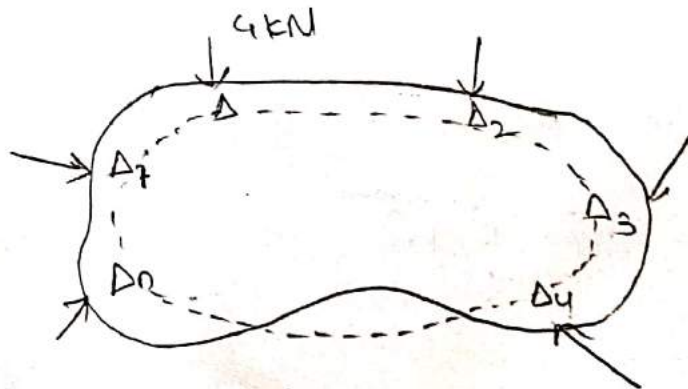
Let the stress produced in the element be 'p' and the strain be 'e'. Then.

$$\text{External work done} = \frac{1}{2} X_1 X_8$$

$$\text{Internal work done} = \int \frac{1}{2} p' e' dv$$

$$\frac{1}{2} X_1 X_8 = \int \frac{1}{2} p' e' dv \quad \text{--- (2)}$$

Now if p system of force is applied to the body shown in figure.



$$\begin{aligned} \text{External work done} &= \frac{1}{2} \Delta_1 P_1 + \frac{1}{2} \Delta_2 P_2 + \dots \\ &+ \frac{1}{2} \Delta_n P_n + \dots \end{aligned}$$

Since unit load is already action internal

$$\text{work done} = \int \frac{1}{2} P e \, dv + \int P e \, dv.$$

Since the stress p is acting through the deformation. Equating internal work to external work.

$$\begin{aligned} \frac{1}{2} \Delta_1 P_1 + \frac{1}{2} \Delta_2 P_2 + \dots + \frac{1}{2} \Delta_n P_n + 1 \times \Delta \\ = \int \frac{1}{2} P \cdot e \, dv + \int P' e \, dv \quad \text{--- (3)} \end{aligned}$$

Sub eq. (1) in (3)

$$1 \times \Delta = \int P' e \, dv$$

$$\Delta = \int P' e \, dy$$

Δ = is the deflection at the point where unit load is applied and is measured in the direction of unit load.

P' - is stress in an element due to unit load and

e' is strain element due to given load system

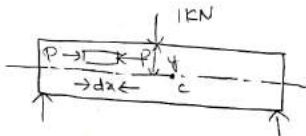
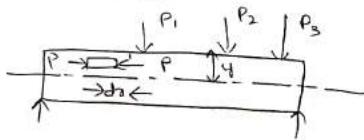
$\Delta = \int p' dx$ → it is the basic tool of the unit load Method.

Deflection of simple beams :-

considers the beam as shown in figure subjected to a system of load 'p' forces

The slope in the element at distance 'x' from neutral axis is

$\theta = \frac{M}{EI} y$, where M is the Moment acting at the section.



∴ Strain in the element due to given system of forces

$$e = \frac{M}{EI} y$$

Let m be the moment at the section where the element is considered due to unit load acting at 'c'

$$\text{stress } p' = \frac{my}{I}$$

$$\Delta = \int p' dx$$

$$= \int \frac{My}{I} \times y \cdot \frac{M}{EI} y dx$$

$$\Delta = \int \frac{Mm}{EI^2 y^2} dx \quad \left[\int_0^A y^2 dA = I \right]$$

$$= \int \frac{Mm}{EI^2} \left[\int_0^A y^2 dA \right] dx$$

$$= \int \frac{Mm}{EI^2} I dx$$

$$\Delta = \int \frac{Mm}{EI} dx$$

Deflection of pin jointed frames:

Castigliano's first theorem can be applicable for the estimating the deflection of pin jointed frames.

If the deflection of only a few joints are required.

Let P_i be the force developed due to loading in any member ' i ' of the truss.

The strain energy stored in the Member is

$$\frac{P_i^2 l_i}{2A_i E}$$

where $l_i \rightarrow$ is length of the Member

$A_i \rightarrow$ is area of cross-section of the Member.

\therefore Total strain energy will be

$$u = \frac{\sum P_i l_i}{2A_i E}$$

Differentiating with respect to u and external load on the structure.

We obtain the deflection due to the force ' W_i '.

$$\Delta = \frac{\partial u}{\partial W_i} = \frac{\sum P_i l_i}{A_i E} \left[\frac{\partial P_i}{\partial W_i} \right]$$

$\therefore \left[\frac{\partial P_i}{\partial W_i} \right]$ is rate of change of force P_i

in any Member ' i ' with respect to external load ' W_i '.

If ' ϵ_i ' denotes the force produced in any bar ' i ' by a unit load in the direction of ' W_i '

then $\frac{\partial P_i}{\partial W_i} = \epsilon_i$.

\therefore The expression for the deflection of a pin joint frame is written in the form of

$$\Delta = \frac{\sum P_i l_i \epsilon_i}{A_i E}$$

(or)

$$\Delta = \sum \frac{P_i l_i}{A_i E}$$

$P \rightarrow$ force in the member

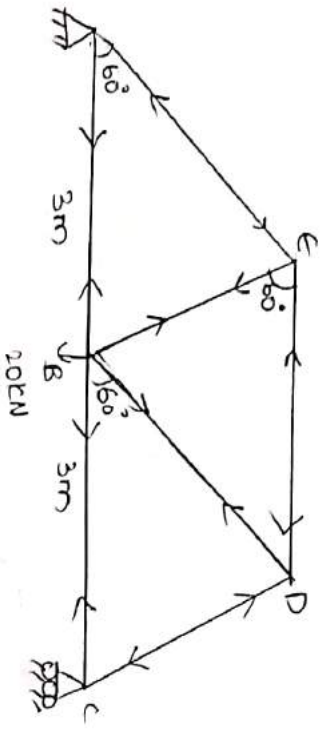
$F \rightarrow$ denotes the force produced in any part of the member

$A \rightarrow$ area of cross-section.

$E \rightarrow$ young's modulus.

Δ calculate the vertical deflection components of joint B of the truss loaded as shown in figure by strain energy Method cross-sectional area of each Member is $500 \times 10^6 \text{ m}^2$ $E = 200 \times 10^6 \text{ N/m}^2$

The forces in the Members are obtained by the Method of joints and taking the advantage of system makes.



Sol:-
$$\Delta_B = \frac{\sum PFL}{AE}$$

$V_A = V_C = \frac{20}{2} = 10 \text{ kN}$

Joint A

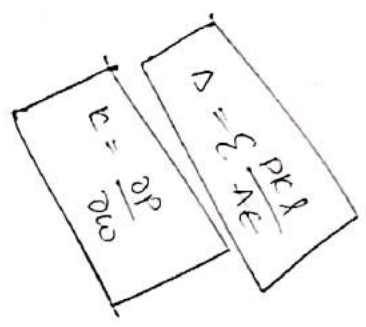
$F_{AB} = 5.78 \text{ kN}$

$F_{AE} = 11.55 \text{ kN}$

Joint E

$F_{BE} = 11.55 \text{ kN}$

$F_{ED} = 11.55 \text{ kN}$



Members	Length	Area (mm ²)	P (kN)	ϵ	$\frac{PL}{A}$
AB	3000	500	5.78	0.289	10.002
BC	3000	500	5.78	0.289	10.002
BCD	3000	500	-11.55	-0.578	40.055
DE	3000	500	-11.55	-0.578	40.055
BE	3000	500	11.55	0.578	40.055
BD	3000	500	11.55	0.578	40.055
AE	3000	500	-11.55	-0.578	8.4055

$$\Delta = \frac{\sum PFL}{A} = \frac{220 \cdot 319}{200}$$

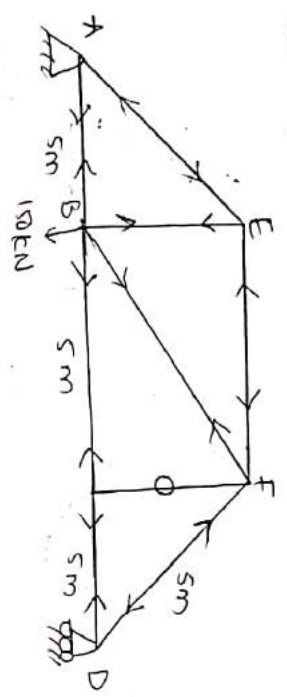
$$\Delta = \frac{220 \cdot 319}{200}$$

$$\Delta = 1.1 \text{ mm}$$

Assignment

A steel truss of span 15m is loaded as shown in figure. the cross section of each members is such that it is subjected to a stress of 100 N/mm^2 . Find the vertical deflection of the joint 'c'.

Take $E = 200 \text{ kN/mm}^2$



$V_A + V_D = 150$
 $\sum MO (5) - 15V_D = 0$

Joint-C
 $F_{CE} = 0$
 $F_{CD} = 50 \text{ kN}$

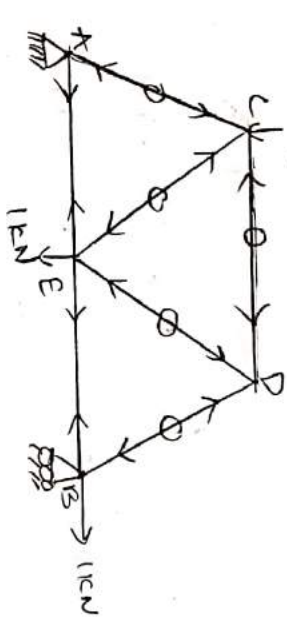
Joint-A
 $V_D = 50 \text{ kN}$
 $V_A = 100 \text{ kN}$

Joint-D
 $F_{DE} = 70.71 \text{ kN}$

Joint-E
 $F_{EB} = 100 \text{ kN}$
 $F_{EC} = 100 \text{ kN}$

Joint-B
 $F_{BF} = 70.71 \text{ kN}$
 $F_{BC} = 50.0 \text{ kN}$

Determine the horizontal Movement of the roller's and the vertical deflection of joint 'c' of the truss shown in figure. Areas of chord members 1000 mm^2 and the area of diagonals are 1300 mm^2 . $E = 200 \text{ kN/mm}^2$



$\sum V = 0 \quad V_A + V_B = 100$

$\sum MO = 0 \quad 100(5) = 20V_B$

$V_B = 25 \text{ kN}$
 $V_A = 75 \text{ kN}$

Joint-A
 $F_{AC} = 75\sqrt{2}$

$F_{AE} = 75 \text{ kN}$

Joint-B

$F_{DB} = 35.355 \text{ kN}$

$F_{DE} = 25 \text{ kN}$

Joint-C

$F_{CE} = -35.355$

$F_{DC} = -35.355 \text{ kN}$

Point-D.

$F_{DB} = 35.355 \text{ kN}$

$F_{CD} = 50 \text{ kN}$

A unit vertical load is applied at E where the deflection is required.

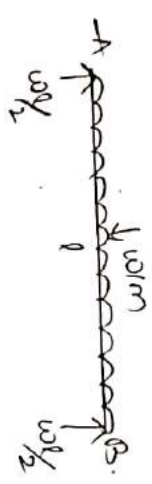
A unit horizontal load is applied at B to calculate the movement of the sloleg

Members	length	Area	P	K	$P \cdot K / A$	$P \cdot K / A$	$P \cdot K / A$
AE	20000	1000	75	0.5	37.5	1	750
EB	10000	1000	25	0.5	12.5	1	250
BD	5000√2	13000	-35.35	0.707	-135.96	0	0
DC	10000	1000	-50.00	-1.000	500	0	0
CA	5000√2	13000	-106.05	-0.707	407.8	0	0
CE	"	"	-35.35	+0.707	-135.96	0	0
DE	"	"	+35.355	0.707	135.96	0	0

$$\Delta_{ve} = \sum \frac{P \cdot K \cdot l}{A \cdot E} = \frac{1543.78}{200} = 7.72 \text{ mm}$$

$$\Delta_{Hb} = \frac{1000}{200} = 5 \text{ mm}$$

For using strain Energy method compute the deflection at midspan of a simply supported beam carrying a uniformly distributed load w kN/m. Assume an uniform flexural rigidity



Members	origin	limits	M	m	I
AC	A	0 - l/2	$\frac{wl}{2}x - \frac{wx^2}{2}$	$x/2$	I
CB	B	0 - l/2	$\frac{wl}{2}x - \frac{wx^2}{2}$	$x/2$	I

$$\Delta_c = \int_0^{l/2} 2x \left(\frac{wl}{2}x - \frac{wx^2}{2} \right) \left(\frac{x}{2} \right) \frac{dx}{EI}$$

$$= \frac{1}{2} \int_0^{l/2} (wlx^2 - wx^3) x \cdot \frac{dx}{EI}$$

$$= \frac{1}{2} \left[\frac{wlx^4}{4} - \frac{wlx^4}{10} \right]_0^{l/2}$$

$$= \frac{1}{2} \left[\frac{wl}{3} \left(\frac{l}{2} \right)^3 - \frac{wl}{10} \left(\frac{l}{2} \right)^4 \right]$$

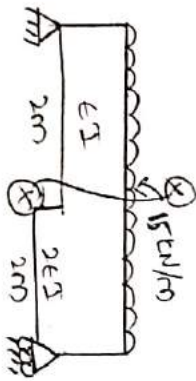
$$= \frac{1}{2} \left[\frac{w_0 x^4}{24} - \frac{w_0 x^4}{64} \right] \cdot \frac{1}{EI}$$

$$= \frac{1}{2} \left[\frac{(8-3) w_0 x^4}{192} \right] \cdot \frac{1}{EI}$$

$$\Delta_c = \frac{5}{384} \cdot \frac{w_0 x^4}{EI}$$

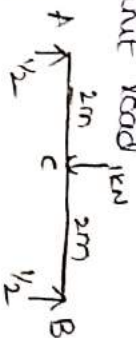
Ex: Determine the deflection for the beam shown in figure at midspan using strain energy Method

$$EI = 3 \times 10^4 \text{ kNm}^2$$



$$V_A = V_B = \frac{\text{total load}}{2} = \frac{w_0 l}{2} = \frac{15 \times 4}{2} = 30 \text{ kN}$$

The relevant bending moment expression due to applied loading and due to unit load



Members	origin	limits	M	m	I
AC	A	0-2	$30x - \frac{15x^2}{2}$	$x/2$	I
CB	B	0-2	$30x - \frac{15x^2}{2}$	$x/2$	I

$$\Delta_c = \int_0^2 (30x - 15x^2) (0.5) x \cdot \frac{dx}{EI}$$

$$+ \int_0^2 \frac{(30x - 15x^2) (0.5) x}{2EI} dx$$

$$= \frac{3}{2EI} \int_0^2 (15x^2 - 3 \cdot 15x^3) dx$$

$$= \frac{3}{2EI} \left[\frac{15x^3}{3} - 3 \cdot \frac{15x^4}{4} \right]_0^2$$

$$= \frac{3}{2EI} \left[\frac{15(2)^3}{3} - 3 \cdot \frac{15(2)^4}{4} \right]$$

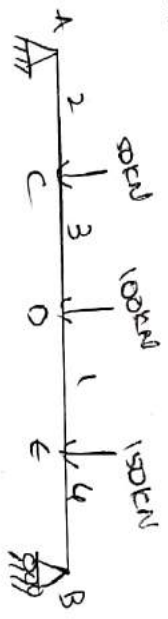
$$= \frac{3}{2EI} \times 25$$

$$= \frac{15}{2EI}$$

$$= \frac{15}{2 \times 3 \times 10^4} \times 1000$$

$$\Delta_c = 1.25 \text{ mm}$$

Ex: A steel beam of UDL is simply supported on a span of 10m and carries concentrated loads of 50, 100 and 150 kN at a distance of 2.5, 5 meters from the left support compute the deflection under 150 kN load $EI = 400 \times 10^4 \text{ kNm}^2$



Sol: The reaction at the supports are found using equation of equilibrium

$$\sum V = 0$$

$$V_A + V_B = 50 + 100 + 150$$

$$V_A + V_B = 300$$

$$\sum M_A = 0$$

$$50(2.5) + 100(5) + 150(7.5) - 10V_B = 0$$

$$\frac{100 + 500 + 900}{10} = V_B$$

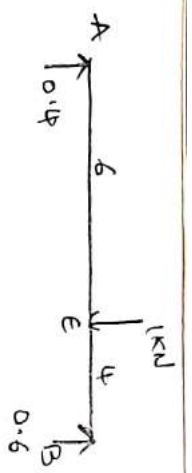
$$V_A + V_B = 300$$

$$V_A = 300 - 150$$

$$V_A = 150 \text{ kN}$$

$$\frac{1500}{10} = V_B$$

$$V_B = 150 \text{ kN}$$



$$R_A + R_B = 1 \text{ kN}$$

$$R_B \times 10 - 1 \times 4 = 0$$

$$R_B = \frac{4}{10} = 0.4$$

$$R_B = 0.4$$

$$R_A = 1 - 0.4$$

$$R_A = 0.6$$

The required expression for bending moment due to applied loads and due to unit load is

Members	Origin	M	m	limit
A-C	A	1.50x	0.4x	0-2
CD	A	150x - 50(x-2)	0.4x	2-5
DE	B	150x - 50(x-2)	0.6x - 1(x-4)	4-5
EB	B	150x	0.6x	0-4

$$\Delta_c = \int_0^2 (150x - 10.4x) x \frac{dx}{EI} + \int_2^5 (150x - 50(x-2)) x \frac{dx}{EI} + \int_4^5 (150x - 150(x-4)) x \frac{dx}{EI}$$

Section	origin	M	m'	limits
AC	A	$25.8x - 5x^2$	$0.5x$	$0-3$
CD	B	$14.2x - 10(x-2)$	$0.5x$	$2-3$
DB	B	$14.2x$	$0.5x$	$0-2$

$$A_C = \int_0^3 (25.8x - 5x^2) \frac{dx}{EI} + \int_2^3 (14.2x - 10(x-2)) \frac{dx}{EI}$$

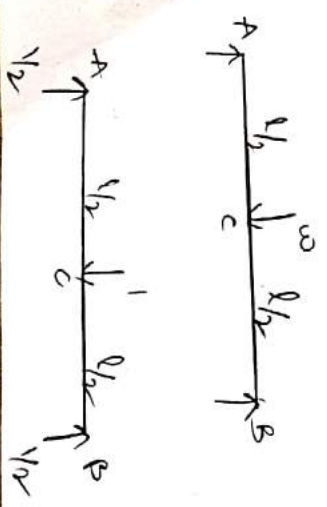
$$+ \int_0^2 14.2x \cdot 10.5x \frac{dx}{EI}$$

$$A_C = \int_0^3 (12.9x^2 - 2.5x^3) \frac{dx}{EI} + \int_2^3 7.1x^2 - 5x^2 + 10 \frac{dx}{EI}$$

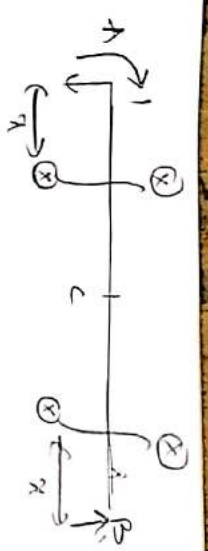
$$+ \int_0^2 7.1x^2 \frac{dx}{EI}$$

$$A_C = \frac{122.7}{EI}$$

→ Determine the maximum slope and deflection of a simply supported beam subjected to a central concentrated load, using unit load Method.



Max deflection occurs at the centre and to obtain the same apply a unit load at 'c'



Section	origin	M	m	limits
AC	A	$w/2x$	$x/2$	$0-l/2$
BC	B	$w/2x$	$x/2$	$0-l/2$

$$A_C = \int_0^{l/2} \frac{Mm}{EI} dx + \int_0^{l/2} \frac{Mm}{EI} dx$$

$$= \int_0^{l/2} (w/2x)(x/2) \frac{dx}{EI} + \int_0^{l/2} (w/2)(x/2) \frac{dx}{EI}$$

$$= 2 \int_0^{l/2} (w/2)(x/2) \frac{dx}{EI}$$

$$= 2 \left(\frac{w}{4EI} \right) \int_0^{l/2} x^2 \cdot dx$$

$$= \frac{w}{2EI} \left(\frac{x^3}{3} \right) \Big|_0^{l/2}$$

$$= \frac{w}{2EI} \left[\frac{(l/2)^3}{3} \right]$$

$$\Delta_c = \frac{\omega l^3}{16EI}$$

A similar procedure is adopted to determine maximum slope at the ends. A unit Moment is applied at 'A' and the bending moment

Expression due to unit moment

$$\theta_A = \int_0^{l/2} \frac{\omega x}{2} \left(1 - \frac{x}{l}\right) \frac{dx}{EI} + \int_0^{l/2} \frac{\omega x}{2} \left(\frac{x}{l}\right) \frac{dx}{EI}$$

$$= \frac{\omega}{2EI} \left[\int_0^{l/2} (lx - x^2) dx + \int_0^{l/2} x^2 dx \right]$$

$$\theta = \frac{\omega l^2}{16EI}$$

4th Unit Slope deflection & Moment distribution

Moment

Slope deflection method:

In the slope deflection, the relationship is established between moments at the ends of members and corresponding ~~and~~ rotations and displacements.

- * The slope deflection method can be used to analyze statically determinant and indeterminate beams
- * When a structure can be solved by using the equations of static equilibrium it is known as "determinant structures".
- * When a structure cannot be solved by using the equations of static equilibrium it is known as "Indeterminant structure".

Ex: continuous beam, propped cantilever etc...



* $\left(\frac{dy}{dx}\right)$ slope at A = 0

y deflection at A = 0

* Simply Supported beam

"For Simply Supported at the middle slope is Zero"

* Case (i)

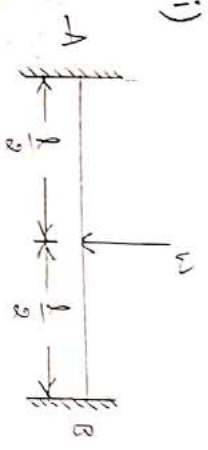


M_{AB} = fixed end moment at A = $-\frac{wL^2}{12}$

M_{BA} = fixed end moment at B = $+\frac{wL^2}{12}$

Bending moment at middle = $\frac{wL^2}{8}$

* case(ii)

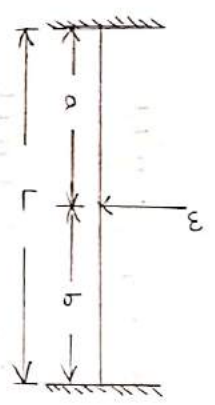


M_{AB} = $-\frac{wL}{8}$

M_{BA} = $\frac{wL}{8}$

B.m at the load = $\frac{wL}{4}$

* case(iii)



M_{AB} = $-\frac{wab^2}{L^2}$

M_{BA} = $\frac{wba^2}{L^2}$

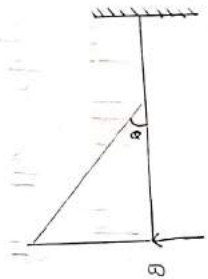
B.m at Load = $\frac{wab}{L}$

Ex: continuous beam, propped cantilever etc...



$\left(\frac{dy}{dx}\right)$ slope at $A=0$

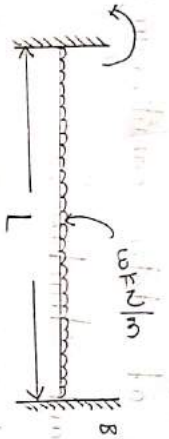
y deflection at $A=0$



* Simply supported beam

"For Simply supported at the middle slope is zero"

* Case (i)

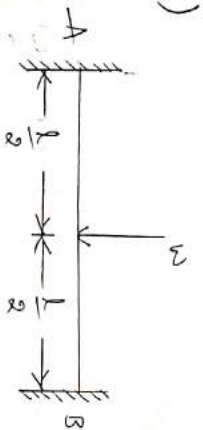


M_{AB} = fixed end moment at $A = -\frac{wL^2}{12}$

M_{BA} = fixed end moment at $B = +\frac{wL^2}{12}$

Bending moment at middle = $\frac{wL^2}{8}$

* case (ii)

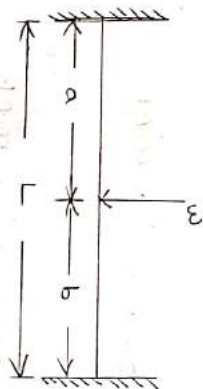


$M_{AB} = -\frac{wl}{8}$

$M_{BA} = \frac{wl}{8}$

B.m at the load = $\frac{wl}{4}$

* case (iii)



$M_{AB} = -\frac{wab^2}{L^2}$

$M_{BA} = \frac{wba^2}{L^2}$

B.m at load = $\frac{wab}{L}$

Step: 1

- To find fixed end moments:

$$m_{AB} = -\frac{wL^2}{12}$$

$$= -\frac{2 \times 6^2}{12}$$

$$= -6 \text{ kN-m}$$

$$m_{BA} = \frac{wL^2}{12}$$

$$= \frac{2 \times 6^2}{12}$$

$$= 6 \text{ kN-m}$$

from B to C it is point load which is acting center to the distance given.

$$m'_{BE} = -\frac{wL}{2}$$

$$= -\frac{12 \times 6}{2}$$

$$= -36 \text{ kN-m}$$

$$m'_{CB} = \frac{wL}{2}$$

$$= \frac{12 \times 6}{2}$$

$$= 36 \text{ kN-m}$$

Step: 2

- To find the Support moments

Slope at A $\Rightarrow \theta_A = 0$

Slope at C $\Rightarrow \theta_C = 0$

$$m_{AB} = \frac{2EI}{L} (\theta_A + \theta_B) + m'_{AB}$$

$$= \frac{2EI}{3} (\theta_B) + (-6)$$

$$m_{BA} = \frac{2EI}{3} (\theta_B - \theta_A)$$

$$m_{BA} = \frac{2EI}{L} (\theta_B + \theta_A) + m'_{BA}$$

$$= \frac{2EI}{6} [\theta_B + 0] + 6$$

$$= \frac{2EI}{3} [\theta_B + 6]$$

$$\begin{aligned}
 m_{BC} &= \frac{2EI}{L} [2i_B + i_C] + m'_{BC} \\
 &= \frac{2EI}{6} [2i_B + 0] - 9 \\
 &= \frac{2EI}{3} i_B - 9 \\
 m_{CB} &= \frac{2EI}{L} (2i_C + i_B) + m'_{CB} \\
 &= \frac{2EI}{6} i_B + 9 \\
 &= \frac{EI}{3} i_B + 9
 \end{aligned}$$

Step: 3:
To find the equations of equilibrium
in the diagram B

$$m_{BA} + m_{BC} = 0$$

$$\frac{2EI}{3} i_B + 6 + \frac{2EI}{3} i_B - 9 = 0$$

$$\frac{4EI}{3} i_B = 3$$

$$i_B = \frac{9}{4EI} \quad \text{--- (1)}$$

Substitute equation (1) in m_{AB} , m_{BA} , m_{BC} , m_{CB} .

$$\begin{aligned}
 m_{AB} &= \frac{EI}{3} \left(\frac{9}{4EI} \right) - 6 \\
 &= -5.85
 \end{aligned}$$

$$\begin{aligned}
 m_{BA} &= \frac{2EI}{3} \left[\frac{9}{4EI} \right] + 6 \\
 &= 7.5
 \end{aligned}$$

$$\begin{aligned}
 m_{BC} &= \frac{2EI}{L} \left[\frac{9}{4EI} \right] - 9 \\
 &= -7.5
 \end{aligned}$$

$$\begin{aligned}
 m_{CB} &= \frac{EI}{3} \left[\frac{9}{4EI} \right] + 9 \\
 &= 9.75
 \end{aligned}$$

Step: 4
To find the bending moment at middle part load:

$$B.m \text{ at } AB = \frac{wL^2}{8} = \frac{2 \times 6^2}{8} = 9 \text{ kN-m}$$

$$B.m \text{ at } BC = \frac{wL}{4} = \frac{12 \times 6}{4} = 18 \text{ kN-m}$$

Step: 5

To find all the reactions and consider

$\Sigma M_B = 0$ on AB span

$$R_B \times 0 + M_{BA} + R_A \times 6 - M_{AB} - 2 \times 6 \times \frac{6}{2} = 0$$

$$7.5 + 6R_A + 5.25 - 36 = 0$$

$$R_A = 3.87 \text{ kN}$$

consider $\Sigma M_B = 0$ on the BC span

$$R_B \times 0 - M_{BC} - R_C \times 6 + M_{CB} + 12 \times 3 = 0$$

$$-7.5 - 6R_C + 9.75 + 36 = 0$$

$$R_C = 6.375 \text{ kN}$$

$$R_A + R_B + R_C = (2 \times 6) + 12 \quad (\text{total load})$$

$$R_B = 12 \text{ kN}$$

Moment Distribution Method

Working process for analysis of continuous beam by using moment distribution method:

Step: 1

To find relative stiffness (K) and modified stiffness factor (K')

$$\text{Relative stiffness } (K) = \frac{\text{moment of inertia}}{\text{Length}}$$

$$\text{modified stiffness factor } (K') = \frac{3}{4} K$$

(either in hing, (or) roller support)

Step: 2

To find distribution factor

$$\text{Distribution factor} = \frac{\text{Relative stiffness factor}}{\text{R.S.F}}$$

ΣK = Sum of K_i for joint

$$DF = \frac{K}{\Sigma K}$$

Note: Distribution factor for fixed support is equal to zero

* Distribution factor for hinged (or) rollersupport is equal to 1.

step: 3
To find the fixed end moments (m^f)

According to table data we can find fixed end moments

step: 4 To find distribution of moment and carry over of moment

D.o.m = Sum of fixed end moment at middle joint) $\times DF$

$$com = \frac{Dom}{2}$$

step: 5 To find end moments (M)

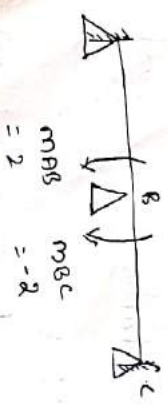
$$End\ moments = m^f + Dom + com$$

step: 6

checking

1) fixed support slope = 0

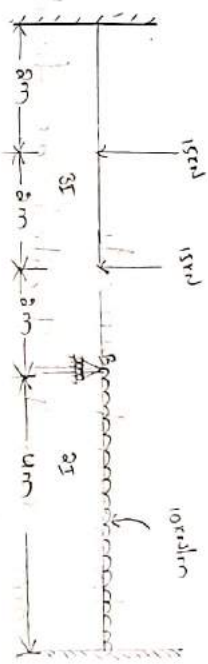
2) End moment at middle joint is equal and opposite direction



$$3) (\theta) slope = \frac{m_{AB} - m_{BA}^f - \frac{1}{2} (m_{AB} - m_{BA}^f)}{\frac{3EI}{L}}$$

Problem:

1) Analyse the two span continuous beam by moment distribution method.



step: 1

To find K & K' (Relative stiffness & modified stiffness)

$$K_{BA} = \frac{m \cdot 0 \cdot I}{L_{AB}} = \frac{8I}{8} = 0.5I$$

$$K_{BC} = \frac{8I}{4} = 0.5I$$

To find Distribution factor

$$DF_{AB} = 0$$

$$DF_{BA} = K_{BA} = \frac{0.5I}{0.5I + 0.5I} = 0.5$$

$$0.5I + 0.5I$$

$$D_{FBC} = \frac{K_{BC}}{EK}$$

$$= \frac{0.5I}{0.5I + 0.5I}$$

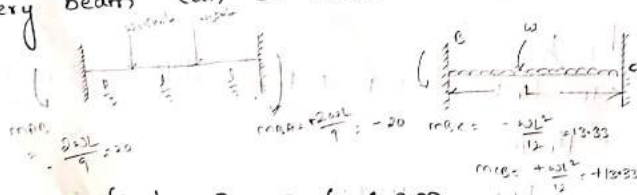
$$= 0.5$$

$$D_{FCB} = 0$$

step: 3

To find fixed end moments

Every beam can be treated as fixed beam



step: 4 To find D.O.M & C.O.M

members	A	B	C
	AB	BA	BC
DF	0	0.5	0
FEM	-20	+20	-13.33
DOM	0	+3.335	+3.335
COM	1.6675	-	-
	21.66	-16.66	16.665

checking step: 5

$$\frac{(m_{AB} - m_{AB}^F) - \frac{1}{2} (m_{BA} - m_{BA}^F)}{\frac{3EI}{L}}$$

$$= \frac{(21.6675 - 20) - \frac{1}{2} (16.6675 - 5.3385)}{0.5I}$$

$$= 1.6675$$

$$\frac{(m_{BC} - m_{BC}^F) - \frac{1}{2} (m_{CB} - m_{CB}^F)}{\frac{2I}{4}} = 0.5I$$

step:

$$\theta_A = \frac{[m_{AB} - m_{AB}^F] - \frac{1}{2} [m_{BA} - m_{BA}^F]}{\frac{3EI}{L}}$$

$$= \frac{[21.6675 - 20] - \frac{1}{2} [-16.665 + 20]}{\frac{3E(3I)}{6}}$$

$$= 0$$

It is correct because "A" is fixed support

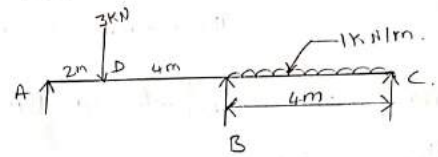
fixed support:

$$\theta_c = (m_{cB} - m_{cB}^F) - \frac{1}{2} (m_{Bc} - m_{Bc}^F)$$

$$= \frac{\frac{3EI}{2} \cdot (-11.6625 + 13.33) - \frac{1}{2} (10.665 - 13.33)}{\frac{3E(2I)}{4}} = 0$$

It is correct because c is fixed support

Ex: A continuous beam ABC 10m long rests on three supports A, B and C at the same level as shown in figure.



Sol

Assume the continuous beam ABC to split up into two fixed beams AB and BC.

Step-I

$$M_{AB}^F = -\frac{wab^2}{l^2} = -\frac{3 \times 2 \times (4)^2}{(6)^2} = -\frac{8}{3} = -2.67 \text{ kN-m}$$

$$M_{BA}^F = \frac{wa^2b}{l^2} = \frac{3 \times 2^2 \times 4}{6^2} = +1.33 \text{ kN-m}$$

$$M_{BC}^F = -\frac{wl^2}{12} = -\frac{1 \times (4)^2}{12} = -\frac{4}{3} = -1.33 \text{ kN-m}$$

$$M_{CB}^F = \frac{wl^2}{12} = \frac{1 \times 4^2}{12} = \frac{4}{3} = 1.33 \text{ kN-m}$$

Calculating Distribution factor at B.

Stiffness factor for BA

$$K_{BA} = \frac{3EI}{l} = \frac{3EI}{6} = \frac{EI}{2}$$

(The beam is hinged at A)

Similarly, stiffness factor for BC

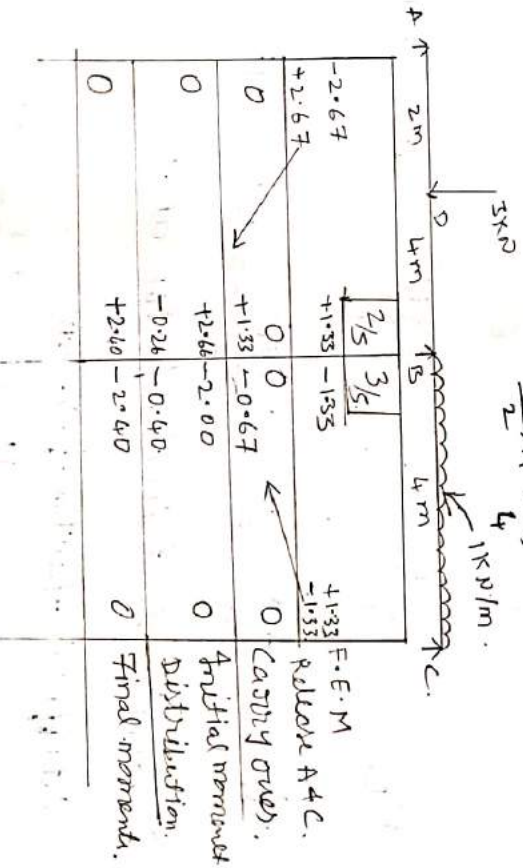
$$K_{BC} = \frac{3EI}{L} = \frac{3EI}{4}$$

(∵ The beam is hinged at C)

⇒ Calculating Distribution factor for BA and BC

$$D.F._{BA} = \frac{EI}{2} \cdot \frac{EI}{2 + \frac{3EI}{4}} = \frac{2}{5}$$

$$D.F._{BC} = \frac{3EI}{4} \cdot \frac{EI}{2 + \frac{3EI}{4}} = \frac{3}{5}$$



⇒ Calculating B.M and S.F

$$B.M \text{ For the Span AB} = \frac{w_0ab}{L} = \frac{3 \times 2 \times 4}{6} = 4.0 \text{ kN-m}$$

$$B.M \text{ For the Span BC} = \frac{w_0b^2}{8} = \frac{1 \times 4^2}{8} = 2.0 \text{ kN-m}$$

S.F.

$$R_A + R_B + R_C = 3 + 1(4) = 7 \text{ kN}$$

Taking moment about B.

$$\sum M_B = 0$$

$$R_A \times 6 - 3 \times 4 = -2.4$$

$$R_A = \frac{-2.4 + 12.0}{6} = \frac{9.6}{6} = 1.6 \text{ kN}$$

Similarly

$$R_C \times 4 - 4 \times 2 = -2.4$$

$$R_C = \frac{-2.4 + 8}{4} = \frac{5.6}{4} = 1.4 \text{ kN}$$

$$R_B = 7$$

$$R_A + R_B + R_C = 7$$

$$R_B = 7 - (1.6 + 1.4)$$

$$R_B = 4.0 \text{ kN}$$

