UNIT-I

Design of Reinforced Concrete Structures

Unit-I: Concept of Rcdesign - working stress method - Limit state method - matetial stress Strain curves. Safety factors - characteristic values, Stress Block parameters - IS-456-2000. Beams: Limit State analysis and design of single reinforced, doubly reinforced, T & L beam section.

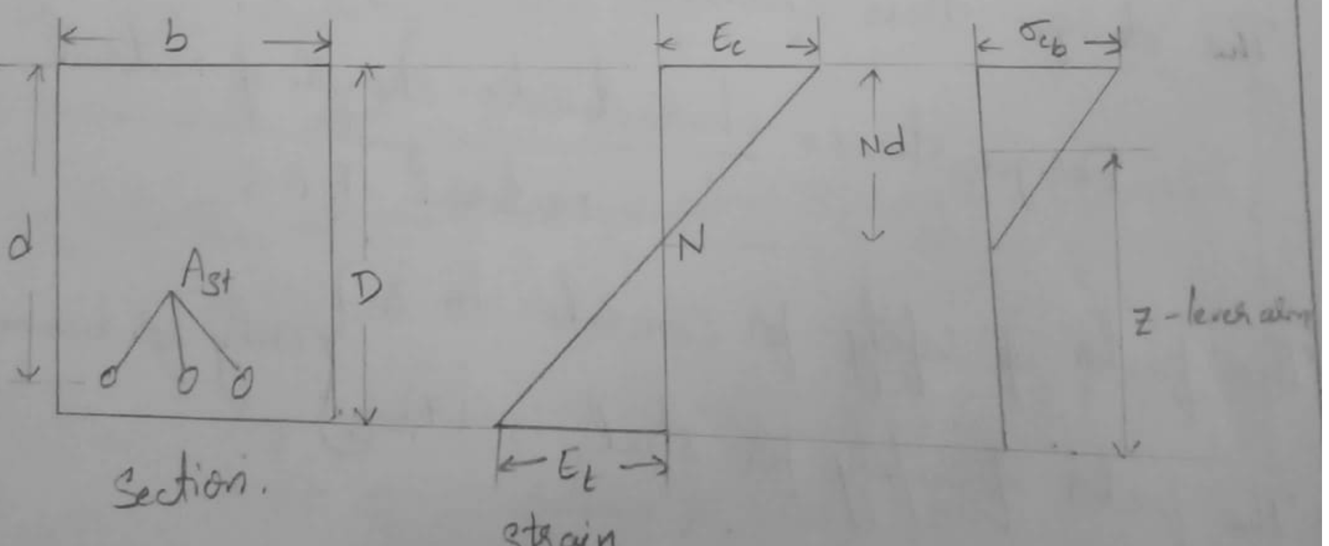
DESIGN OF REINFORCED CONCRETE STRUCTURES

Methods of Design:

1. Working stress method
2. Ultimate load method
3. Limit state method

Assumptions of Working stress method:

* The concrete is elastic, the steel and concrete act together in elastic nature.
* The tensile strength of concrete is ignored.
* The concrete is elastic, i.e., the stress in concrete varies nearly zero at the neutral axis to a maximum at the extreme fibre.
* A section is a plane before remains plane after bending.
* The modular ratio, 'm' value is $\frac{280}{3\sigma_{cb}}$. The σ_{cb} is the Permissible stresses in bending.



A_{st} - Area of steel

d - Effective depth

b - width of the section

C - Total compression

D - Depth of the section

Z - lever arm [the distance b/w point of application of forces of compression and the force of tension.]

ND - Depth of Neutral axis

T - The total force of tension

σ_{cb} - The permissible compressive stresses in concrete

σ_{st} - Permissible tensile stress in steel

E_c - Compressive strain in concrete

E_{st} - Tensile strain in steel.

The design stress also called Permissible stress.

The design stress characteristic strength.

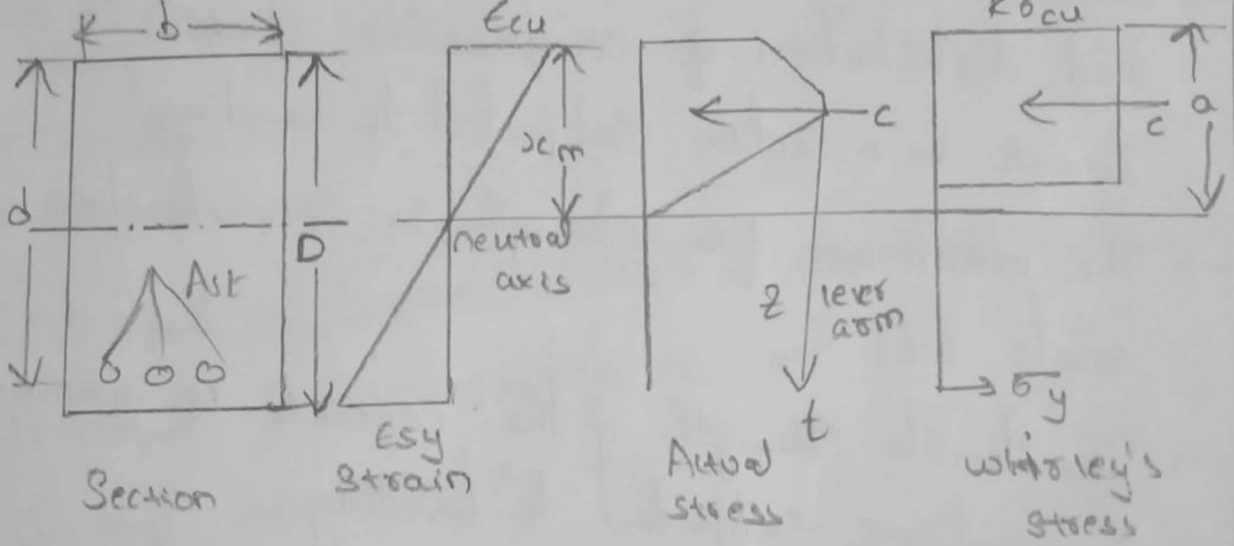
$$\text{Design stress} = \frac{\text{characteristic strength of material}}{\text{Material F.O.S}}$$

$\left.\begin{array}{l}\to \text{The factor of safety for concrete is } 3 \\ \to \text{The factor of safety for steel is } 1.78\end{array}\right\}$ only for beams.

✩ Ultimate Load method:-

The Ultimate load method, the working loads are increased by suitable factor to obtain ultimate loads. These factors are called load factors.

This method takes non linear stress strain behaviour of concrete.



d - depth of Rectangular stress block.

x_m - The depth of neutral axis at failure

z - lever arm.

σ_{cu} - The ultimate compressive strength of concrete cubes at 28 days.

$k.\sigma_{cu}$ - Avg. stresses.

$0.85\ \sigma_{cu}$ according to whitney's

$0.55\ \sigma_{cu}$ according to IS: 456 : 2000.

σ_y - yielding stresses in steel
ϵ_{cu} - The ultimate strain in concrete
ϵ_{sy} - The yield strain in steel

Assumptions :- (according to IS : 456 - 1964)

→ A section which is plane before bending remains plain after bending.

→ ultimate stresses and strains are not proportional and distribution of compressive stresses is non-linear in a section subjected to bending.

→ The maximum fire strength in concrete does not exists 0.68 σ_{cu}.

→ The tensile strength of the concrete is ignored in sections subjected to bending.

✡ Limit state method:-

Limit state method (or) limit state design (or) plastic design.

The limit state concept is to achieve an acceptable probability that a structure will not become unserviceable in its life time for the use of which it is intend, that is it will not reach a limit state.

The most important of these limit state which must be explained in design as follows.

→ Limit state of collapse

→ Limit state of serviceability

Limit state collapse:- The structures are designed for a limit state of collapse

i) flexure

ii) Shear

iii) Compression.

iv) Torsion.

Limit state of Serviceability:- A structure designed for limit state of Serviceability.

i) Control on deflection

ii) control on Corrosion

iii) Control on cracking.

iv) Control on Abrasion.

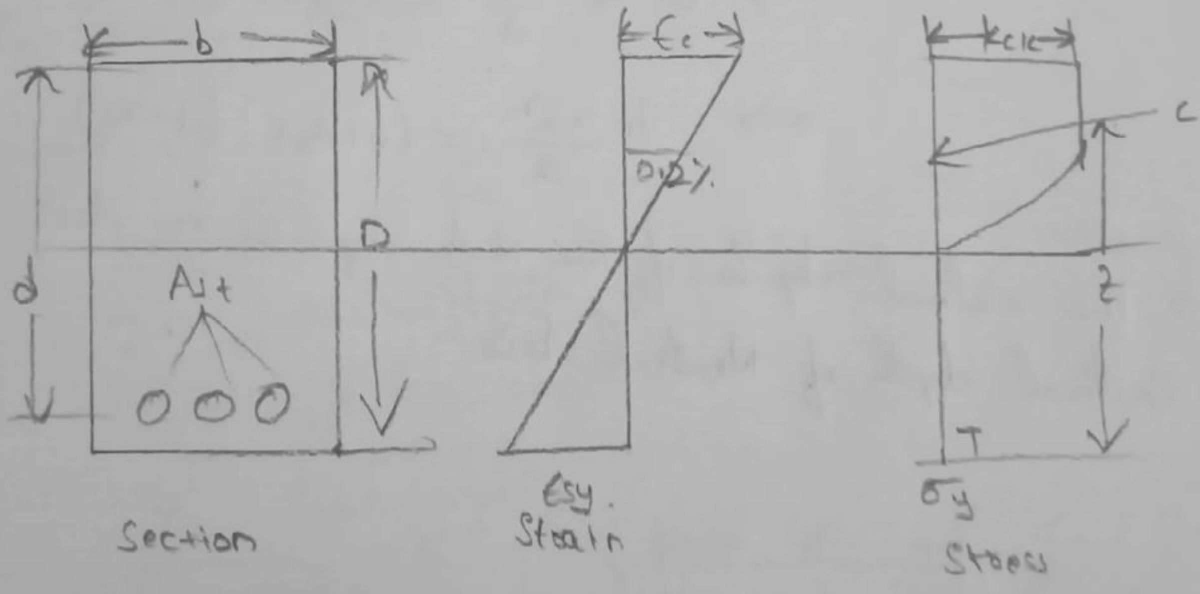
v) Actual behaviour of structure.

Assumptions in analysis and design of limit state method as per IS : 456 - 2000.

a) plane sections normal to the axis remain plane after bending.

Limit	Material	Loads.
collapse	$\gamma_c = 1.5$ concrete $\gamma_s = 1.15$ steel	$UL = 1.5(DL + LL)$ $UL = 1.5(D.L + WL)$ (or) $(0.9 DL + 1.5 W.L)$
Serviceability	$\gamma_c = 1.0$ concrete $\gamma_s = 1.0$ steel	$SL = 1.0(DL + LL)$ $SL = 1.0(DL + WL)$ $SL = (1.0 D.L) + (0.8 LL)$ $(0.8 WL)$

stress - strain diagram:-



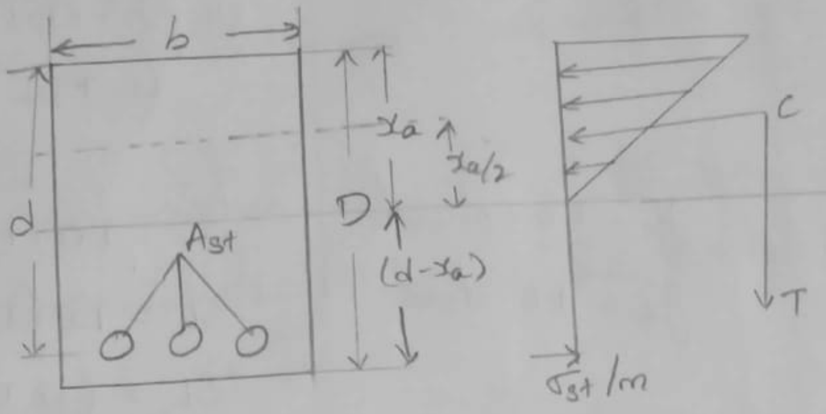
Where
σ_{ck} - characteristic compressive strength.
k - Factor of safety.

LSL can be expressed as

$$\frac{\delta}{L} \leq \frac{1}{2}$$

Depth of Neutral axis :-

a) Actual depth of neutral axis (x_a)



stress Profile

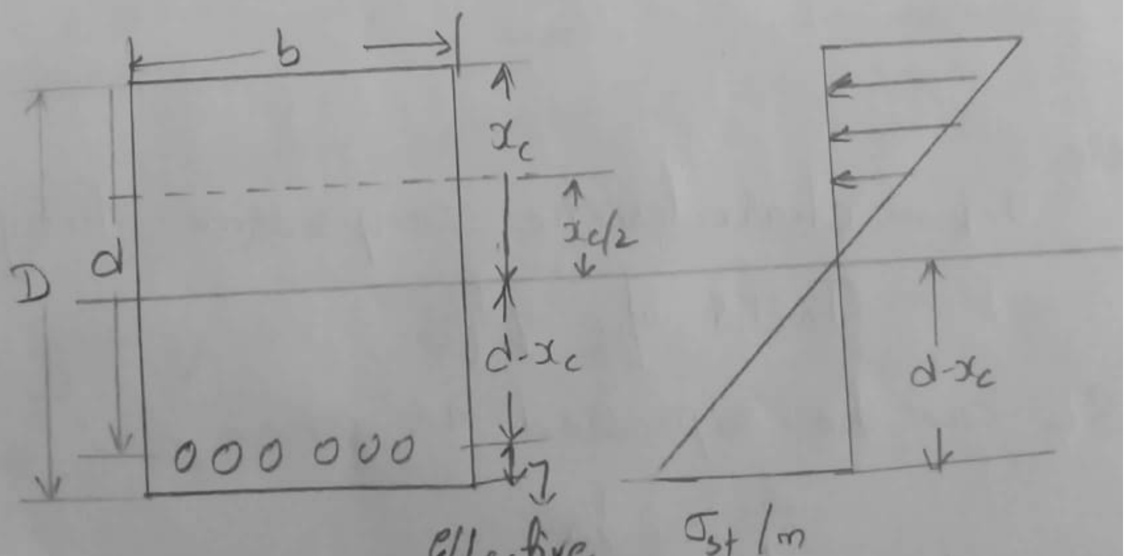
Area of concrete = area of steel × distance

$$\Rightarrow b \cdot x_a \cdot \frac{x_a}{2} = (m A_{st})(d - x_a)$$

$$\Rightarrow b \cdot \frac{x_a^2}{2} = (m A_{st})(d - x_a)$$

→ The actual depth of the N.A depends on Ast.

b) Critical depth of Neutral Axis :-



stress profile

By similar triangular property

$$\frac{\sigma_{cbc}}{x_c} = \frac{\sigma_{st}/m}{(d-x_c)}$$

$$x_c = \frac{d}{\left[1+\frac{\sigma_{st}}{m.\sigma_{cbc}}\right]}$$

$$x_c = k.d \qquad \left[\because k = \frac{1}{1+\frac{\sigma_{st}}{m.\sigma_{cbc}}}\right]$$

Note:- The depth at which the max. permissible stresses in concrete and steel are obtained at same time is the critical depth.

Where,

b = width of the beam

D = overall depth of the beam

d = effective depth

Z - lever arm

clear cover = $D-d$

$x_a \rightarrow$ Actual depth of N.A

$x_c \rightarrow$ Critical depth of N.A

k = F.O.S.

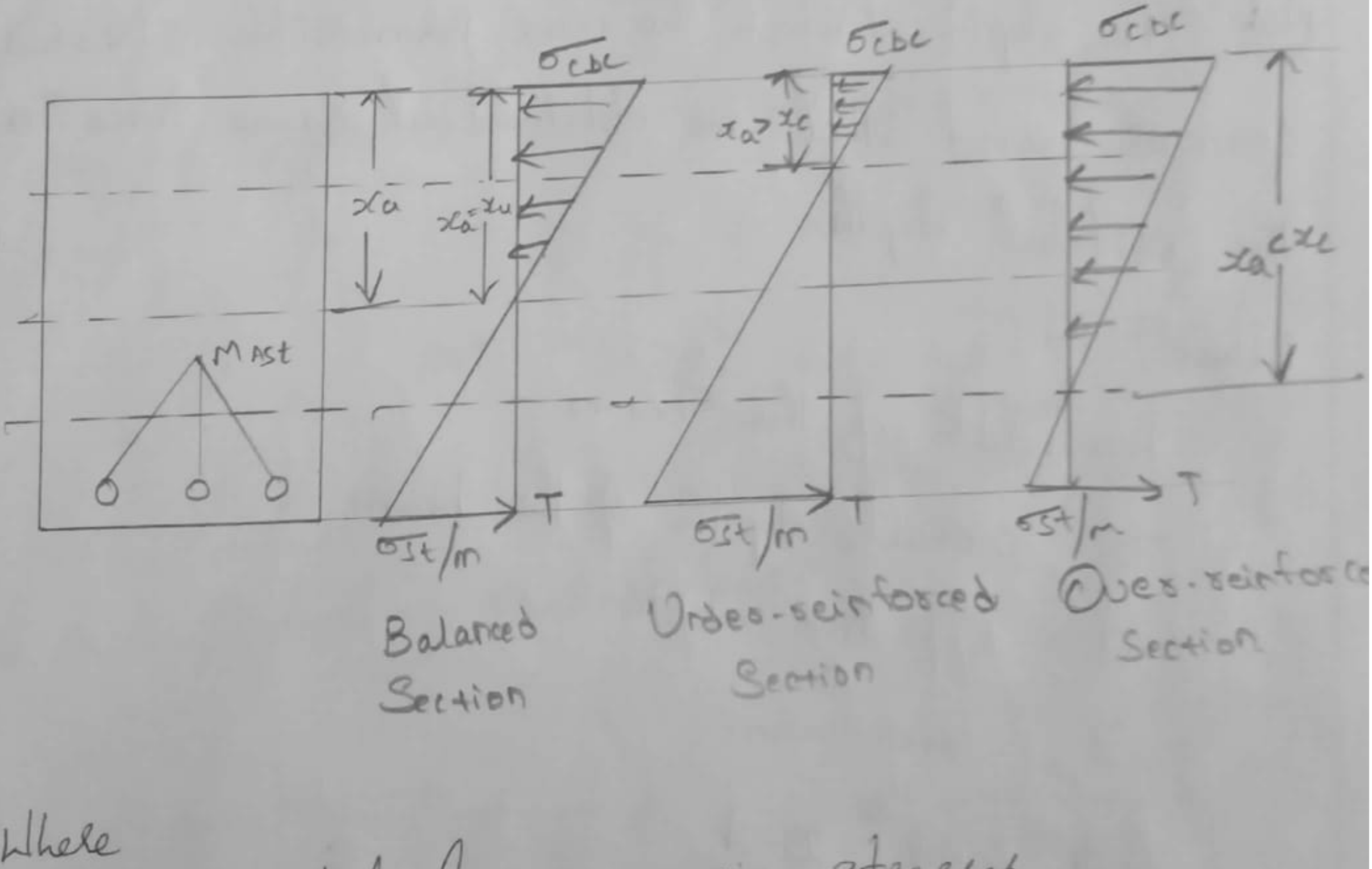
Q) Different types of Sections & compare the Actual depth x_a to x_c (critical depth).

→ under reinforced section

→ Balanced Section ($x_a = x_c$)

→ over reinforced section.

*) Balanced Section :-



Balanced Section　　Under-reinforced Section　　Over-reinforced Section

Where

σ'_{cbc} - Actual compressive stresses

σ_{cbc} - Permissible compressive stresses

σ'_{st} - Actual Compressive stresses in steel

σ_{st} - Permissible compressive stress in steel

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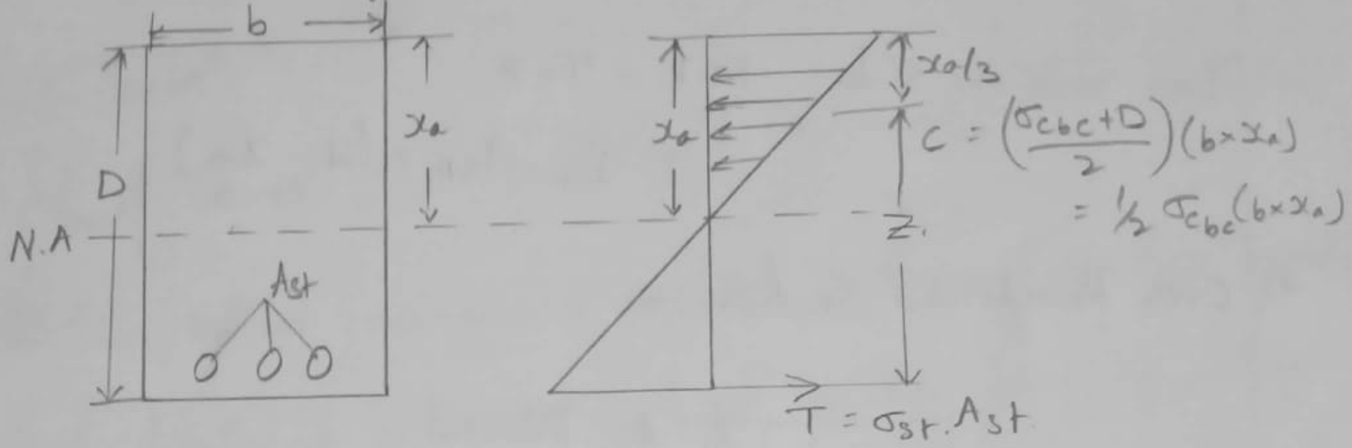
Moment Resistance:-

The max value of Bending moment that any beam can resist safely.



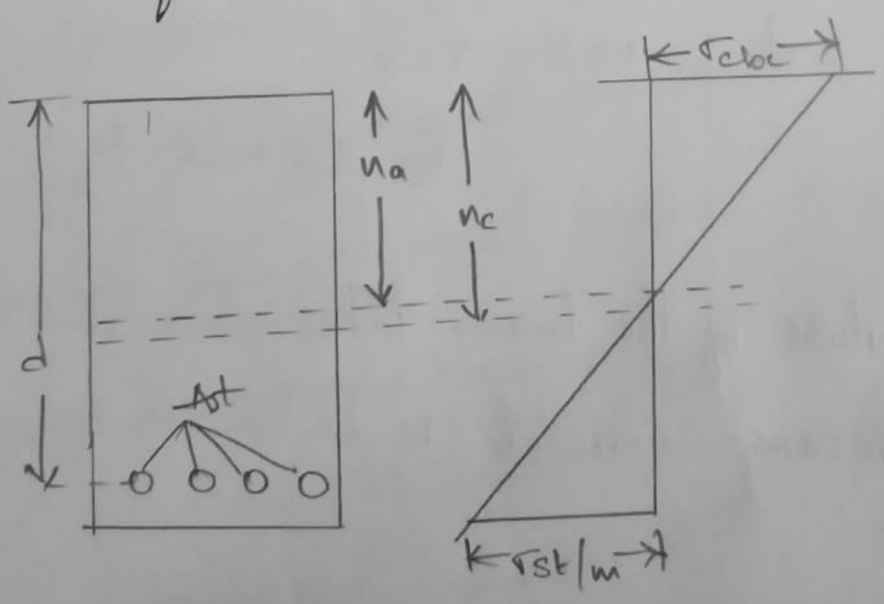
Moment Resistance for compression side:

$$M.R = C \times Z$$
$$= \frac{1}{2} \times \sigma_{cbc} \times b \times x_a \times \left(d - \frac{x_a}{3}\right)$$

Moment Resistance for tension side:

$$M.R = T \times Z$$
$$= \sigma_{st} \times A_{st} \times \left(d - \frac{x_a}{3}\right)$$

*) Under Reinforced section:-



Moment of Resistance :-

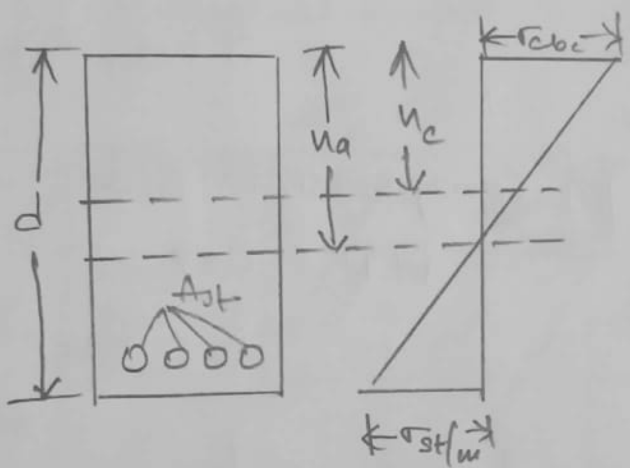
For Compression Side : $M.R = C \times Z$

$$= \frac{1}{2} \sigma'_{cbc} \cdot b x_a \times \left(d - \frac{x_a}{3}\right)$$

For Tension Side : $M.R = T \times Z$

$$= \sigma_{st} \cdot A_{st} \times \left(d - \frac{x_a}{3}\right).$$

*) Over Reinforced section :-



Moment of Resistance :-

For Compression Side : $M.R = C \times Z$

$$= \frac{1}{2} \sigma_{cbc} b x_c \left(d - \frac{x_c}{3}\right)$$

For Tension Side : $M.R = T \times Z$

$$= \sigma_{st}' A_{st} \times \left(d - \frac{x_c}{3}\right)$$

Note :- The failure of the beam takes place due to failure of Concrete because Concrete is brittle material

* Design of Singly reinforced beam in W.S.M :-

step 1 :- Write given data, Loads, grade of Concrete and steel.

step 2 :- Write given design constants.

→ Modular ratio (m)

→ Permissible stress (σ_{st} , σ_{cbc})

→ Critical coefficient (k_c)

→ lever arm co-efficient (j)

→ M.O.R co-efficient (Q)

Step 3 :- Find c/s^n dimensions.

→ Assume overall depth (D - provided) = $\frac{1}{10} \times L_{eff}$.

Depth (required) = effective depth + effective cover

* Effective depth :

→ Under Reinforced Section - M.O.R > B.M → $d > \sqrt{\frac{B.M}{Q.b}}$

→ Balanced Section - B.M = M.R = Qbd^2

$$= d = \sqrt{\frac{B.M}{Q.b}} = \sqrt{\frac{M.R_{balance}}{Q \times b}}$$

→ $b = \frac{Span}{30} + 8$ = ____ cm. [width of section]

<u>Note</u>:- Thumb rule of breadth;

* The width of column on which beam is going to rest (or) the width of masonry wall which is to be supported (or)

$b = 250$ to $300\ mm$.

→ $D_{req.} < D.provided$.

Step: 4:- Find out the area of steel

$$M.R = (\sigma_{st} \cdot A_{st})\left(d - \frac{x_a}{3}\right) = (\sigma_{st} \cdot A_{st}) j \cdot d$$

$$A_{st} = \frac{M.R}{\sigma_{st} \cdot j \cdot d} = \frac{B.M}{\sigma_{st} \cdot j \cdot d}$$

Step 5:- check for Minimum steel

$$\frac{A_{st}.min}{bd} = \frac{0.87}{f_y}$$

$$A_{st}.min = \frac{0.87 \times bd}{f_y}$$

$$\text{No. of steel bars} = \frac{\text{Area of steel}}{\text{area of one bar}}$$

Problems:-

1) A critical concrete beam of rectangular section 300 mm width of 650 mm over all depth is reinforced with four bars 32 mm dia at an effective depth of 600 mm. Using M_{20} grade concrete and Fe 415, HYSD bars, estimate the moment of resistance of the section.

Sol:- Width, $b = 300mm$

Overall depth, $D = 650mm$

Effective depth, $d = 600mm$

$f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$

Area of tension steel, $A_{st} = 4 \times \frac{\pi}{4} d^2$

$= 4 \times \frac{\pi}{4} \times (32)^2$

$= 3216.9 mm^2$

Permissible stresses:-

$\sigma_{cbc} = 7 N/mm^2$

$\sigma_{st} = 230 N/mm^2$

Design constants for M_{20} Concrete and Fe 415 steel:

Modular ratio, $m = 13.33$.

i.e, $m = \frac{280}{3\sigma_{cbc}} \Rightarrow \frac{280}{3 \times 7} = 13.33$.

$$k = \frac{1}{1 + \frac{\sigma_{st}}{m.\sigma_{cbc}}}$$

$$= \frac{1}{1 + \frac{230}{13.33 \times 7}} = 0.288$$

Critical depth of Neutral axis, $x_c = k.d$

$= 0.288d$

Depth of Neutral axis :-

$$\frac{b.x_a^2}{2} = m.A_{st}(d - x_a)$$

$$\frac{300 \times x_a^2}{2} = 13.33 \times 3216.9 (600 - x_a)$$

$$\boxed{x_a = 295.16m.}$$

Critical depth of neutral axis :

$$x_c = \left[\frac{1}{1 + \left[\frac{\sigma_{st}}{m.\sigma_{cbc}}\right]}\right] \times d$$

$$\boxed{x_c = 172.8mm.}$$

Moment of Resistance :-

$\because x_a > x_c$, it is a over reinforced beam.

$$M.R = \frac{1}{2}\sigma_{cbc} \cdot b.x_a\left(d - \frac{x_a}{3}\right)$$

$$M.R = \frac{1}{2} \times 7 \times 300 \times 295.16\left(600 - \frac{295.16}{3}\right)$$

$$\boxed{M.R = 155.46 \ kN/m}$$

* Limit state Method :-

Procedure:

- Limiting depth of Neutral axis:

From the strain diagram, based on assumptions,

$$\frac{0.0035}{x_u\,lim} = \frac{\frac{0.087}{E_s} + 0.002}{d - x_u\,lim}$$

$$\frac{x_u\,lim}{d} = \frac{0.0035}{\frac{0.87 f_y}{E_s} + 0.0055}$$

$$\frac{x_u\,lim}{d} = \frac{700}{1100 + 0.87 f_y}$$

values for xulim/d for different grades of steel.

$f_y (N/mm^2)$	250	415	500
$x_u\,lim/d$	0.53	0.48	0.46

- Actual depth of Neutral axis (x_u):

$$\bar{x} = 0.42 x_u.$$

Force in compression C_u = Avg. stress x area of beam in comp.

$$= 0.36 f_{ck} \cdot b \cdot x_u.$$

Force in tension = T_u = Design yield stress x area of steel

$$= 0.87 f_y \cdot A_{st}.$$

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Force of Compression should be equal to force of tension.

$$0.36 f_{ck} \cdot b \cdot x_u = 0.87 f_y \cdot A_{st}$$

$$\boxed{x_u = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b}}$$

* **Moment of Resistance :-**

For compression zone : $M_u = C_u \times z$

$$= 0.36 \times f_{ck} \cdot b \cdot x_u (d - 0.42 x_u)$$

For Tension zone : $M_u = T \times z$

$$= 0.87 \times f_y \times A_{st} (d - 0.42 x_u).$$

Limiting moment of Resistance ($M_{u lim}$) :-

$$x_u = x_{u lim}.$$

→ From Compression side

$$M_{u lim} = 0.36 f_{ck} \cdot b \cdot x_{u lim} (d - 0.42 x_{u lim})$$

$$= f_{ck} \cdot b \cdot d^2 \left[\left(0.36 \frac{x_{u lim}}{d} \left(1 - 0.42 \frac{x_{u lim}}{d}\right)\right]$$

$$\frac{M_{u lim}}{f_{ck} \cdot bd^2} = 0.36 \cdot \frac{x_{u lim}}{d} \left(1 - 0.42 \times \frac{x_{u lim}}{d}\right)$$

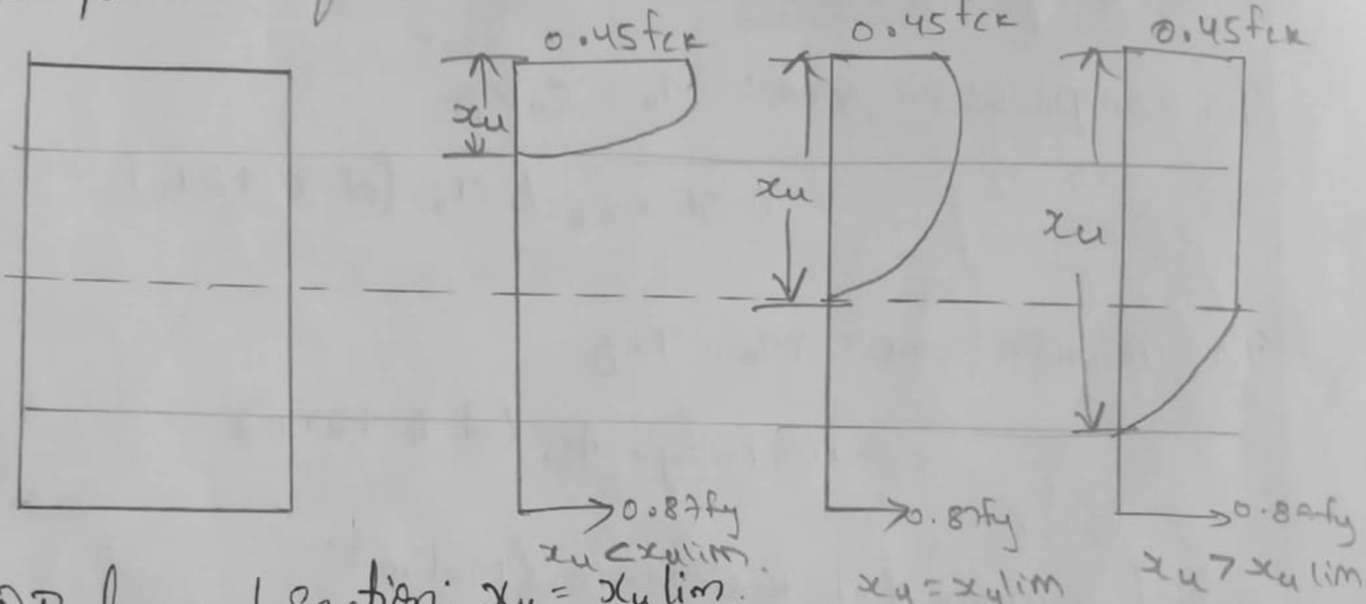
Limiting percentage of steel:

$$P_t \text{ limit} = \frac{A_{st} \text{ lim}}{bd} \times 100$$

$$= \frac{0.36 f_{ck}}{0.87 f_y} \times \frac{x_{u\,lim}}{d} \times 100$$

$$= 41.37 \left[\frac{f_{ck}}{f_y}\right]\left[\frac{x_{u\,lim}}{d}\right]$$

Comparision of x_u and x_u limit:-



① Balanced Section: $x_u = x_{u\,lim}$.

$M_{ulim} = 0.36 f_{ck} \cdot b \cdot x_{u\,lim} (d - 0.42 x_u)$ - Compression

$M_{ulim} = 0.87 f_y \cdot A_{st} (d - 0.42 x_{u\,lim})$ - Tension

② Underreinforced Section: $(x_u < x_{u\,lim})$

$$x_u = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b}$$

③ Over Reinforced Section: $(x_u > x_{u\,lim})$.

$M_{ulim} = 0.36 f_{ck} \cdot b \cdot x_{u\,lim} (d - 0.42 x_{u\,lim})$ - Compression

$$f_{st} \neq 0.87 f_y.$$

Problems:-

1) A singly reinforced concrete beam section 300 mm × 550 mm is reinforced with 5 bars of 16mm diameter with an effective cover of 50mm. The beam is simply supported over a span of 5m. Find the safe uniformly distributed load the beam can carry. Use M20 grade concrete and Fe-415 steel.

Sol:-

$b = 300mm$

$d = 550 - 50 = 500mm$

$f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$

$A_{st} = 5 \times \frac{\pi}{4} \times 16^2 = 1005.3 mm^2$

i) Depth of neutral axis

$$C = T$$

$$0.36 f_{ck} . b . x_u = 0.87 f_y . A_{st}$$

$$x_u = \frac{0.87 f_y . A_{st}}{0.36 . f_{ck} . b}$$

$$x_u = \frac{0.87 \times 415 \times 1005.3}{0.36 \times 20 \times 300}$$

$$x_u = 168 mm.$$

$x_{u\,lim} = 0.48d$

$= 0.48 \times 500$

$= 240mm$

$x_u < x_{u\,lim}$

∴ the section is Under reinforced section

ii) Moment of Resistance

$$M_u = 0.87 f_y \cdot A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 1005.3 (500 - 0.42 \times 168)$$

$$= 155.87 \times 10^6 \text{ N-mm}$$

$$= 155.87 \text{ kN-m}$$

iii) Safe load:

$$\text{Factored B.M} = \frac{W_u l^2}{8}$$

$$155.87 = \frac{W \times 5^2}{8} = 3.125 W$$

$$W = \frac{155.87}{3.125} = 49.88 \text{ kN/m}$$

Safe Working load of beam

$$w = \frac{W}{\text{load factor}} = \frac{49.88}{1.5}$$

$$= 33.25 \text{ kN/m}$$

Self weight of the beam $= 0.3 \times 0.55 \times 1 \times 25 = 4.125$ kN/m

Net superimposed load the beam can carry $= 33.25 - 4.125$

$= 29.125$ kN/m

Doubly Reinforced beam:-

Beams which are reinforced in both compression an tension sides are called as doubly reinforced beam.

Situations under which doubly reinforced beams are used.

* When the depth of the beam is restricted due to architectural or any construction Problems.
* At the Supports of a continous beam where B.M changes its sign
* In precast members (during handling B.M changes its sign)
* In bracing members of a frame due to changes in the direction of wind loads.
* To reduce long term deflections or to increase stiffness of the beam.

Design of Doubly Reinforced beams:-

→ Find the ultimate moment of resistance and area of tension and compression reinforcement.

→ Assume $x_u = x_{umax}$

→ calculate the strain in compression steel, $\epsilon_{sc} = 0.0035\left[1-\frac{d'}{x_u}\right]$ and the corresponding stress f_{sc}. from the stress strain curve of steel or from table 3.4

→ Determine the depth of neutral axis. x_u.

$$x_u = \frac{0.87 f_y . A_{st} - f_{sc} . A_{sc}}{0.36 f_{ck} . b}$$

→ Moment of resistance of the section is given by

$$M_u = 0.36 f_{ck} . b . x_u (d - 0.42 x_u) + f_{sc} . A_{sc} (d - d')$$

If $x_u > x_{umax}$, x_u is limited to x_{umax}.

$$M_u = 0.36 f_{ck} . b . x_{umax} (d - 0.42 x_{umax}) + f_{sc} . A_{sc} (d - d')$$

Problems :-

1. Design a rectangular reinforced concrete beam for a clear span of 4000mm. The Super imposed load is 35 kN/m and the size of the beam is limited to 250mm x 400mm. Use M_{20} grade concrete and Fe 415 steel.

Sol:-

$b = 250mm$

$d = 400 - 40 = 360mm$

$d' = 40mm$

$f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$

Effective span : least of

centre to centre of supports $= 4 + 0.3 = 4.3m$.

clear span + d $= 4 + 0.36 = 4.36m$

Hence, Effective span $= 4.3m$.

Loads:-

Self wt of the beam = $0.25 \times 0.4 \times 25 = 2.5 kN/m$

Super imposed load = $35 kN/m$

Total load = $35 + 2.5 = 37.5 kN/m$

Factored load $W_u = 1.5 \times 37.5 = 56.25 kN/m$

Factored B.M. $M_u = \frac{W_u l^2}{8} = \frac{56.25 \times 4.3^2}{8}$

$= 130 kN\text{-}m$

Limiting M.O.R of the given section as a singly reinforced section

$M_{u\,lim} = 0.138 f_{ck} . bd^2$

$= 0.138 \times 20 \times 250 \times 360^2$

$= 89.42 \times 10^6 N\text{-}mm$

$= 89.42 kN\text{-}m$

As $M_u > M_{ulim}$, the section should be designed as a doubly reinforc

Area of Tension steel corresponding to $M_{u,lim}$ (A_{st1})

$0.87 f_y . A_{st} = 0.36 f_{ck} . b . x_{u\,max}$.

$A_{st1} = \frac{0.36 f_{ck} . b . x_{u\,max}}{0.87 f_y}$

$= \frac{0.36 \times 20 \times 250 \times 0.48 \times 360}{0.87 \times 415} = 861.5 mm^2$

Compression Reinforcement (A_{sc}):

For $\frac{d'}{d} = \frac{40}{360} = 0.11$,

$M_{u2} = M_u - M_{ulim}$

$= 130 - 89.42 = 40.58$ kN-m

$M_{u2} = f_{sc} \cdot A_{sc} (d - d')$

$40.58 \times 10^6 = 351 \times A_{sc} (360 - 40)$

$A_{sc} = \frac{40.58 \times 10^6}{351 (360 - 40)} = 361.29 \text{ mm}^2$

Additional Tensile stress (A_{st2}):

$0.87 f_y . A_{st2} = f_{sc} . A_{sc}$

$A_{st2} = \frac{f_{sc} . A_{sc}}{0.87 f_y} =$

$= \frac{351 \times 361.29}{(0.87 \times 415)} = 351.23 \text{ mm}^2$

Total tension steel $A_{st} = A_{st1} + A_{st2}$

$= 861.5 + 351.2$

$= 1212.7 \text{ mm}^2$

Provide 4-20mm bars in tension ($A_{st} = 1256 \text{ mm}^2$) and 2-16mm bars in Compression ($A_{sc} = 402 \text{ mm}^2$).

Using SP-16:

$$M_u/bd^2 = \frac{130\times10^6}{250\times360^2} = 4.01$$

$$\frac{d'}{d} = \frac{40}{360} = 0.11$$

Refer table 50 of SP-16 corresponding to $f_y = 415\ N/mm^2$ and $f_{ck} = 20\ N/mm^2$. Read the corresponding values of percentages of reinforcements P_t and P_c.

$P_t = 1.342$ and $P_c = 0.408$

$$A_{st} = P_t \times \frac{bd}{100}$$

$$A_{st} = \frac{1.342}{100}\times250\times360$$

$$\boxed{A_{st} = 1208\ mm^2}$$

$$A_{sc} = P_c \times \frac{bd}{100}$$

$$A_{sc} = \frac{0.408}{100}\times250\times360$$

$$\boxed{A_{sc} = 367\ mm^2}$$

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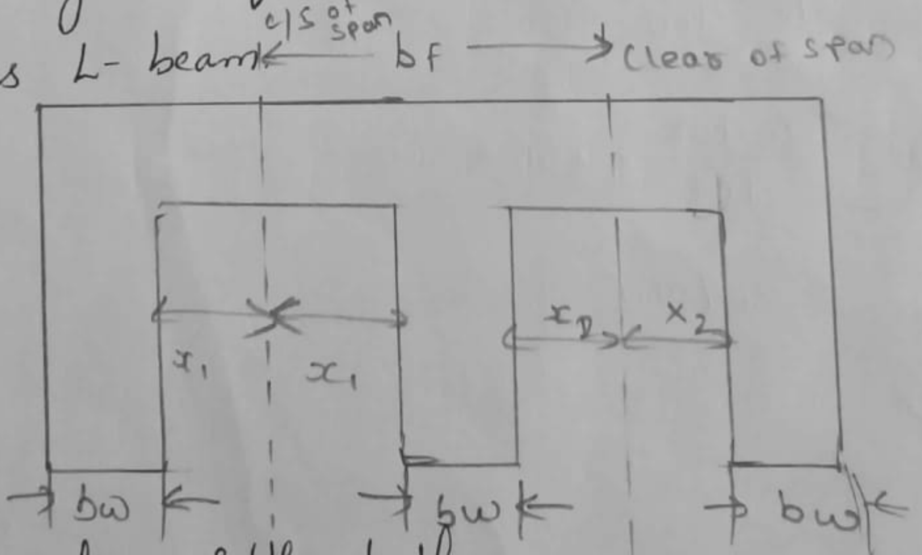
* Advantages of T- beam:-

- As the slab being monolithic with the beam is also compressed and shares the compressive force with the beam, which significantly increases the moment of resistance of the beam.
- As most of the compressive force is shared by the flange, the depth of the beam required is less and hence the maximum deflections are also less.

* Flanged Sections of beams:-

- R.C.C beams and slabs can be cast together to form a monolithically construction.
- When the slabs occurs structure contruction.
- When the slab occurs on both sides of the beam. (Intermediate) the beam is known as 'T' beam. When the slab is only one of the beam (End beam). the beam is known as L- beam



Where,

b_f = Effective width of flange

d = Effective depth

D_f = Depth of the flange (or) slab

D_w = Width of the web (or) rib.

Design of 'T' beam using (IS : 456-2000) :-

- Effective flange width (b_f)
- Flange thickness (d_f)
- Breadth of the rib (b_w)
- Depth of the rib (D_w)
- For T-beams, $b_f = \frac{l_0}{6} + b_w + 6D_f$
- For L-beams, $b_f = \frac{l_0}{12} + b_w + 3D$
- For isolated 'T' beam

$$b_f = \frac{l_0}{\left[\frac{l_0}{b}\right] + 4} + b_w$$

Where

l_0 = distance b/w points of zero moments in the beam

b = actual width of the flange.

- For continous beam, l_0 may be taken as 0.7 times of Effective span.

Thickness of flange :(D_f):-

- It is equal to the total thickness of the slab, the flange Provided compressive resistance to the tension.
- It is generally ½ to ⅔ of the width of the web.

→ It is the distance from the top of the flange to the centre of tensile reinforcement.

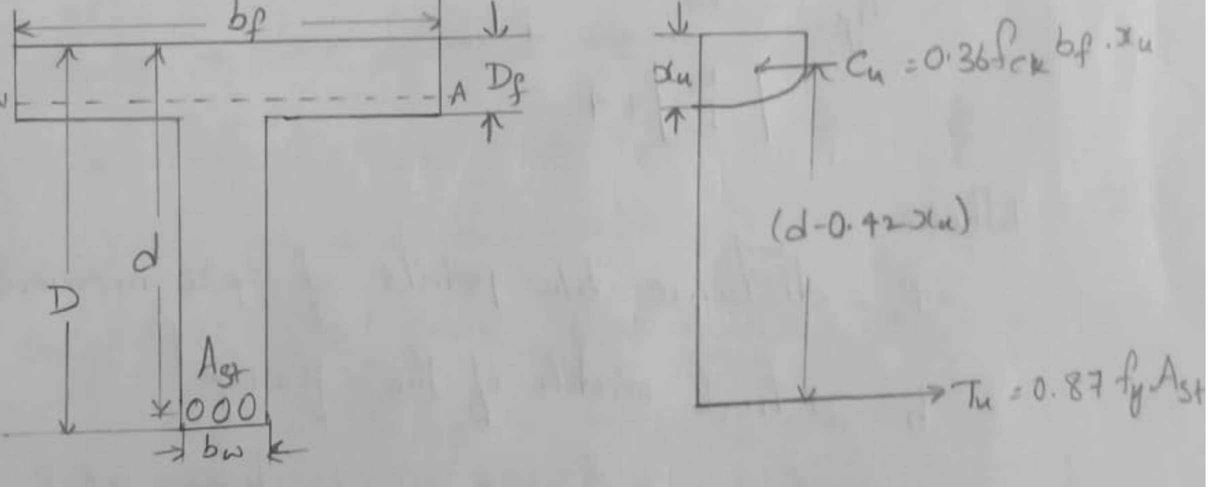
→ The depth may vary from $l/12$ to $l/15$ for simply supported beam.

→ The continous beam based on stiffness is $l/15$ to $l/20$.

Moment of Resistance:-

The Moment of Resistance of the flanged section depends on the depth of the neutral axis. There are 3 cases.

Case 1: Neutral axis is within the flange ($x_u \le D_f$)



$C = 0.36\, f_{ck} \cdot b_f \cdot x_u$

$T = 0.87\, f_y \cdot A_{st}$

Depth of Neutral Axis:

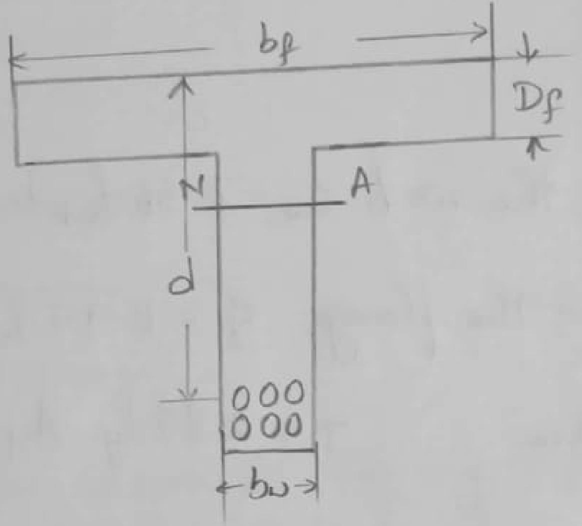
$$x_u = \frac{0.87\, f_y \cdot A_{st}}{0.36\, f_{ck} \cdot b_f}$$

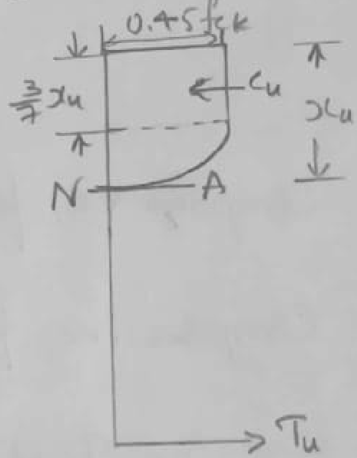
Moment of Resistance :-

$$M_u = 0.36\, f_{ck} \cdot b_f \cdot x_u\,(d - 0.42\, x_u)$$

$$M_u = 0.87\, f_y \cdot A_{st}\,(d - 0.42\, x_u)$$

Case 2: Neutral axis is below the flange ($x_u > D_f$)





Depth of Neutral Axis :

$$x_u = \frac{0.87 f_y\, A_{st} - 0.45\, f_{ck}\,(b_f - b_w)\, D_f}{0.36 f_{ck} \cdot b_w}$$

Moment of Resistance:

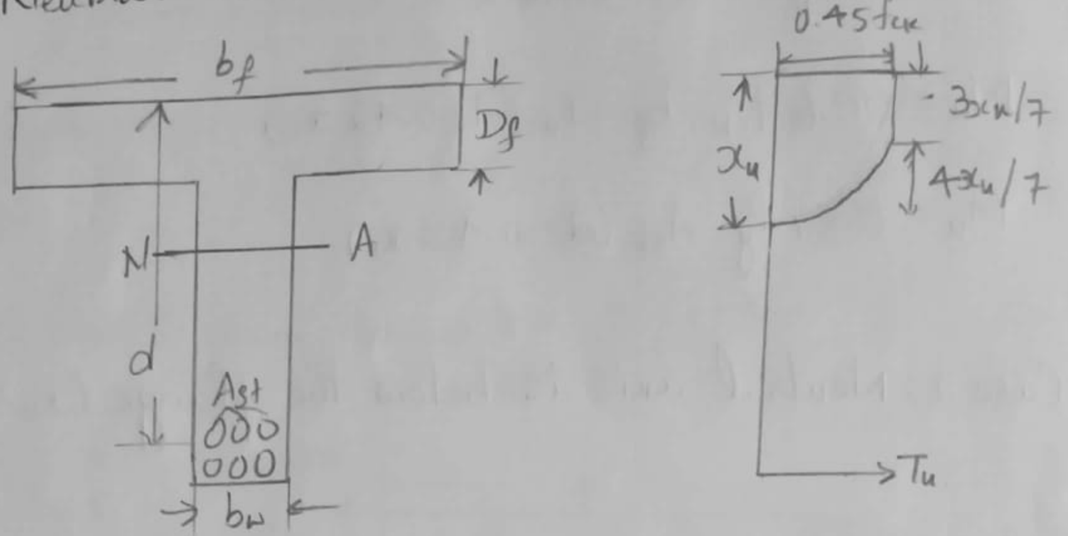
$$M_u = M_{u\,web} + M_{u\,flange}$$

$$M_u = C_w\,(d - 0.42\, x_u) + C_f\,\left(d - \frac{D_f}{2}\right)$$

Substitute C_w & C_f values:

$$M_u = 0.36 f_{ck} \cdot b_w \cdot x_u\,(d - 0.42\, x_u) + 0.45\, f_{ck}\,(b_f - b_w)\, D_f\left(d - \frac{D_f}{2}\right)$$

Case 3: Neutral axis is below the flange ($x_u > D_f$)



Compressive force in the web $C_w = 0.36 f_{ck} b_w \cdot x_u$

Compressive force in the flange $C_f = 0.45 f_{ck} (b_f - b_w) D_f$

Tensile force $T = 0.87 f_y \cdot A_{st}.$

Depth of Neutral Axis:

$$x_u = \frac{0.87 f_y A_{st} - 0.45 f_{ck} (b_f - b_w) D_f}{0.36 f_{ck} \cdot b_w}$$

Moment of Resistance:

$$M_u = M_{u\,web} + M_{u\,flange}$$

$$M_u = C_w (d - 0.42 x_u) + C_f \left(d - \frac{D_f}{2}\right)$$

$$M_u = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right.$$

Problems:-

1) Find the effective flangewidth of the following simply supported isolated T-beam. Effective span = 5.0m, Breadth of the web = 230mm. Thickness of slab = 110mm. Width of the Support = 230mm. Actual width of the flange = 750mm.

Sol:-

$l = 5m$

$b_w = 230mm$

$D_f = 110mm$

$b = 750mm$

Since the beam is Simply Supported, the distance b/w the Points of zero moments.

$l_o = l = 5m.$

For isloted T-beams, effective width of the flange is the least of the following.

1) $b_f = \dfrac{l_o}{(l_o/b + 4)} + b_w$

$= \dfrac{5000}{\left(\frac{5000}{750}\right) + 4} + 230 = 698.88mm$

2) b_f = actual width of the flange = 750mm

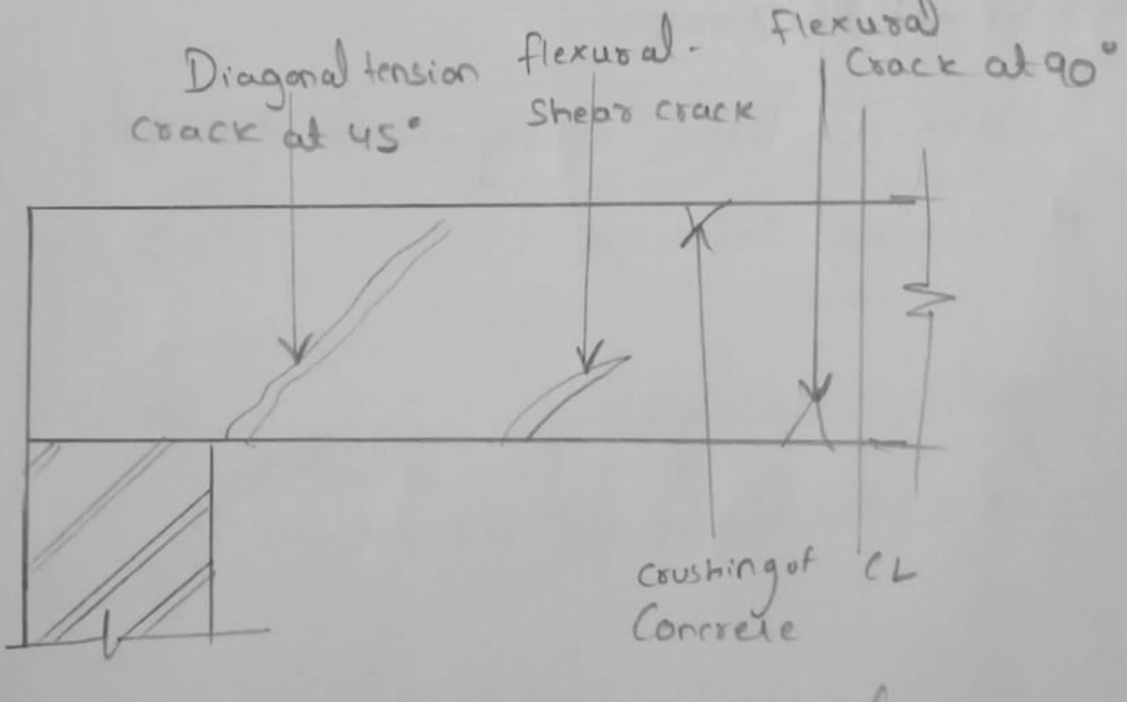
Hence, $b_f = 698.8mm$

UNIT - 2

SHEAR.

Shear:

The Combination of Shear and bending stressess. Produces the principle stress which causes diagonal tension in the beam section. The diagonal tensile Stress caused by the shear and combination of Shear and bending is likely to cause failure of the section by providing cracks. This should be resisted by providing shear reinforcement in the form of vertical stirrups or bent up bars.



CRACK PATTERN IN BEAMS (Simply supported beam)

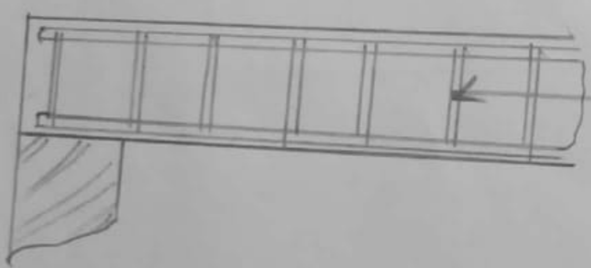
Design of Shear Reinforcement:

Shear reinforcement has to be provided against diagonal tensile stresses caused by the shear force. The longitudinal bars do not prevent the diagonal tension failure. The inclined shear crack starts at the bottom near the support and extend towards compression zone. The shear reinforcement can be provided in any of the following forms

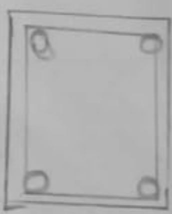
⟶ Vertical stirrups

⟶ Bent-up bars along with stirrups

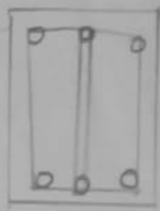
⟶ Inclined stirrups.



Vertical Stirrups



Two-legged

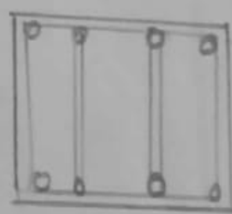


Four-legged

Longitudinal view

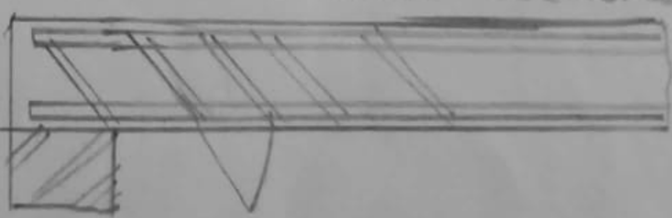
Bent up bars

One-legged



Multi-legged

Cross sectional view



Inclined stirrups

Different forms of Stirrups.

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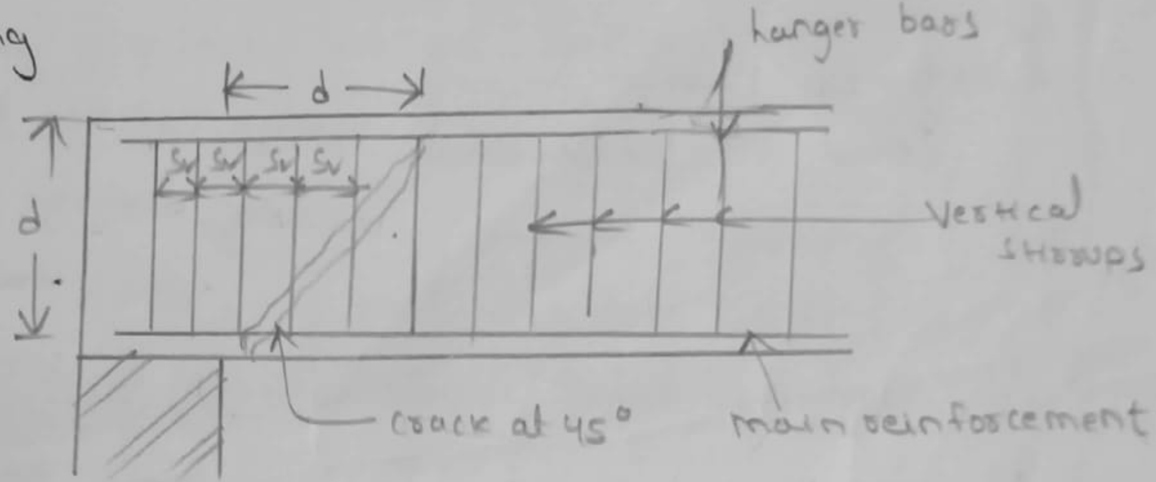
Vertical stirrups:

They are provided as two or four legged stirrups bend round the tensile reinforcement and taken to the compression zone and anchored to the hanger bars. Hanger bars are provided to keep vertical stirrups in position otherwise they may get displaced while concreting



Arrangement of Vertical stirrups

Shear to be resisted by shear reinforcement is given by

$$V_{us} = V_u - V_{uc}$$
$$= V_u - \tau_c\, b d$$

where, V_{uc} = Shear resistance of Concrete $= \tau_c b d$

Let A_{sv} = Total area of legs of vertical stirrups.

S_v = Spacing of stirrups

d = Effective depth of section

No of stirrups cut by 45° crack line is

$$n = \frac{d}{S_v}$$

Total shear resistance of vertical stirrups is given by

S_F = Force resisted by each stirrup × No of stirrups

$$V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{S_v}$$

$$S_v = \frac{0.87 f_y A_{sv} \cdot d}{V_{us}}$$

Bent-up Bars :-

Some of longitudinal bars can be bent up near the supports as the bending moment to be resisted near the supports is very ~~tiner~~ little. Such bent up bars resist diagonal tension.

If all the bars are bent up at the same c/s at an angle of α, the shear resistance of bent up bars is given by

$$V_{usb} = 0.87 f_y A_{sb} \sin\alpha$$

where, V_{usb} = Shear resistance of bent up bars

A_{sb} - Total area of bent up bars

α = Angle b/w the bent up bars and axis of memb ($>45°$)

If the bent up bars or inclined stirrups are provided at a spacing of S_v, the shear resistance of the bent up bars

$$V_{usb} = 0.87 f_y A_{sb} (\sin\alpha + \cos\alpha) \frac{d}{S_v}$$

The shear resistance of bent up bars shall not exceeds 50% of the total shear to be resisted by the shear reinforcement. Because bent up bars alone are not effective in preventing shear failure.

Minimum Shear reinforcement :-

The minimum quantity of shear reinforcement that should be provided for all beams except those of minor importance like lintels by the revised equation

$$\frac{A_{sv}}{b\,S_v} = \frac{0.4}{0.87 f_y} \quad \left\{ \text{By code } \left(\text{IS 456:2000} \right) \right.$$

Maximum Spacing of Shear reinforcement

Spacing of vertical stirrups should not exceed 0.75d (or) 300mm which ever is less. and dia should not less than 6mm.

For incined stirrups at 45°. the maximum spacing is d or 300 mm which ever is less.

Hence, Spacing should be least of the following

a, Spacing Calculated to resist V_{us}

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

b, Spacing calculated from minimum shear reinforcement

$$S_v = \frac{0.87 f_y A_{sv}}{0.4 b}$$

c, 0.75d

d, 300 mm

Procedure for Design of Shear reinforcement:-

* Calculate the factored shear force V_u in beam
* Calculate the nominal shear stress

$$\tau_v = \frac{V_u}{bd}$$

* Calculate the % of tension reinforcement at section P_t and obtain the design shear strength of Concrete τ_c from table 19 of IS:456.

Case a:- $\tau_v < \tau_c$, Provide min shear reinforcement

Case b:- $\tau_v > \tau_c$, Design shear reinforcement

Case c:- $\tau_v > \tau_{c\,max}$, the section must be redesigned such that the nominal shear stress falls with in the maximum limit

Problem

1. Determine the spacing of 8 mm 2 legged stirrups for RCC beam of 230 mm width and 450 mm Effective depth to resist a factored shear force of 85 kN. Use M_{20} concrete and Fe 250 steel

Sol:- Given data:-

$b = 230\,mm$

$d = 450\,mm$

$V_{us} = 85\,kN = 85 \times 10^3\,N.$

$A_{st} = 2 \times \frac{\pi}{4} \times 8^2 = 100.5\,mm^2$ for 2-legged

Spacing of stirrups to resist V_{us}

$$S_v = \frac{0.87\, f_y\, A_{sv} \cdot d}{V_{us}}$$

$$S_v = \frac{0.87 \times 250 \times 100.5 \times 450}{85000} = 115.7\,mm$$

Spacing from minimum shear reinforcement

$$\frac{A_{sv}}{b s_v} = \frac{0.4}{0.87\, f_y}$$

$$S_v = \frac{0.87\, f_y\, A_{sv}}{0.4 b} = \frac{0.87 \times 250 \times 100.5}{0.4 \times 230}$$

$$= 237.6\,mm$$

Maximum allowed spacing $= 0.75d = 0.75 \times 450 = 337.5\,mm$ or 300 mm which ever is less.

Spacing should be least of the above

Hence provide 2 legged 8mm stirrups @ 115 mm c/c.

Design problem:

An Rcc beam 230 mm wide and 450mm deep is reinforced with 4 bars of 16mm ø and grade of fe 415 on tension side. if design shear force is 60 KN. Design the shear reinforcement consisting only of Vertical stirrups. The grade of Concrete used in M_{20}.

Sol:- Given data :-

$b = 230\ mm$

$D = 450\ mm$

Design shear force $V_u = 60\ kN = 60000\ N$

Area of steel $= A_{st} = 4 \times \frac{\pi}{4} \times 16^2 = 804.3 mm^2$

Assuming an effective cover of 50 mm, effective depth,

$d = 450 - 50 = 400 mm$.

1) Nominal shear stress :

$$\tau_v = \frac{V_u}{bd}$$

$$\tau_v = \frac{60 \times 10^3}{230 \times 400} = 0.65\ N/mm^2$$

2) Shear resistance of Concrete

% of tension steel at Support

$$P_t = \frac{A_{st} \times 100}{bd}$$

$$P_t = \frac{804.3 \times 100}{230 \times 400} = 0.874\ \%$$

Referring to the table-19 ; IS: 456 Shear strength of Concrete is

$$\tau_c = 0.59\ N/mm^2$$

(Interpolation has to be done for intermediate Valued of P_t)

P_t %	τ_c N/mm^2
0.75	0.56
1.0	0.62
0.875	?

By interpolation.

$$\tau_c = 0.56 + \frac{(0.62 - 0.56)}{(1.0 - 0.75)} \times (0.875 - 0.75)$$

$$\tau_c = 0.59\ N/mm^2$$

Maximum shear stress in Concrete

$$\tau_{c\,max} = 2.8\ N/mm^2$$

As $\tau_v < \tau_{c\,max}$, and $\tau_v > \tau_c$, Shear reinforced has to be designed.

3) Design of Vertical stirrups;

Shear to be resisted by shear reinforcement.

$$V_{us} = V_u - \tau_c b d$$

$$V_{us} = 60 \times 10^3 - 0.59 \times 230 \times 400$$

$$V_{us} = 5720\ N$$

Spacing of 2-legged 6mm stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 6^2 = 56.5\ mm^2$$

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$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$S_v = \frac{0.87 \times 415 \times 56.5 \times 400}{5720}$$

$$S_v = 1426.5 \text{ mm}$$

Spacing from minimum shear reinforcement

$$\frac{A_{sv}}{b S_v} = \frac{0.4}{0.87 f_y}$$

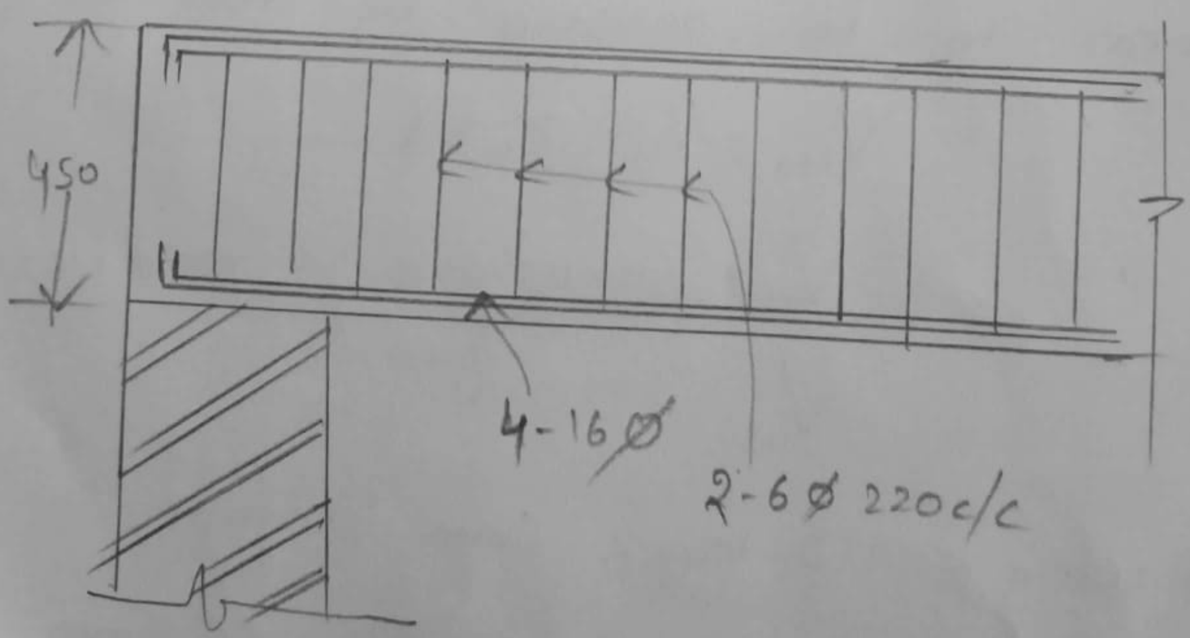
$$S_v = \frac{0.87 f_y A_{sv}}{0.4 \times b} = \frac{0.87 \times 415 \times 56.56}{0.4 \times 230}$$

$$= 221.7 \text{ mm}$$

Maximum allowed spacing $= 0.75 d = 0.75 \times 400 = 300$ mm (or) 300 mm which ever is less

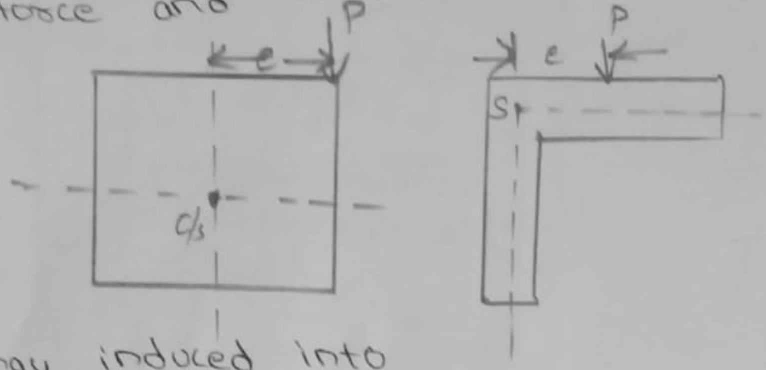
Spacing should be least of above

Hence provide 2-legged 6mm stirrups @ 220 mm c/c through out the span of the beam.



TORSION

If line of action of force is not passing through 'shear centre then torsion develops in addition to shear force and bending moment.



Torsion may induced into

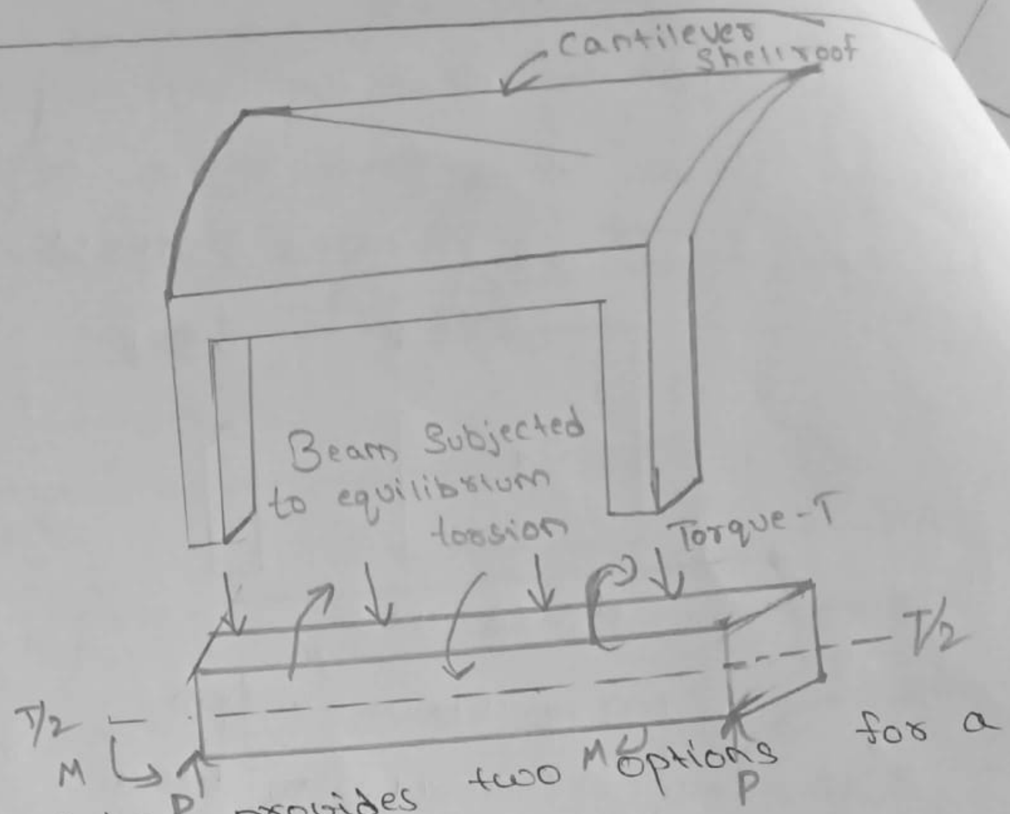
→ Primary (or) equilibrium torsion.

→ Secondary (or) Compactability torsion

Primary Torsion:

This is associated with twisting moments that are developed in a structural member to maintain static equilibrium with a external load directly applied on the member and are independent of the torsional stiffness of the member

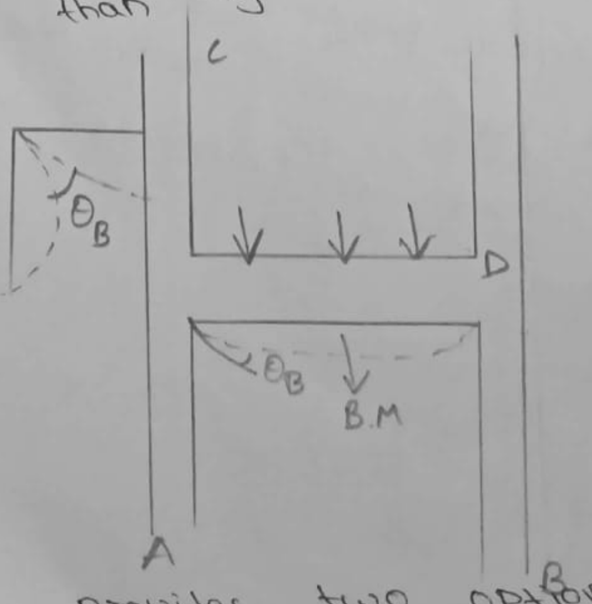
→ Primary torsion is induced in beam supporting lateral overhanging projections and is caused by eccentricity of the loads such torsion is induced in beams curved in plan. and subjected to gravity loads in beams.



The Code provides two options for a design of statically

Secondary Torsion:

The type of torsion induced in a structural member has a secondary effect by the rotations applied in one (or) more points along the length of the member through interconnected members, the rather than by directly applied loads on it



The Code provides two options for statically

→ If torsional stiffness of members is not considered in the analysis, the structure may be designed for zero torsion and the resulting moment and shear. The nominal shear reinforcement is expected to take care of any torsional cracking

→ If the torsional stiffness of member is considered in the analysis, the member must design for compatibility torsion.

→ The equivalent shear force is calculated from the emperical relation of the code (Page 75)

$$\boxed{V_e = V_u + 1.6\frac{T_u}{b}}$$

where,

V_e = Equivalent shear, V_u = Shear

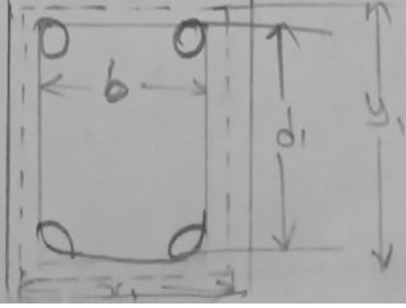
T_u = Torsional moment

b = breadth of beam (or) least lateral dimensi

In T. beams $b = b_w$

The equivalent nominal shear stress $\boxed{\tau_e = \frac{V_e}{bd}}$

→ τ_e should not exceed the maximum shear stress



Problem:-

Determine the shear stress in a 25cm x 40cm Effective rectangular Section if the shear force is and torsional moment is 4KN.m at factored load Assume M_{20} mix and 0.25 % tension steel at the given section. State whether reinforcement is required

Sol:-

$V_u = 10\ kN, \quad T_u = 4kN.m$

Equivalent shear force $V_e = V_u + 1.6\ T_u/b$

$= 1000 + 1.6 \times \frac{4 \times 10^5}{25} = 35600N$

Equivalent nominal shear stress $\tau_e = \frac{V_e}{bd} = \frac{35600}{250 \times 400}$

$= 0.36\ N/mm^2$

Shear strength of M_{20} concrete at 0.2 % tension steel is equal to 0.36 N/mm^2 from table 8.2.

the observations can be made;

i, $\tau_e = \tau_c$, no torsional reinforcement is required

ii, c/s dimension are less than 450mm, no side reinforcement is required, and

iii, minimum shear reinforcement should be provided,

that is, $\frac{A_o}{bx} \geq \frac{0.4}{0.87\ \sigma_y}$; $A_o = 2 \times 50mm^2$; $\sigma_y = 415\ N/mm^2$

Use 8mm - 2 legged stirrups of fe415 steel

$x \leq \frac{2 \times 50 \times 0.87 \times 415}{0.4 \times 250} = 360\ mm$; $x < 0.75d$, $x < 300mm$

Hence provide 8mm - 2 legged fe415 grade stirrups @ 300 mm c/c

DESIGN PROCEDURE:-

→ When a member designed for torsion the torsion reinforcement should be provided as follows

i, The longitudinal reinforcement should be placed as near to the corners of the cross section as possible.

ii, Their must be atleast one longitudinal bar in each corner of the ties.

iii, If the cross section dimensions of the members exceeds 450mm, then additional bars must be Provided along the two faces of the member. The total of such reinforcement should not be less than 0.1% of the web area and must be distributed equally on two phases at a spacing not exceeding 300mm or the web thickness, which ever is less.

iv, The transverse reinforcement for torsion consists of the rectangular closed stirrups placed perpendicular to the axis of the member. The spacing stirrups should not exceed

i, $S_v < x_1$

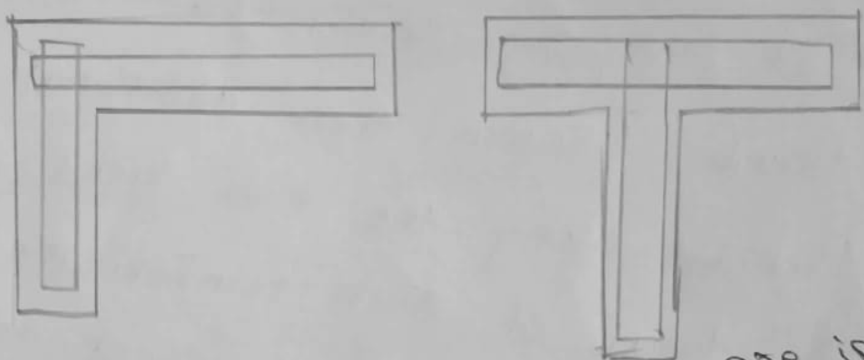
ii, $S_v < (x_1 + y_1)/4$

iii, $S_v < 300mm$

x_1 = short dimension of the stirrup

y_1 = long dimension of the stirrup.

v, In T, & L-beam, if the main reinforcement of the slab is parallel to the beam, the transverse reinforcement should be provided in the flange. Such a reinforcement should not be less than 60% of the reinforcement at mid span of the slab.



vi, In L&T-beam, where flanges are in tension the part of the main torsion reinforcement must be distributed over the effective flange width (or) a width equal $\frac{1}{10}^{th}$ of the span, whichever is smaller.

→ The effective flange width exceed $\frac{1}{10}^{th}$ of the span the nominal longitudinal reinforcement must be provided in the outer portion of the flange.

Design problem :-

Design a Section of a ring beam 50cm wide and 70cm deep Subjected to bending moment of 200 kN.m, twisting moment of 15 kNm and a Shear force of 150 kN ultimate. Use M20 mix and fe415 grade steel.

$\therefore$ $V_u = 150\,kN$, $T_u = 15\,kNm$, $M_u = 200\,kN.m$

Equivalent Shear $V_e = V_u + 1.6\,\frac{T_u}{b}$

$= 150 + 1.6 \times \frac{15}{0.5}$

$= 198\,kN$

Let effective cover be 40mm

Equivalent nominal shear stress $\tau_e = \frac{V_e}{bd}$

$= \frac{198 \times 1000}{500 \times 665} = 0.60\,N/m$

maximum shear stress in M_{20} mix Concrete $\tau_{cm} = 2.8\,N/mm^2$
$> 0.60\,N/mm^2$

Let us assume that tension steel is 0.25%.

shear strength of Concrete $\tau_c = 0.36\,N/mm^2$ (Table 8.2)

$\tau_c < \tau_e$

$\therefore$ Torsion reinforcement is required in the form of longitudinal and transverse steel.

Longitudinal reinforcement !

Equivalent Bm $M_{e1} = M_u + M_t$

$= M_u + T_u\,\frac{1 + D/b}{1.7}$

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$= 200 + 15 \frac{1+70/50}{1.7} = 221.1 \, kN/m$

Since $M_t > M_u$ there is no need of compression reinforcement due to the resist moment

Maximum depth of N.A $= 0.48 d = 0.48 \times 665 = 319 mm$

Factored equivalent BM = force of tension × 2

$$221.1 \times 10^6 = 0.87 \sigma_y A_t \left(d - \frac{\sigma_y A_t}{\sigma_{ck} b}\right)$$

$$221.1 \times 10^6 = 0.87 \times 415 A_t \left(660 - \frac{415 A_t}{20 \times 500}\right)$$

$$A_t = 981 mm^2$$

Depth of neutral axis $x = \frac{0.87 \sigma_y A_t}{0.36 \sigma_{ck} b}$

$$= \frac{0.87 \times 415 \times 981}{0.36 \times 20 \times 500}$$

$$= 98.4 mm < x_u$$

minimum tension reinforcement $A_0 = \frac{0.85 bd}{\sigma_y}$

$$= \frac{0.85 \times 500 \times 600}{415}$$

$$= 680 mm^2 < A_t$$

Provide 5-16mm bars $A_t = 1005 mm^2 > 981 mm^2$

Transverse reinforcement

$$0.87 \sigma_y A_{sv} = \frac{T_u x}{b_1 d_1} + \frac{V_u x}{2.5 d_1}$$

b_1 = c/c distance b/w corner bars in the direction of width

$b_1 = 500 - 25 - 25 - 8 - 8 - 16/2 - 16/2 = 418 mm$

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d_1 = c/c distance b/w corner bars in the direction of depth

$= d - 25 - 8 - 12/2 = 600 - 25 - 8 - 6 = 621\,mm$

A clear cover of 25mm is assumed, all around th Shear Stirrups. Use 8mm - 2 legged vertical stirrups. $A_{sv} = 100.5\,mm^2$. Spacing of shear reinforcement % is give by

$$0.87 \times 415 \times 100.5 = \left[\frac{15 \times 10^6}{418 \times 621} + \frac{150 \times 1000}{2.5 \times 621}\right] X$$

$X = 217\,mm$

$x_1 = 500 - 25 - 25 - 8 = 442\,mm$

$y_1 = 700 - 25 - 25 - 8 = 642\,mm$

$X < (x_1 + y_1)/4 = 271\,mm > 237\,m$

Adopt a spacing of 200 mm c/c

minimum shear reinforcement $A_{sv} \geq \frac{\tau_e - \tau_c\, bx}{0.87\sigma_y}$

% tension steel $p = \frac{100\,A_t}{bd} = \frac{100 \times 1005}{500 \times 660} = 0.30$

Shear Strength of concrete $\tau_c = 0.38\,N/mm^2$

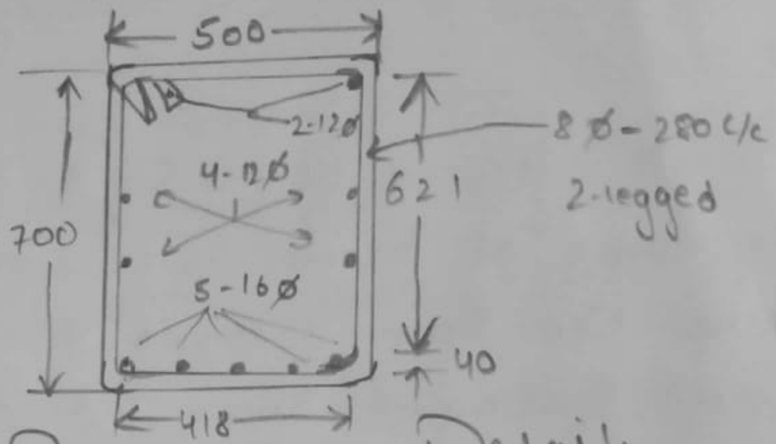
$$A_{sv} = \frac{(0.60 - 0.38)\,500 \times 200}{0.87 \times 415} = 61\,mm^2$$

$61 < 100.5\,mm^2$

Use 8mm - 2 legged vertical stirrups at 200mm c/c

Since the depth exceeds 450mm, provide 0.1% steel that is 4-12 mm bars along the vertical sides, that is 4-12 mm bars

The reinforcement details are in Fig.



Reinforcement Detail.

BOND.

DEFINATION:-

The theory of reinforced concrete is that it
is perfect bond between steel and concrete.

Bond stress is the shear stress acting parallel to
the bar on the interface b/w the reinforcing
bar and the surrounding concrete. Hence, it is
the stress developed b/w the contact surface of
steel and concrete to keep them together. it
resists any force that tries to pull out the
rods from the concrete.

Mechanism of bonding effect

→ Chemical addition force

→ frictional resistance

→ Mechanical interlocking.

It depends on grade of Concrete, dia of bar, bar Profile condition, nature of force in the bar, grouping of bars, bends and hooks in the bar.

The values of design bond stress by IS 456 : 2000 table 3.8 for plan round bars in tension.

Grade of Concrete	M_{20}	M_{25}	M_{30}	M_{35}	M_{40} above
Design bond stress τ_{bd}, N/mm^2	1.2	1.4	1.5	1.7	1.9

Note :-

for deformed bars the values may increase by 60%.

for bars in compression, above values may increase by 25%.

for deformed Compression bars, the above values mayb be multiplied by 1.25 × 1.6

Types of BOND :-

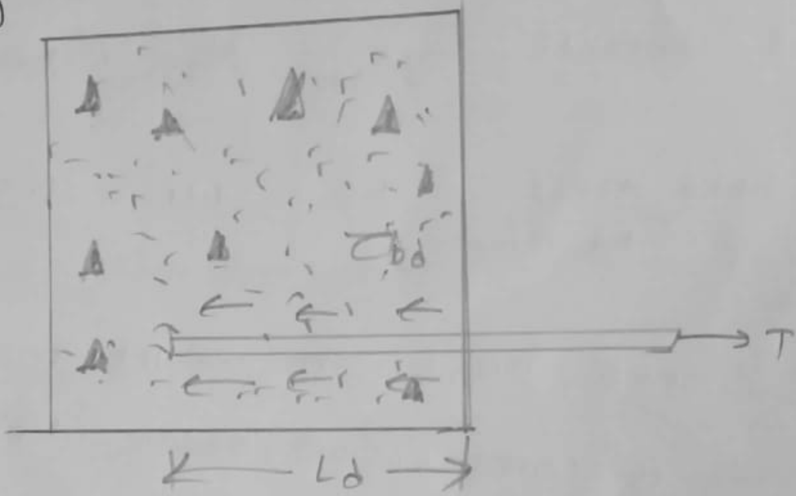
In design of RCC sections with respect to bond failures. the following two cases of bond

→ Anchorage Bond

→ Flexural Bond.

Anchorage Bond:

It arises when a bar carrying certain force is terminated. In such cases, it is necessary to transfer this force in the bar to the surrounding concrete over a certain length.



Anchorage bond

The length of the bar L_d required to transfer the force in the bar to the surrounding concrete through bond is called development length.

It can be easily ~~detvar~~ determined by pull out test.

$$T = \text{Design stress} \times \text{area of bar}$$

$$T = 0.87\, f_y \left(\frac{\pi}{4} \times \phi^2\right)$$

where,

T = subjected to be pull of the test

The force must be transferred from steel to concrete through bond acting over the perimeter of bar over a length L_d

τ_{bd} is the average design bond stress, for equilibrium

Ultimate Bond force = Put out force

$$\tau_{bd}(\pi \phi) L_d = 0.87 f_y \left(\pi \frac{\phi^2}{4}\right)$$

$$L_d = \frac{0.87 f_y \left(\pi \frac{\phi^2}{4}\right)}{\tau_{bd} \pi \phi}$$

$$\boxed{L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}}$$

where ϕ is the dia of the bar.

Hence all bars should extend at a distance L_d beyond the section, where they are required to take full design force.

Flexural Bond :-

Flexural Bond at a point is the rate of change of tension in the steel at a given location in a reinforced concrete member due to variation of bending moment.

Problem :-

A Simply supported beam is 6m in span and carries a uniformly distributed load of 60 kN/m. If 6 No's of 20 mm bars are provided at the centre of span and 4 no's of these bars are continued in to the supports, Check the development at the supports assuming M_{20} grade concrete and fe 415 steel

Sol :-

$f_{ck} = 20 N/mm^2$

$f_y = 415 mm^2$

Deformed bar. $\tau_{bd} = 1.6 \times 1.2 = 1.92 N/mm^2$ (from table 2.8)

factored load $= 1.5 \times 60 = 90 kN/m = W_u$

factored shear force $= \frac{W_u l^2}{8} = \frac{90 \times 6^2}{8} = 405 kN.m$

Moment of resistance of the bars continued in to the support (4 bars)

$M_1 = 405 \times \frac{4}{6} = 270 kN.m$

Development length of 20mm dia bar with M_{20} Concrete fe 415 Steel.

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 20}{4 \times 1.92}$$

$= 940 mm$

→ Depth of the beam; In range of $\frac{1}{12}$ to $\frac{1}{15}$ based on stiffness

$$d = \frac{6000}{15} = 400\,mm$$

adopt $d = 400\,mm$ $= 0.4d$

$D = 450\,mm$

Cover = 50mm.

→ Effective Span:

C/c Supports $= 6 + 0.23 = 6.23\,m$ } whichever is least

Clear span + d $= 6 + 0.4 = 6.4\,m$ }

∴ Effective span = 6.23m

→ Loads.

Self weight of beam $= 0.3 \times 0.45 \times 1 \times 25$

$= 3.375\ kN/m$

Imposed load $= 12\ kN/m$

Total load $= 15.375\ kN/m$

Factored load $w_u = 1.5 \times 15.375 = 23.06\ kN/m$

Factored BM $M_u = \frac{w_u l^2}{8} = \frac{23.06 \times 6.23^2}{8}$

$= 111.9\ kN.m$

Factored S.F $V_u = \frac{wl}{2} = \frac{23.06 \times 6.23}{8}$

$= 71.83\ kN$

→ Depth required:

minimum depth required to resist Bm

$$M_u = 0.138\, f_{ck}\, bd^2$$

$$1119 \times 10^6 = 0.138 \times 20 \times 300 \times d^2$$

$$d = \sqrt{\frac{111.9 \times 10^6}{0.138 \times 20 \times 30}}$$

$d = 367.6$ mm < 400mm providth (d)

Hence provided depth is adequate.

→ Hence Tension Reinforcement

$$M_u = 0.87\, f_y\, A_{st}\, d\left(1 - \frac{f_y\, A_{st}}{f_{ck}\, b \cdot d}\right)$$

$$111.9 \times 10^6 = 0.87 \times 415 \times A_{st} \times 400\left(1 - \frac{415 \times A_{st}}{20 \times 300 \times 400}\right)$$

$$\cancel{A_{st}} \times \left(1 - \frac{A_{st}}{5783.1}\right) A_{st} = 774.8$$

$$A_{st} = 921.7 \text{ mm}$$

Provide 3-20mm bars, A_{st} provided $= 942.5$ mm^2

→ Design of Shear reinforcement :—

Nominal shear stress $\tau_v = \dfrac{V_u}{bd} = \dfrac{71.83 \times 10^3}{300 \times 400}$

$= 0.598$ N/mm^2

% of tension steel at support

$$P_t = \frac{A_{st} \times 100}{bd} = \frac{942.5 \times 100}{400 \times 300} = 0.785\%$$

Referring table 19 of IS 456; Shear strength of Concrete

$$\tau_c = 0.57 \text{ N/mm}^2$$

Maximum shear stress in concrete $\tau_{c\,max}$ from 20 table

$$\tau_{c\,max} = 2.8\ N/mm^2$$

As $\tau_v > \tau_c$ shear reinforcement designed

Shear resistance of concrete

$$V_{uc} = \tau_c\, bd$$
$$= 0.57 \times 300 \times 400$$
$$= 68400\,N$$
$$= 68.4\ kN$$

Shear to be resisted by shear reinforcement

$$V_{us} = V_u - V_{uc}$$
$$= 71.83 - 68.4 = 3.43\ kN$$

Using 8mm, 2 legged ~~fe 415~~ fe 250 steel stirrups.

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$$
$$= 100.5\ mm^2$$

$$\text{Spacing},\ S_v = \frac{0.87\, f_y\, A_{sv}\, d}{V_{us}}$$

$$= \frac{0.87 \times 250 \times 100.5 \times 400}{3430}$$

$$= 2549.1\,mm$$

UNIT - 3

Column : cl 25.1 Pg-41

A column may be defined as an element used primarily to support axial compressive loads and with a height of at least three times its least lateral dimensions.

(3)

41, 42, 43
44, 41, 42 A vertical member whose effective length is greater than 3 times its least lateral dimension carrying compressive loads is called as column.

→ The strength of a column depends on the strength of materials, shape and size of the cross-section, length and the degree of positional and directional restraints at its ends.

→ The maximum concrete compressive strain at crushing has been observed in various tests to vary from 0.003 to higher than 0.008 under special conditions.

→ A column may be classified based on different criteria.

(a) Shape of cross-section

(b) Slenderness ratio

(c) type of loading

(d) pattern of lateral reinforcement.

→ A column may be Rectangular, Square, circular (or) polygon in cross-section.

→ Strut :- The inclined member carrying compressive loads as in case of frames and trusses is called as struts.

→ Column transfer the load from the beam (or) slabs to footing (or) foundation.

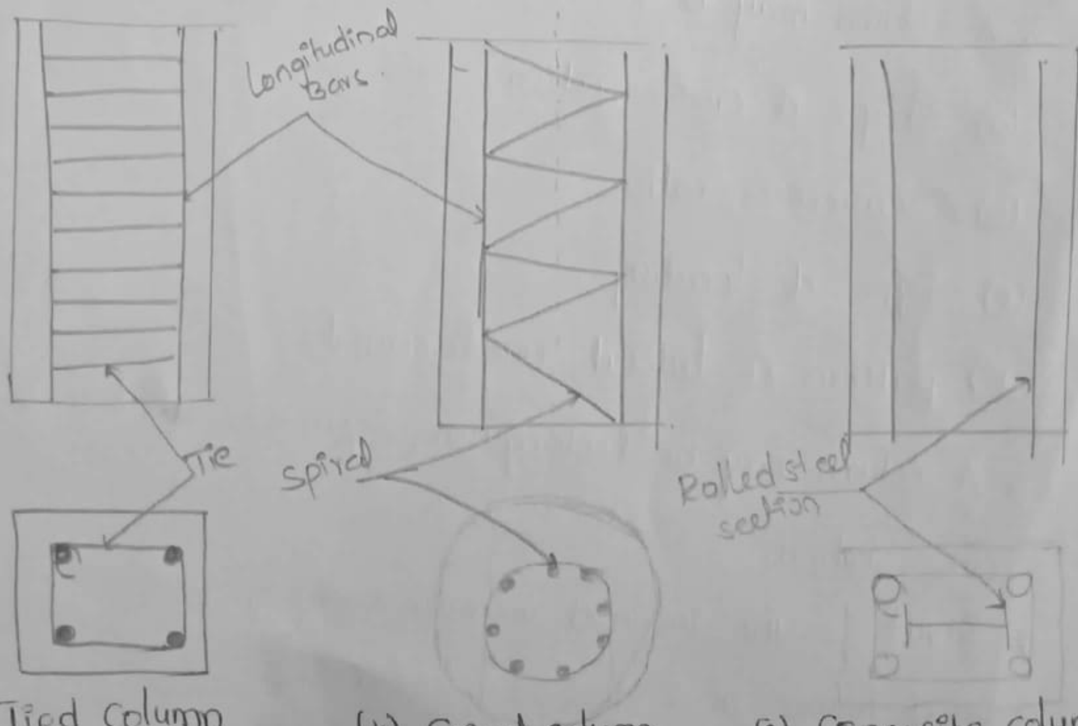
Types of columns

1) Based on Type of Reinforcement :- Depending up on the type of reinforcement use reinforced columns are classified into.

(a) **Tied column** := When the main longitudinal bars of the column are confined with in closely spaced lateral ties, it is called as tied column.

(b) Spiral column := When the main longitudinal bars of the column are enclose with in closely spaced and continuosisly wound spiral reinforcement, it is called as spiral column.

(c) Composite column : When the longitudinal reinforcement is in the form structural steel section (or) pipe with (or) with out longitudinal bars, it is called composite column.



(a) Tied Column (b) Spiral column (c) Composite column

2) Based on type of loading: Depending upon the type of loading, columns may be classified into the following three types.

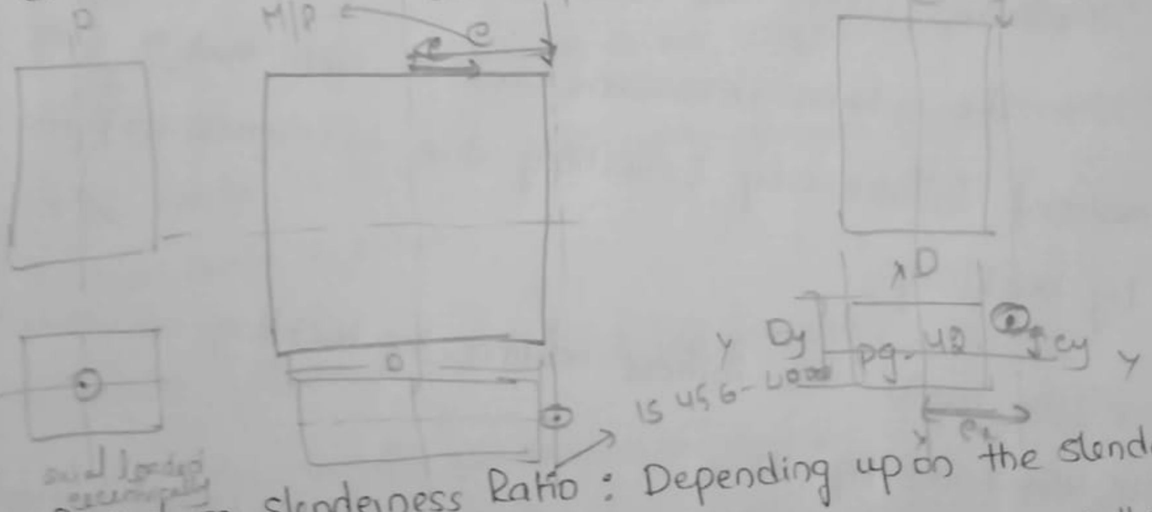
(a) Axially loaded column:

When the line of action of the resultant compressive force coincides with the center of gravity of the cross section of the column, it is called as axially loaded column.

(b) Eccentrically loaded columns (Uniaxial & Biaxial):

When the line of action of the resulant compressive force doesn't coincide with the center of gravity of the cross section of the column, it is called as eccentrically loaded column.

Eccentrically loaded columns have to be designed for combined axial force and bending moment.



3) Based on Slenderness Ratio: Depending upon the slenderness ratio (ratio of effective length to least lateral dimension of the column) the columns are classified as.

(a) Short Column: When the ratio of effective length of the column to the least lateral dimension is less than 12.

→ A short column fails by crushing (pure compression failure)

(b) Long Column :-

If the ratio effective length of the column to the least lateral dimension exceeds 12.

→ A long column fails by bending (or) buckling.

Effective length of the column → Table 28 pg-44

Effective length of a column is the distance b/w the points of zero bending moments of a buckled column.

The effective length of the column depends up on the unsupported length (distance b/w the lateral connections) and the end conditions (free, fixed (or) hinged).

Slenderness Limits for columns :

The column dimensions shall be such that it fails by material failure only (crushing due to compression) and not by buckling.

→ To avoid the failure column by buckling. recommends IS 456-2000 Pg - 42.

(a) The unsupported length (distance b/w the lateral connections) shall not exceed 60 times the least lateral dimension of the column.

$$L < 60b$$

(b) If one end of the column is unrestrained (unsupported)

$$L < \frac{100\,b^2}{D}$$

∴ b = width of the cross-section

D = depth of the cross-section.

Short Column Under Axial Compression :-

1) Short Column with Lateral Ties :-

The ultimate load on the short column with lateral ties, when min eccentricity does not exceed 0.05 times D the lateral dimension.

IS 456-2000 - Pg-71, clause 39.3 code permits the design of short ~~columns~~ axially loaded compression members by following Eqn.

$$\boxed{P_u = 0.4 f_{ck} \cdot A_c + 0.67 f_y \cdot A_{sc}}$$

$P_u \rightarrow$ factored axial load on the member

$A_c \rightarrow$ area of concrete, may be taken equal to the gross area.

$A_{sc} \rightarrow$ Total area of longitudinal reinforcement for columns.

$f_{ck} \rightarrow$ characteristic compressive strength of the concrete

$f_y \rightarrow$ " strength of the compression reinforcement.

$\rightarrow P_u$ should be based on stresses in concrete & steel corresponding to max. strain of 0.002.

2) Short Column with Helical Reinforcement :

The strength of compression member with helical reinforcement shall be 1.05 times the strength of similar member with lateral ties.

The ratio of the volume of helical reinforcement to the volume of the core shall not be less than $0.36\left(\frac{A_g}{A_c} - 1\right)\frac{f_{ck}}{f_y}$.

$A_g \rightarrow$ gross area of the section

$A_c \rightarrow$ area of the core of the helical reinforced column measured to the outside dia of the helix.

$f_y \rightarrow$ characteristic strength of the helical reinforcement but not exceeding 415 N/mm^2.

(3) Minimum Eccentricity

A truly axially loaded column is rare, if not nonexis[tent]. every column should be designed for certain minimum eccentrici[ty].

This accidental eccentricity may occur due to end conditio[n], inaccuracy during construction. (or) variation in materials even when the load is theoretically axial.

The code requires min eccentricity should be IS456-2000-P[g]

$$e_{min} \geq \frac{l}{500} + \frac{D}{30}$$

$$> 20mm$$

$l \rightarrow$ unsupported length of column in mm.

$D \rightarrow$ lateral dimension of column in the dir^n under consideration in mm.

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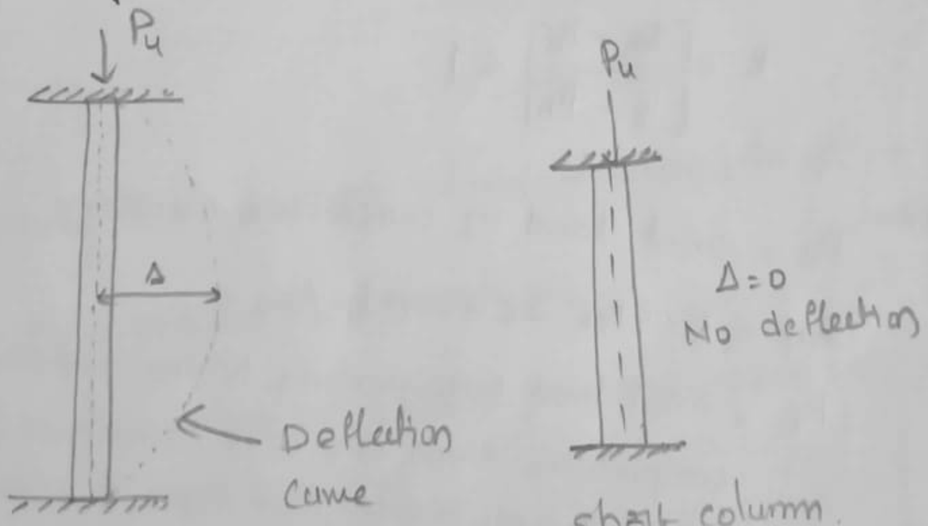
Long columns (or) slender columns :-

If the ratio of effective length of the column to its least lateral dimension is more than 12. the columns are called as long columns.

A long column under a action of axial loads deflects laterally causing max. lateral deflection at the centre (Δ). the load eccentric at the central section of the column by a distance Δ. a bending moment $P \times \Delta$ in addition to the axial load P. Hence in long column the moment produced by the lateral deflection should be considered in the design.



(a) Long column

short column.

→ According to IS : 456-2000 pg-71, the additional moments M_{ax} and M_{ay} due to the lateral deflection shall be calculated by the Eqn.

$$M_{ax} = \frac{P_u D \left(\frac{l_{ex}}{D}\right)^2}{2000}$$

$$M_{ay} = \frac{P_u b \left(\frac{l_{ey}}{b}\right)^2}{2000}$$

where P_u = factored axial load on the member

l_{ex} = effective length in respect of the major axis

l_{ey} = effective " " " " minor axis

D = depth of the cross-section at right angles to the major axis

b = width of the cross-section.

The above expressions are applicable to a balance design of a slender column subjected to uniaxial bending as well as biaxial bending.

As the axial load ↑ from zero, the tensile stress in the steel decreases to zero and change to a compressive stress. As this occurs, the curvature and deflection ↓. cl: 39.7.1.1 of the code permits a reduction in the additional moments by factor 'k' given by

$$k = \left[\frac{P_{uz} - P_u}{P_{uz} - P_b}\right] \leq 1$$

where

P_u = axial load on compression member

P_{uz} = $0.45 f_{ck} \cdot A_c + 0.75 f_y \cdot A_{sc}$

P_b = axial load corresponding to the condition of max. compressive strain of 0.0035 in concrete and tensile strain of 0.002 in outer most layer of tension steel.

Note:

The CEB-FIP recommendations for buckling in compression and bending advise a check for columns with effective slenderness ratio more than 35.

The effective slenderness ratio should not exceed 140 for normal aggregate concrete and 80 for light weight aggregate.

However, clause 25.3.1 of the code restricts maximum slenderness ratio of a column to 60.

A short column 400mm x 400mm is reinforced with 4 numbers of 25mm dia. Find the axial factored load that the column can carry. the materials are M_{20} grade concrete and HYSD reinforcement of Fe 415

sol.

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

Area of steel $A_{sc} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.5\ mm^2$

Area of concrete A_c = gross-area − area of steel

$= 400 \times 400 - 1963.5$

$= 158036.5 mm^2$

For axially loaded short column, factored load is given by

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$= 0.4 \times 20 \times 158036.5 + 0.67 \times 415 \times 1963.5$

$= 1810.24 \times 10^3 N$

$= 1810.24 KN$

A short circular column of dia 400mm is reinforced with 6 numbers of 16mm dia. Find the axial factored load on the column if M_{20} grade concrete and Fe415 grade steel is used.

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

Area of steel $A_{sc} = 6 \times \frac{\pi}{4} \times 16^2 = 1206.4 mm^2$

Area of concrete A_c = gross area − area of steel

$= \frac{\pi}{4} \times 400^2 - 1206.4$

For axially loaded short columns, factored load is given by

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$= 0.4 \times 20 \times 124457.3 + 0.67 \times 415 \times 1206.4$$

$$= 1331.1 \times 10^3 N.$$

$$= 1331.1 KN$$

Design a short column square in s/n to carry an axial load of 800kN using M20 grade concrete and Fe415 Steel.

Sol: Factored load $P_u = 1.5 \times 800 = 1200KN = 1200 \times 10^3 N.$

Assuming 1% of steel $A_{sc} = 1\% \, A_g = 0.01 A_g$

Area of concrete $A_c = A_g - A_{sc}$

$$= A_g - 0.01 A_g = 0.99 A_g.$$

For axially loaded short columns.

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$1800 \times 10^3 = 0.4 \times 20 \times 0.99 A_g + 0.67 \times 415 \times 0.01 A_g$$

$$A_{g\,required} = 112149.5 mm^2$$

Size of the square column $= \sqrt{112149.5} = 334.9mm \approx 350mm$

adopt 350×350 mm square column

$$A_{sc} = 0.01 \times A_{g,required}$$

$$= 0.01 \times 112149.5$$

$$= 1121.5 mm^2$$

Provide 6 bars of 16 mm diameter

A_{sc} provided $= 1206.4 mm^2$

UNIT-4

Footings:-

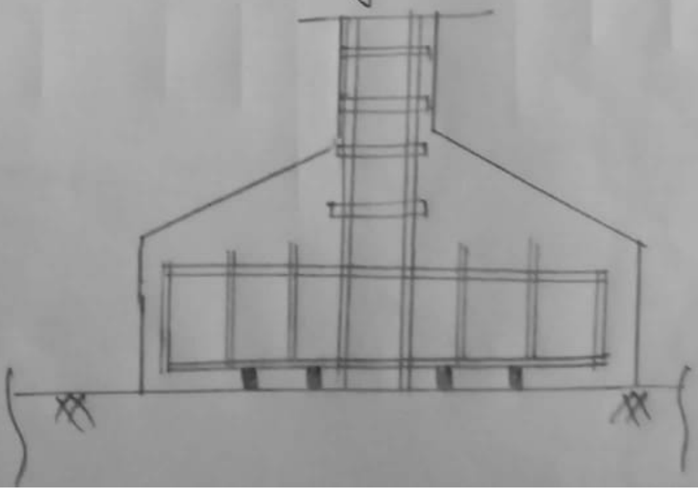
Footings are shallow foundations which are provided when the soil of adequate bearing capacity is available at a relatively short depth below the ground level. Footings may be of masonry P.C and R.C.

Types of footings:-

* Isolated footings
* combined footings
* strap footings
* Raft (or) mat footings
* Wall footings / strip footings
* Spread footings
* stepped footings
* pile foundation.

→ Isolated footings:-

Footings which are provided under each column independently are called as "Isolated footings." They may be square, rectangular or circular is plan.



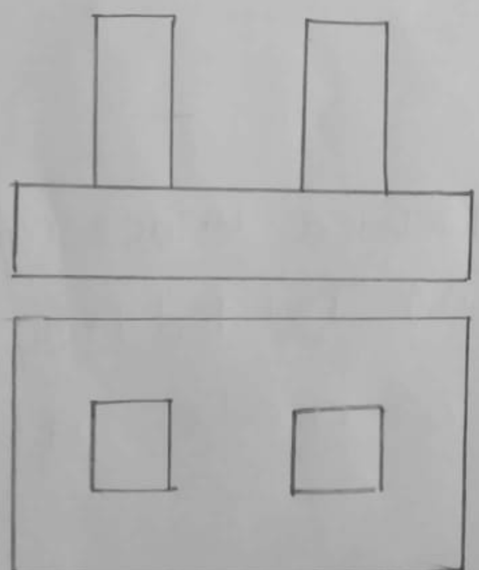
Combined footings:-

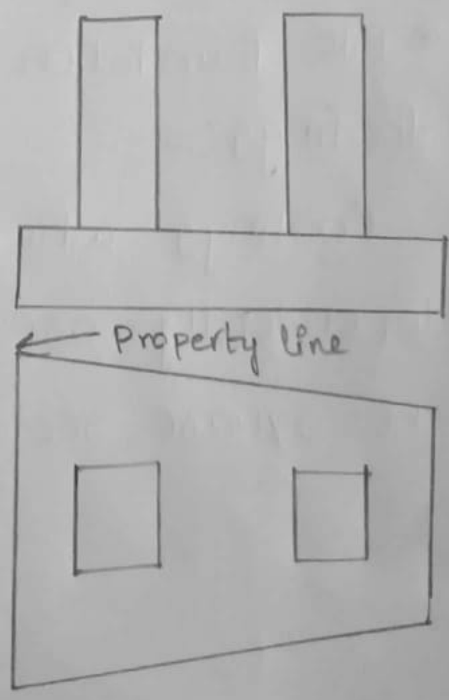
When two or more columns are supported by a footing, it is called as combined footing. This footing may be of rectangular (or) trapezoidal in plan. This type of footing is provided under footing situations.

→ When column are close to each other and their individual footings overlap.

→ Soil having low bearing capacity and requires more area under individual footing.

→ The column end is situated near the property line and the footing can not be entended.





→ Strap footing :-

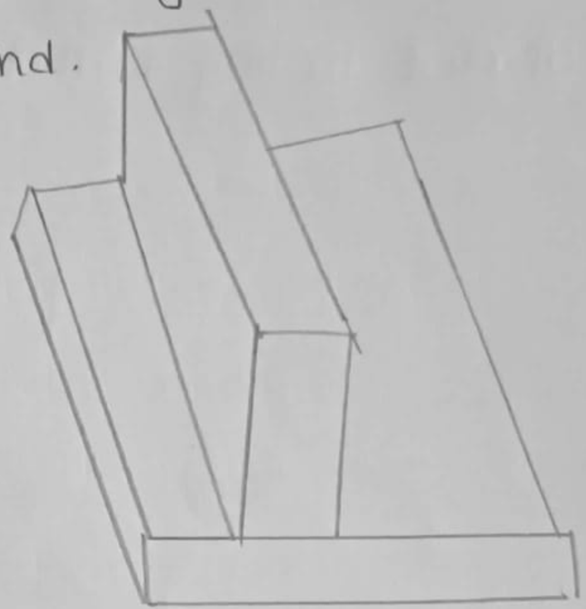
In such footing, the outer and inner col--umn is connected by a strap beam, does not tran--sfer any load to the soil. The individual footing areas of ~~any~~ the columns are so arranged that C.G of the combined loads of the two columns pass through the C.G of the two footing areas. Once this criterion is achieved, the pressure distribution below each individual footing will be uniform.

→ Raft (or) Mat footing :-

This foundation covers the entire area under the structure. This foundation has only RCC slab covering the whole area or slab and beam toge--ther. Mat foundation is adopted when heavy stru--cture are to be constructed on soft made-up ground or marshy siter with uncertain behaviour.

→ wall / strip footing:-

It is a component of shallow foundation which distributes the weight of a load bearing wall across the area of the ground.



→ spread footing:-

As the name suggests, a spread is given under the base of the foundation so that the load of the structure is distributed on wide area of the soil in such a way that the safe bearing capacity of the soil is not exceeded.

→ stepped footing:-

The main purpose of using stepped footing is to keep the metal column away from the direct contact with soil to save them from corrosive effect. They are used to carry the load of metal column

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4. Determine the area of Reinforcement in width B.

5. Check for one way shear.

6. Check for Two way shear.

7. Check for Bond length.

8. Check for Bearing stress.

1, Size of the footing:-

Determination of size of the footing is based on service loads (or) working loads and not for the factored loads. Take 10% of load as self weight

$$\text{Area of the footing} = \frac{1.1P}{\text{SBC of soil}}$$

where,

P = working load

SBC = Safe bearing capacity.

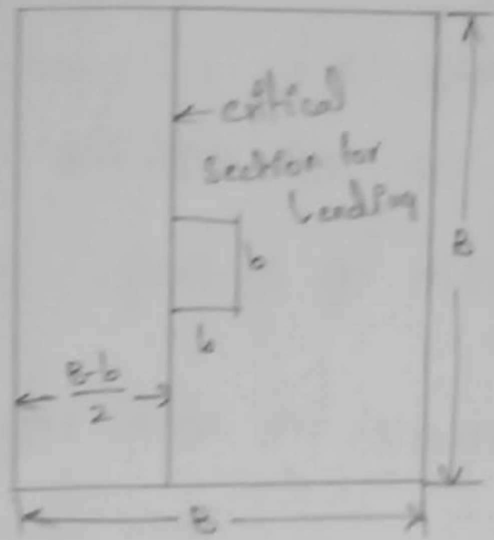
2, Determine the upward soil reaction for the factored load:-

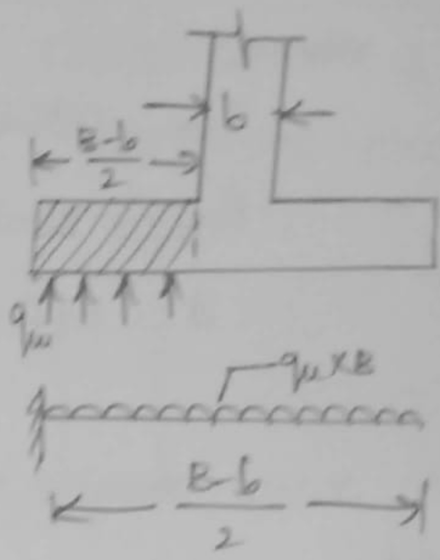
$$q_u = \frac{P_u}{A} = \frac{1.5P}{A}$$

3, Determine the minimum depth required to resist B.M:-

calculate the depth required to resist B.M and is kept uniform, if the footing size is small for check for single shear and check for double shear. and the depth is made sloping, if the footing is large.

The max. B.M is calculated at the face of the column by passing a section extends completely across the footing.





Projection of the footing = $\frac{B-b}{2}$

The B.M about x-x is $\frac{wl^2}{2}$ (cantilever beam)

$$M_u = \frac{q_u B\left[\frac{B-b}{2}\right]^2}{2}$$

$$\boxed{M_u = q_u \frac{B(B-b)^2}{8}}$$

where,

q_u = upward soil pressure

B = width of footing

b = width of column.

→ Determine the area of required reinforcement in width B:-

$$M_u = 0.87 f_y A_{st}\left[1 - \frac{f_y A_{st}}{f_{ck} B d}\right]$$

The bar used is should not be less than 10mm.

$$\text{spacing of bars} = \frac{B\, a_{st}}{A_{st}}$$

where,

a_{st} = area of bars used

A_{st} = Total area of steel required

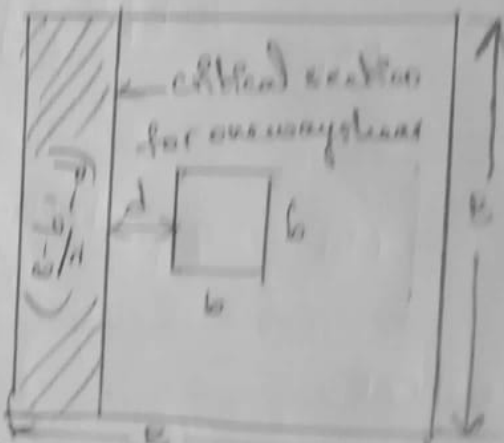
d = effective depth of footing

Note:-

* Provide same require reinforcement in both the directions.

5, check for one way shear:-

The critical section for one way shear at a distance 'd' from the column extending the full width of the footing



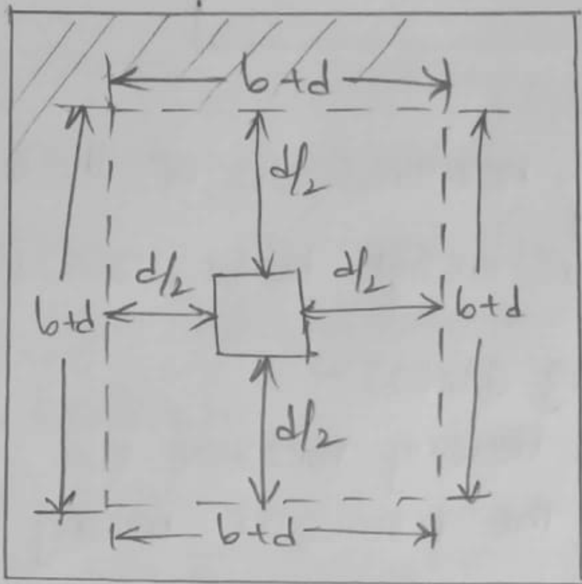
V_u = soil Pressure from the shaded areas

$$V_u = q_u \cdot B\left(\frac{B-b}{2} - d\right)$$

$$\tau_v = \frac{V_u}{B\,d} < \tau_c$$, Permissible shear stress in concrete.

6, Check for two way shear:-

Two way shear is also known as Punching shear. If the depth of footing is less. the column may punch through the footing because of the shear stress in the footing around the perimeter of the column. As per IS 456-2000, the critical section for two way shear is at a distance of $\frac{d}{2}$ from the periphery of the column.



Perimeter of the punching area = $4(b+d)$

Area of concrete resisting punching force = Perimeter of Punching area × depth

$$A = 4(b+d)d$$

Force of punching $S = q_u \times$ shaded area

$$S = q_u\left[B^2 - (b+d)^2\right]$$

Punching shear stress

$$\tau_p = \frac{S}{A} < \text{Permissible value}$$

Note:-

Permissible value of punching shear stress is

$$\tau_p = 0.25\sqrt{f_{ck}}$$

7, Check for Bond length:-

Since the footing is designed i.e. as a cantilever with reinforcement subjected to design strength at the column face, sufficient bond length should be available from the face of the column.

$$\boxed{L_d = \frac{0.87 f_y \phi}{4 \cdot \tau_{bd}}}$$

where,

ϕ = nominal dia of the barea

τ_{bd} = design bond stress [from table 26.2.1.1 Pg. No - 4.3]

8, Check for bearing stress:-

The bearing pressure on the loaded area shall not exceed the permissible bearing stress.

$$\text{Actual bearing pressure} = \frac{P_u}{\text{area of column}} < \text{permissible values.}$$

As per clause 34.4.

The permissible bearing stress is

$$= 0.45 f_{ck} \sqrt{\frac{A_1}{A_2}} \quad \left[\because \sqrt{\frac{A_1}{A_2}} \text{ should not exceedd } 2\right]$$

where,

A_1 = supporting area for bearing of footing.

A_2 = loaded area at the column face.

Problem:-

Q Design a reinforced concrete footing of uniform thick--ness for a reinforced concrete column of 400mm x 400mm size carrying an axial load of 1200KN using M_{20} grade concrete and fe415 steel. The safe bearing capacity of soil is 220 KN/m^2.

Given data,

Axial load $P = 1200$ KN

Size of the column $= 400 \times 400$ mm

SBC of soil $= 220$ KN/m^2

$f_{ck} = 20$ N/mm^2

$f_y = 415$ N/mm^2

1) Size of the footing:-

$P = 1200$ KN

consider self wt of footing = 10% of column load

$$= \frac{P}{10} = \frac{1200}{10} = 120 \text{ KN}$$

Total load on the soil = 1200 + 120

= 1320 KN

$$\text{Area of the footing} = \frac{\text{Total load}}{\text{SBC of soil}}$$

$$= \frac{1320}{220}$$

$A = 6$ m^2

Size of $\square^{lar}$ footing.

$$\boxed{B = \sqrt{6} = 2.45 \text{ m}}$$

Adopt size of footing : 2.5×2.5 m.

2) Upward soil pressure :-

Factored loaded (P_u):

$P_u = 1.5 \times 1200$

$= 1800$ KN

soil pressure at ultimate load

$$q_u = \frac{P_u}{A}$$

$$= \frac{1800}{2.5 \times 2.5} = 288 \ kN/m^2$$

$$\boxed{q_u = 0.288 \ N/mm^2}$$

3, Depth of footing from bending moment consideration:-

$$M_u = q_u \frac{B(B-b)^2}{8}$$

$$= 0.288 \times \frac{2500(2500-400)^2}{8}$$

$$M_u = 396.9 \times 10^6 \ N\text{-}mm$$

WKT

$$M_u = 0.138 f_{ck} Bd^2$$

$$396.9 \times 10^6 = 0.138 \times 20 \times 2500 \times d^2$$

$$d = \sqrt{\frac{369.6 \times 10^6}{0.138 \times 20 \times 2500}}$$

$$\boxed{d = 239.836 mm}$$

provide 500mm effective depth and 550mm overall depth Increase in depth is due to shear consideration.

4) Area of reinforcement:-

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} B d}\right)$$

$$396.9 \times 10^6 = 0.87 \times 415 \times A_{st} \times 500 \left[1 - \frac{415 \times A_{st}}{20 \times 2500 \times 500}\right]$$

$$396.9 \times 10^6 = 180525 \times A_{st} \left[1 - 1.66 \times 10^{-5} A_{st}\right]$$

$$396.9 \times 10^6 = 180525 A_{st} - 2.99 \times A_{st}^2$$

$$2.99 A_{st}^2 - 180525 A_{st} + 396.9 \times 10^6 = 0$$

$$\boxed{A_{st} = 2285.07 \, mm^2}$$

Using 16mm ϕ,

$$\text{spacing of bars} = \frac{a_{st} \times B}{A_{st}}$$

$$a_{st} = \frac{\pi}{4} \times 16^2 = 201.06 mm^2$$

$$S = \frac{201.06 \times 2500}{2285.07}$$

Hence, Provide spacing of bars 220mm c/c on both directions.

5) check for one way shear:-

$$V_u = q_u \cdot B \left[\frac{B-b}{2} - d\right]$$

$$= 0.288 \times 2500 \left[\frac{2500 - 400}{2} - 500\right]$$

$$\boxed{V_u = 396 \, KN}$$

$$\tau_v = \frac{V_u}{Bd} = \frac{396 \times 10^3}{2500 \times 500} = 0.316 \, N/mm^2$$

for τ_c,

$$\text{Percentage of steel } P_t = \frac{A_{st}}{b \times d} \times 100$$

$$= \frac{201.06}{220 \times 500} \times 100$$

$= 0.182\%$

For 0.182% of steel, for M_{20} grade concrete

$\tau_c = 0.32\ N/mm^2$ [from Pg.No.73]

$\therefore \tau_c > \tau_c$

Hence it is safe against one way shear

6, Check for two way shear:-

The critical section is at a distance of $d/2$ from face of column.

perimeter of critical section $= 4(b+d)$

$= 4(400+500)$

$= 3600\ mm.$

Area of critical section

$A = 3600 \times d$

$= 3600 \times 500$

$A = 1.8 \times 10^6\ mm^2$

Two way shear $V_{u_2} = q_u \times$ Area of shaded portion.

$= 0.288\ (2500 \times 2500 - 900 \times 900)$

$= 1566.72 \times 10^3\ N.$

Two way shear stress

$$= \frac{V_{u2}}{A} = \frac{1566.72 \times 10^3}{1.8 \times 10^6}$$

$\tau_{P_0} = 0.87\ N/mm^2$

permissible punching stress

$\tau_{P_P} = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.11\ N/mm^2$

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$\tau_{ps} < \tau_{pp}$

Hence, it is safe against two-way shear

7) check for development length:-

$$L_d = \frac{0.87 f_y \phi}{4 \cdot \tau_{bd}}$$

$$\tau_{bd} = 1.6 \times 1.2 = 1.92 \ N/mm^2$$

$$= \frac{0.87 \times 415 \times 16}{4 \times 1.92}$$

$$= 752.18 mm$$

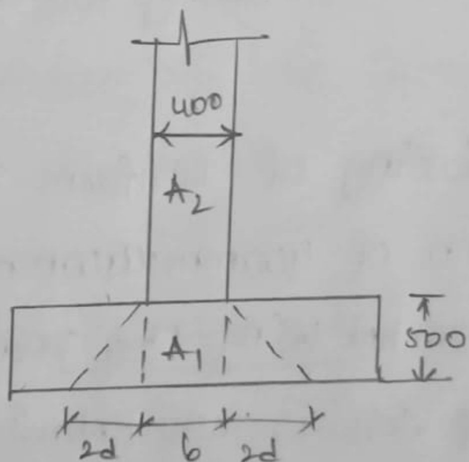
length available beyond the column face

$$= \frac{2500 - 400}{2}$$

$$= 1050 > L_d$$

Hence it is safe.

8) check for bearing pressure:-



supporting area of footing

$$A_1 = 2.4 \times 2.4 = 5.76 m^2$$

loaded area of column base

$$A_2 = 0.4 \times 0.4 = 0.16 m^2$$

As per clause 34.4 IS 456:2000, the permissible bearing stress is

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$= 0.45 f_{ck} \sqrt{\frac{A_1}{A_2}}$

$= 0.45 \times 20 \sqrt{\frac{5.76}{0.16}}$

$= 6$, but it is limited to 2.

so, the allowable bearing pressure is

$= 0.45 \times 20 \times 2$

$= 18 \, N/mm^2$.

But, the actual bearing pressure is : $\frac{P_u}{A} = \frac{1800 \times 10^3}{400 \times 400}$

$= 11.25 \, N/mm^2 <$ allowable value.

Hence, it is safe with respect to bearing.

→ Design of isolated footing for rectangular column:-

For rectangular columns, the design is same as follows as square footing for square column.

Problem:-

Q:- Design of a footing of uniform thickness for a reinforced column of 400mm x 600mm size carrying an axial load of 1500 KN using M_{20} grade concrete and Fe_{415} steel. the SBC of soil is 200 KN/m³.

Given data,

column size = 400 x 600mm

axial load P = 1500 KN

$f_{ck} = 20 \, N/mm^2$

$f_y = 415 \, N/mm^2$.

$S.B.C = 200 \, KN/m^3$.

→ size of footing :-

$P = 1500\ KN$

consider 10% of load as self weigth

$$= \frac{1500}{10}$$

$$P = 150\ KN$$

Total load on the column $= 1500 + 150$

$$= 1650\ KN$$

Area of the footing $= \frac{\text{Total load}}{SBC}$

$$= \frac{1650}{200}$$

$$A = 8.25 m^2$$

as it is rectangular footing. the area of Rectangular $= b \times d$.

$$A = 2.8 m \times 3.0 m.$$

→ upward soil pressure for the factored load :-

$$P_u = 1.5 \times P$$

$$= 1.5 \times 1500$$

$$\boxed{P_u = 2250\ KN}$$

soil pressure at ultimate load.

$$q_u = \frac{P_u}{A} = \frac{2250 \times 10^3}{2800 \times 3000}$$

$$\boxed{q_u = 0.26\ N/mm^2}$$

$$561.6 \times 10^6 = 180525\, A_{st}(1 - 1.48 \times 10^{-5} A_{st})$$

$$561.6 \times 10^6 = 180525\, A_{st} - 2.67\, A_{st}^2$$

$$2.67\, A_{st}^2 - 180525\, A_{st} + 561.6 \times 10^6 = 0$$

$$\boxed{A_{st} = 3268.97\, mm^2}$$

*spacing :- provide 16mm ⌀ bars.

$$a_{st} = \frac{\pi}{4} \times 16^2 = 201.06\, mm^2$$

$$S = \frac{a_{st} \times B}{A_{st}} = \frac{201.06 \times 2800}{3268.97} = 172.21\, mm$$

Provide 16mm ⌀ bars @ distance of 172mm c/c in both directions.

→ check for one way shear:-

$$V_u = q_u \cdot L\left[\frac{B-b}{2} - d\right]$$

$$= 0.26 \times 3000 \left[\frac{(2800-400)}{2} - 500\right]$$

$$\boxed{V_u = 546\, kN}$$

$$\tau_v = \frac{V_u}{Bd} = \frac{546 \times 10^3}{2800 \times 500} = 0.39$$

For τ_c :-

$$P_t = \frac{a_{st}}{S \times d} \times 100$$

$$= \frac{201.06}{172 \times 500} \times 100 = 0.23\%$$

For 0.23% steel, for M_{20} grade concrete

$$\tau_c = 0.32 > \tau_v$$

Hence, it is not safe for the one way shear, so increase the depth of footing.

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Design of circular column of square footing:-

Q. Design a square footing of uniform thickness for a reinforced concrete column of dia 400mm carrying an axial load of 1000KN. The safe bearing capacity of soil is 200 KN/m³. Use M_{20} grade concrete and Fe415 steel?

Sol:-

Axial service load = 1000 KN.

size of the column = 400mm ∅

SBC of soil = 200 KN/m²

f_{ck} = 20 N/mm²

f_y = 415 N/mm².

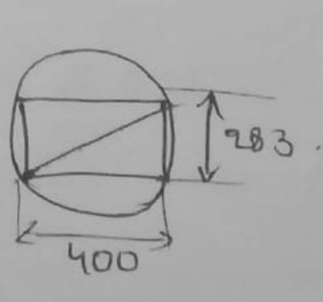
According to clause 34.2.2 of IS:456-2000, for the purpose of computing stresses in footing for a circular column, the face of the column shall be taken as the side of square inscribed with in the perimeter of a circular column as shown in fig.

Hence, the size of the equivalent

square column = d sin45° = 0.707 d

= 0.707 × 400

= 283 mm.



Size of the footing:-

The design procedure is same as the design of footing for a square column. (Take column dimension as 283 mm)

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$P = 1000\,KN$

Self wt of footing = 10% of column load = $\frac{1000}{10} = 100\,KN$

Total load on the soil = 1100 KN

Area of the footing = $\frac{\text{Total load}}{\text{SBC of soil}} = \frac{1100}{200} = 5.5\,m^2$

Size of the square footing.

$$B = \sqrt{5.5} = 2.34\,m$$

Adopt 2.4m x 2.4m square footing.

2, upward soil pressure :-

Factored load $P_u = 1.5 \times 1000 = 1500\,KN$

Soil pressure at ultimate load

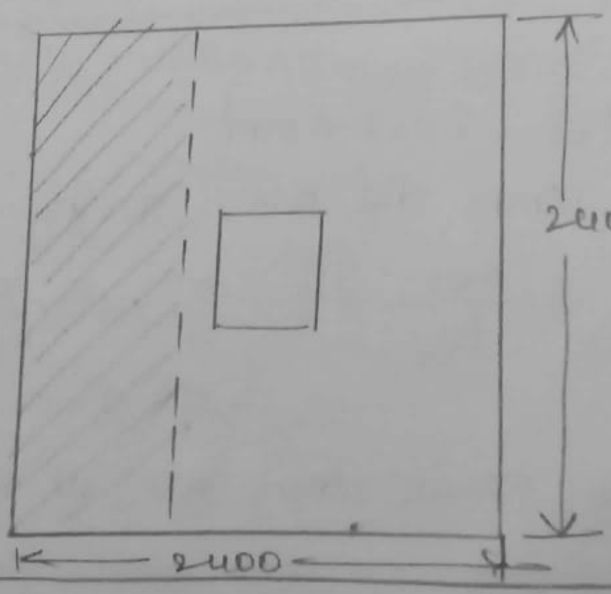
$$q_u = \frac{P_u}{\text{area of footing}}$$

$$q_u = \frac{1500}{2.4 \times 2.4}$$

$$= 260.4\,KN/m^2 = 0.26\,N/mm^2$$

3, Depth of footing from B.M consideration :-

The critical section for B.M will be at the face of the equivalent square column as shown in fig.



$$M_u = q_u \frac{B(B-b)^2}{8}$$

$$= 0.26 \times 2400 \frac{(2400-283)^2}{8}$$

$$= 349.6 \times 10^6 \ N\text{-mm}$$

$$M_u = 0.138 f_{ck} B d^2$$

$$349.6 \times 10^6 = 0.138 \times 20 \times 2400 \times d^2$$

$$d = \sqrt{\frac{349.6 \times 10^6}{0.138 \times 20 \times 2400}}$$

$$d = 229.73 \text{mm}.$$

Provide 400mm effective depth and 450mm overall depth. Increased depth is taken due to shear consideration.

4. Reinforcement :-

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} B d}\right]$$

$$349.6 \times 10^6 = 0.87 \times 415 \times A_{st} \times 400 \left[1 - \frac{415 \times A_{st}}{20 \times 2400 \times 400}\right]$$

$$2420.7 = A_{st} \left[1 - \frac{A_{st}}{46265}\right]$$

$$A_{st}^2 - 46265 A_{st} + 46265 \times 2420.7 = 0.$$

$$A_{st} = \frac{46265 - \sqrt{46265^2 - 4 \times 46265 \times 2420.7}}{2}.$$

$$A_{st} = 2562.6 \text{mm}^2.$$

using 16mm dia bars, spacing of bars.

$$s = \frac{a_{st}}{A_{st}} \times B = \frac{\pi}{4} \times 16^2 \times \frac{2400}{2562.6}$$

$$= 188.3$$

Hence, provide 16mm bars at 180mm c/c in both directions.

Check for one way shear :-

The critical section for one way shear is at a distance 'd' from the face of the equivalent square column

Factored S.F

V_u = soil pressure from the shaded area

$= q_u \cdot B \left[\frac{B-b}{2} - d \right]$

$= 0.26 \times 2400 \left[\frac{2400-283}{2} - 400 \right]$

$= 0.26 \times 2400 \times 658.5 = 410904\ N$

$\tau_v = \frac{V_u}{Bd} = \frac{410904}{2400 \times 400} = 0.42\ N/mm^2$

% of steel, $P_t = \frac{\pi}{4} \times 16^2 \times \frac{100}{180 \times 400} = 0.28\%$

For 0.28% of steel, for M_{20} grade concrete

$\tau_c = 0.38\ N/mm^2 < \tau_v$

Hence it is not safe with respect to one way shear. so increase depth of footing to 450mm.

$V_u = q_u \cdot B \left[\frac{B-b}{2} - b \right]$

$= 0.26 \times 2400 \left[\frac{2400-283}{2} - 450 \right]$

$= 379704\ N$.

$\tau_v = \frac{V_u}{Bd} = \frac{379704}{2400 \times 450} = 0.35\ N/mm^2$.

% of steel, $P_t = \frac{\pi}{4} \times 16^2 \times \frac{100}{180 \times 450} = 0.25\%$

For 0.25% of steel, for M20 grade concrete τ_c ...
$> \tau_v$

Hence it is ~~sta~~ safe with respect to one way shear.

6. Check for two way shear:-

The critical section is at a distance of $\frac{d}{2}$ from the face of the equivalent square column

Perimeter of the critical section $= 4(b+d)$

$= 4(283+450)$

$= 2932\,mm$

Area of critical section $A = 2932 \times d$

$= 2932 \times 450$

Two way shear $V_{u2} = q_u \times$ area of the shaded portion

$= 0.26\ (2400 \times 2400 - 733 \times 733)$

$= 1357.9 \times 10^3\ N$

Two way shear stress $= \frac{V_{u2}}{A} = \frac{1357.9 \times 10^3}{2832 \times 450} = 1.03\ N/mm^2$

Permissible punching stress

$\tau_p = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.12\ N/mm^2 > 1.03\ N/mm^2$

~~$\tau_p = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.12\ N/mm^2 >$~~

Hence, it is safe w.r.t two way shear

7) Check for development length:-

$\tau_{bd} = 1.6 \times 1.2 = 1.92\ N/mm^2$

$L_d = \frac{0.87 f_y \phi}{4 \cdot \tau_{bd}}$

$$= \frac{0.87 \times 415 \times 16}{4 \times 1.92}$$

$$\boxed{L_d = 752.2 mm}$$

Length available beyond the column face

$$= \frac{(2400-283)}{2}$$

$$= 1058.5 mm > L_d$$

Hence ok.

→ Design of combined footing :-

Q:- Design of rectangular combined footing with a central beam for supporting two columns 400 x 400mm size to carry a load of 1000kN each. centre to center distance b/w the column is 3.5 m. The projection of the footing on either side of the column with respect to center is 1m. safe bearing capacity of the soil can be taken as 190 kN/m². use M20 concrete and fe415 steel.

Sol:- Given data,

$f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$

$SBC = 190 kN/m^2$

column A = 400 x 400mm

column B = 400 x 400mm

c/c spacing of columns = 3.5

$P_A = 1000 kN$ and $P_B = 1000 kN$

Required :- To design combined footing with cen
beam joining the two columns.

Ultimate loads.

$P_{uA} = 1.5 \times 1000 = 1500\,KN$,

$P_{uB} = 1.5 \times 1000 = 1500\,KN$.

1, Proportioning of base size:-

working load carried by column $A = P_A = 1000\,KN$.

working load " " " $B = P_B = 1000\,KN$.

self wt of footing $10\% \times (P_A + P_B) = 2200\,KN$.

Total working load $= 2200\,KN$.

Required area of footing $= A_f = \frac{\text{Total load}}{SBC}$

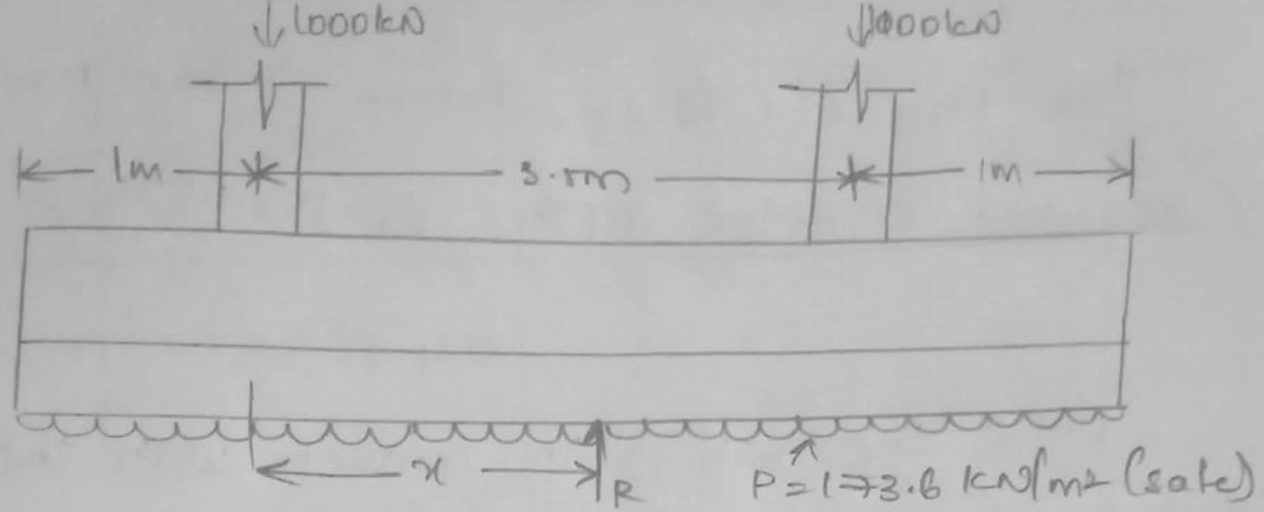
$= \frac{2200}{190} = 11.57\,m^2$.

Length of the footing $L_f = 3.5 + 1 + 1 = 5.5\,m$

Required width of footing $= b = \frac{A_f}{L_f} = \frac{11.57}{5.5} = 2.1\,m$

provide footing of size $= 5.5 \times 2.1\,m$

For uniform pressure distribution the C.G of the footing should coincide with C.G of column loads. As the footing and columns loads are symmetrical, this condition is satisfied. The details are shown in fig.



Total load from columns = P = (1000+1000)

= 2000 KN.

upward intensity of soil pressure = column loads

$= \frac{P}{A_f} = \frac{2000}{5.5 \times 2.1}$

$= 173.16\ KN/m^2 < SBC.$

Design of slab:-

· Intensity of upward pressure P = 173.16 KN/m²

consider one meter width of slab b = 1m.

load per m run of slab at ultimate = 173.16×1

= 173.16×1 kN/m.

cantilever projections of the slab

(for smaller column).

= 1050 − 400/2

= 850mm

maximum ultimate moment = $173.16 \times 0.850^2/2$

= 62.55 KN-m (working condition).

For M_{20} and $fe415$, $q_{umax} = 2.76\ N/mm^2$

Required effective depth $= \sqrt{62.15 \times 1.5 \times 10^6 / (2.76 \times 1050)}$

$= 184.28mm$.

since the slab is in contact with the soil clear cover of the 50mm is assumed.

using 20mm dia, bars, effective cover $= \frac{20}{2} + 5$

say $= 75mm$.

Required total depth $= 184.28 + 75$

$= 259.4mm$

However provide 300mm from shear conside-ration as well. Provide effective depth $= d = 300 - 75$

$= 225mm$.

To find steel:-

$$\frac{M_u}{bd^2} = \frac{1.5 \times 62.15 \times 10^6}{1000 \times 225^2} = 1.84 < 2.76, \text{ URS}$$

$P_t = 0.584\%$

$A_{st} = 1314 mm^2$

use 20mm dia bars at spacing.

$= 1000 \times 314 / 1314 = 238.96mm$

say $= 230$ mm c/c

Area provided $= 1000 \times 314 / 230 = 1365mm$.

Hence safe. This steel is required for the entire length of the footing.

Check the depth from one-way shear consideration:-

Design S.F $= V_u = 1.5 \times 173.16 \times (0.850 - 0.225)$

$= 162.33 \text{ kN}$

Nominal shear stress $= \tau_v = \frac{V_u}{bd} = \frac{162330}{(1000 \times 225)}$

$= 0.72 \text{ MPa}$

Permissible shear stress:-

$P_t = 100 \times 1365 / (1000 \times 225)$

$= 0.607\%$

$\tau_{uc} = 0.51 \text{ N/mm}^2$

value of k for 300 mm thick slab = 1

Permissible shear stress $= 1 \times 0.51$

$= 0.51 \text{ N/mm}^2$

$\tau_{uc} < \tau_v$ and hence unsafe.

The depth may be increased to 400 mm so that

$d = 325 \text{ mm}$

$\frac{M_u}{bd^2} = \frac{1.5 \times 62.15 \times 10^6}{1000 \times 325^2} = 0.883 < 2.76$, URS

$P_t = 0.26\%$, $A_{st} = 845 \text{ mm}^2$

use 16 mm dia, bar at spacing $= 1000 \times 201 / 845$

$= 237.8 \text{ mm}$

say $= 230 \text{ mm c/c}$

Area provided $= 1000 \times 201 / 230 = 874 \text{ mm}^2$

check the depth from one-way shear consideration:-

Design shear force $V_u = 1.5 \times 173.16 \times (0.850 - 0.325)$

$= 136.36 \text{ kN}$

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...n the depth from one-way shear consideration:-

Design S.F : $V_u = 1.5 \times 173.16 \times (0.350 - 0.225)$

$= 162.33 kN$

Nominal shear stress $\tau_v = \frac{V_u}{bd} = \frac{162330}{(1000 \times 225)}$

$= 0.72 MPa$.

Permissible shear stress:-

$P_t = 100 \times 1365 / (1000 \times 225)$

$= 0.607\%$

$\tau_{uc} = 0.51 N/mm^2$

value of K for 300 mm thick slab = 1

Permissible shear stress = 1×0.51

$= 0.51 N/mm^2$

$\tau_{uc} < \tau_v$ and hence unsafe

The depth may be increased to 400 mm so that

$d = 325 mm$

$\frac{M_u}{bd^2} = \frac{1.5 \times 62.15 \times 10^6}{1000 \times 325^2} = 0.883 < 2.76$, URS

$P_t = 0.26\%$, $A_{st} = 845 mm^2$

use 16 mm dia. bar at spacing $= 1000 \times 201 / 845$

$= 237.8 mm$

say $= 230 mm$ c/c

Area provided $= 1000 \times 201 / 230 = 874 mm^2$

check the depth from one-way shear consideration:-

Design shear force $V_u = 1.5 \times 173.16 \times (0.850 - 0.325)$

$= 136.36 kN$.

Nominal shear stress: $\tau_v = \frac{V_u}{bd} = \frac{136360}{(1000 \times 325)} = 0.42$ Mpa

Permissible shear stress

$P_t = 100 \times 875 / (1000 \times 325) = 0.269\%$, $\tau_{uc} = 0.38$ N/mm²

value of K for 400mm thick slab = 1

permissible shear stress = $1 \times 0.38 = 0.38$ N/mm²

Again $\tau_{uc} < \tau_v$, and hence slightly unsafe.

Howevere, provide steel at closure spacing,

ϕ 16 @ 150mm c/c.

$A_{st} = 201 \times 1000/150 = 1340$ mm² and $P_t = 0.41\%$

Hence,

$\tau_{uc} = 0.45$ Mpa and safe.

check for development length:-

$L_{dt} = 47$ times dia, $= 47 \times 16 = 768$mm

Available length of bars = $850 - 25 = 825$ mm > 768 mm

Hence safe.

Transverse reinforcement:-

Required $A_{st} = 0.12 b D/100 = \frac{0.12 \times 1000 \times 400}{100}$

$= 480$mm

using 10mm bars, spacing $= 1000 \times 79/480 = 164.58$mm.

Provide distribution steel of #10mm at 160mm c/c

Design of Longitudinal Beam:-

Two columns are joined by means of a beam monolithic with the footing slab. The load from the slab will be transfered to the beam. As the width of the footing is 2.1m, the net upward soil pressure per meter length of the beam under service.

$w = 173.16 \times 2.1$

$= 363.64$ KN/m

Shear force and B.M at service condition:-

$V_{AC} = 363.64 \times 1 = 363.14$ KN.

$V_{AB} = 1000 - 363.14 = 636.36$ KN

$V_{BD} = 363.14$ KN

$V_{BA} = 636.36$ KN

Point of zero shear is at the center of footing at $\frac{1}{2}$ i.e, at E.

Max. B.M occurs at E

$M_E = 363.64 \times 2.75^2/_2 - £1000(2.75-1)$

$= -375.15$ KN-m.

B.M under column $A = M_A = 363.64 \times 1^2/_2$

$= 181.82$ Kn-m

Parallel B.M under column $A = M_A = 363.64 \times 1^2/_2$

$= 181.82$ Kn-m

Let the point of contraflexure be at a distance of x

Then $M_x = \frac{363.63 x^2}{2} - 1000(x-1) = 0$

Therefore $x = 1.30$m and 4.2m from C.

Depth of beam from B.M consideration:-

$M_u = q_u \cdot Bd^2$. $\quad B = 400$mm.

$d = \sqrt{\frac{M_u}{q_u \cdot B}}$

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$$= \sqrt{\frac{375.15 \times 10^6}{2.76 \times 400}}$$

$d = 713.8mm$

Provide depth = 800mm

Assuming 2 rows of c.c = 75mm

effective depth provided 'd' = 800 - 75

= 725mm.

Check the depth for two way shear :-

In this case $b = D = 400mm$, $d_b = 725mm$, $d_s = 325mm$ Area resisting two way shear.

$= 2(b \times d_b + d_s \times d_s) + 2(D + d_b) d_s$

$= 2(400 \times 725 + 325 \times 325) + 2(400 + 725) 325$

$= 1522500 mm^2$.

Design shear = P_{ud} = column load − $w_u \times$ area at critical section.

$= 1500 - 173.16 \times 1.5 \times (b + d_s) \times (D + d_b)$

$= 1500 - 173.16 \times 1.5 \times (0.400 + 0.325) \times (0.400 + 0.725)$

$= 1288.14 KN$.

$$\tau_v = \frac{P_{ud}}{bod} = \frac{1288.14 \times 1000}{1522550} = 0.845 Mpa$$

shear stress resisted by concrete = $\tau_{uc} = \tau_{uc} \times K_s$

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where, $\tau_{uc} = 0.25\sqrt{f_{ck}}$

$= 0.25\sqrt{20}$

$= 1.11\ N/mm^2$

$K_s = 0.5 + d/D = 0.5 + \frac{400}{400} = 1.5 \neq 1$ Hence $K_s = 1$

$\tau_{uc} = 1 \times 1.11 = 1.11\ N/mm^2$

$\therefore$ safe.

Area of reinforcement :-

cantilever portion BD and AC

Length of cantilever from the face of column = 0.8m

ultimate moment at the face of column $= 363.64 \times 1.5 \times \frac{0.8^2}{2}$

$= 177.53\ KN\text{-}m$

$M_{umax} = 2.76 \times 400 \times 725^2 \times 10^{-6}$

$= 580.29\ KN.m > 177.53\ KN\text{-}m$

$\therefore$ section is singly reinforced.

$\frac{M_u}{bd^2} = \frac{177.53 \times 10^{+6}}{(400 \times 725)^2} = 0.844 < 2.76$, URS.

$P_t = 0.248\%$ $A_{st} = 719.2\ mm^2$

Provide 4-16mm at bottom face, Area provide:

$: 804\ mm^2$

$P_t = 0.278\%$.

$L_d = 47 \times 16 = 752\ mm$

curtailment :-

All bottom base will be continued upto the end of cantilever for both column. If required two bottom bars of 2-16mm will be curtailed at a distance (d = 725mm) from the point of contraflexure in the portion BE as shown in fig.

Region AB b/w point of contra flexures:—

The beam acts as an isolated T-beam.

$$b_f = \left[\frac{l_0}{l_0/b + 4}\right] + b_w$$

$l_0 = 4.2 - 1.3 = 2.9m = 2900mm$.

b = actual width of flange

= 2100mm.

$b_w = 400mm$

$$b_f = \left(\frac{2900}{2900/2100 + 4}\right) + 400$$

$= 938.9mm < 2100mm$.

$D_f = 400mm$, $M_u = 1.5 \times 375$

$= 562.5$ KN-m.

Moment of Resistance M_{uf} of a beam for $x_u = D_f$ is

$$M_{uf} = \left[0.36 \times 20 \times 938.9 \times 400(725 - 0.42 \times 400)\right] \times 10^{-6}$$

$= 1506$ KN-m $> M_u = 562.5$ KN-m.

$\therefore x_u < D_f$

$$M_u = 0.87 f_y A_{st} \left(d - \frac{f_y A_{st}}{f_{ck} b_f}\right)$$

$A_{st} = 2334mm^2$.

Provide 4 bars of 25mm and 2 bars of 16mm Area provided = $2354 mm^2 > 2334mm^2$

...llement:-

cuetailement can be done as explained in the Previous problem. However extend all bars upto a dischage 'd' from the paint of contra flexure i.e, upto 225mm from the outer faces of the columns. Extend 2-16mm only upto the end of the footing.

Design of shear reinforcement portion b/w column AB:-

In this case the crack due to diagonal tension occur at the point of contraflexure beacause the distance of the point of contraflexure from the column is less than the effective depth (d = 725 mm).

i, Max. shear force at A or B = V_{max}

$= 1.5 \times 636.36$

$= 954.54 KN$

shear at the P.C $= 954.54 - 1.5 \times 363.64 \times 0.3$

$= 790.9 KN.$

$\tau_v = 790900/(400 \times 725) = 2.73 Mpa < \tau_{cmax} (2.8Mpa)$

Area of steel available $= 2354 mm^2, 0.805\%.$

$\tau_c = 0.59 Mpa, \tau_v > \tau_c$

Design of shear reinforcement is required using 12mm dia, 4-legged stirrups.

Spacing $= 0.87 \times 415 \times (4 \times 113) / (2.73 - 0.59) \times 400$

$= 190.6 mm$ say 190mm c/c

Zone of shear reinforcements is b/w τ_v to τ_c

Scanned by CamScanner

cantilever Portion BD and AC:-

$V_{max} = 363.64 \times 1.5 = 545.45$ KN

shear from face at distance d = V_{uD}

$d = V_{uD} = 545.45 - 363.64 \times 1.5\,(0.400/2 + 0.725)$

$= 40.90$ KN.

$\tau_v = 40900/(400 \times 725) = 0.14\,MPa < \tau_{c\,max}$

(This is very small)

steel at this section is 4-16mm.

Area provided = $804\,mm^2$, $P_t = 0.278\%$

$\tau_c = 0.38\,N/mm^2$ [Table IS:450:2000]

NO shear steel is needed

provide minimum steel

using 12mm dia, 2-legged stirrups.

spacing $= 0.87 \times 415 \times (2 \times 113) / 0.4 \times 400$

$= 509.9$ mm say 300mm c/c.

1 ①

UNIT 5

Slab

A beam is a horizontal structural elements that is capable of withstanding load primarily by resisting bending.

The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a bending moment.

Types of slabs

(a) Based on support conditions:

31,36 37,38,39, 48,45

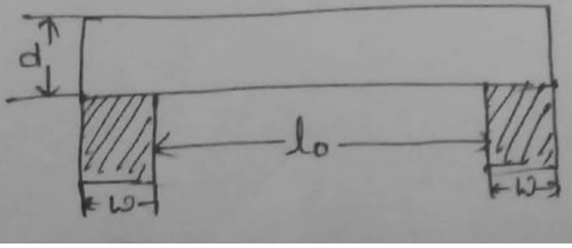
* Simply supported slab
* Cantilever slab
* Restrained slabs (with fixed (or) continuous slab)
* Continuous slabs
* Flat slabs (slabs resting directly on columns)

(b) Based on spanning directions

* Oneway slabs - spanning in one direction
* Two way slabs - spanning in two directions

Simply Supported Slab : (IS 456 - Cl-22.2

The effective span of a member that is not built integrally with its supports shall be taken as clear span + the effective depth of slab (or) beam (or) centre of support



l_{eff} (Effective span) = $l_0 + d$

$$l_{eff} = l_0 + \frac{w}{2} + \frac{w}{2}$$

$d \rightarrow$ effective depth

$w \rightarrow$ width of support

$l_0 \rightarrow$ clear span

$l_{eff} \rightarrow$ effective span.

(b) Continuous slab (or) beams :-

if the width of support is less than $\frac{1}{12}$ of the clear span,

case (i): If width of support < $\frac{span}{12}$ $\left(w < \frac{l_0}{12}\right)$ than effective span is calculated same as for simply supported case.

$$l_{eff} = l_0 + d$$

(or)

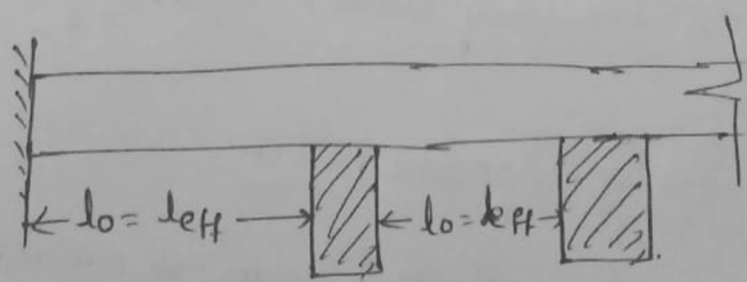
$$l_{eff} = l_0 + \frac{w}{2} + \frac{w}{2}$$

which ever in less.

case (ii): If width of support > $\frac{Span}{12}$ $\left(w > \frac{l_0}{12}\right)$

(a) (i) For one end fixed other continuous

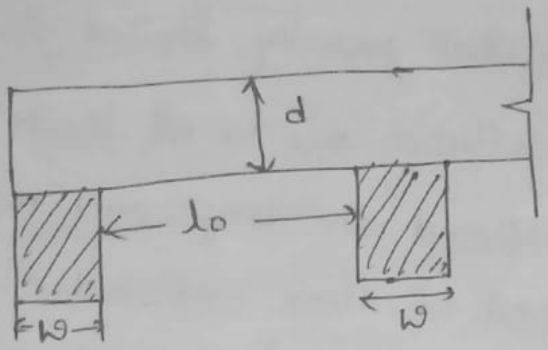
(ii) Both end conditions (Intermediate span)



$$l_{eff} = l_0 = \text{clear span}$$

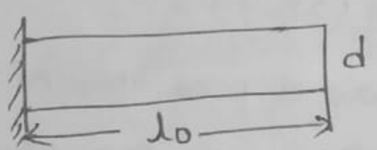
(2)

(b) One end discontinuous other continuous (simply supported)

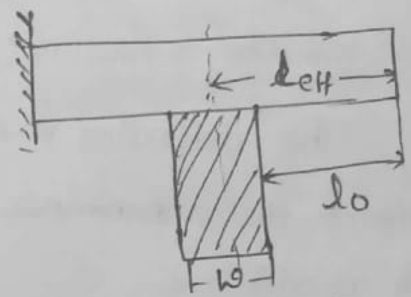


$l_{eff} = l_o + \frac{d}{2}$
$l_{eff} = l_o + \frac{w}{2}$
} which ever is less.

Case (iii): Cantilever

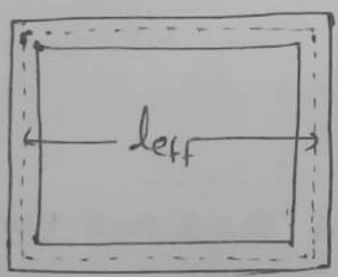


$l_{eff} = l_o + \frac{d}{2}$



$l_{eff} = l_o + \frac{w}{2}$

case (iv) :- For frame it's centre to centre distance b/w member.



l_{eff} = Centre to centre distance b/w members.

case(v): In the case of spans with roller. (or) rocket bearings, the distance b/w the centre the effective span shall always be the distance b/w the centre of bearings.

Check for deflection (cl : 23.2 / P-37)

The deflection shall generally limited to following.

1) The final deflection due to all loads including the of temp, creep, and shrinkage and measured from a level of support of floor, roof and all Hz. Deflection sh not normally exceed span / 250.

2) The deflection including the effect of creep, temp & shrinkage occuring after erection of partition and application of finishes ≯ $\frac{\text{Span}}{350}$ (or) 20mm which ever is less.

control of deflection :- (cl : 23.2.1)

The deflection of structure of part there of shall not adversely affect the appearance (or) efficiency of the structure (or) finishes of partitions.

→ For beam and slabs, the vertical deflection limits may generally be assumed to be satisfied, provided that the span to depth ratio are not greater that the values obtained as below.

Cantilever - 7

Simply supported - 20

continuous - 26.

→ Depending upon the type of steel and percentage of steel, the above values have to be modified as per fig. 4 IS 456.

→ For two way slabs, the shorter span should be used for calculating the span to effective depth ratio.

One way slab :-

(3) If the ratio of longer span to shorter span (l_y/l_x) is greater than 2. is called as one way slab.

One way slab bends only in one dir^n across the span, and acts like a wide beam.

The analysis and design of one way slab is same as that of beam of 1 m width.

Design Procedure

① Assume the suitable depth based on the stiffness consideration and calculate the effective span.

$$\text{Required effective depth} = \frac{\text{Span}}{\text{Basic value} \times \text{modification factor}}$$

• $\frac{\text{Span}}{\text{depth}}$ ratio can safely be selected in range of 25 to 30 S.S.S. 28

② Considering 1 m width of slab, calculate the loads acting on the slab. Find the factored moment and shear force.

For simply supported slabs.

$$M_u = \frac{w_u l^2}{8} \qquad l = \text{length of shorter span.}$$

$$V_u = \frac{w_u l}{2}$$

③ Determine the min. depth required to resist the bending moment by equating.

$$M_u = M_{u,lim} = K f_{ck} b d^2 \quad , \; b = 1000\text{mm}$$

$K = 0.138$ for Fe 415 steel

$K = 0.148$ " Fe 250 "

Provided depth should be more than this value, otherwise ↑ the depth.

(4) Calculate the area of steel per meter width of sla

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} b.d}\right]$$

(5) Find the spacing of bars using

$$S = \frac{a_{st}}{A_{st}} \times 100$$

a_{st} = area of bar used

A_{st} = total area of steel required

Spacing should not be more than 3d (or) 300mm which ever is less.

(6) Distribution Steel:

Provide distribution reinforcement at 0.12%. (for HYSD bars) of gross cross sectional area.

If mild steel bars are used, provide 0.15% of gross area as distribution steel.

Spacing of distribution steel should not be more than 5d (or) 450mm which ever is less.

(7) Check for deflection:

calculate the % P_t corresponding to max mid span moment. take the modification factor (F_1) from fig 4. of IS456.

$$\left(\frac{l}{d}\right)_{provided} < \left(\frac{l}{d}\right)_{max} = \text{basic value} \times F_1$$

(8) Check for shear

Max. shear force at the edges of one way slab is

$$V_u = \frac{wl}{2}$$

$$\tau_v \leq \tau_c$$

→ for solid slabs, the shear strength of concrete shall be $\tau_c k$. k value as per IS 456 - Cl: 40.2.1.1 — Pg-72

overall depth	300 more	275	250	225	200	175	150 (or) less
k	1	1.05	1.10	1.15	1.20	1.25	1.30

Design a rectangular beam simply supported over a cl. span of 6m if superimposed load is 30kN/m and support width is 500mm each. Use M15 mix and HYSD steel

sol: simply supported beam $d = \frac{span}{10} = \frac{6000}{10} = 600mm$.

take clear cover for beam is 25mm & 20mm dia. bars

$D = 600 + 25 + \frac{20}{2} = 635mm$.

$\frac{l}{d} = 10$

adopt D = 650mm.

$d = 650 - 25 - \frac{20}{2} = 615mm$

$b = 0.5D = 0.5 \times 650 = 325mm$.

$\frac{6000}{20} = d$

(ii) Effective span is minimum of

(i) $6 + \frac{0.5}{2} + \frac{0.5}{2} = 6.5m$,

(ii) $6 + 0.615 = 6.615m$

∴ Effective span = 6.5m.

(iii) Load calculation

Dead load of beam $= b \times D\gamma$

$= 0.325 \times 0.650 \times 25$

$= 5.28 kN/m$

Superimposed load = 30kN/m.

total load $= 30 + 5.28 = 35.28 kN/m$,

$B.M = \frac{wl_{eff}^2}{8} = \frac{35.28 \times (6.5)^2}{8} = 186.32 kN/m$

Factored B.M $= 1.5 \times 186.32 = 279.48 kN\text{-}m$

(iv) check for deflection.

take $M_u = M_{u\,limit}$

$M_{u\,limit} = 0.36 f_{ck} x_{u\,lim} b(d - 0.42x_u)$

$279.48 \times 10^6 = 0.36 \times 15 \times 0.48 \times d \times 325 (d - 0.42 \times 0.48d)$

$d = \sqrt{\frac{279.48 \times 10^6}{672.57}}$

(5)

$d = 644.62\ mm > d_{provided}$ (615mm)

let us revised the section. take D = 700mm.

410

$d = 700 - 25 - \frac{20}{2} = 665mm.$

$b = 0.\cancel{0}5D = 0.5 \times 700 = 350mm$

(v) Dead load = $b \times D \times \gamma$

$= 0.350 \times \overset{0.7}{\cancel{0.665}} \times 25$

$= 6.125\ kN/m$

Superimpose load = 30 kN/m

total load = $30 + 6.125 = 36.125\ kN/m$

(vi) Effective span is min of

(i) $6 + \frac{0.5}{2} + \frac{0.5}{2} = 6.5m$

(ii) $6 + 0.665 = 6.665m$

$l_{eff} = 6.5m$

$B.M = \frac{w\, l_{eff}^2}{8} = \frac{36.125 \times 6.5^2}{8} = 190.78\ kN\text{-}m$

Factored B M = $1.5 \times 190.78 = 286.18\ kN\text{-}m$

(vii) Check for deflection.

$M_{u,lim} = 0.36 f_{ck}\, x_{u,limit}\, b\, (d - 0.42 x_{u,limit})$

$286.18 \times 10^6 = 0.36 \times 15 \times 0.48 d \times 350\ (d - 0.42 \times 0.48 d)$

$d = \sqrt{\frac{286.18 \times 10^6}{724.30}} = 628.57\ mm.$

$D = 628.57 + 25 + \frac{20}{2} = 663.57\ mm <$ assumed value (0 700m

adopt D = 665mm

$d = 665 - 25 - \frac{20}{2} = 630mm$

(viii) Area of steel

$M_{u,limit} = 0.87 f_y A_{st}\ (d - 0.42 x_{u,limit})$

$286.18 \times 10^6 = 0.87 \times 415\, A_{st}\ (630 - 0.42 \times 0.48 \times 630)$

$A_{st} = \frac{286.18 \times 10^6}{181605.25} = 1575.8\ mm^2$

(8)

Equate C = T

$0.36 f_{ck}\, x_{u,limit}\, b = 0.87\, f_y\, A_{st}$

$$A_{st} = \frac{0.36 f_{ck}\, x_{u\,limit}\, b}{0.87 f_y}$$

$$= \frac{0.36 \times 15 \times 0.48 \times 630 \times 350}{0.87 \times 415}$$

$A_{st} = 1582.98\,mm^2$

$$A_{st,min} = \frac{0.85}{f_y} \cdot bd$$

$$= \frac{0.87}{415} \times 630 \times 350$$

$= 415.62\,mm^2$

$A_{st}\ (1582.98\,mm^2) > A_{st\,min}\ (451.62\,mm^2)$ o.k

Provide 4nos 20mm & 20nos 6mm dia bars given steel area = 1659 mm²

max tension steel = 0.04bD

$= 0.04 \times 350 \times 665$

$= 9310\,mm^2 > 1659\,mm^2$

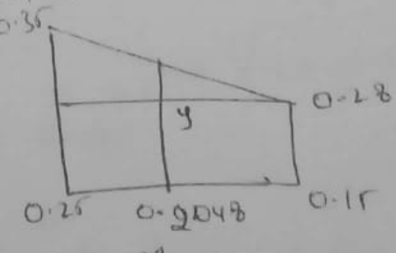
check for shear:

$$\tau_v = \frac{V_u}{bd} = \frac{162.56 \times 10^3}{350 \times 630} = 0.737 = 0.74\,N/mm^2$$

$$\text{min \% of steel} = \frac{0.85}{f_y} \times 100 = \frac{0.85}{415} \times 100 = 0.2048\%$$

τ_c corresponding to 0.2048% of steel is - Table 20 IS456-2000

$$\frac{0.35 - 0.28}{0.25 - 0.15} = \frac{y - 0.28}{0.2048 - 0.15}$$



$y = \tau_c = 0.318\,N/mm^2$

$\tau_v > \tau_c$ so shear reinforcement is required

$V_{us} = V_u - V_c$

$= 162.56 \times 10^6 - 0.136 \times 300 \times 630$

$= 92.44\,kN$

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Use 8mm bar 2 legged

$$S_v = \frac{0.87 f_y d A_{st}}{V_{us}} = \frac{0.87 \times 415 \times 630 \times 2 \times \frac{\pi}{4} \times 8^2}{92.44 \times 10^3}$$

$$S_v = 247.37 \text{ mm} < 300\text{mm} < 0.75d\ (472.5)$$

from Min shear Reinforcement

$$\frac{A_{st}}{b \cdot S_v} \geq \frac{0.4}{f_y}$$

$$S_{v,max} = \frac{A_{st} f_y}{b \times 0.4} = \frac{2 \times \frac{\pi}{4} \times 8^2 \times 415}{350 \times 0.4} = 298\text{mm}$$

$247.37 \text{ mm} < S_{v,max}$ (298 mm) ok

Provide 2 legged 8mm dia stirrups @ 240mm c/c.

Check for devlopment length.

$$l_d \leq 1.3 \frac{M_1}{V} + l_0$$

$$l_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 20}{4 \times 1.6 \times 1} = 1128.3 \text{ mm}$$

l_0 = max of 12ϕ (or) d

l_0 = Max. of $12 \times 20 = 240$mm (or) 630mm

l_0 = 630 mm

$M_1 = 0.87 f_y A_{st} (d - 0.42 x_u)$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$M_1 = 0.87 \times 415 \times 1659 \left(630 - \frac{0.87 \times 415 \times 1659}{0.36 \times 15 \times 350}\right)$$

$M_1 = 187.52$ kN-m

$$V = 1.5 \times \frac{w.l}{2} = \frac{1.5 \times 36.125 \times 6}{2} = 162.56 \text{ kN}$$

$$= 1.3 \frac{M_1}{V} + l_0$$

$$= 1.3 \times \frac{187.52 \times 10^6}{162.56 \times 10^3} + 630$$

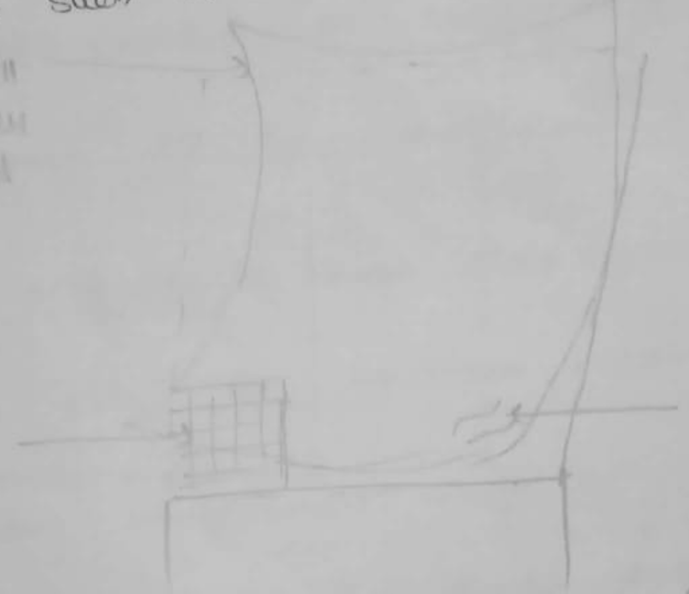
$$= 2129.6 \text{ mm}$$

l_d (1128.3 mm) < 2129.6 mm Ok

Design of two way slabs:

when the slab is supported on all the four edges and if the ratio of longer span to shorter span is less than (or) Equal to 2.

The slab is likely to bend along the two directions and such slabs are called as two way slabs.



Torsion effects in two way slabs.

Two way slabs can be divided into the following categories depending on support conditions.

1) Slabs simply supported on all the four edges and corners free to lift.

2) Restrained slab i.e... slabs with fixed (or) continuous edges.

Recommenations of IS:456 for design of Restained slabs:

1. The Max. bending moment per unit width in a slab are given by following eqn.

$$M_x = \alpha_x w l_x^2$$

$$M_y = \alpha_y w l_x^2$$

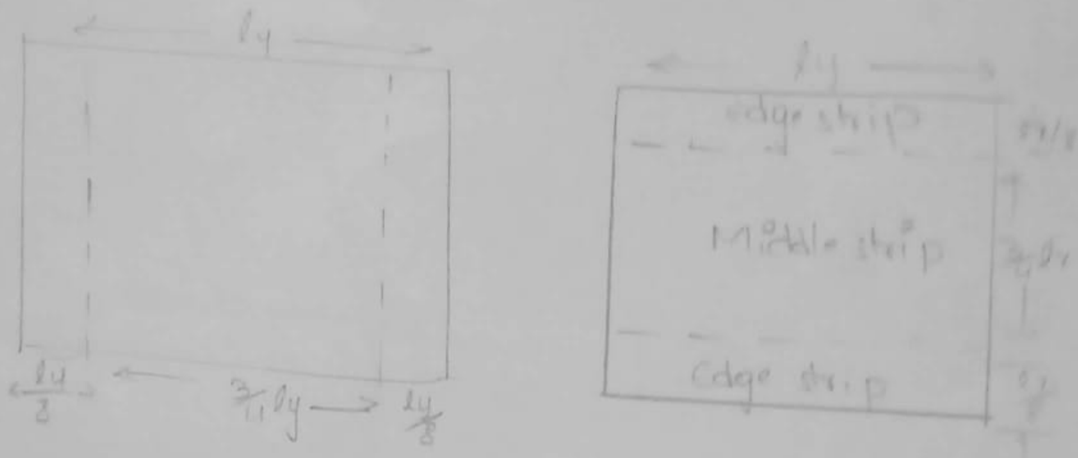
where M_x & M_y are the design moment along short and long spans

w = total design load on the slab

l_x & l_y are the length of short and long spans

α_x & α_y are the moment coefficient given in table 26 IS 456

2) Slabs are considered as divided in each direction in to middle strips and edge strips as shown in fig 4.10. The middle strip being 3/4 of the width and edge strip of $\frac{1}{8}$ width of the slab.



for span lx

for span ly

fig 4.10 : Division of slab into middle and edge strip

3) The maximum moment calculated (i) apply only to the middle strips only

4) Tension reinforcements provided at the mid span in the middle strip shall extend in to the lower part of the slab to with in 0.25 l of a continuous edge or 0.15 l of a discontinuous edge.

5) Over the continuous edges of a middle strip, the tension reinforcement shall extend in the upper part of the slab a distance of 0.15 l from the support and

at least 50% shall extend a distance of 0.3 l

6) At a discontinuous edge, negative moment may arise. They depend on the degree of fixity at the edge of the slab, but in genaral, tension reinforcement eaual to 50% of that provided at mid span extending 0.1 l in to the span will be sufficient.

7) Reinforcement in edge strip, parallel to that edge, shall comply with the minimum reinforcement

8) Torsion reinforcement shall be provided at any corner where the slab is simply supported on both the edges meeting at that corner. It shall consists of top and bottom reinforcement, each with layers of bars placed parallel to the side of the slabs and extending from the edges a minimum distance of 1/5 of the shorter span, The area of reinforcement in each of these four layers shall be 3/4 of the area required for the maximum mid span moment in the slab.

9) Torsion reinforcement eaual to half that described in (8) shall be provided at a corner contained by edges over only one of which the slab is continuous.

10) Torsion reinforcement need not be provided at any corner contained by edges over both of which the slab is continuous.

11) Where $\frac{l_y}{l_x}$ is greater than 2, the slab shall be designed as one way slab.

Design procedure for two way slab

1) Assume the depth of the slab based on stiffness

a) for two way slabs with shorter span less than 3.5m and LL < 3 KN/m², the allowable $\frac{l_x}{d}$ ration is

	Fe 250	Fe 415
simply supported slabs	35	28
fixed or continuous slabs	40	32

b) If $l_x > 3.5$ m and LL > 3 KN/m², the allowable $\frac{l_x}{d}$ ratio is same as that of one way slabs.

2) Find the effective spans l_x and l_y.

3) calculate the ultimate load considering 1 m width of the slab.

4) obtain the design moment co-efficients along short and long span depending on the boundary conditions given in

table 26 of IS : 456 as applicable. Calculate the bending moments by multiplying the co-efficients by $w\, l_x^2$

5) Calculate the minimum depth required to resist the absolute maximum design moment (M_x or M_y) which should be less than the depth provided. Otherwise increase the depth.

6) Calculate the area of steel at mid span (and at support if the slab is continuous) in both the directions using

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} \cdot b \cdot d}\right]$$

The short span bars are provided in the bottom layer and long span bars are provided above the short span bars in the mid span regions

Thus for short span $d = D - \text{clear cover} - \phi/2$

long span $d_1 = (D - \text{clear cover} - \phi/2) - \phi = d - \phi$.

The main reinforcement shall be provided in the middle strips of width equal to 3/4 of slab width.

7) Torsion steel :

a) At corners where slab is discontinuous over both the edges $A_t = 3/4\, A_{stx}$

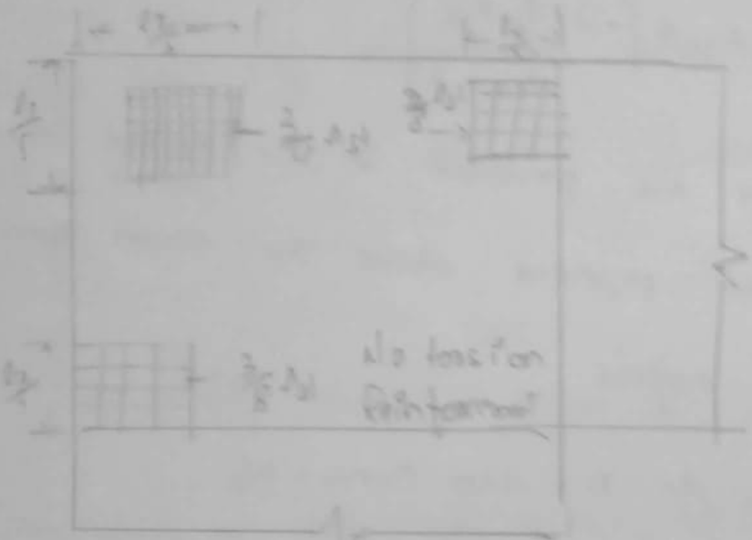
b) At corners where slab is discontinuous over one edge

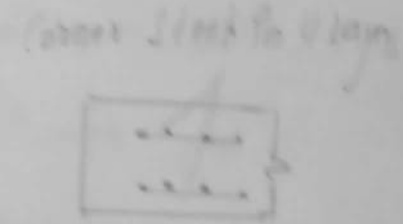
$$A_t = \frac{3}{8} A_{stx}$$

c) At corners where slab is continuous over both the edges $A_t = 0$ i.e., no torsion steel is required

Where $A_{st x}$ = area of steel for maximum mid span moment

This area of torsion reinforcement will be provided at corners in the form of mesh, one at top and the other at bottom for a length $\frac{l_x}{5}$ in each orthogonal direction, parallel to the sides of the slab as shown in figure





8) Check for Deflection:

Calculate the P_t % corresponding maximum mid span moment

Take the modification factor (MF) from fig 4.4 of IS 456-2000

(l/d) provided < $(l/d)_{max}$ = basic value × MF

9) Check for shear:

Maximum shear force at the edges of two way slab is given by $V_{ux} = w_u \left[\frac{r^4}{1+r^4} \right] \frac{l_x}{2}$ where $r = \frac{l_y}{l_x}$

$\tau_v < \tau_c$

10) Check for Development length:-

$$L_d \le \frac{M_1}{V} + l_0$$

The check for shear and check for development length are mostly satisfied in all case. Slabs subjected to uniformly distributed loads and therefore omitted in design calculations.

Prblm:-

1) Design a two way slab for a room 4000mm x 3500mm clear in size, if the super imposed load is 3 KN/m² and floor finish of 1 KN/m². The edges of the slab are simply supported and corners are not held down. Use M20 grade concrete and Fe 415 steel..

Soln:- $\frac{l_y}{l_x} = \frac{4}{3.5} = 1.14 < 2$

Hence, the slab is to be designed as a two way slab.

1) Data:-

short span, $l_x = 3.5$ m

long span $l_y = 4$m

live load = 3 KN/m²

Floor finish = 1 KN/m²

$f_{ck} = 20$ N/mm², $f_y = 415$ N/mm².

2) Thickness of Slab:

Assume effective depth $d = \frac{span}{28} = \frac{3500}{28} = 125 \text{ mm}$

Adopt effective depth $d = 125$ mm

Overall depth $D = 150$mm.

3) Effective span:

$l_x = 3.5 + 0.125 = 3.625$ m

$l_y = 4.0 + 0.125 = 4.125$ m

$\frac{l_y}{l_x} = \frac{4.125}{3.625} = 1.14.$

4) Loads: per unit area of slab

Self weight of the slab $= 0.15 \times 25 = 3.75$ KN/m²

live load $= 3$ KN/m²

Floor finish $= 1$ KN/m²

Total load $= 7.75$ KN/m²

Factored load $w_u = 1.5 \times 7.75 = 11.625$ KN/m².

5) Design Moments and shear force: The slab is simply supported on all the four sides. The corners are not held down. Hence moment co-efficients are obtained from

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$\alpha_x = 0.074 + (0.084 - 0.074) \times 4/10 = 0.078$

$\alpha_y = 0.061 - (0.061 - 0.059) \times 4/10 = 0.06$

$M_{ux} = \alpha_x w l_x^2 = 0.078 \times 11.625 \times 3.625^2 = 11.92$ KN-m

$M_{uy} = \alpha_y w l_x^2 = 0.06 \times 11.625 \times 3.625^2 = 9.17$ KN-m

$$V_u = \frac{W_u l}{2} = \frac{11.625 \times 3.625}{2} = 21.07 \text{ KN}$$

6) Minimum Depth Required :- The minimum depth required to resist Bending Moment

$$M_u = 0.138 \cdot f_{ck} \cdot bd^2$$

$$11.92 \times 10^6 = 0.138 \times 20 \times 1000 \times d^2$$

$$d = \sqrt{\frac{11.92 \times 10^6}{0.138 \times 20 \times 1000}} = 65.7 \text{ mm} < 125 \text{ mm, provided depth}$$

Hence provided depth is adequate.

7) Reinforcement :- along x-direction

$$M_{ux} = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} \cdot b \cdot d}\right]$$

$$11.92 \times 10^6 = 0.87 \times 415 \times A_{st} \times 125 \left[1 - \frac{415 \times A_{st}}{20 \times 1000 \times 125}\right]$$

$$264.1 = A_{st} \left[1 - \frac{A_{st}}{6024.1}\right]$$

$$A_{st}^2 - 6024.1 A_{st} + 6024.1 \times 264.1 = 0$$

$$A_{st} = \frac{6024.1 - \sqrt{6024.1^2 \times 6024.1 \times 264.1}}{2}$$

$A_{st} = 276.6 \text{ mm}^2$

Scanned by CamScanner

using 8mm diameter bars, Spacing of bars

$$S = \frac{a_{st}}{A_{st}} \times 1000 = \frac{\frac{\pi}{4} \times 8^2}{276.8} \times 1000 = 181.6 \text{ mm}$$

Maximum spacing is (i) $3d = 3 \times 125 = 375$ mm, (ii) 300 mm which ever is less. Hence, provide 8 mm bars at 180 mm c/c.

Along y- direction:-

These bars will be placed above the bars in x- direction

Hence $d = 125 - 8 = 117$ mm

$$M_{uy} = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} \cdot b \cdot d}\right]$$

$$9.17 \times 10^6 = 0.87 \times 415 \times A_{st} \times 117 \left[1 - \frac{415 \times A_{st}}{20 \times 1000 \times 117}\right]$$

$$217.1 = A_{st}\left[1 - \frac{A_{st}}{5638.6}\right]$$

$$A_{st}^2 - 5638.6 A_{st} + 5638.6 \times 217.1 = 0$$

$$A_{st} = 5638.6 - \frac{5638.6 - \sqrt{5638.6^2 - 4 \times 5638.6 \times 217.1}}{2} = 226.2\text{mm}$$

Using 8mm diametere bars, spacing of bars

$$S = \frac{a_{st}}{A_{st}} \times 1000 = \frac{\frac{\pi}{4} \times 8^2}{226.2} \times 1000 = 222.2 \text{mm}$$

Maximum Spacing is (i) $3d = 3 \times 117 = 351$ mm (ii) 300 mm which ever is less

Hence provide 8mm bars at 220 mm c/c