

INFORMATION THEORY

Source coding:-

Properties of Mutual Information:-

1) Mutual information is always positive (Non-negative) i.e. $I(X; Y) \geq 0$.

2) Mutual information is symmetric i.e. $I(X, Y) = I(Y, X)$

3) $I(X, Y) = H(Y) - H(Y/X)$

Source coding:-

→ Source coding is defined as process of conversion of the o/p of a discrete memoryless source (DMS) into sequence of binary symbols i.e. (Binary code word).

→ The device that performs conversion is called as source encoder.

coding Length:-

Discrete memory less source emits equal length binary symbols. (0's & 1's) one's every T_s Sec's.

The code length L_k is assigned to symbol X_k occurs with probability P_k . The code word length L is given as

$$L = \sum_{k=1}^K P_k L_k = \frac{\text{avg no. of bits}}{\text{Symbol}}$$

coding efficiency:-

coding efficiency of the source encoder is defined as the ratio of the minimum possible value of the code word length $L_{\min}$ to avg code word length L .

$$\eta = \frac{L_{\min}}{L}$$

where $L_{\min}$ is the minimum possible value of L .

→ The codeword efficiency can be written in terms of $H(X)$

$$\eta = \frac{H(X)}{L}$$

→ The source encoder is said to be eff when $\eta = 1$.

Shannon Fano Algorithm:-

The designed of a variable length code such that its avg codeword length approaches entropy of DMS is oftenly referred as entropy coding. In this we have two types of entropy coding.

① Shannon Fano coding

② Huffman coding.

Algorithm of Shannon Fano coding:-

→ The Shannon Fano can be obtained by the following procedure

→ Step 1: List the source symbols in the order of decreasing probabilities.

Step 2: Partition the symbol set into two sub-sets that are close (assign 2w(0)). For upper set contained in the sub-set & 1 to the each of the symbol in the other set. Thus the sum of the probabilities of the in both partition is equal.

Step 3:- continue this process till there is only one msg in the partition and further partition is not possible.

Ex. 1 A DMS has 4 symbols x_1, x_2, x_3 & x_4 with probabilities $P(x_1) = \frac{1}{2}$, $P(x_2) = \frac{1}{4}$, $P(x_3) = \frac{1}{8}$ & $P(x_4) = \frac{1}{8}$. Consider a Shannon Fano code and show that this code has 100% efficiency.

Sol:

Symbols x_1, x_2, x_3, x_4 .

Given

Symbols (or msg.)	PB $P(x)$	I	II	III	Code	length
x_1	$\frac{1}{2}$	0	0	0	0	1
x_2	$\frac{1}{4}$	1	0	0	10	2
x_3	$\frac{1}{8}$	1	1	0	110	3
x_4	$\frac{1}{8}$	1	1	1	111	3

efficiency $\eta = \frac{L_{min}}{H(X)}$ (or) $\frac{H(X)}{L}$

$$\frac{H(X)}{L} = \frac{\sum_{k=1}^K P_k \log_2(1/P_k)}{\sum_{k=1}^K P_k L_k}$$

$$= \frac{\sum_{k=1}^4 P_k \log_2(1/P_k)}{\sum_{k=1}^4 P_k L_k} = \frac{\frac{1}{2} \log_2(2) + \frac{1}{4} \log_2(4) + \frac{1}{8} \log_2(8) + \frac{1}{8} \log_2(8)}{\frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 + \frac{1}{8} \cdot 3}$$

$$\frac{H(X)}{L} = \frac{0.75}{1.75} = 0.4286$$

$$H(X) = 1.75 \text{ Bits for symbols} \rightarrow \text{B/s}$$

$$L = \sum_{k=1}^4 P_k L_k$$

$$L = \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{8}(3)$$

$$L = 1.75 \text{ BPS/symbol}$$

$$\eta = \frac{H(X)}{L} = \frac{1.75}{1.75} = 1$$

2) A DMS (Discrete Memoryless source) has 5 symbols x_1, x_2, x_3, x_4, x_5 with probability $P(x_1) = 0.40$, $P(x_2) = 0.10$, $P(x_3) = 0.16$, $P(x_4) = 0.15$ & $P(x_5) = 0.10$.

Sol:

Given that $P(x_1) = 0.40$

$$P(x_2) = 0.10$$

$$P(x_3) = 0.16$$

$$P(x_4) = 0.15$$

$$P(x_5) = 0.10$$

Symbols	Prob $P(x)$	I	II	III	code	code length
x_1	0.40	0	0	0	00	2
x_2	0.10	0	1	0	01	2
x_3	0.16	1	0	0	10	2
x_4	0.15	1	1	0	110	3
x_5	0.10	1	1	1	111	3

3) $P(X_1) = 0.4$, $P(X_2) = 0.2$ $P(X_3) = 0.2$ $P(X_4) = 0.1$
 $P(X_5) = 0.1$

Sol.

Symbols	$P(X)$	<u>I</u>	<u>II</u>	<u>III</u>	<u>code</u>	code length
X_1	<u>0.4</u>	0	—	—	0	1
X_2	<u>0.2</u>	1	0	—	10	2
X_3	<u>0.2</u>	1	1	0	— 110	3
X_4	<u>0.1</u>	1	1	0	0 1110	4
X_5	<u>0.1</u>	1	1	1	1 1111	4

4) $P(X_1) = 0.4$ $X_2 = 0.15$ $X_3 = 0.15$ $X_4 = 0.15$ $X_5 = 0.15$

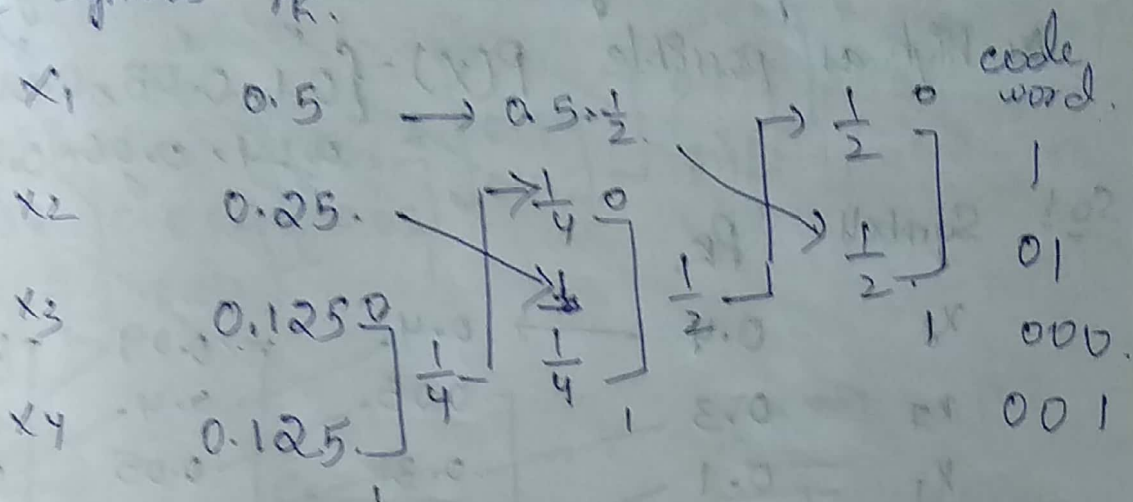
Sol.

Symbols	$P(X)$	<u>I</u>	<u>II</u>	<u>III</u>	code	code length
X_1	<u>0.4</u>	0	—	—	0	1
X_2	<u>0.15</u>	1	0	—	01	2
X_3	<u>0.15</u>	1	0	—	10	2
X_4	<u>0.15</u>	1	1	0	110	3
X_5	<u>0.15</u>	1	1	1	111	3

Huffman's

→ A discrete memoryless source has 4 symbols x_1, x_2, x_3, x_4 with probabilities $P(x_1) = 1/2$ $P(x_2) = 1/4$ $P(x_3) = P(x_4) = 1/8$.
consider Huffman code for x and calculate the efficiency.

Sol: Symbols. P_k



$$\eta_c = \frac{L_{\min}}{L} \approx \frac{H(x)}{L}$$

$$H(x) = \sum_{k=1}^K P_k \log_2 \left(\frac{1}{P_k} \right)$$

$$= \sum_{k=1}^4 P_k \log_2 \left(\frac{1}{P_k} \right)$$

$$= 0.5 \log_2(2) + \frac{1}{4} \log_2(4) + \frac{1}{8} \log_2(8) + \frac{1}{8} \log_2(8)$$

$$H(x) = 1.75 \text{ B/s}$$

$$L = \sum_{k=1}^4 P_k L_k$$

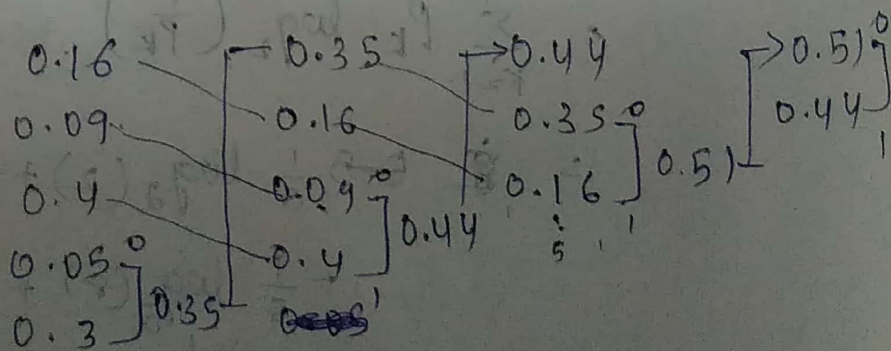
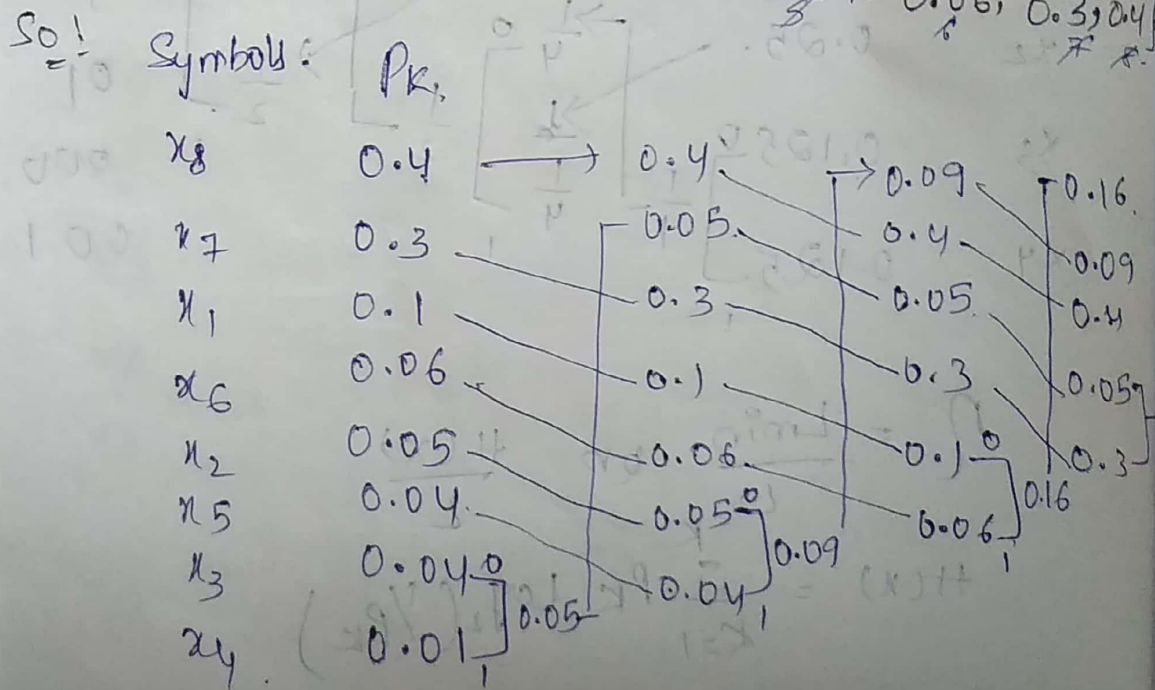
$$= 0.5 \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 + \frac{1}{8} \cdot 3 = 1.75 \text{ B/s}$$

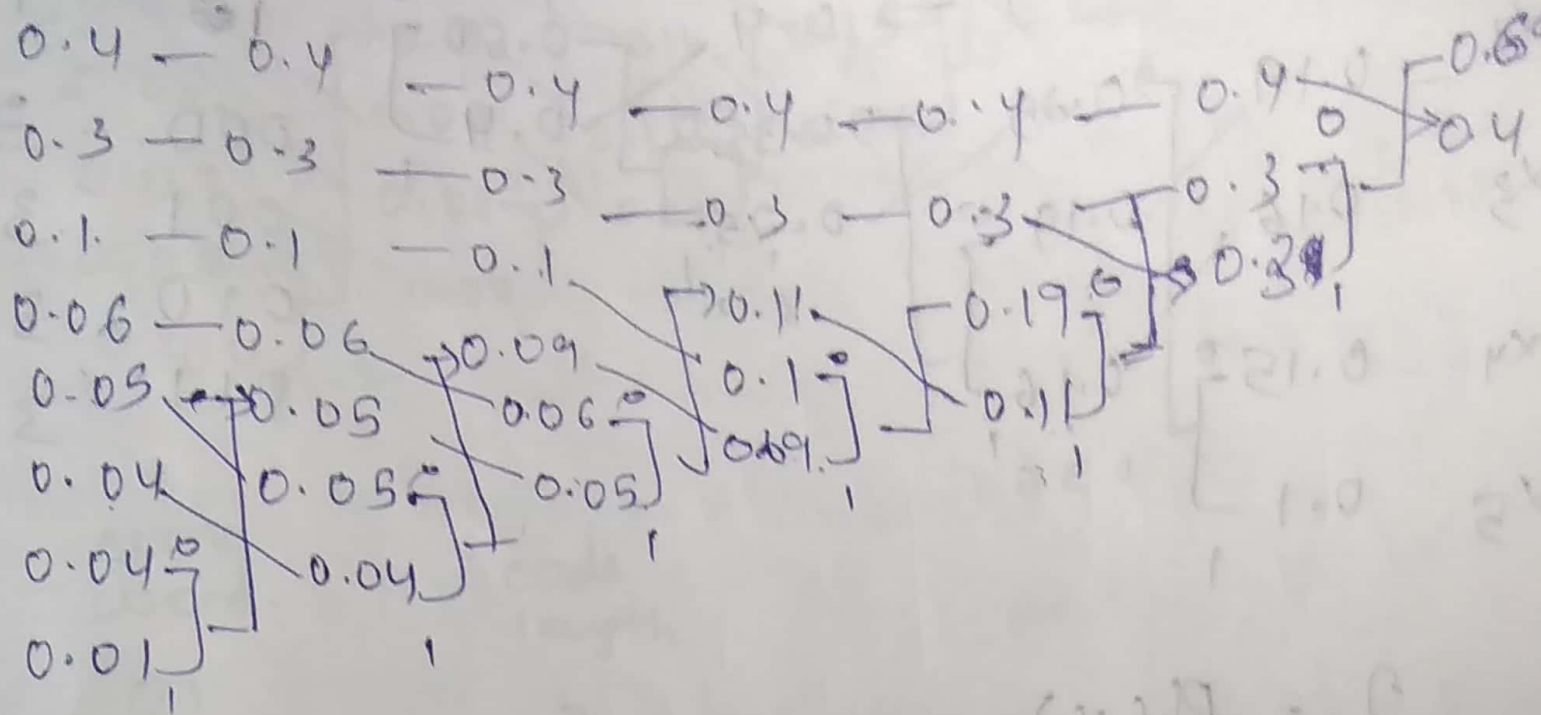
$$\eta = \frac{H(X)}{L}$$

$$= \frac{1.75}{1.75}$$

$$= 1.$$

1) Encode the following msg said with Huffman method by moving the combined symbols as high as possible. $P(X) = \{0.1, 0.05, 0.04, 0.01, 0.04, 0.06, 0.3, 0.4\}$





cod
word.

Co. dec

1
0001.

2

2/8 PNI.6

$$H(x) = \frac{H(x)}{2}$$

0000

4

0010.

4

$$H(x) = \sum_{k=1}^8 P_k \log_2 \left(\frac{1}{P_k} \right)$$

00010

5

$$= 0.4 \cdot \log_2 \left(\frac{1}{0.4} \right) +$$

00011

5

$$+ 0.3 \log_2 \left(\frac{1}{0.3} \right) + 0.1 \log_2 \left(\frac{1}{0.1} \right) + \dots$$

00110

5 PNI.6

00111

5 5.8

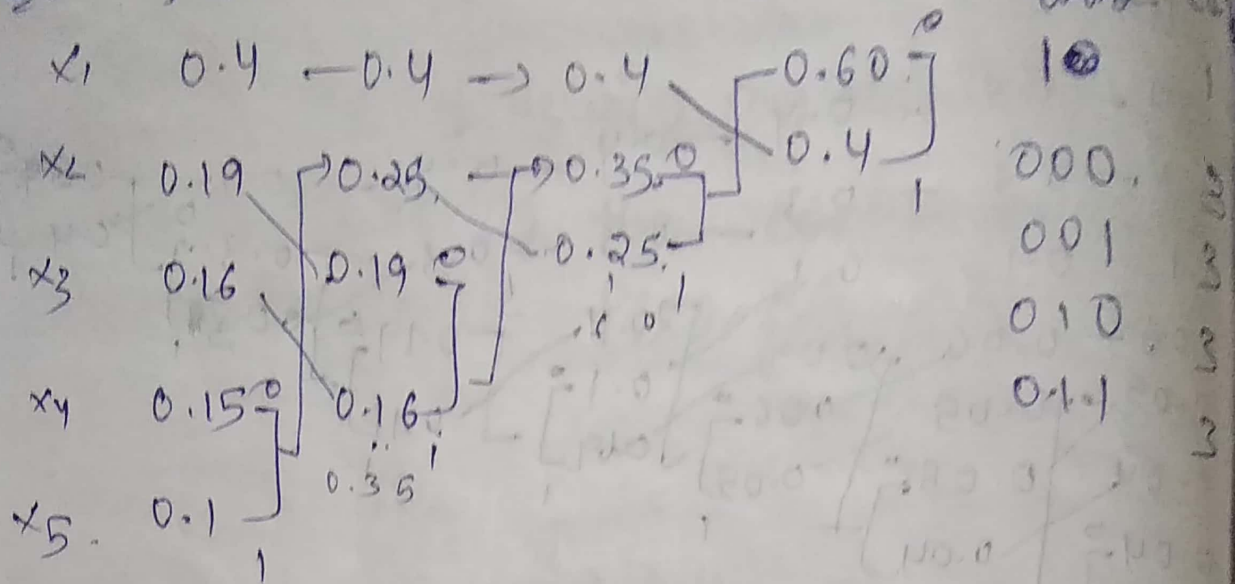
$$= 2.24 \text{ b/s.}$$

$$L = \sum_{k=1}^8 P_k L_k$$

$$= 2.34 \text{ b/s.}$$

Q) A DMS 5 symbols x_1, x_2, x_3, x_4, x_5 with
 PB $P(x_1) = 0.4, P(x_2) = 0.19, P(x_3) = 0.16,$
 $P(x_4) = 0.15, P(x_5) = 0.1$.

Sol: Symbols.



$$\eta = \frac{H(x)}{L}$$

$$H(x) = \sum_{k=1}^5 p_k (\log_2 (1/p_k))$$

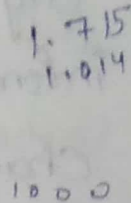
$$= 2.149 \text{ B/s}$$

$$L = \sum_{k=1}^5 p_k l_k$$

$$= 2.2$$

$$\eta = \frac{H(x)}{L} = \frac{2.149}{2.2} = 0.97$$

Redundancy = $1 - \eta$



1101

$$\approx 0.99$$

0.01

Bandwidth Signal to Noise Trade off:-

Trade off blw Bandwidth & Signal to Noise
ratio:-

→ The amount of information that can be transmitted over a channel is called capacity of a channel. The maximum information that can be transmitted over channel in a given time depends on the ch/s (Bandwidth & signal to noise ratio) of the channel.

Trade off:- The Bandwidth of channel increases, it is possible to increase the information range.

→ As the signal to noise ratio increases information rate also ↑, with preventing errors due to noise.

→ For no noise the S/N ratio is '∞' & an ∞ information rate is possible irrespective of B.W.

→ Thus we can trade off B.W for S/N

Hartley shannon laws:-

* The information rate 'R' & the channel capacity 'C' are inter related by shannon's theory on channel capacity.

* (The channel capacity of)

According to hartley law the info capacity is $\propto$ B.W of the channel

$$\therefore C \propto B.W$$

→ The greater the B.W of channel, the greater will be the amount of information that can be transmitted in a given time

→ But there is a limit to the amount of data that can be sent in a given bandwidth. This limit is given by Hartly Shannon law or theorem.

$$\text{i.e. } C = B \log_2 \left(1 + \frac{S}{N} R \right)$$

where C is channel capacity (bit/sec)

B is B.W (Hz.)

R is info rate. (Bits/sec.)

S/N is signal to noise ratio.

→ If the signal to noise ratio is 7 & B.W is 4 KHz. Then the channel capacity

$$C = 4 \log_2(1+7)$$

$$C = 4 \times 10^3 \log_2(1+7)$$

$$C = 12 \times 10^3 \text{ B/sec}$$

→ If S/N ratio rises to 15 & B.W ↓ to 3 KHz. Then C is

$$C = 3 \times 10^3 \log_2(1+15)$$

$$= 12 \times 10^3 \text{ B/sec}$$

→ However as B.W becomes one (1) the channel capacity does not become infinity

Pr

→ If the S/N ratio is 30 and bandwidth is 5 KHz. Then what is the channel capacity.

Sol.

$$C = B \log_2(1 + \frac{S}{N})$$

$$C = 5 \times 10^3 \log_2(1+30)$$

$$C = 24770.9 \text{ B/sec}$$

Lossy source coding:-

→ In Huffman coding & Shannon Fano coding technique the source information is recovered exactly without any loss. They are lossless source coding.

→ In lossy source coding some of the info is lost during the coding and that is never recovered. This techniques are 'MPEG' & 'JPEG' used for videos and images.

MPEG coding:-

MPEG stands for motion pictures expert group. It was formed by ISO. MPEG has developed that the standards for compression of video with audio. MPEG audio codes are used for compression of audio. This is used for "PERCEPTUAL".

JPEG coding for images:-

The JPEG stands for Joint photography experts Group. This Group is working on international compression standard for colour and monochrome continuous tone to the

image (or) photograph etc. This group came up with a compression standard which is widely known as JPEG standard. This is also known as ISO standard 10918.

→ There are two versions of JPEG algorithm: Lossy version and Lossless version.

→ In Lossy version the DCT-compression won't produce the original image perfectly. It uses discrete cosine transform and quantization.

→ The Lossless version is based on arithmetic coding.

cyclic codes are sub-class of linear block codes.

→ A linear code is called cyclic code if every cyclic shift of the code vector produces some other code vector.

→ These definition includes two fundamental properties of cyclic code

Properties of cyclic code:-

They are two fundamental properties.

1) Linearity Property:- A code is said to be linear, if some of any two code words is also a code word.

Ex: Let x_1 & x_2 are two code words then $x_3 = x_1 \oplus x_2$ be a code word. i.e

x_3 is also a code word. These property states that the cyclic codes are linear block codes.

Cyclic Property:-

A code word is said to be cyclic, if any cyclic shift of a code word results another code word.

Ex: [1 bit right shift].

1011 → 1110 → 0111 → 1011

→ It can be seen that the code word 1011, 1101, 1110, 0111 is obtained by cyclic shift of $n=4$. The code obtained by rearranging the four words is also a cyclic code.

Algebraic structure of cyclic codes:-

The code word can be represented by a polynomial. To ~~define~~ ^{derive} the cyclic code for example consider the n -bit code word $x = \{x_{n-1}, x_{n-2}, \dots, x_1, x_0\}$.

These code word can be represented by a polynomial of a degree less than or equal to $(n-1)$.

$$X(P) = \{x_{n-1} P^{n-1}, x_{n-2} P^{n-2}, \dots, x_1 P + x_0\}$$

Here $X(P)$ is the polynomial of degree $(n-1)$. P is the ordinary variable of the polynomial.

The power of P represents the position of the code word bits i.e. P^{n-1} represents MSB. P^0 is represents LSB. $\{P^1\}$ represents 2nd bit from LSB side.

Generation of code vector in non-symmetric form:-

→ If $G(x)$ is a polynomial of the degree $(n-k)$ and its factor of $x^{n+1} + 1$. Then $g(x)$ has a degree Q for an (n, k) cyclic code where $Q = (n-k)$ represents the no. of parity bits.

$$G(x) = p^0 + p^{Q-1} G(Q-1)$$

$$G(x) = p^0 + p^{Q-1} G(Q-1) + \dots + G_1 p + 1$$

where $G(Q-1), G(Q-2)$ are parity bits.

→ Let $D = \{D_{k-1} + D_{k-2} + \dots + D_1, D_0\}$

Be k bits code of msg vector then it can be represented by the polynomial, as

$$D(P) = D_{k-1} P^{k-1} + D_{k-2} P^{k-2} + \dots + D_0 P^0$$

→ The cyclic code in which the code polynomial $V(t)$ for a data vector 'D' is generated by $V(P) = D(P) \cdot G(P)$.

→ Summary of Generating code vector in non-symmetric form.

Step-1:- consider the msg vector 'D'.

Step 2: Convert the msg vector to msg polynomial i.e D to $D(p)$

Step 3:- Calculate the code polynomial using
 $V(p) = D(p) \cdot G(p)$

Step 4: Convert code polynomial i.e $V(p)$ to

p.b) The generator poly

NOTE: The generator polynomial $G(p)$ remains the same for all code vectors.

Q. The generator polynomial of a 7,4 cyclic code is (7,4) given as $g(x) = 1+x+x^3$. Find the code word vector for the data 0101.

Sol: Step 1:
 $D = d_0, d_1, d_2, d_3$
0 1 0 1

Step 2: $D(p) = 0p^0 + 1(p) + 0(p^2) + 1(p^3)$

Step 3: $D(p) = p + p^3$

$V(p) = D(p) \cdot G(p)$

$G(p) = 1 + p + p^3$

$$= (p + p^3)(1 + p + p^3)$$

$$= p + p^2 + p^4 + p^3 + p^4 + p^6$$

$$= p + p^2 + p^3 + p^4 + p^6$$

$$= p + p^2 + p^3 + p^6$$

Step 4: $V(P) = 0 \times P^0 + 1 \times P^1 + 1 \times P^2 + 1 \times P^3 + 0 \times P^4 + 0 \times P^5 + 1 \times P^6$
 $V = 0111001$

Messages.	$V(R) = D(P) \cdot G(P)$
0000	0000000
0001	0001101
0010	0011010
0011	0010111
0100	0110100
0101	0111001
0110	0101110
0111	0100011
1000	1101000
1001	1100101
1010	1110010
1011	1111111
1100	1011100
1101	1010001
1110	1000110
1111	1001011

$$\begin{aligned} 8 &= 1000 \rightarrow 1101000 \text{ (Right shift)} \\ &\quad 0110100 \end{aligned}$$

cyclic Encoder:-

To design n/k cyclic encoder, first we ~~find~~ require to design Q no. of registers from $r_0, r_1, \dots, r_{Q-1}$.

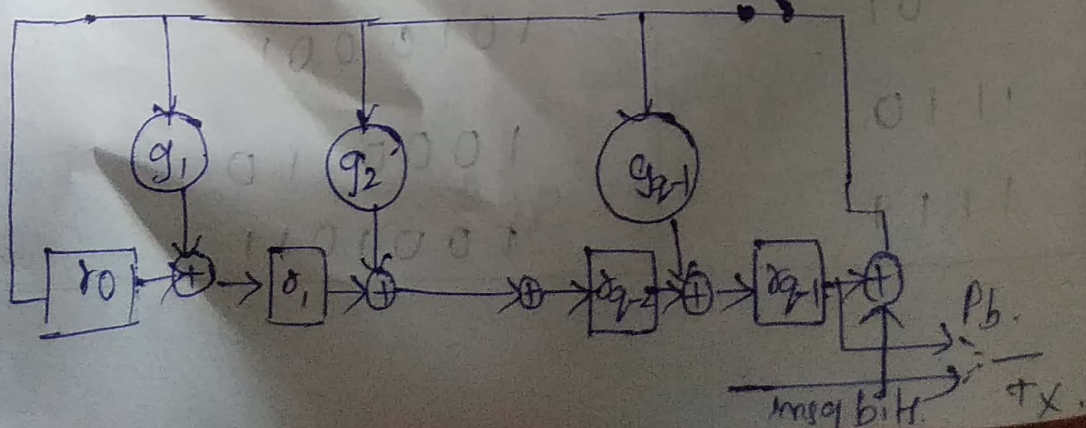
→ we need modulo-2 adder in btw 2 registers and these register will be acting as shift registers.

• then $Q-1$ no. of gates are required from g_1 to g_{Q-1} and the gate value is directly derived from generator polynomial Eqn.

$$G(P) = p^Q \oplus g_{Q-1} p^{Q-1} \oplus g_{Q-2} p^{Q-2} \oplus \dots \oplus g_1 p^1 \oplus 1$$

→ If $g=1$, then the closed path is formed else that is $g=0$. Open path is formed.

Now connect the registers with adders and as well as gates by including a switch. So that the encoder will form



Initially the switch is closed, the msg bits is connected to register as well as to the parity gate. The msg bits gets shifted to the transmitter as well as to the shift register after k msg bits get shifted, the register contains q no. of check bits. and the switch will be opened at the output initially the transmitter is connected to the msg bits then it is connected to the parity bits when switch is open.

∴ The generating code words contains msg bits followed by parity (or) check bits.

Ex:- Design an encoder $(7,4)$ cyclic code generated by a polynomial $G(P) = P^3 \oplus P \oplus 1$ and verify its operation using msg vector 1100.

Sol:- General eqn for generator polynomial of (n, k) cyclic code is

$$G(P) = P^q \oplus g_{q-1} P^{q-1} \oplus g_{q-2} P^{q-2} \oplus \dots \oplus g_1 P \oplus 1.$$

Given $(7,4)$ cyclic code in which $n = 7$,

$$k = 4$$

$$q = n - k \Rightarrow 7 - 4 = 3.$$

$$g = q - 1 \Rightarrow 3 - 1 = 2.$$

msg bits 1100 are applied one by one. when we apply first msg bit one to the encoder shifting process will be takes place after the new value of Registers

M	r_0	r_1	r_2				
1	0	0	0	$r_0' = r_2 \oplus m$	$r_1' = r_2 \oplus m \oplus r_0$	$r_2' = r_1$	
1	1	1	0	1	1	0	
0	1	0	1	1	0	1	
1	1	0	0	0	0	0	
	0	1	0	0	1	0	

0 1 0 →
Parity bits.

→ For the msg bits 1100 the check bits generated are 010. so the transmitted codeword is 1100 010.

→ 1010, $g(p) = p^3 \oplus p \oplus 1$.

$Q(p) = p^3 \times 1 \oplus 0 \times p^2 \oplus 1 \times p \oplus 1 \times p^0$

$Q(p) = p^3 \oplus p^2 \oplus p \oplus 1$

$g_1 = 1, g_2 = 0$

$$M = m_0 \quad m_1 \quad m_2 \quad r_0 = r_2 \oplus m_1 = 0 \oplus 0 = 0 \quad r_2 = 0$$

$$1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0$$

$$0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1$$

$$1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1$$

$$0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 0$$

$$\hline 1 \quad 1 \quad 0$$

Generation of code vector in systematic form

→ The code can be kept into a systematic form by using the following steps.

Systematic ^{code} involves 3 steps.

Step 1:- Multiply the msg polynomial $D(x)$ by x^{n-k} . This multiplication is equivalent to shifting the msg bits by $(n-k)$ bits.

Step 2:- Divide the shifted msg polynomial $D(x)x^{n-k}$ by the generator polynomial $G(x)$ to get remainder $R(x)$.

Step 3:- Finally add the remainder polynomial to the shifted msg polynomial to obtain code word polynomial $P(x)$.

$$\therefore V(x) = R(x) \oplus D(x)x^{n-k}$$

Find the codeword for the data 1110 for the given generator polynomial of a CRC cyclic code given as $G(x) = x^3 + x + 1$

$$G(x) = x^3 + x + 1 \quad (\text{Systematic})$$

$$= 0x^5 + 0x^4 + 1x^3 + 1x^2 + 1x^1 + 1x^0$$

Given msg vector $D = 1110$

$\therefore$ The msg polynomial $D(x) = 1 + x + x^2 + x^3$

$D(x) = \text{by } x^{n-k} = x^{7-4} = x^3$

$$n=7, k=4$$

$$D(x) \times x^{n-k} = (1 + x + x^2) \times x^3$$

$$= x^3 + x^4 + x^5$$

$$\begin{array}{r} x^3 + x + 1 \overline{) x^5 + x^4 + x^3} \\ \underline{x^5 + x^3 + x^2} \\ x^4 - x^2 \\ \underline{x^4 + x^2 + x} \\ x \end{array}$$

$$\therefore R(x) = x$$

$$V(x) = R(x) \oplus D(x) \times x^{n-k}$$

$$= x \oplus x^3 + x^4 + x^5$$

$$= x + x^3 + x^4 + x^5$$

$$= 0101110 \Rightarrow 0111010$$

NOTE: The Subtraction same as addition in modulo two arithmetic.

$$\Rightarrow 0101 \quad 1 \quad G(x) = x^3 + x + 1$$

$$D(x) = 0x^0 + 1x^1 + 0x^2 + 1x^3 \\ = x^1 + x^3 \Rightarrow x + x^3$$

$$D(x) x^{n-k} \Rightarrow (x + x^3) x^{7-4} \\ = (x + x^3) (x^3) \Rightarrow x^4 + x^6$$

$$\begin{array}{r} x^3 \\ x^3 + x + 1 \overline{) x^6 + x^4} \\ \underline{x^6 + x^4 + x^3} \end{array}$$

$$R(x) = x^3$$

$$v(x) = R(x) \oplus D(x) x^{n-k}$$

$$= x^3 \oplus (x^4 + x^6)$$

$$= x^3 + x^4 + x^6$$

$$= 0001101 \Rightarrow 1011000 \quad \begin{matrix} \text{(Right side)} \\ \text{(Left side)} \end{matrix}$$

→ verify the operation of the encoder for msg vector 1001 for a generator polynomial $1+x+x^3$ for a (7,4) cyclic code.

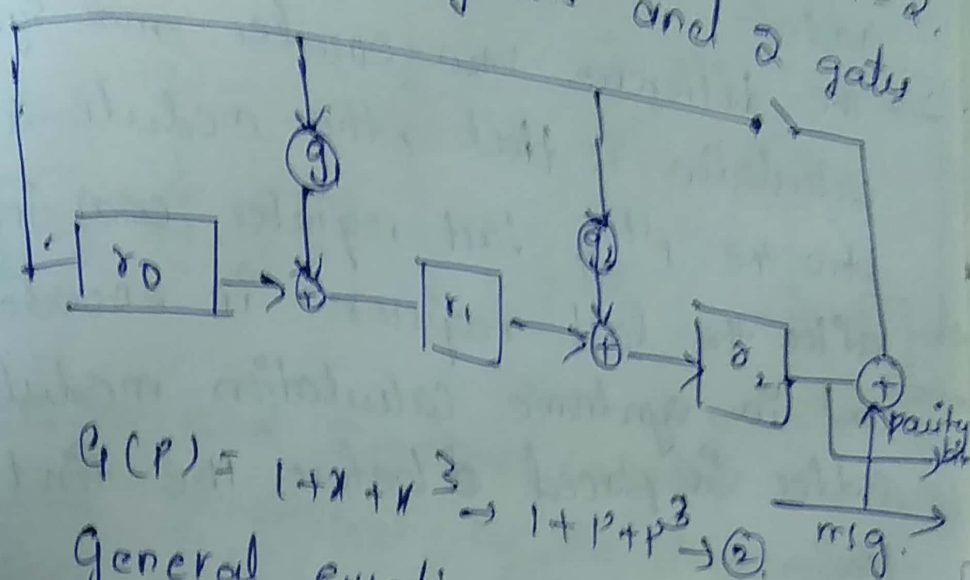
1001011

General eqn for cyclic code generator polynomial of (n,k)

$$G(P) = p^q + g_{q-1} p^{q-1} + g_{q-2} p^{q-2} + \dots + g_1 p + g_0$$

Given (7,4) cyclic code in which $n=7, k=4$
 $q = n - k = 7 - 4 = 3$

we require 3 Registers and 2 gates



$$G(P) = 1 + x + x^3 \rightarrow 1 + p + p^3 \rightarrow (2)$$

General equation for generator polynomial for $q=3$ is

$$G(P) = 1 + x + x^3 = p^3 + g_2 p^2 + g_1 p + 1 \rightarrow (3)$$

comparing eq (2) & eq (3), we get the value of g_1 & g_2 .

$$g_1 = 1 \quad g_2 = 0$$

Since $g_2 = 0$ the gate g_2 will be opened

Syndrome calculation for (n, k) cyclic code

→ In order to design syndrome calculation
Q no. of registers are required and the
registers are numbered as $s_0, s_1, \dots, s_{q-1}$.

These registers are acting as shift-registers.

In b/w registers modulo-2 adder is added.

→ The difference b/w encoder and syndrome calculation is that, the modulo-2 adder b/w the 1st & last register was included after the last register in encoder.

and in syndrome calculation modulo-2 adder is placed before the first register.

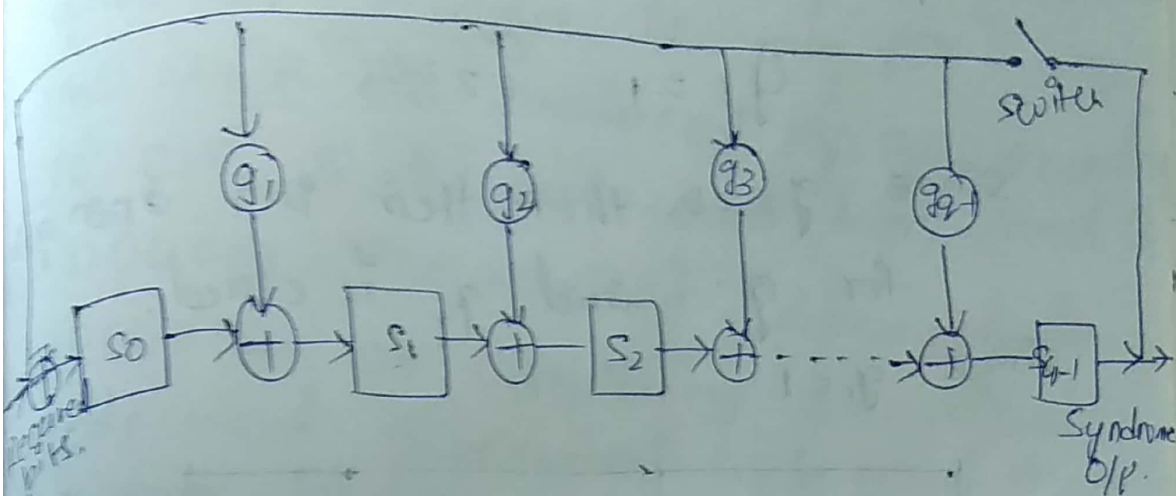
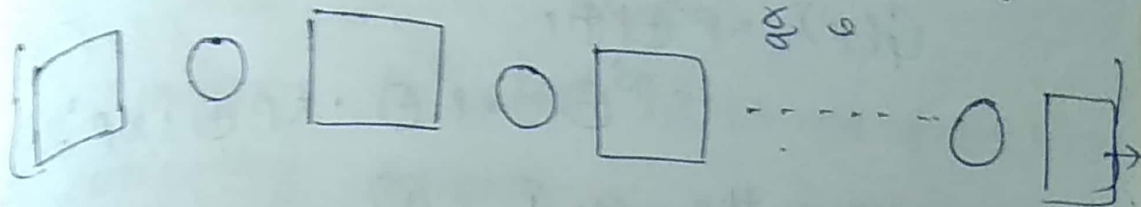
→ $(q-1)$ no. of gates are required and these gates are numbered $G_1, G_2, \dots, G_{q-1}$. These values are derived from generator polynomial

$$G(P) = P^q \oplus g_{q-1} P^{q-1} \oplus g_{q-2} P^{q-2} \oplus \dots \oplus g_1 P \oplus g_0$$

→ If $g = 1$ then the switch on gate is closed.
if $g = 0$ then switch on gate = opened.

→ Initially switch is closed and the gate is connected to received bits. So the receiver

bits gets shifted to the shift register. after all n -bits gets shifted the register contains the syndrome. when the switch is open the output is connected to the syndrome.



Design and syndrome calculator for $(7,4)$ cyclic generated by the polynomial $G(P) = P^3 \oplus P \oplus 1$ & calculate the syndrome for received bits $Y = 1000010$.

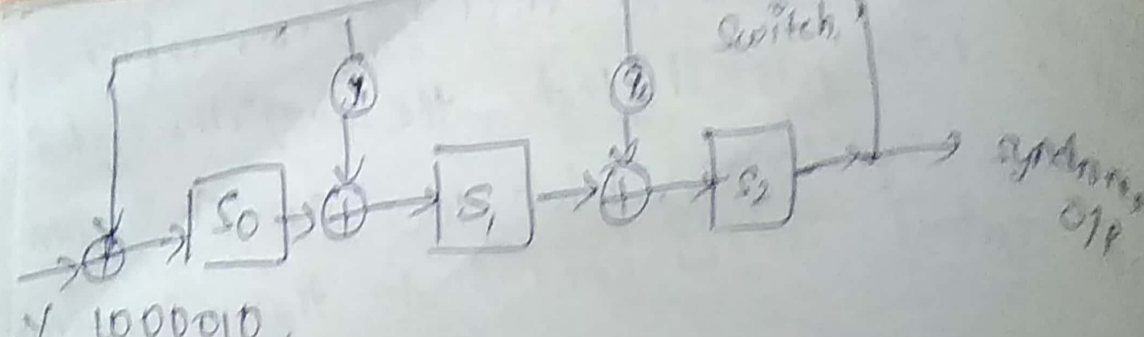
Given
 $G(P) = P^3 \oplus P \oplus 1$

$$G(P) = P^q \oplus g_{q-1}P^{q-1} \oplus g_{q-2}P^{q-2} \dots \oplus g_1P \oplus 1$$

$$\therefore q = n - k = 7 - 4 = 3$$

Since $q = 3$, we required 3 registers. i.e s_0, s_1, s_2 .

we required 2 gates. i.e $g_{q-1} = g_{3-1} = 3-1 = 2$.



$$G(x) = x^3 + x + 1$$

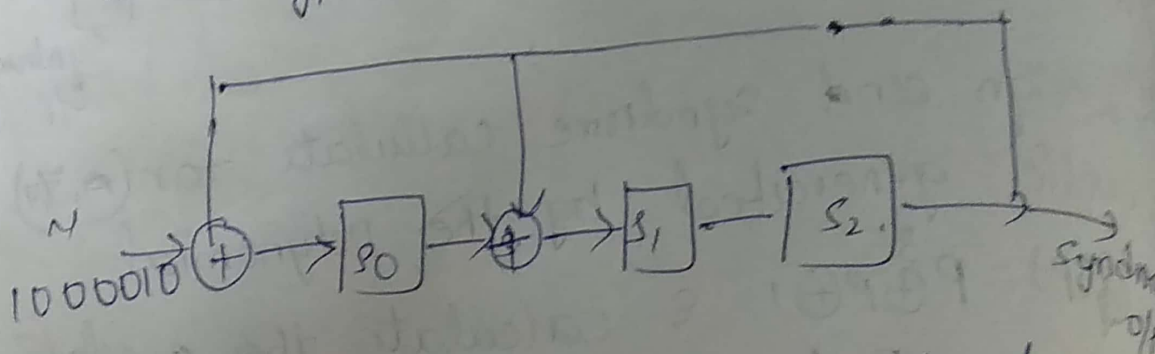
$$= 1 \times x^3 + 0 \times x^2 + 1 \times x + 1 \times x^0 = 0$$

compare the q_1 and q_2 .

$$q_1 = 1 \quad q_2 = 0$$

Since $q_2 = 0$ then their is a open for q_2 and q_1 is closed.

Since $q_1 = 1$



Received bits.

y

register value before shifted.

$s_0 \quad s_1 \quad s_2$

register value after shifted.

$$s_0' = y \oplus s_2 \quad s_1' = s_0 \oplus s_2$$

1	0	0	0	1	0
0	1	0	0	0	1
0	0	1	0	0	0
0	0	0	1	1	1
0	1	1	0	0	1
1	0	1	1	0	1
0	0	1	1	1	1

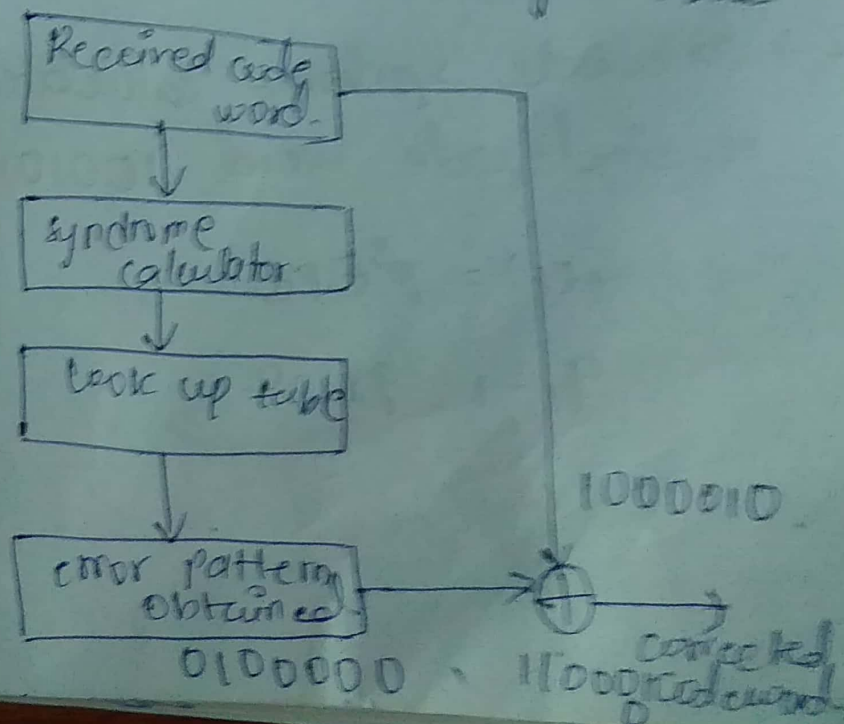
→ Syndrome value

The syndrome calculated value is 111.
If syndrome value is available, then there is a error in the received code.

Syndrome Decoder:-

In syndrome decoder we are detecting & correcting error. The msg bits value given for encoder is msg bits = 1101.
The check bits produced by the encoder is 010 & the code word transmitted is 1100010. The received code word is 1000010. On comparing transmitted code word & received code word there is a error in second bit.

The procedure to detect error & correct the error is procedure Right now.



lookup table is nothing but a error pattern followed by syndrome based on syndrome values the error pattern will be obtained from the lookup table. the error pattern and received codeword is given to exclusively 'OR' gate which produces corrected word.

Error pattern	Syndrome
00000000	000
10000000	101
01000000	111
00100000	011
00010000	110
00001000	001
00000100	010
00000010	100

→ calculate syndrome calculation for received code word 1001011 $\gamma = 1011011$

$$\therefore Q(P) = P^3 \oplus P \oplus 1$$

$$g_1 = 1 \quad g_2 = 0$$

Received
bits.

s_0

s_1

s_2

$$s_0' = y \oplus s_2$$

$$s_1' = s_0 \oplus s_2$$

$$s_2' = s_1$$

y

1

0

1

1

0

1

1

0

1

0

1

0

1

1

0

0

0

1

0

0

1

0

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0

0

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1

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0

0

0

0001000

$y = 1011011$

1010011

Convolutional Codes:-

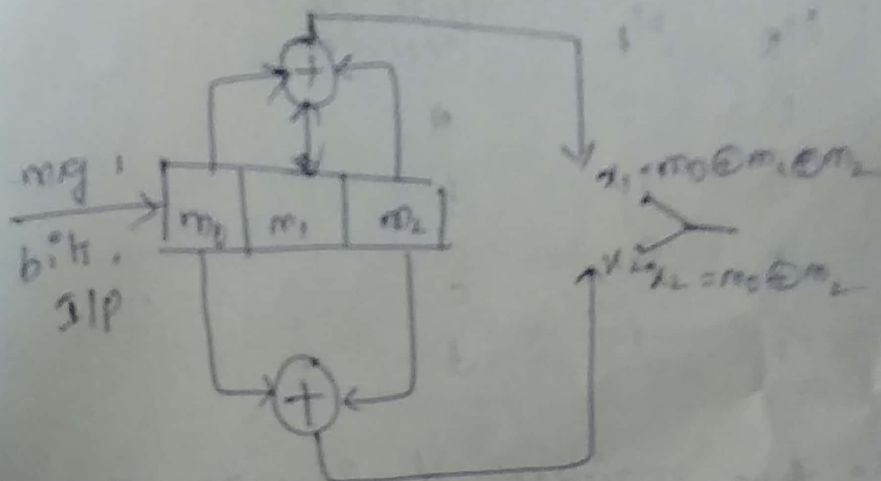
A convolution coding is done by combining the fixed no. of i/p bits. The i/p bits are stored in the fixed length shift register and they are combined with the help of modulo-2 adders.

- convolution encoding process 'K' bits in a serial manner.
- convolution encoding consists of shift registers (made up of FF) & Modulo-2 adder.
- Constraint length 'K':- It is defined as the no. of shifts over which a single msg bit can influence the encoder o/p.
- It is expressed in terms of msg bits.
- The convolution encoder or code takes one input msg bit at a time and generates 2 or more encoder bits.
- convolution codes are used to achieve reliable data transfer in numerical applications such as digital video transmission, radio transmission, mobile communications and satellite communications.

The generator polynomial for x_1 is $G_1(x) = x^2 + x + 1$
 The generator polynomial for x_2 is $G_2(x) = x + 1$
 Initially all registers are 0,0,0 when
 the i/p bit is applied the bits in the
 registers will shifted to next position.
 The values of x_1 & x_2 are generated depend-
 ing upon m_0, m_1, m_2 . M_1 & M_2 stores
 the previous 2 msg bits and current
 bit is present in m_0 .

$$\therefore x_1 = m_0 \oplus m_1 \oplus m_2$$

$$x_2 = m_0 \oplus m_1$$



$$x_1 \text{ is } 1, 1, 1$$

$$x_2 \text{ is } 1, 0, 1$$

→ For every single input bit two output
 encoded bits are generated.

∴ the code rate of this encoder is

$$R = k/n \quad \therefore k=1, n=2$$

$$R = 1/2 \quad k = \text{i/p} \quad n = \text{o/p}$$

i) code tree:-

Each branch of the tree represents an input with its corresponding value of its binary system indicated on the branch.

Input '0' specifies the upper branch & '1' the lower branch.

→ In the encoder, the 1st state the register, the values remaining two registers (containing) decides the state of encoder.

ii) State of the Encoder:-

M_0	M_1	State of encoder
0	0	a
0	1	b
1	0	c
1	1	d

→ The two registers decides the state of the encoder to produce 4 different states. Initially, the state of encoder is 'a'. The encoder can either accept a binary one (or) '0' as '1' (1)

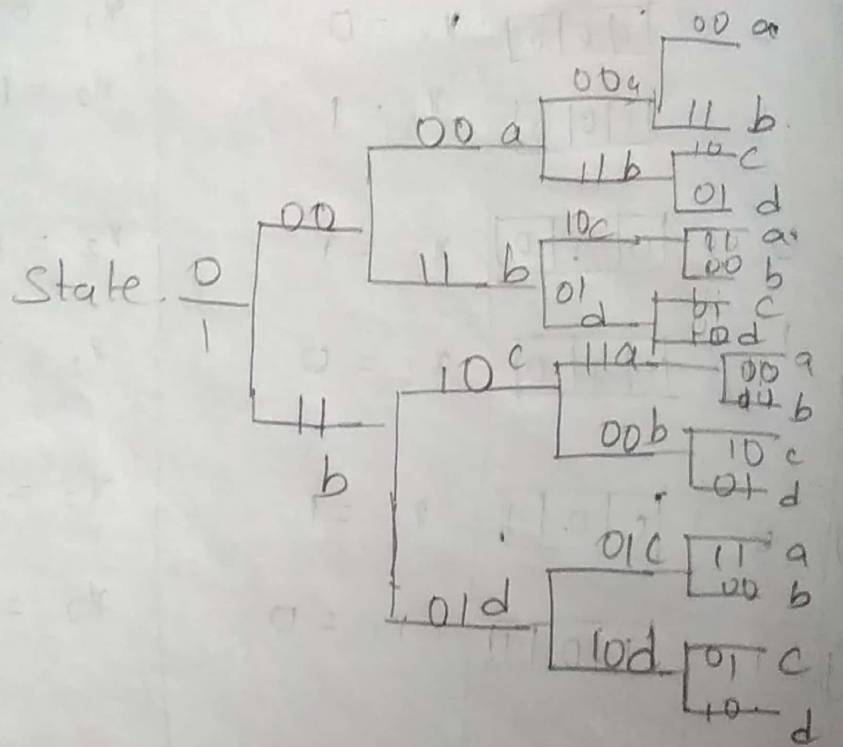
Current state	I/P	Register content	O/P $x_1 = m_1 \oplus m_2$ $x_2 = m_1 \oplus m_2$
000	0	000	$x_1 = 0$
	1	100	$x_1 = 1$
010	0	010	$x_1 = 1$
	1	110	$x_1 = 0$
100	0	001	$x_1 = 1$
	1	1101	$x_1 = 0$
101	0	011	$x_1 = 0$
	1	1111	$x_1 = 1$

Next state

000	a
010	b
101	c
111	d
000	a
010	b
101	c
111	d

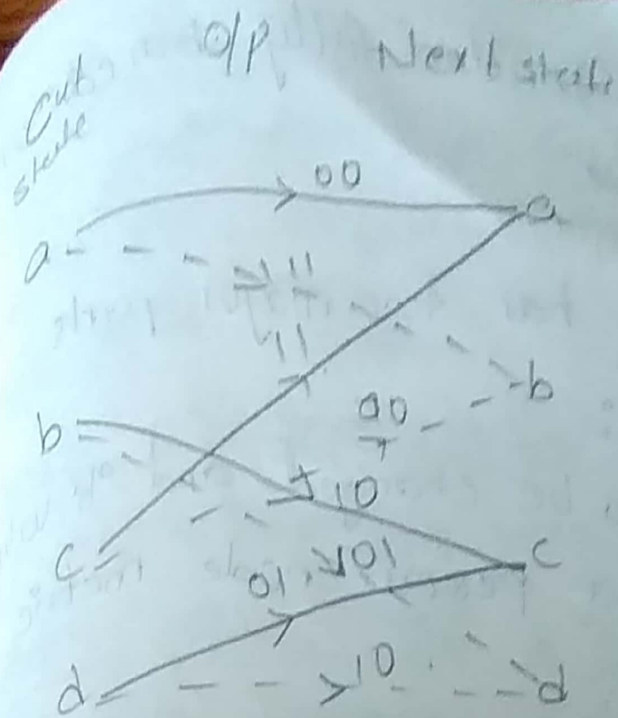
Current state	I/P	O/P x_1, x_2	Next state
a	0	00	a
	1	11	b
b	0	10	c
	1	01	d
c	0	11	a
	1	00	b
d	0	01	c
	1	10	d

tree diagram:-

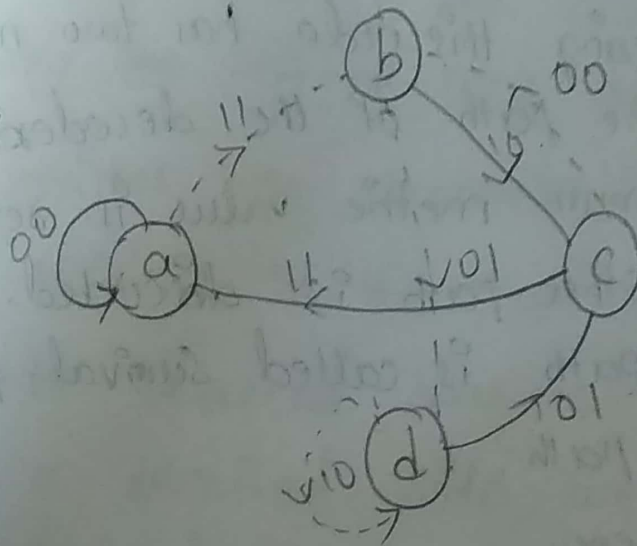


Code Trellis:-

Current state	I/p	O/p	Next state
a	0	00	a
a	1	11	b
b	0	10	c
b	1	01	d
c	0	11	a
c	1	00	b
d	0	01	c
d	1	10	d



state diagram:-



Decoding Method of convolution code:-

There are several different approaches to decoding convolution code.

Viterbi:- or Max like wood decoding:-

It is used for the decoding of convolution code (corrects the error). It is also known as Max like wood decoding.

→ The viterbi algorithm fully based on trellis structure.

→ The decoding has two major parts

i) Metric value :-

no. of bits to be changed and its value added to in a previous node metric value.

ii) Survival path (or active path) :- In each node after the 2nd stage two paths to be entering this node has two metric values. The path of the decoder signal with minimum metric value is retained and the other path is discarded. These retained path is called survival path (or active path).

Algorithm steps:-

→ First & second stage is like a trellis.

→ Find the metric value for each nodes (metric value will be find with the help of received bits will be compared with the coded bits).

→ In initial node metric value becomes 0.

step 2:-
each path having a single metric value &
after the second stage each node have
two metric value.

step 3:- Find the survivor path with the help
of minimum metric value.

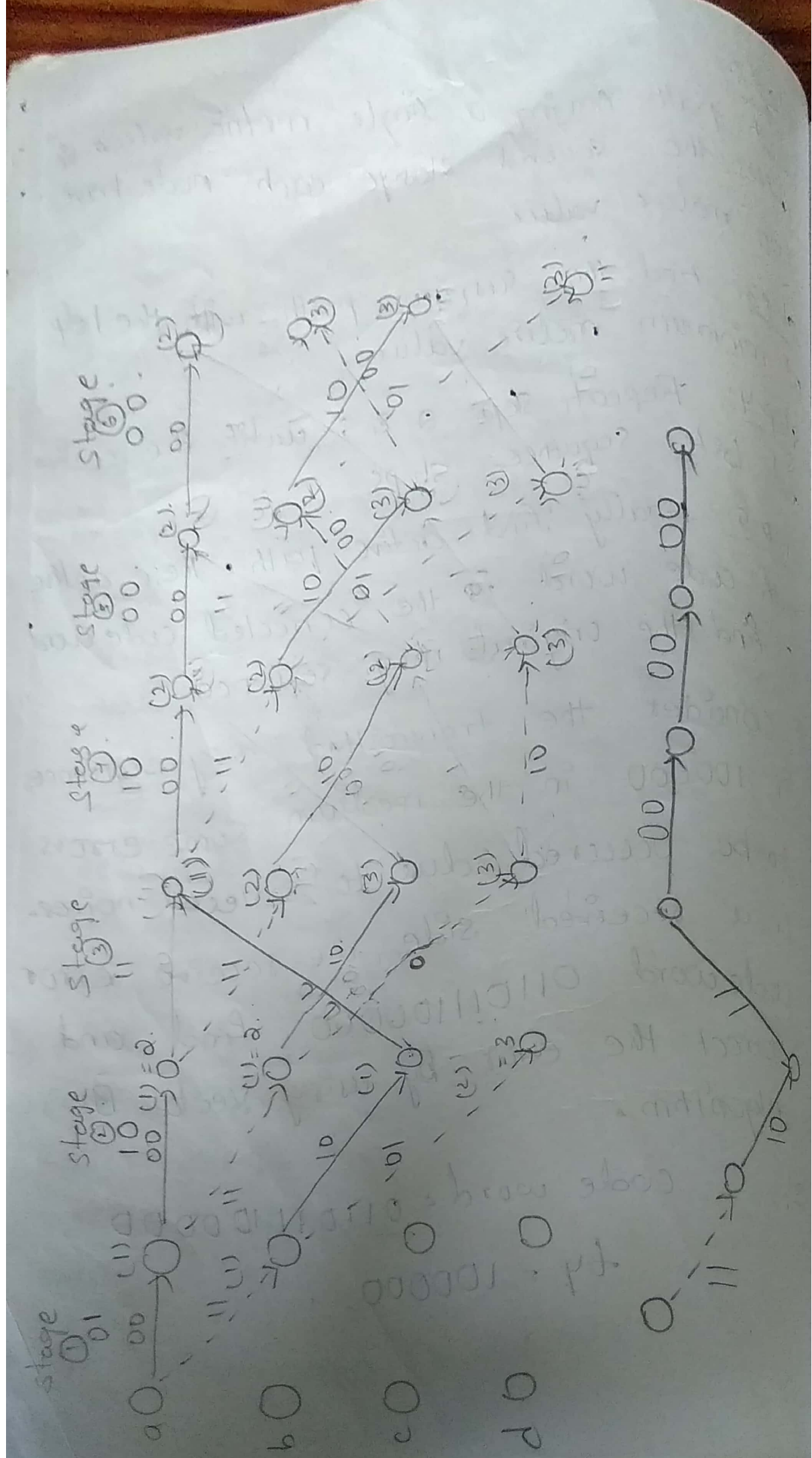
step 4:- Repeat step 2 & 3 until the received
bits sequence stops.

step 5:- Finally find active path. These active
path code word is the corrected code word
& find the original msg sequence.

→ Consider the transmitted msg sequence
is 100000 in the medium some errors
to be occurred due to several noises.
In a received side, we receive error
code word 011011100000 find and
correct the error by using vector B
Algorithm.

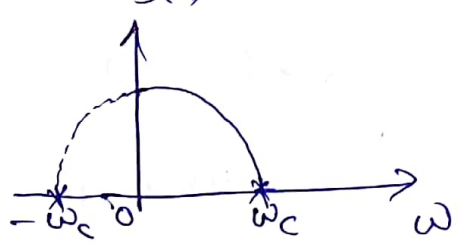
Sol:- code word = 011011100000.

$t_y = 100000.$

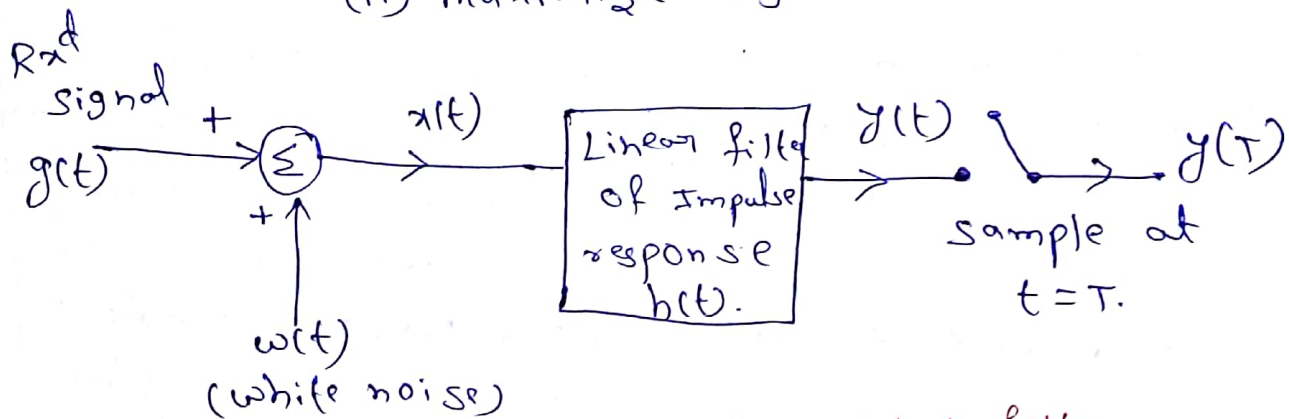


unit - III

Base band pulse transmission & Digital pass band transmission

Base band transmission	passband transmission
1) Base band signals are transmitted without modulation. 2) No carrier signal required 3) Ex Audio, video, txt etc. 4) frequency of base band signals lies between 0 to some 100's of MHz. 0 to 4 kHz $\rightarrow$ audio signal 0 to 55 MHz $\rightarrow$ video signal 5) frequency domain representation. $S(f)$  lies at '0'. 6) Unmodulated transmission.	1) pass band signals are transmitted with modulation. 2) High frequency carrier signal used for transⁿ. 3) Ex ASK, FSK signals etc 4) frequency of passband signals lies > 100 kHz to > 400 MHz's. 550 - 1650 kHz $\rightarrow$ for AM 88 - 110 MHz $\rightarrow$ for FM. 5) frequency domain representation. $S(f)$  lies apart from '0' 6) modulated transmission.

- (*) Matched filter :- Matched filter is a linear filter, used to (i) minimize effect of noise in the received signal.
(ii) maximize signal to noise ratio (S/N).



Block diagram of matched filter

- Matched filter is a type of LTI filter hence it is called linear filter. Impulse response of LTI filter is $h(t)$.
- Received signal is denoted by $g(t)$. and $w(t)$ is additive white gaussian noise. This $w(t)$ is also called as front end noise at the receiver.
- $x(t)$ is the input of matched filter. It is given as,

$$x(t) = g(t) + w(t) \quad 0 \leq t \leq T.$$

Labels: $g(t)$ is the received signal (Rxd sgl), $w(t)$ is the white gaussian noise (w. g. n.).

Rxd sgl + noise sgl $w(t)$

After Linear filtering.

$$o/p \Rightarrow y(t) = g(t) + n(t)$$

filtered noise.

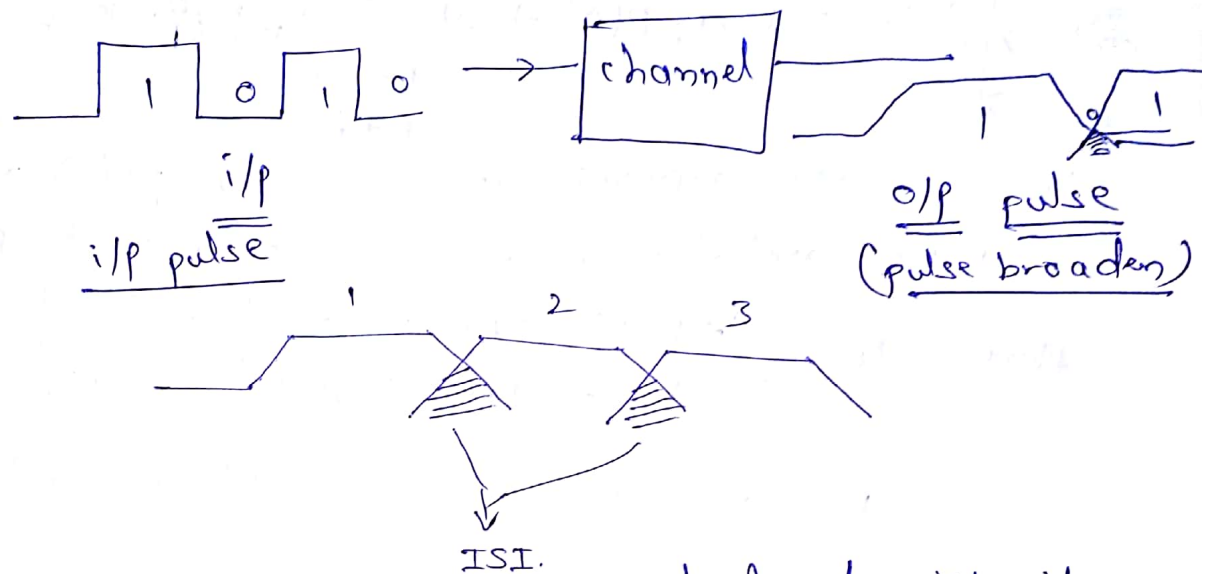
Linear o/p corresponding to received signal $g(t)$.

* Inter symbol Interference (ISI) :-

→ The interference caused between the symbols which are being transmitted in consecutive manner is called "Inter symbol interference".

→ when bit transmission rate increases $\rightarrow$ the bits will come together causes inter symbol interference.

→ Due to channel also ISI occurs.



→ At the input the pulse is well defined with its timing. (shape is perfect).

→ when it is passed to the channel, some gaussian noise will present, some other channel parameters will present. Due to these parameters of channel the pulse will be broaden at the output.

→ i.e the pulse will take more time to complete its wave shape. Hence when many bits are transmitted the timing of next bit will interfere with the previous bit as shown in above fig. It is called "Inter symbol Interference (ISI)".

→ ISI can be reduced by "pulse shaping". By pulse shaping pulse broadening will decrease hence ISI also reduces.

* Nyquist's criterion for Zero ISI :=

(or) Nyquist's criterion for distortionless transmission

→ The output of matched filter $y(t)$ { generalized eqⁿ } is given as,

$$\text{eqn ①} \Rightarrow y(t) = \mu \sum_{k=-\infty}^{\infty} a_k p(t - kT_b) + n(t)$$

by rearran-
-ging
(neglecting
noise)

$$\text{eqn ②} \Rightarrow y(t_i) = \mu a_i p(0) + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p(t_i - kT_b)$$

$i = 0, \pm 1, \pm 2, \dots$

→ The first term represents the value of $y(t_i)$ when $i = k$. $p(t)$ and it is normalized such that $p(0) = 1$.

Here, $a_k = \begin{cases} +1 & \text{if input msg bit is '1'} \\ -1 & \text{if input msg bit is '0'} \end{cases}$

$k = 0, \pm 1, \pm 2, \pm 3, \dots$ (variable parameter).

μ = scaling factor.

$p(t - kT_b)$ = shape difference b/w input signal & received signal. $\rightarrow p(t)$

$n(t)$ = filtered noise.

→ from equation ② the second term $\mu \sum_{k=-\infty}^{\infty} a_k p(t - kT_b)$

must be zero to eliminate effect of $k \neq i$ intersymbol interference.

→ This is possible if the received pulse $p(t)$ is controlled such that,

$$p[(i - k)T_b] = \begin{cases} 1 & ; \text{for } i = k \\ 0 & ; \text{for } i \neq k. \end{cases} \quad \text{--- ③}$$

↑ nyquist criteria for '0' ISI.

→ If $p(t)$ satisfies the above condition then we get a signal which is free from ISI. That signal is given as, $y(t_i) = MA_i$ — (4)

→ Hence the equation (3) gives the condition for Zero ISI. The eqn (3) is the condition in time domain. This condition gives more useful criterion in frequency domain.

In frequency domain := Let $p(nT_b)$ represents the impulses at which $p(t)$ is sampled. Fourier spectrum of these impulses is given as, — (5)

$$p_s(f) = f_b \sum_{n=-\infty}^{\infty} p(f - n f_b).$$

This means the spectrum of $p(t)$ are periodic with period f_b . Here f_b is sampling frequency. $p_s(f)$ is the spectrum of $p(nT_b)$ and $p(f)$ is the spectrum of $p(t)$.

→ $p(nT_b)$ is the infinite length of impulses with period of T_b , which are weighted with amplitudes of $p(t)$. i.e.,

$$p_s(t) = \sum_{n=-\infty}^{\infty} p(nT_b) \delta(t - nT_b) \quad \text{--- (6)}$$

→ Fourier transform of $p_s(t)$ becomes,

$$p_s(f) = \int_{-\infty}^{\infty} p_s(t) \cdot e^{-j2\pi ft} dt.$$

$$= \int_{-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} p(nT_b) \delta(t - nT_b) \right] e^{-j2\pi ft} dt$$

Let $n = i - k$ in above eqn.

$$p_s(f) = \int_{-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} p[(i-k)T_b] \delta[t - (i-k)T_b] \right] e^{-j2\pi ft} dt.$$

Now let us apply the condition in eqn (3) in above eqn

$$p_s(f) = \begin{cases} \int_{-\infty}^{\infty} p(0) \delta(t) e^{-j2\pi ft} dt & \text{for } i = k \\ \int_{-\infty}^{\infty} 0 \cdot \delta(t) e^{-j2\pi ft} dt & \text{for } i \neq k \end{cases}$$

→ becomes 0

$$\therefore p_s(f) = \int_{-\infty}^{\infty} p(t) \cdot s(t) \cdot e^{-j2\pi ft} dt \text{ for } i=k.$$

$$p_s(f) = p(t) \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} dt.$$

→ An integration in above eqⁿ is the fourier transform of $s(t)$ {impulse} which is = 1.

Hence, $p_s(f) = p(t) (1)$

$$p_s(f) = 1.$$

→ Hence eqⁿ (5) becomes { with $p_s(f) = 1$ }.

$$\Rightarrow 1 = f_b \sum_{n=-\infty}^{\infty} p(f - nf_b).$$

$$\Rightarrow 1/f_b = \sum_{n=-\infty}^{\infty} p(f - nf_b).$$

$$\Rightarrow T_b = \sum_{n=-\infty}^{\infty} p(f - nf_b).$$

frequency domain.

$$\begin{cases} p(i-k)T_b = 1; \text{ for } i=k \\ = 0; \text{ for } i \neq k \end{cases}$$

time domain

→ This is the frequency domain condition for zero ISI. Above eqⁿ is called Nyquist pulse shaping criteria for 0 ISI.

* M-ary PAM base band transmission :-

→ If $M=2$ then PAM is called binary PAM.

→ In binary PAM amplitude levels are given by the generalized eqⁿ

$$A_m = \frac{2^m - 1}{M};$$

modulation order (or) no. of possible signals.
 $m = 1, 2, 3, \dots, M.$

→ for binary PAM, $M=2$.
 Hence $m = 1, 2$ { 2 values }.

for BPAM $M=2$

$$\rightarrow \therefore A_m = 2^m - 1 - m$$

$$= 2^m - 1 - 2$$

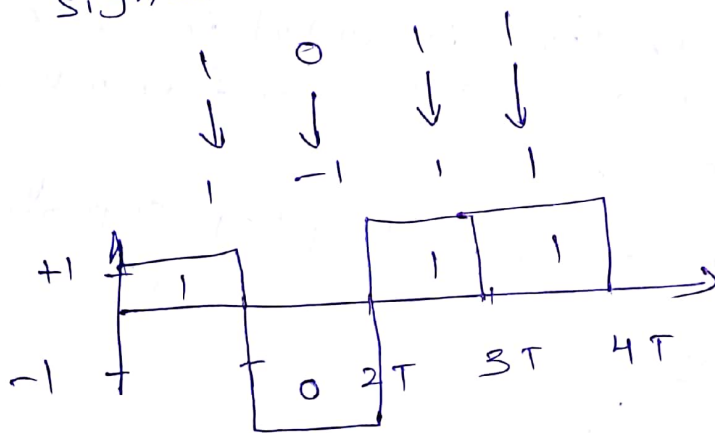
$$A_m = 2^m - 3 \quad \{m=1, 2\}$$

$$\text{when } m=1 \Rightarrow A_1 = 2(1) - 3 = -1$$

$$\text{when } m=2 \Rightarrow A_2 = 2(2) - 3 = 1.$$

Bits	levels
1	→ 1
0	→ -1

Ex if data sequence is 1011 generate 2-ary PAM signal.



4-ary PAM : Here $M=4$ (modulation order)
No. of bits in a symbol is, $b = \log_2 M$

$$b = \log_2 4$$

$$b = 2 \text{ bits/symbol.}$$

→ Amplitude levels $e^{j\theta^n}$, $A_m = 2^m - 1 - m$

$M=4$ here $\therefore m=1, 2, 3, 4$.

$$\therefore A_m = 2^m - 1 - 4$$

$$A_m = 2^m - 5$$

$$\begin{aligned}
 m=1 &\Rightarrow A_1 = 2(1) - 5 = -3 \\
 m=2 &\Rightarrow A_2 = 2(2) - 5 = -1 \\
 m=3 &\Rightarrow A_3 = +1 \\
 m=4 &\Rightarrow A_4 = +3
 \end{aligned}
 \left. \vphantom{\begin{aligned} m=1 \\ m=2 \\ m=3 \\ m=4 \end{aligned}} \right\} \text{amplitude levels.}$$

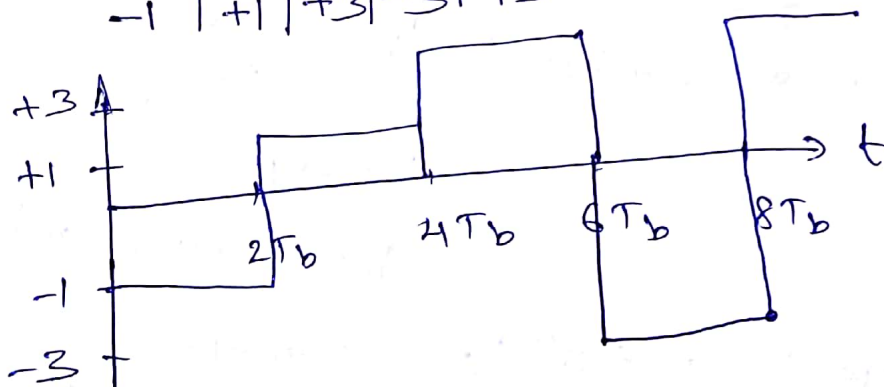
Level	Natural coding	Gray coding
-3	00 (symbol)	00
-1	01	01
+1	10	11
+3	11	10

→ In gray coding each level is coded in such a way that there will be a 1 bit difference b/w the successive levels.

Ex 11 0110110011 generate 4-PAM signal.

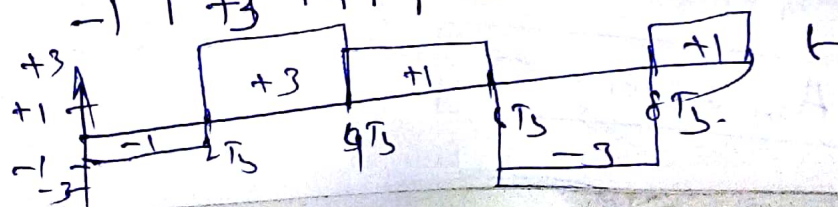
(i) Natural coding.

$$\begin{array}{c|c|c|c|c}
 0 & 1 & 1 & 0 & 11 & 00 & 11 \\
 \hline
 -1 & +1 & +3 & -3 & +3
 \end{array}$$



(ii) Gray coding

$$\begin{array}{c|c|c|c|c}
 01 & 10 & 11 & 00 & 11 \\
 \hline
 -1 & +3 & +1 & -3 & +1
 \end{array}$$



⊛ Digital subscriber Line (DSL) :=

→ DSL technology uses existed twisted pair of lines to carry high bandwidth applications like multimedia, videos, etc.

→ DSL services dedicated for point to point.

→ Dial up service is limited to 53.3 kbps whereas DSL service can be enable upto 8.122 mbps.

→ Types of DSL.

(i) HDSL (High speed digital subscriber line).

(ii) SDSL (slow speed dgtl sub line).

(iii) IDSL (ISDN Digital sub line).

(iv) VDSL (very high speed dgtl sub line).

(v) ADSL (Adaptive DSL).

i) HDSL := It is not commercially used because it require two twisted pair of lines and not supported for normal telephone systems.

ii) SDSL := Uses only one twisted pair of lines. And is used for transmission of voice. The problem with SDSL is it is slow and used for short distance only.

iii) IDSL := It is similar to ISDN technology. It is not suitable for present voice transmission. data suitable for analog voice transmission only. data transmission only 28 kbps. This is also disadvantage of IDSL.

iv) VDSL := It provides very high data rate. Data rate is upto 52 MBPS. But VDSL requires shorter connection range.

v) ADSL := It is most widely used DSL. used for personal broadband, allows same channel for T^n & R^n .

vi) Rate adaptive DSL (RADSL) := It is the advancement for ADSL. It is automatically optimize the data rate for the data in a line.

(*) optimum Receiver (or) optimum filter :=

→ The generalized filter for receiving binary coded signals is called "optimum receiver" (or) "optimum filter".

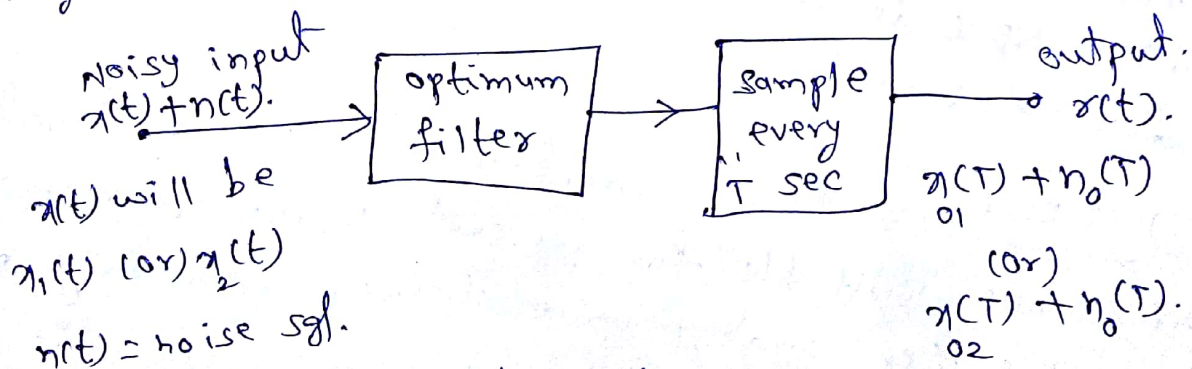
→ Let us assume the received signal is a binary waveform. Let us say that the polar NRZ signal is used to represent binary 1 and 0. i.e

for binary 1; $x_1(t) = +A$ → for one bit period T .

for binary 0; $x_2(t) = -A$ → " " " "

→ Hence the input signal $x(t)$ will be either $x_1(t)$ (or) $x_2(t)$ depending upon the polarity of NRZ signal.

→ The following fig shows block diagram for such binary coded signal.



Block dgm of optimum Rx.

→ As shown in above fig. noise $n(t)$ is added to the signal $x(t)$ at the channel during the transmission. Hence input to the optimum filter $= x(t) + n(t)$.

→ In the absence of noise $n(t)$, the output of optimum receiver will be,

$$\left\{ \begin{array}{l} r(T) = r_{01}(T) \text{ ; if } x(t) = x_1(t) \\ \text{and } r(T) = r_{02}(T) \text{ ; if } x(t) = x_2(t) \end{array} \right.$$

→ Thus in the absence of noise, decision will be taken clearly. But if noise is present then, select $x_1(t)$ if $r(T)$ is closer to $r_{01}(T)$ than $r_{02}(T)$ and, select $x_2(t)$ if $r(T)$ is closer to $r_{02}(T)$ than $r_{01}(T)$. Therefore the decision boundary will be midway between $r_{01}(T)$ and $r_{02}(T)$. It is given as,

$$\text{Decision boundary} = \frac{r_{01}(T) + r_{02}(T)}{2}$$

* Probability of Error (P_e) for optimum receiver / filter =

→ Suppose that $x_2(t)$ was transmitted, but $r_{01}(T)$ is greater than the $r_{02}(T)$. If the noise $n_0(T)$ is positive & larger in magnitude than the voltage difference $\frac{1}{2} [r_{01}(T) + r_{02}(T)] - r_{02}(T)$, then incorrect decision will be taken at the receiver. Hence error will be generated if,

$$n_0(T) \geq \frac{r_{01}(T) + r_{02}(T)}{2} - r_{02}(T)$$

$$\therefore n_0(T) \geq \frac{r_{01}(T) + r_{02}(T) - 2r_{02}(T)}{2}$$

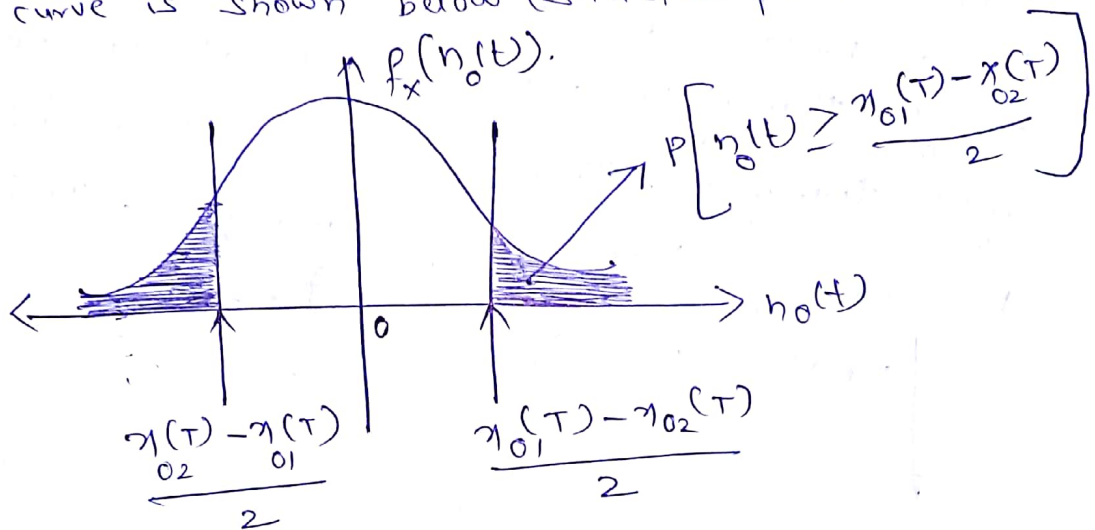
$$\therefore n_0(T) \geq \frac{r_{01}(T) - r_{02}(T)}{2} \quad \text{--- (1)}$$

→ The probability density function (PDF) for $\eta_0(t)$ is given by,

$$f_x(\eta_0(t)) = \frac{1}{\sigma\sqrt{2\pi}} e^{-[\eta_0(t)]^2/2\sigma^2} \quad (2)$$

Here, $\eta_0(t)$ = random noise function,
 σ = standard deviation.

→ To obtain the probability of error (P_e), we should integrate area under the PDF curve. This portion of the curve is shown below (shaded part).



$$\therefore P_e = P\left[\eta_0(\tau) \geq \frac{\eta_{01}(\tau) - \eta_{02}(\tau)}{2}\right] = \int_{\frac{\eta_{01}(\tau) - \eta_{02}(\tau)}{2}}^{\infty} f_x(\eta_0(t)) d[\eta_0(t)]. \quad (3)$$

substitute (2) in (3) we get,

$$P_e = \int_{\frac{\eta_{01}(\tau) - \eta_{02}(\tau)}{2}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-[\eta_0(t)]^2/2\sigma^2} d[\eta_0(t)]. \quad (4)$$

Let, $\frac{[\eta_0(t)]^2}{2\sigma^2} = y^2$
 $\eta_0(t) = \sigma\sqrt{2} y$

$$d[n_0(t)] = a\sqrt{2} dy.$$

when $n_0(t) = 0$ then $y = 0$.

$$\text{when, } n_0(t) = \frac{\gamma_{01}(T) - \gamma_{02}(T)}{2}, \text{ then } y = \frac{\gamma_{01}(T) - \gamma_{02}(T)}{2\sqrt{2}a}.$$

substitute above values in eqⁿ (4)

$$\therefore P_e = \int_{\frac{\gamma_{01}(T) - \gamma_{02}(T)}{2\sqrt{2}a}}^{\infty} \frac{1}{a\sqrt{2\pi}} e^{-y^2} \cdot a\sqrt{2} dy.$$

$$P_e = \frac{1}{\sqrt{\pi}} \int_{\frac{\gamma_{01}(T) - \gamma_{02}(T)}{2\sqrt{2}a}}^{\infty} e^{-y^2} dy.$$

Let us rearrange the above eqⁿ as follows,

$$P_e = \frac{1}{2} \left[\frac{2}{\sqrt{\pi}} \int_{\frac{\gamma_{01}(T) - \gamma_{02}(T)}{2\sqrt{2}a}}^{\infty} e^{-y^2} dy \right]. \quad \text{--- (5)}$$

we have the standard relation that,

$$\boxed{\frac{2}{\sqrt{\pi}} \int_u^{\infty} e^{-y^2} dy = \text{erfc}(u).}$$

$\therefore$ eqⁿ (5) will be,

$$\boxed{P_e = \frac{1}{2} \text{erfc} \left[\frac{\gamma_{01}(T) - \gamma_{02}(T)}{2\sqrt{2}a} \right]} \quad *$$

$\rightarrow$ This is the required expression for error probability of optimum receiver/filter. The 'erfc' function is a monotonically decreasing function.

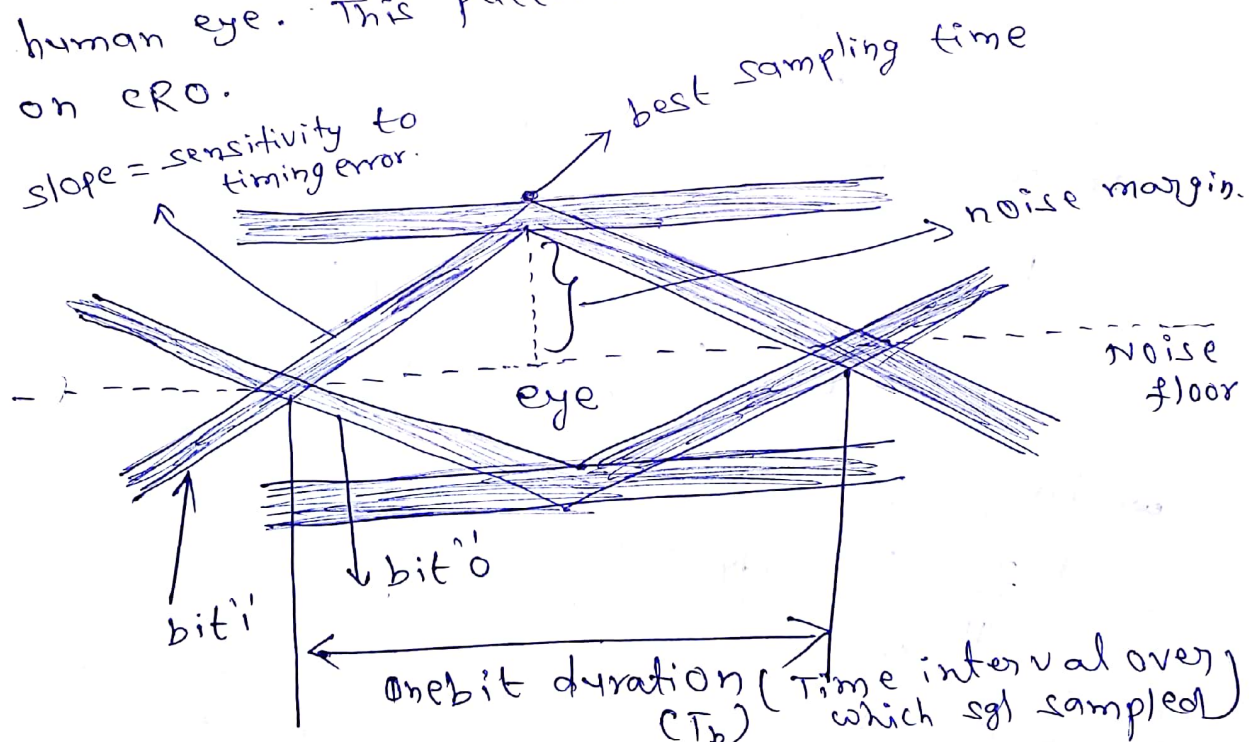
Therefore p_e decreases as the difference $\gamma_{01}(T) - \gamma_{02}(T)$ becomes larger.

→ The optimum filter has to maximize the ratio $\gamma_{01}(T) - \gamma_{02}(T)$ such that the probability of error (p_e) is minimum.

* Eye patterns (or) Eye diagrams :-

→ The eye patterns (or) Eye diagrams are used to study the effect of Inter symbol Interference (ISI) in base band transmission.

→ The name 'eye' is given because it looks like human eye. This pattern can also be obtained on CRO.



→ dotted line is a threshold value.

→ At the middle point the ISI is very less. Hence it is suggested to sample at that time. At that point (best sampling time) eye opening is high hence ISI effect is less.

→ The amplitude above noise floor is called noise margin.

→ The time for rising & falling of a pulse is represented by slope. i.e. timing error.

→ At any instance of time if eye opening is high then ISI is less. (best time). If eye opening is less then ISI is very high. (not suggested to sample at that time).

i) width of the eye opening := Time interval over which the received distorted signal can be sampled with minimum error. (ISI).

ii) Sensitivity to timing error (slope) := Rate of the closure of eye pattern when the sampling time is varied.

iii) Noise margin := Height of the eye opening for a specified sampling time. with respect to noise floor.

(*) Adaptive Equalization := If the channel is varying channel, then the weights used in the equalizer that should be updated as per the nature of varying channel. such equalizers are called "adaptive equalizers".

→ Adaptive equalizers work in 2 modes.

i) Training mode.

ii) Decision directed mode.

→ In training mode adaptive equalizers don't send original information. Instead of original information they send 'pilot signal' (or) 'Reference signals' (or) 'Training signals'

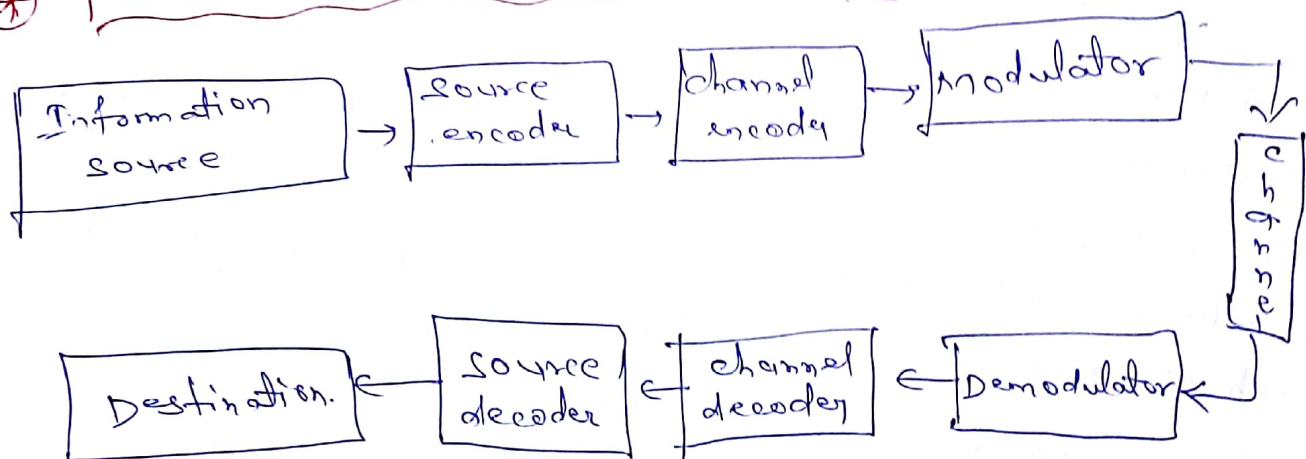
→ This pilot signal is nothing but it is known for the transmitter as well as receiver. what is transmitted that receiver also knows. lets call this signal as $d(n)$.

→ Now equalizer makes some decision and it obtains $\hat{d}(n)$. Now the difference $d(n) - \hat{d}(n)$ gives error signal $e(n)$. i.e. $e(n) = d(n) - \hat{d}(n)$

→ This error signal $e(n)$ is used to update the weights of the adaptive equalizer. This updation will be stopped when $d(n) = \hat{d}(n)$. i.e. $e(n) = 0$.

→ when $d(n) = \hat{d}(n)$ error becomes zero then equalizer will go to decision directed mode. Now transmitter starts sending original information instead of sending pilot information.

* pass band transmission model :-



* Gram - schmidt procedure :- This procedure is used to represent a "signal" in the form of "orthogonal basis function".

→ Let us suppose if we have a signal $s_m(t)$ where
 $m = 1, 2, 3, \dots, M$
 i.e. $s_1(t), s_2(t), \dots, s_M(t)$.

→ Now our aim is to find orthogonal basis functions.

i.e $\phi_1(t), \phi_2(t) \dots \phi_M(t)$.

$$\rightarrow \phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} \quad \text{energy of a sig. } s_1(t)$$

$$\text{where } E_1 = \int_{-\infty}^{\infty} |s_1(t)|^2 dt$$

$$\therefore \phi_1(t) = \frac{s_1(t)}{\sqrt{\int_{-\infty}^{\infty} |s_1(t)|^2 dt}}$$

$$\rightarrow \phi_2(t) = ?$$

first we have to subtract projection of $s_2(t)$ on $\phi_1(t)$.

Then we have to calculate energy of a sig E_2 .

$$\text{i.e } y_2(t) = s_2(t) - \underbrace{c_{21}}_{\text{projection of } s_2(t) \text{ on } \phi_1(t)} \phi_1(t)$$

$$\oint c_{21} = \langle s_2(t) \phi_1(t) \rangle = \int_{-\infty}^{\infty} s_2(t) \phi_1^*(t) dt$$

$$E_2(t) = \int_{-\infty}^{\infty} |y_2(t)|^2 dt$$

$$\phi_2(t) = \frac{y_2(t)}{\sqrt{E_2}}$$

$$\vdots$$
$$\phi_K(t) = \frac{y_K(t)}{\sqrt{E_K}}$$

$$\phi_3(t) = \frac{y_3(t)}{\sqrt{E_3}}$$

$$\text{Here, } y_3(t) = s_3(t) - \langle c_{31} \rangle \phi_1(t) - \langle c_{32} \rangle \phi_2(t)$$

$$\oint c_{31} = \langle s_3(t) \phi_1(t) \rangle = \int_{-\infty}^{\infty} s_3(t) \phi_1^*(t) dt$$

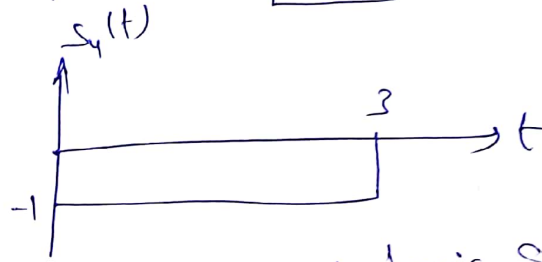
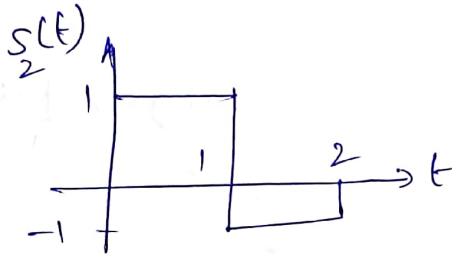
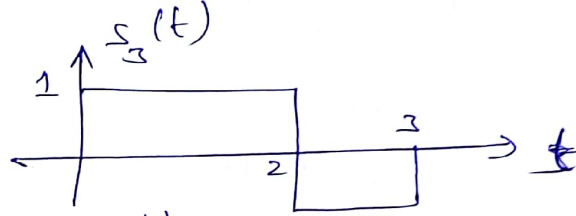
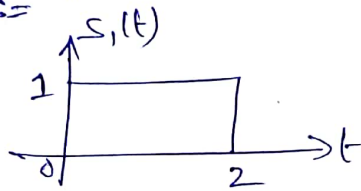
$$\oint c_{32} = \langle s_3(t) \phi_2(t) \rangle = \int_{-\infty}^{\infty} s_3(t) \phi_2^*(t) dt$$

Similarly, $y_3(t) = s_3(t) - \sum_{i=1}^2 c_{ki} \phi_i(t)$.

$$c_{ki} = \langle s_k(t), \phi_i(t) \rangle = \int_{-\infty}^{\infty} s_k(t) \phi_i^*(t) dt$$

$$E_k = \int_{-\infty}^{\infty} y_k^*(t) \cdot dt.$$

Ex 11 find orthogonal basis of following signals?



→ four signals are given hence orthogonal basis can be ≤ 4 .

→ we have to find $\phi_1(t)$ first.

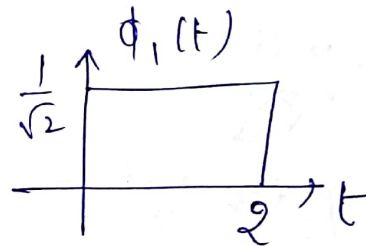
$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

$$\text{where, } E_1 = \int_{-\infty}^{\infty} |s_1(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} |1|^2 dt$$

$$E_1 = \int_0^2 1 dt = (2-0) = \underline{\underline{2}}$$

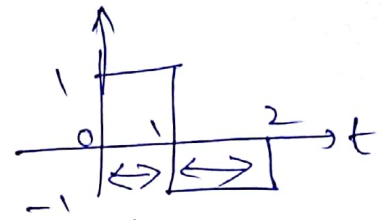
$$\phi_1(t) = \frac{s_1(t)}{\sqrt{2}}$$



Next $\phi_2(t)$

$$\phi_2(t) = \frac{y_2(t)}{\sqrt{E_2}} = \frac{s_2(t) - c_{21}\phi_1(t)}{\sqrt{E_2}}$$

$$c_{21} = \int_{-\infty}^{\infty} \phi_1(t) s_2(t) dt$$



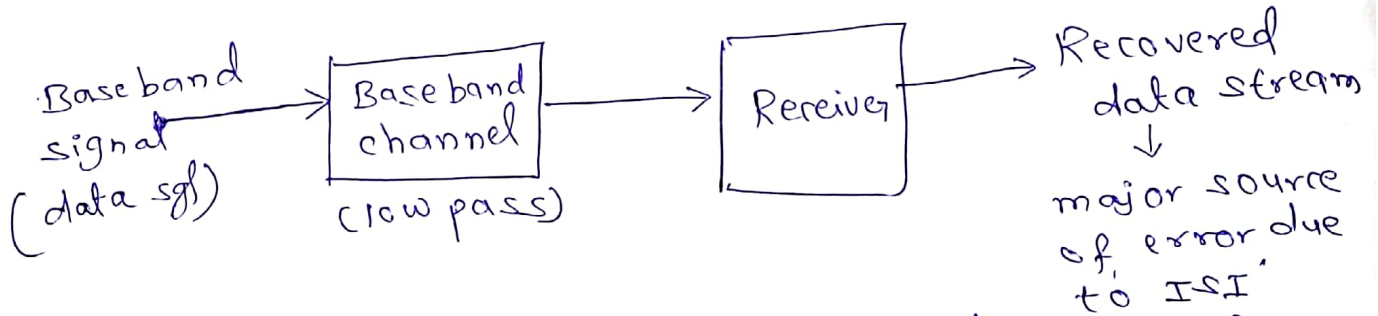
$$c_{21} = 0$$

$$= \int_0^1 \frac{1}{\sqrt{2}} dt + \int_1^2 \left(\frac{-1}{\sqrt{2}}\right) dt$$

$$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

$$= 0$$

* Baseband Transmission Model ::



- The baseband sgl (or) message signal in the form of pulses is directly transmitted through the baseband channel. This is a low pass channel.
- That signal is received by receiver. At the receiver the recovered data stream will be affected by Internal symbol Interference (ISI) and channel noise.
- At the receiver to reduce ISI, to get optimum detection the receiver must be a LTI filter. Here at the receiver matched filter is used.

- In this section we will study a little different type of receiver which is called *correlator*. Fig. 4.7.1 shows the block diagram of this correlator.

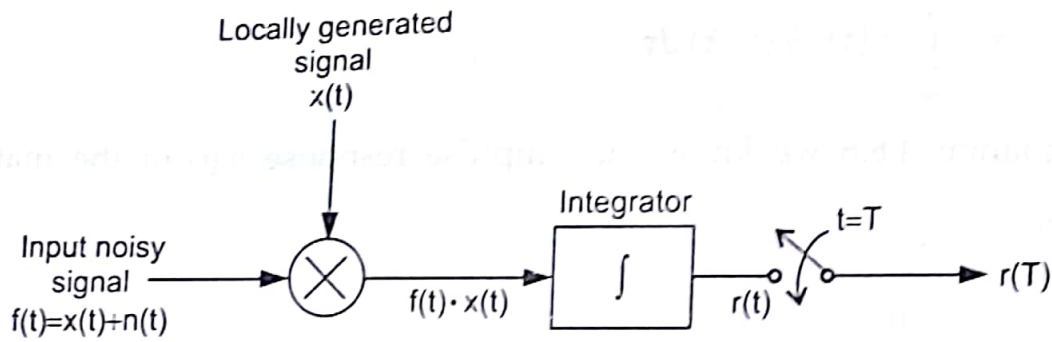


Fig. 4.7.1 Block diagram of the correlator

In the above figure $f(t)$ represents input noisy signal, i.e., $f(t) = x(t) + n(t)$. The signal $f(t)$ is multiplied to the locally generated replica of input signal $x(t)$. This result of multiplication $f(t) \cdot x(t)$ is integrated.

- The output of the integrator is sampled at $t = T$ (i.e. end of one symbol period). Then based on this sampled value, decision is made. This is how the correlator works. It is called *correlator* since it *correlates the received signal $f(t)$ with a stored replica of the known signal $x(t)$* . In the block diagram of above figure, the product $f(t) x(t)$ is integrated over one symbol period, i.e. T . Hence output $r(t)$ can be written as,

$$r(t) = \int_0^T f(t) x(t) dt$$

At $t = T$, the above equation will be,

$$\text{Output of correlator : } r(T) = \int_0^T f(t) x(t) dt \quad \dots (4.7.1)$$

- Now let us consider the matched filter as shown in Fig. 4.7.2 below.

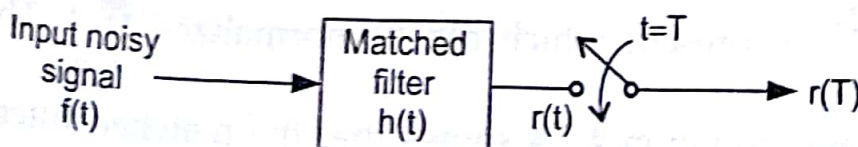


Fig. 4.7.2 Block diagram of a matched filter receiver

In the above block diagram observe that the matched filter does not need locally generated replica of input signal $x(t)$. The output of the matched filter can be obtained by convolution of input $f(t)$ and its impulse response $h(t)$. i.e.,

$$\begin{aligned} r(t) &= f(t) * h(t) \\ &= \int_{-\infty}^{\infty} f(\tau) \cdot h(t-\tau) d\tau \end{aligned} \quad \dots (4.7.2)$$

- From equation 4.6.6 we know that impulse response $h(t)$ of the matched filter is given as,

$$h(t) = \frac{2k}{N_0} x(T-t) \quad \dots (4.7.3)$$

$$\therefore h(t-\tau) = \frac{2k}{N_0} x(T-t+\tau)$$

Putting this value of $h(t-\tau)$ in equation 4.7.2 we get,

$$r(t) = \int_{-\infty}^{\infty} f(\tau) \frac{2k}{N_0} x(T-t+\tau) d\tau$$

Since the integration is performed over one bit period, we can change integration limits from 0 to T. i.e.,

$$r(t) = \frac{2k}{N_0} \int_0^T f(\tau) x(T-t+\tau) d\tau$$

At $t=T$, the above equation will be,

$$r(T) = \frac{2k}{N_0} \int_0^T f(\tau) x(T-T+\tau) d\tau = \frac{2k}{N_0} \int_0^T f(\tau) x(\tau) d\tau$$

Let us put $\tau=t$ just for convenience of notation,

$$\text{Output of matched filter : } r(T) = \frac{2k}{N_0} \int_0^T f(t) x(t) dt \quad \dots (4.7.4)$$

- This equation gives the output of matched filter. Observe that this equation and equation 4.7.1 (which gives output of correlator) are identical. In above equation the constant $\frac{2k}{N_0}$ is present which can be normalized to 1. The similarity between equation 4.7.1 and equation 4.7.4 shows that the matched filter and correlator gives same output. Therefore we can state,

The matched filter and correlator are two distinct, independent techniques which give the same result. These two techniques are used to synthesize the optimum

4.8.2 Error Probability of ASK

In Amplitude Shift Keying (ASK), some number of carrier cycles are transmitted to send '1' and no signal is transmitted for binary '0'. Thus,

Binary '1' $\Rightarrow x_1(t) = \sqrt{2P_s} \cos(2\pi f_0 t)$ and

Binary '0' $\Rightarrow x_2(t) = 0$ (i.e. no signal)

... (4.8.7)

Here P_s is the normalized power of the signal in 1Ω load. i.e. power $P_s = \frac{A^2}{2}$. Hence $A = \sqrt{2P_s}$. Therefore in above equation for $x_1(t)$ amplitude 'A' is replaced by $\sqrt{2P_s}$.

We know that the maximum signal to noise ratio is given by equation 4.6.13 as,

$$\rho_{\max} = \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 = \frac{2}{N_0} \int_{-\infty}^{\infty} |X(f)|^2 df \quad \dots (4.8.8)$$

Parseval's power theorem states that,

$$\int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-\infty}^{\infty} x^2(t) dt$$

Hence equation 4.8.8 becomes,

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 = \frac{2}{N_0} \int_{-\infty}^{\infty} x^2(t) dt$$

We know that $x(t)$ is present from 0 to T. Hence limits in above equation can be changed as follows :

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 = \frac{2}{N_0} \int_0^T x^2(t) dt \quad \dots (4.8.9)$$

We know that $x(t) = x_1(t) - x_2(t)$. For ASK $x_2(t)$ is zero, hence $x(t) = x_1(t)$. Hence above equation becomes,

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 = \frac{2}{N_0} \int_0^T x_1^2(t) dt$$

Putting equation of $x_1(t)$ from equation 4.8.7 in above equation we get,

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 = \frac{2}{N_0} \int_0^T [\sqrt{2P_s} \cos(2\pi f_0 t)]^2 dt = \frac{4P_s}{N_0} \int_0^T \cos^2(2\pi f_0 t) dt$$

We know that $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$. Here applying this formula to $\cos^2(2\pi f_0 t)$ we get,

$$\begin{aligned} \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 &= \frac{4P_s}{N_0} \int_0^T \frac{1 + \cos 4\pi f_0 t}{2} dt \\ &= \frac{4P_s}{N_0} \cdot \frac{1}{2} \left\{ \int_0^T dt + \int_0^T \cos 4\pi f_0 t dt \right\} = \frac{2P_s}{N_0} \left\{ [t]_0^T + \left[\frac{\sin 4\pi f_0 t}{4\pi f_0} \right]_0^T \right\} \\ &= \frac{2P_s}{N_0} \left\{ T + \frac{\sin 4\pi f_0 T}{4\pi f_0} \right\} \quad \dots (4.8.10) \end{aligned}$$

Digital Comm.

We know that T is the bit period and in this one bit period, the carrier has integer number of cycles. Thus the product $f_0 T$ is an integer. This is illustrated in Fig. 4.8.2.

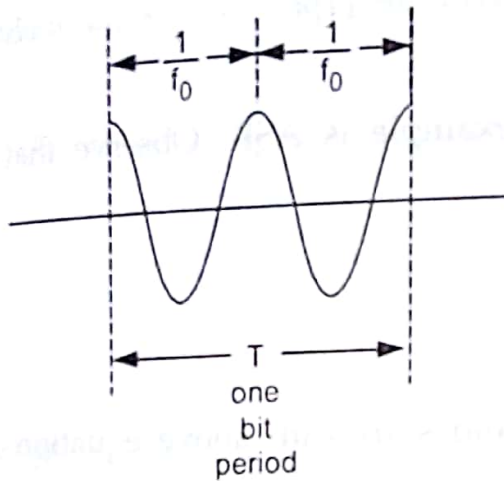


Fig. 4.8.2 In one bit period T , the carrier completes its two cycles. The carrier has frequency f_0 . From figure we can write,

$$T = \frac{1}{f_0} + \frac{1}{f_0}$$

$$\text{i.e. } T = \frac{2}{f_0}$$

$$\therefore f_0 T = 2 \quad (\text{integer no. of cycles})$$

As shown in above figure, the carrier completes two cycles in one bit duration. Hence

$$f_0 T = 2$$

Therefore, in general if carrier completes ' k ' number of cycles, then,

$$f_0 T = k \quad (\text{Here } k \text{ is an integer}) \quad \dots (4.8.11)$$

Therefore the sine term in equation 4.8.10 becomes, $\sin 4\pi k$ and k is integer.

For all integer values of k , $\sin 4\pi k = 0$. Hence equation 4.8.10 becomes,

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 = \frac{2P_s T}{N_0} \quad \dots (4.8.12)$$

$$\therefore \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max} = \sqrt{\frac{2P_s T}{N_0}} \quad \dots (4.8.13)$$

Putting this value in equation 4.5.6 we get error probability of ASK using matched filter detection as,

$$P_e = \frac{1}{2} \operatorname{erfc} \left\{ \frac{1}{2\sqrt{2}} \cdot \sqrt{\frac{2P_s T}{N_0}} \right\} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{P_s T}{4N_0}}$$

Here $P_s T = E$, i.e. energy of one bit hence above equation becomes,

$$\text{Error probability of ASK : } P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E}{4N_0}} \quad \dots (4.8.14)$$

This is the expression for error probability of ASK using matched filter detection.

4.8.3 Probability of Error for Coherently Detected BPSK

In Binary PSK (BPSK), the phase of the carrier is shifted by 180° for two symbols. These two symbols (bits) are represented by two signals as follows :

$$\text{Binary '1'} \Rightarrow x_1(t) = \sqrt{2P} \cos(2\pi f_0 t) \quad \dots (4.8.15)$$

$$\text{and Binary '0'} \Rightarrow x_2(t) = -\sqrt{2P} \cos(2\pi f_0 t) \quad \dots (4.8.16)$$

Here P is normalized power of the carrier; and $P = \frac{A^2}{2}$, where A is amplitude of the carrier. From the above two equations we can write,

$$x_2(t) = -x_1(t) \quad \dots (4.8.17)$$

The probability of error of the matched filter is given as (By equation 4.6.12),

$$P_e = \frac{1}{2} \operatorname{erfc} \left\{ \frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\sigma} \right\} \quad \dots (4.8.18)$$

where,

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 = \frac{2}{N_0} \int_0^T x^2(t) dt \quad \text{By equation 4.6.13 and 4.6.14} \quad \dots (4.8.19)$$

Here we know that $x(t) = x_1(t) - x_2(t)$. For PSK, $x_2(t) = -x_1(t)$ as we have seen (equation 4.8.17).

$$\therefore x(t) = x_1(t) - [-x_1(t)] = 2x_1(t)$$

Hence equation 4.8.19 becomes,

$$\begin{aligned} \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 &= \frac{2}{N_0} \int_0^T 4x_1^2(t) dt \\ &= \frac{8}{N_0} \int_0^T x_1^2(t) dt \end{aligned} \quad \dots (4.8.20)$$

As in the previous subsection it can be shown very easily that,

$$\begin{aligned} \int_0^T x_1^2(t) dt &= \int_0^T 2P \cos^2(2\pi f_0 t) dt \\ &= 2P \cdot \frac{1}{2} \int_0^T [1 + \cos 4\pi f_0 t] dt = P \left[\int_0^T dt + \int_0^T \cos 4\pi f_0 t dt \right] \end{aligned}$$

In the above equation second integration will be zero since it is integration of cosine wave over one bit period. This we have proved in last subsection (i.e. sec 4.8.2). Hence above equation becomes,

$$\begin{aligned} \int_0^T x_1^2(t) dt &= P \int_0^T dt && \text{Since second term is zero.} \\ &= P [t]_0^T = PT \\ &= E && \dots (4.8.21) \end{aligned}$$

Thus energy $E = \text{Power (P)} \times \text{bit duration (T)}$. Putting the above result in equation 4.8.20 we get,

$$\begin{aligned} \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 &= \frac{8}{N_0} \cdot E \\ \therefore \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max} &= \sqrt{\frac{8E}{N_0}} && \dots (4.8.22) \end{aligned}$$

Putting this result in equation 4.8.18 we get,

$$P_e = \frac{1}{2} \operatorname{erfc} \left\{ \frac{1}{2\sqrt{2}} \sqrt{\frac{8E}{N_0}} \right\}$$

On simplification of above equation we get,

$$\therefore \text{Error probability in PSK : } P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E}{N_0}} \quad \dots (4.8.23)$$

This is the expression which gives error probability of PSK using matched filter detection.

4.8.4 Probability of Error for Coherently Detected Binary Orthogonal FSK

In the binary FSK transmission, two different carrier frequencies are used to transmit two binary levels. As we have seen these two signals are as follows :

$$\begin{aligned} \text{Binary '1'} \Rightarrow \quad x_1(t) &= \sqrt{2P} \cos(2\pi f_0 + \Omega) t \\ \text{and Binary '0'} \Rightarrow \quad x_2(t) &= \sqrt{2P} \cos(2\pi f_0 - \Omega) t \end{aligned} \quad \dots (4.8.35)$$

From above equation we can write,

$$x_1(t) - x_2(t) = \sqrt{2P} \{ \cos(2\pi f_0 + \Omega)t - \cos(2\pi f_0 - \Omega)t \}$$

$$[x_1(t) - x_2(t)]^2 = 2P [\cos(2\pi f_0 + \Omega)t - \cos(2\pi f_0 - \Omega)t]^2$$

Here let us use $2 \sin(x) \sin(y) = \cos(x-y) - \cos(x+y)$, then above equation becomes,

$$[x_1(t) - x_2(t)]^2 = 2P [-2 \sin 2\pi f_0 t \sin \Omega t]^2$$

$$= 2P [4 \sin^2 \omega_0 t \sin^2 \Omega t]$$

By putting $2\pi f_0 = \omega_0$

$$= 2P \{ (2 \sin^2 \omega_0 t) (2 \sin^2 \Omega t) \}$$

By rearranging the equation

Here let us use $2 \sin^2(x) = 1 - \cos(2x)$, then above equation becomes,

$$[x_1(t) - x_2(t)]^2 = 2P \{ (1 - \cos 2\omega_0 t) (1 - \cos 2\Omega t) \}$$

$$= 2P \{ 1 - \cos 2\Omega t - \cos 2\omega_0 t + \cos 2\omega_0 t \cos 2\Omega t \}$$

In the above equation let us use $\cos(x) \cos(y) = \frac{1}{2} [\cos(x-y) + \cos(x+y)]$ then above equation will be,

$$[x_1(t) - x_2(t)]^2 = 2P \left\{ 1 - \cos 2\Omega t - \cos 2\omega_0 t + \frac{1}{2} [\cos 2(\omega_0 - \Omega)t + \cos 2(\omega_0 + \Omega)t] \right\}$$

Let us take integration of both the sides from 0 to T then we get,

$$\begin{aligned} \int_0^T [x_1(t) - x_2(t)]^2 dt &= \int_0^T 2P \{ 1 - \cos 2\Omega t - \cos 2\omega_0 t \\ &\quad + \frac{1}{2} [\cos 2(\omega_0 - \Omega)t + \cos 2(\omega_0 + \Omega)t] \} dt \\ &= 2P \left\{ \int_0^T dt - \int_0^T \cos 2\Omega t dt - \int_0^T \cos 2\omega_0 t dt \right. \\ &\quad \left. + \frac{1}{2} \int_0^T \cos 2(\omega_0 - \Omega)t dt + \frac{1}{2} \int_0^T \cos 2(\omega_0 + \Omega)t dt \right\} \\ &= 2P \left\{ T - \frac{\sin 2\Omega T}{2\Omega} - \frac{\sin 2\omega_0 T}{2\omega_0} + \frac{1}{2} \frac{\sin 2(\omega_0 - \Omega)T}{2(\omega_0 - \Omega)} \right. \\ &\quad \left. + \frac{1}{2} \frac{\sin 2(\omega_0 + \Omega)T}{2(\omega_0 + \Omega)} \right\} \end{aligned}$$

$$= 2PT \left\{ 1 - \frac{\sin 2\Omega T}{2\Omega T} - \frac{\sin 2\omega_0 T}{2\omega_0 T} + \frac{1}{2} \frac{\sin 2(\omega_0 - \Omega) T}{2(\omega_0 - \Omega) T} + \frac{1}{2} \frac{\sin 2(\omega_0 + \Omega) T}{2(\omega_0 + \Omega) T} \right\} \quad \dots (4.8.37)$$

We know that the frequency shift ' Ω ' is very small in comparison with the carrier frequency ω_0 . Then the last three terms in above equation will be of the form $\frac{\sin 2\omega_0 T}{2\omega_0 T}$.

Here ω_0 is the angular frequency of the carrier signal and T is the period of one bit. Actually many cycles of carrier are completed in one bit period, i.e. $\omega_0 T \gg 1$. Hence the ratio $\frac{\sin 2\omega_0 T}{2\omega_0 T}$ approaches to zero as $\omega_0 T$ increases. Therefore we can neglect last three terms in equation 4.8.37. Then the equation becomes,

$$\int_0^T [x_1(t) - x_2(t)]^2 dt = 2PT \left\{ 1 - \frac{\sin 2\Omega T}{2\Omega T} \right\} \quad \dots (4.8.38)$$

We know that error probability of the matched filter is given as,

$$P_e = \frac{1}{2} \operatorname{erfc} \left\{ \frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\sigma} \right\} \quad \text{By equation 4.6.12} \quad \dots (4.8.39)$$

And for a matched filter detection, in presence of white gaussian noise,

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 = \frac{2}{N_0} \int_0^T x^2(t) dt \quad \text{By equation 4.6.13 and 4.6.14} \quad \dots (4.8.40)$$

Here we know that $x(t) = x_1(t) - x_2(t)$. Therefore above equation becomes,

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 = \frac{2}{N_0} \int_0^T [x_1(t) - x_2(t)]^2 dt$$

Putting the value of integral of RHS from equation 4.8.38 we get,

$$\begin{aligned} \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 &= \frac{2}{N_0} \cdot 2PT \left\{ 1 - \frac{\sin 2\Omega T}{2\Omega T} \right\} \\ &= \frac{4PT}{N_0} \left\{ 1 - \frac{\sin 2\Omega T}{2\Omega T} \right\} \quad \dots (4.8.41) \end{aligned}$$

The above ratio obtains largest value when $2\Omega T = \frac{3\pi}{2}$

Putting this value in above equation we get,

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max}^2 = h \frac{4PT}{N_0} \left\{ 1 - \frac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}} \right\}$$

$$= \frac{4.84 PT}{N_0} \quad \dots (4.8.42)$$

$$\therefore \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]_{\max} = \sqrt{\frac{4.84 PT}{N_0}} \quad \dots (4.8.43)$$

Putting this value of $\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma} \right]$ in equation 4.8.39 we get probability of error as,

$$P_e = \frac{1}{2} \operatorname{erfc} \left\{ \frac{1}{2\sqrt{2}} \cdot \sqrt{\frac{4.84 PT}{N_0}} \right\} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{0.6 PT}{N_0}}$$

We know that the product $PT = E$ (energy of one bit). Hence above equation becomes,

$$\text{Error probability of FSK : } P_e = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{0.6 E}{N_0}} \quad \dots (4.8.44)$$

This is the expression for probability of error of FSK which uses matched filter detection.

Unit - II

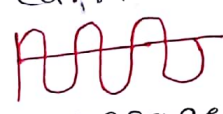
Digital modulation Techniques

→ The process of varying (or) shifting (or) switching (or) keying one of the characteristics of carrier signal in accordance with message signal is called "modulation".

→ The process of varying one of the characteristics of carrier signal in accordance with message signal is called "modulation".

→ There are basically 3 types of digital modulation techniques. They are,

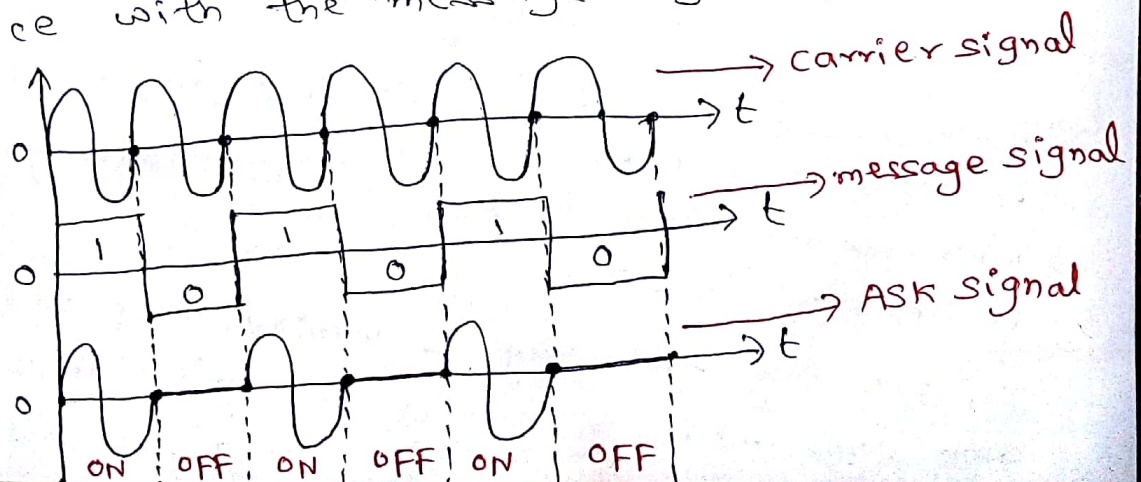
- * i) Amplitude shift keying (ASK) (or) Binary ASK (BASK)
- * ii) Frequency shift keying (FSK) (or) Binary FSK (BFSK)
- * iii) phase shift keying (PSK) (or) Binary PSK (BPSK)

Note := i) In digital modulation techniques carrier signal is a sine wave (or) cos wave. 

ii) In digital modulation techniques message signal is a pulse signal. Because our message is transferred in binary 0 and 1. Hence pulse is used. 

binary '1' is represented with +ve pulse and
binary '0' is " " -ve " " .

i) Amplitude shift keying (ASK) := The amplitude of the carrier signal (sine wave) is varied (or) shifted in accordance with the message signal (pulse signal).



→ when message bit is '1' then carrier present in Ask signal.
when message bit is '0' then carrier is Absent. Hence the amplitude of the Ask signal is shifted between two levels (present and absent).

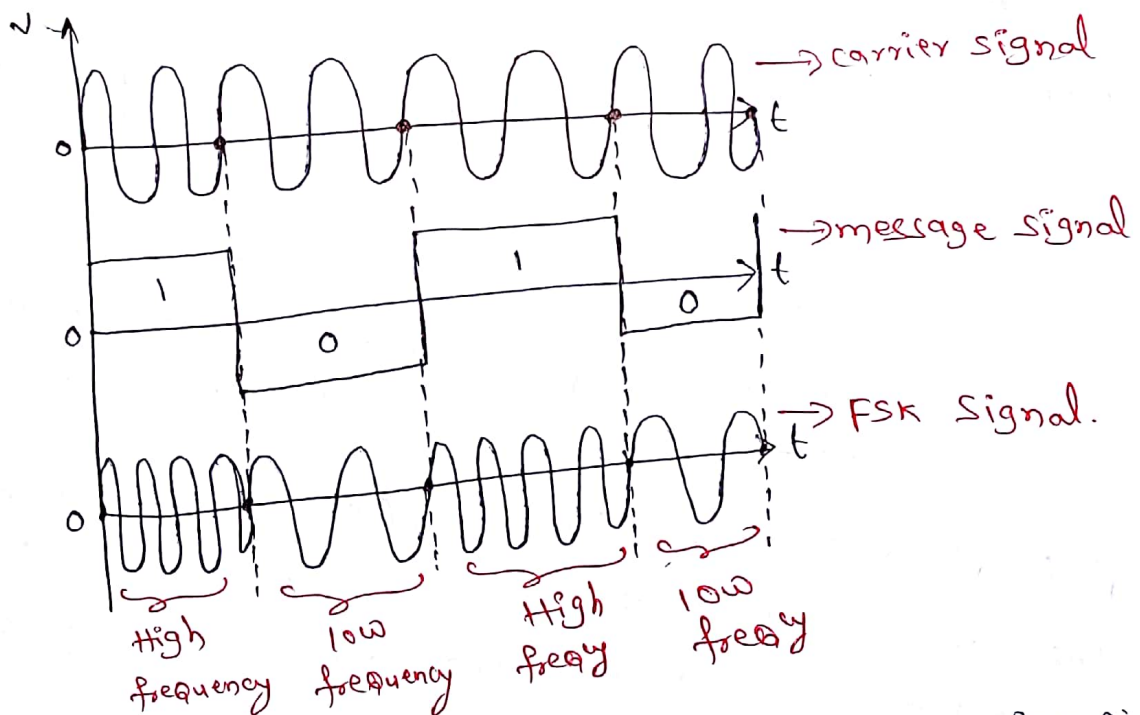
→ The amplitude of Ask signal looks like ON and OFF. Hence Ask is also called as "ON-OFF keying".

→ In Ask only amplitude is varied. Frequency and phase are constant.

In shortcut ⇒ when msg sig bit = 1 then amplitude same
when msg sig bit = 0 the amplitude = 0.

ii) Frequency shift keying := The process of varying (or) shifting frequency (f) of the carrier signal in accordance with the message signal is called "Frequency shift keying".

→ FSK waveforms are shown below.

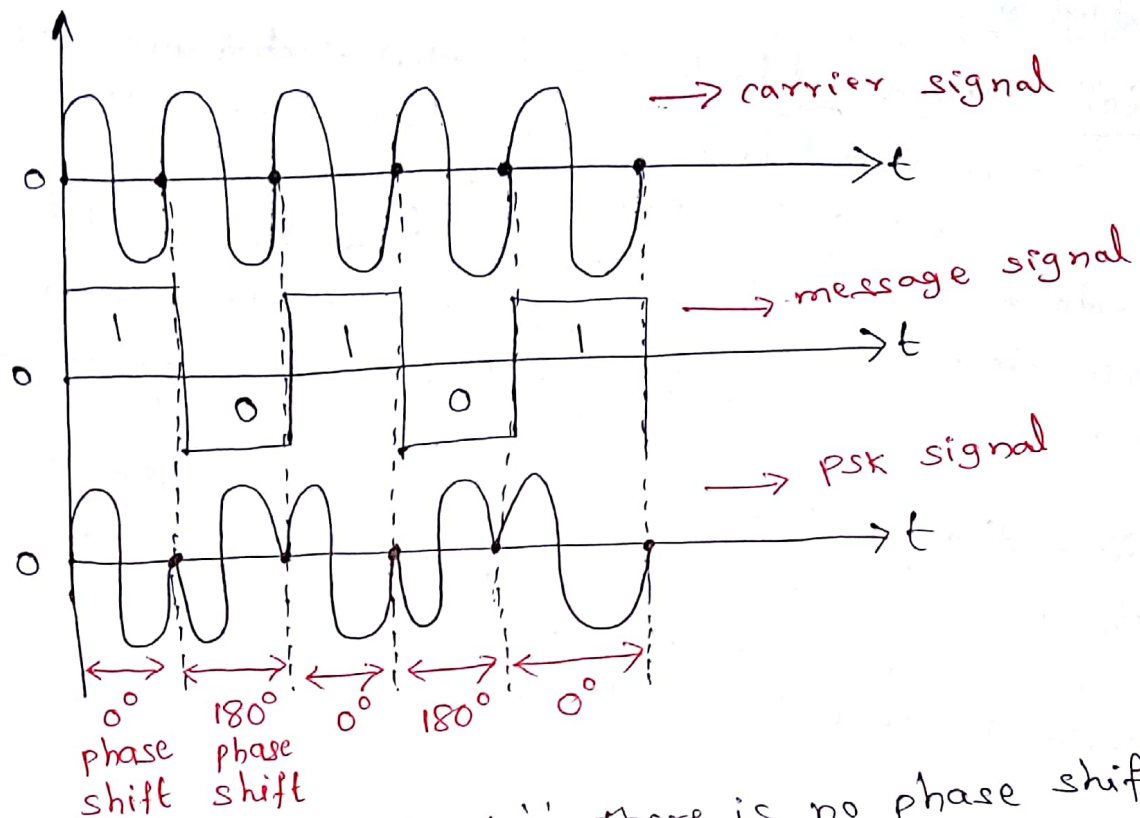


→ when message bit is '1' then High frequency carrier signal is appeared in FSK.

→ when message bit is '0' then Low frequency carrier signal is appeared in FSK.

→ In FSK only frequency is varied. Amplitude and phase are constant. In shortcut ⇒ msg '1' = High freq.
msg '0' = Low freq.

iii) phase shift keying (PSK) := The process of varying (or) shifting phase of the carrier signal in accordance with the message signal is called "phase shift keying".



- During the message bit '1' there is no phase shift in psk signal. (i.e. 0° phase shift, same carrier signal appears in psk o/p).
 - During the message bit '0' there is 180° of phase shift i.e. carrier signal with 180° phase shift appears in psk.
 - In psk only phase is varied. Amplitude and frequency are constant.
- In shortcut $\Rightarrow$ msg bit '1' = 0° phase shift.
 msg bit '0' = 180° phase shift.

* Types of Demodulation (or) Detection Techniques :=

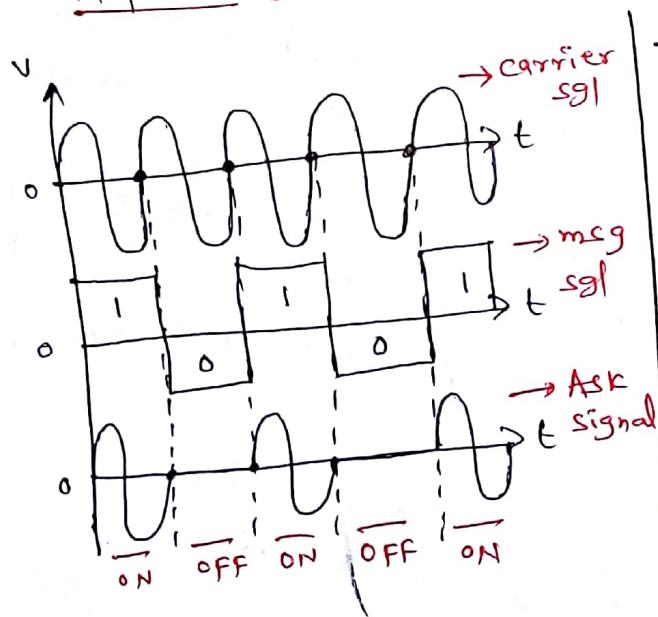
→ The process of reconstructing (or) regenerating message signal back from modulated signal (i.e. from ASK signal, FSK signal, PSK signal) is called "Demodulation" (or) "Detection".

→ There are '2' types of demodulation (or) detection techniques.

i) coherent detector := In this technique the carrier signal at the receiver is phase locked with carrier signal at the transmitter. This technique also called as "synchronous detection".

ii) Non-coherent detection := In this technique the carrier at the receiver doesn't phase locked with the carrier at the transmitter. i.e phase locking is not required. This technique is also called as "Asynchronous detection". This detection implementation is simple, but probability of occurrence is very high. Hence it is not widely used.

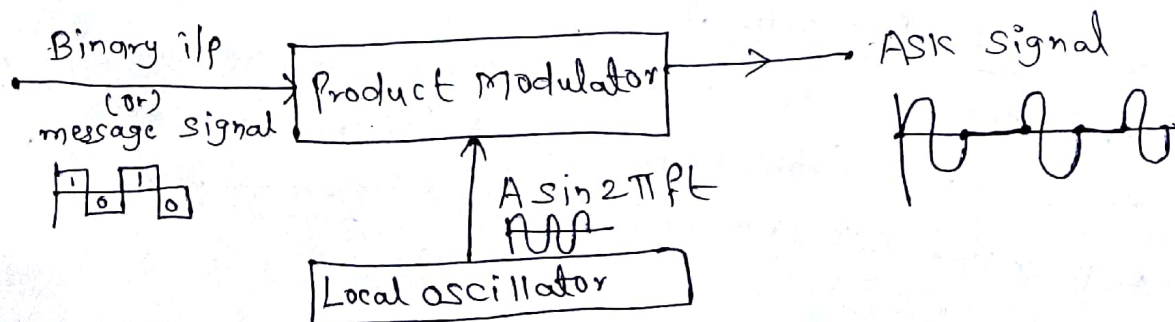
* Generation of ASK (or) ASK modulator (or) ASK Transmitter :=
 → The process of varying (or) shifting amplitude of the carrier signal in accordance with message signal is called "Amplitude shift Keying (ASK)".



→ In ASK binary '1' (or) bit '1' is considered as carrier present. And binary '0' (or) bit '0' is considered as carrier absent.

→ The mathematical representation of ASK signal is given as,

$$S(t)_{ASK} = \begin{cases} A \sin 2\pi f t; & \text{for binary '1'} \\ 0; & \text{for binary '0'}. \end{cases}$$



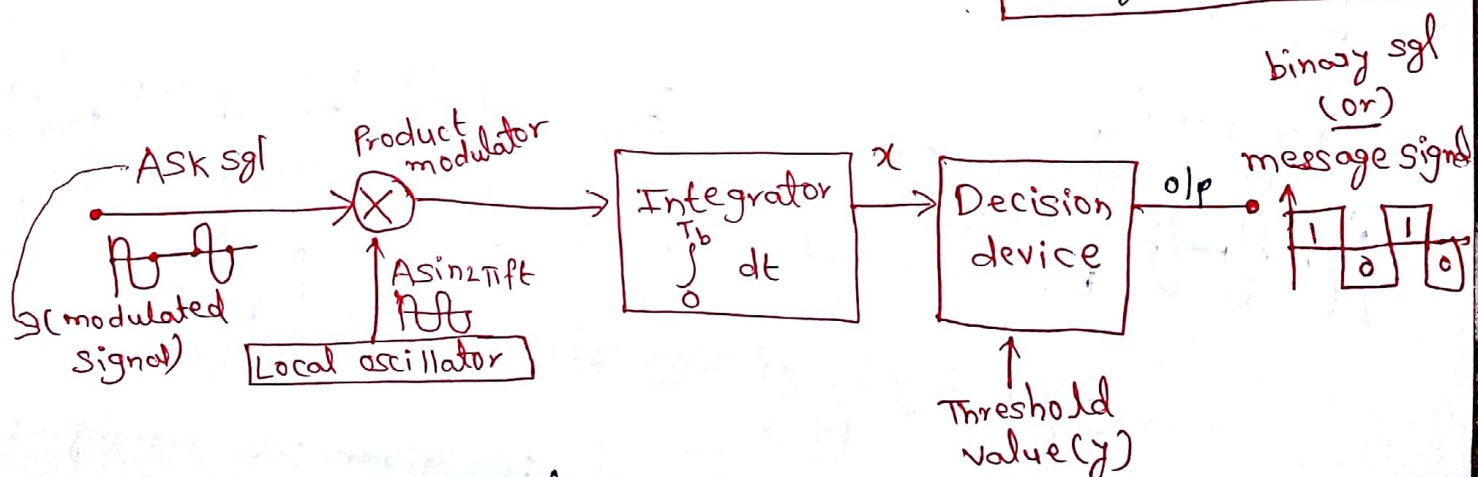
Block diagram of ASK generator..

- The Local oscillator generates a carrier signal $\sin 2\pi ft$ whose equation is $A \sin 2\pi ft$.
- The message signal and carrier signal from local oscillator are given to product modulator.
- product modulator is a device which multiplies two signals and it gives product of two signals as a o/p. (i.e it multiplies carrier signal and message signal).
- when message bit is '1' then o/p of product modulator becomes $A \sin 2\pi ft$ (i.e $1 * A \sin 2\pi ft = A \sin 2\pi ft$). Hence during msg bit '1' of message signal the ASK o/p is $A \sin 2\pi ft$ (i.e carrier present).
- when message bit '0' then o/p of product modulator becomes '0'. (i.e $0 * A \sin 2\pi ft = 0$). Hence during msg bit '0' of a message signal the ASK o/p is '0' (i.e carrier absent).

* Coherent ASK detector (or) demodulator (or) Receiver :-

- The process of reconstructing (or) regenerating original message signal (pulse signal) from modulated signal is called "demodulation" (or) "detection".

If $x > y \rightarrow$ bit '1' transmitted
If $x < y \rightarrow$ bit '0' transmitted



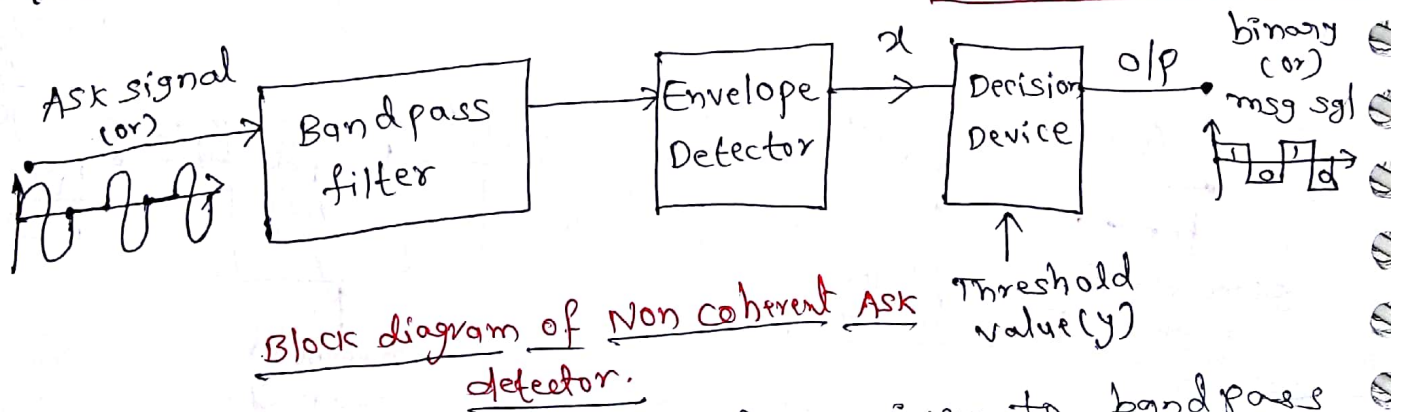
Block diagram of ASK coherent detector

- The local oscillator generates a carrier signal $A \sin 2\pi ft$ whose phase frequency is locked with carrier signal which is used in ASK generator.

- The ASK signal (modulated signal) and carrier signal from local oscillator is given to 'product modulator'.
- product modulator multiplies ASK signal and carrier.
- The output of product modulator is given to Integrator.
- Integrator integrates the output of product modulator over one bit time period ' T_b '. The o/p of integrator is denoted by ' x '.
- The o/p of integrator is given to 'Decision device'. The decision device consists of threshold value ' y '. The decision device takes a decision to transmit bit '0' to the output (or) bit '1' to the output.
- The decision device compares ' x ' and ' y ' values.
- If $x > y$ then decision device decides to transmit bit '1' to the output. If $x < y$ it transmits bit '0' to the o/p.
- Hence the o/p of coherent detector is a original message signal (pulse sgl).

(*) Non-coherent ASK detector (or) demodulation (or) Receiver

if $x > y \Rightarrow$ bit '1' transmitted
 if $x < y \Rightarrow$ bit '0' transmitted



- ASK signal (i.e. modulated signal) is given to bandpass filter.
- Bandpass filter allows only specific band of frequencies from ASK signal. And rejects all other frequencies.
- The output of BPF is given to envelope detector.

→ Envelope detectors are widely used in non coherent detection.
If the input is high for envelope detector then it generates high out.

→ The envelope detector generates high output voltage when carrier is present. i.e. 'x' value is high. The envelope detector generates low output (or) '0' output when carrier is absent. i.e. 'y' value is '0'.

→ The output of envelope detector i.e. 'x' is given to decision device. The decision device consists of threshold value 'y'. The decision device takes a decision to transmit bit '0' to the output (or) bit '1' to the output.

→ The decision device compares 'x' and 'y' values to take a decision.

→ If $x > y$ then decision device decides to transmit bit '1' to the output. If $x < y$ it transmits bit '0' to o/p.

→ Hence the o/p of non-coherent ASK detector is a original message signal.

Note := Non-coherent detector doesn't use carrier synchronization hence local oscillator is not required in this type of demodulation technique.

Advantages of ASK := i) ASK generation & ASK detection is simple.
ii) ASK needs less bandwidth.

Disadvantages of ASK := i) Noise is High.
ii) As Noise (N) is high, signal to noise ratio (S/N) is less.
iii) Probability of error is High.
iv) power efficiency (η) is Low.

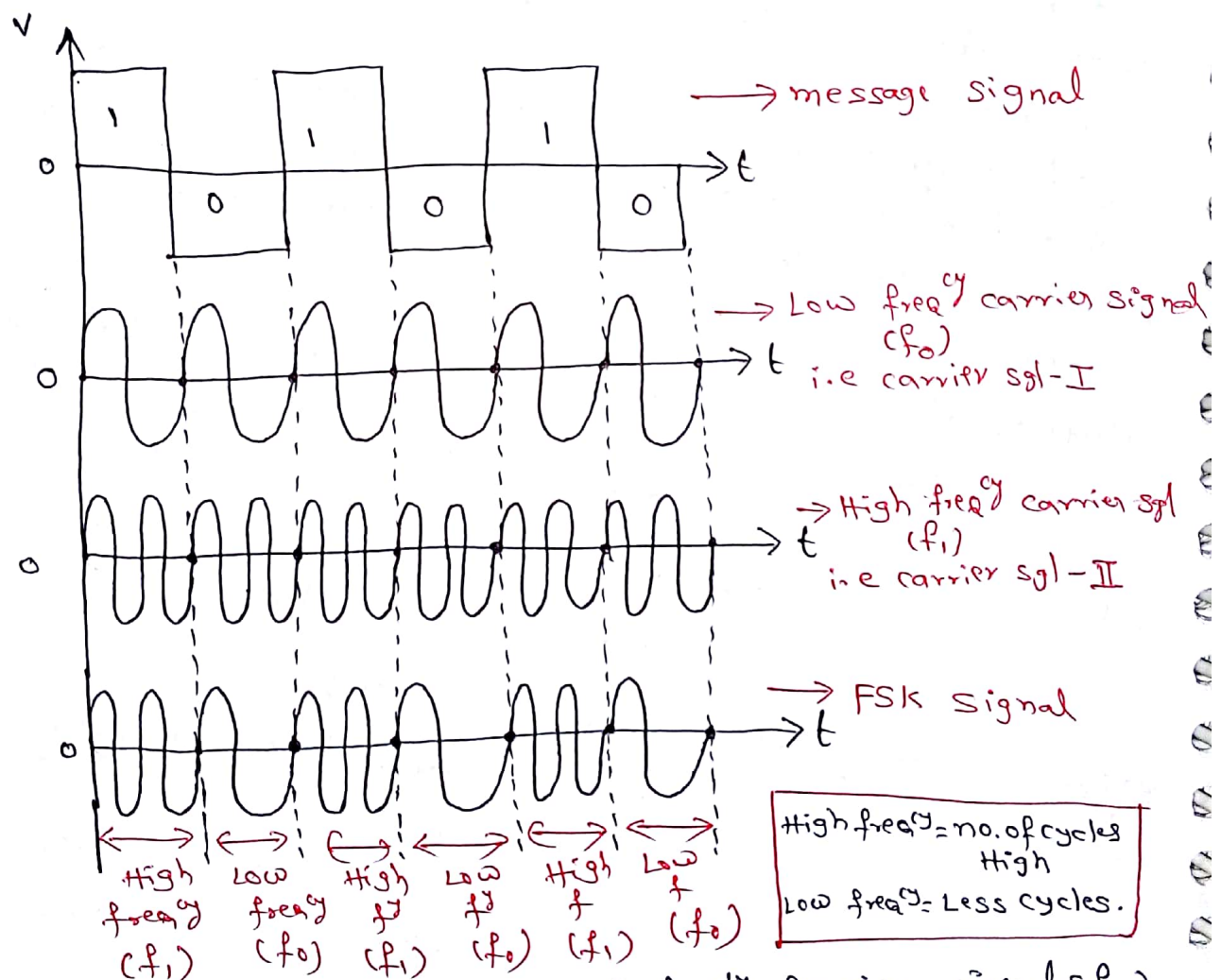
Applications of ASK := i) ASK used in telephone modems.
ii) ASK used to transmit digital data (0,1) through the optical fiber cables.

* Frequency Shift Keying (FSK) := The process of varying (or) shifting frequency of carrier signal in accordance with message signal is called "frequency shift keying".

→ In fsk two carrier signals of same amplitude, same phase but different frequencies are used to represent bit '0' and bit '1'.

→ In fsk, when message bit is '1' then high frequency carrier signal is transmitted, when message bit is '0' then low frequency carrier signal is transmitted.

→ The waveforms of FSK are shown below.

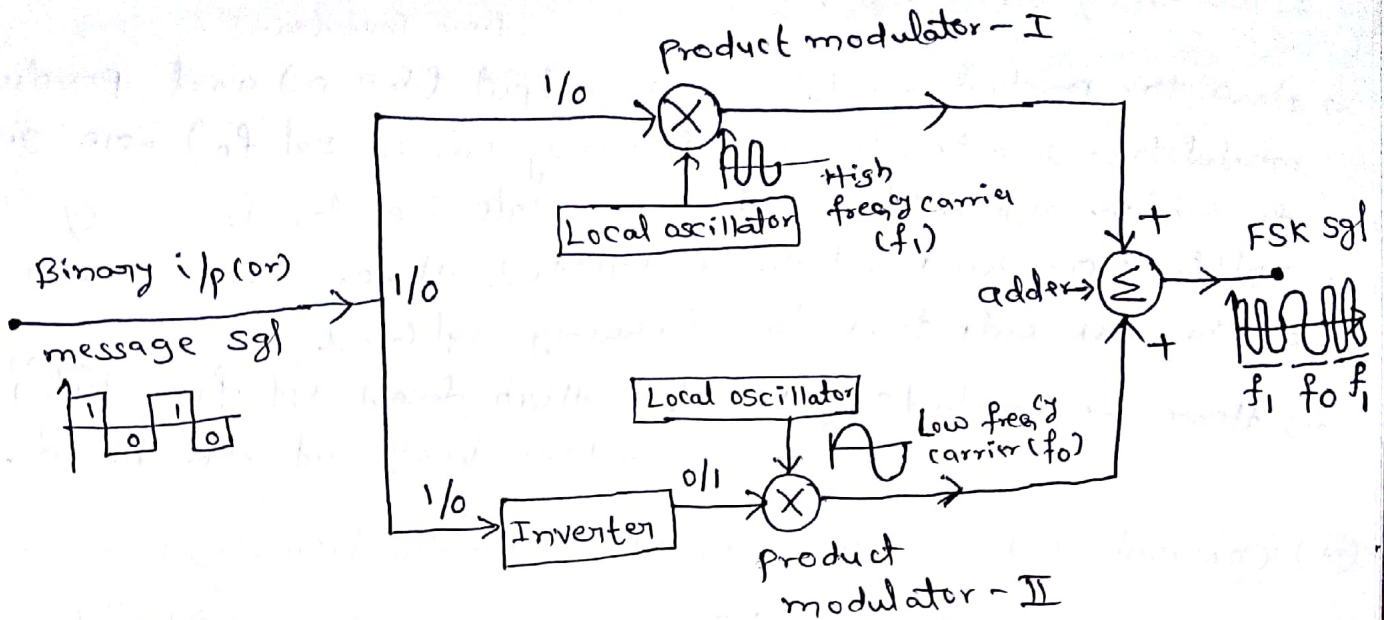


→ when message bit is '1' then High freq^y carrier signal (f_1) is appeared in o/p. when message bit is '0' then low freq^y carrier signal (f_0) is appeared in o/p.

→ The mathematical expression for fsk is given as,

$$S(t)_{\text{FSK}} = \begin{cases} A \sin 2\pi f_1 t & \text{for bit '1'} \\ A \sin 2\pi f_0 t & \text{for bit '0'}. \end{cases}$$

* FSK Generator (or) FSK modulator (or) FSK transmitter :-



Block diagram of FSK generator

- Local oscillator - I generates High frequency carrier signal whose frequency is f_1 . And local oscillator - II generates low frequency carrier signal whose frequency is f_0 .
- Inverter is a device which inverts the bit. If i/p is bit '0' it gives o/p as bit '1'. If i/p is bit '1' it gives o/p as bit '0'.
- The message signal is applied to both the product modulator - I and product modulator - II.
- product modulator - I multiplies message signal and High frequency carrier signal (f_1) and produces o/p. product mod - II multiplies message signal and Low frequency carrier signal (f_0) and produces output.
- when message bit is '1' the product modulator - I output produces high frequency signal (f_1). For the same message bit '1' product modulator - II output becomes '0'. (i.e. $0 \times \text{low frequency signal} = 0$).
- Now the product modulator - I output (High frequency signal) and product modulator - II output (i.e. 0) is given to adder. Adder will add two signals i.e. High frequency signal + 0. Hence we get high frequency signal as a FSK signal o/p.
- similarly, when message bit is '0' the product modulator - I output becomes '0'. (i.e. $0 \times \text{High frequency carrier signal} = 0$).
- for the same message bit '0' the product modulator - II

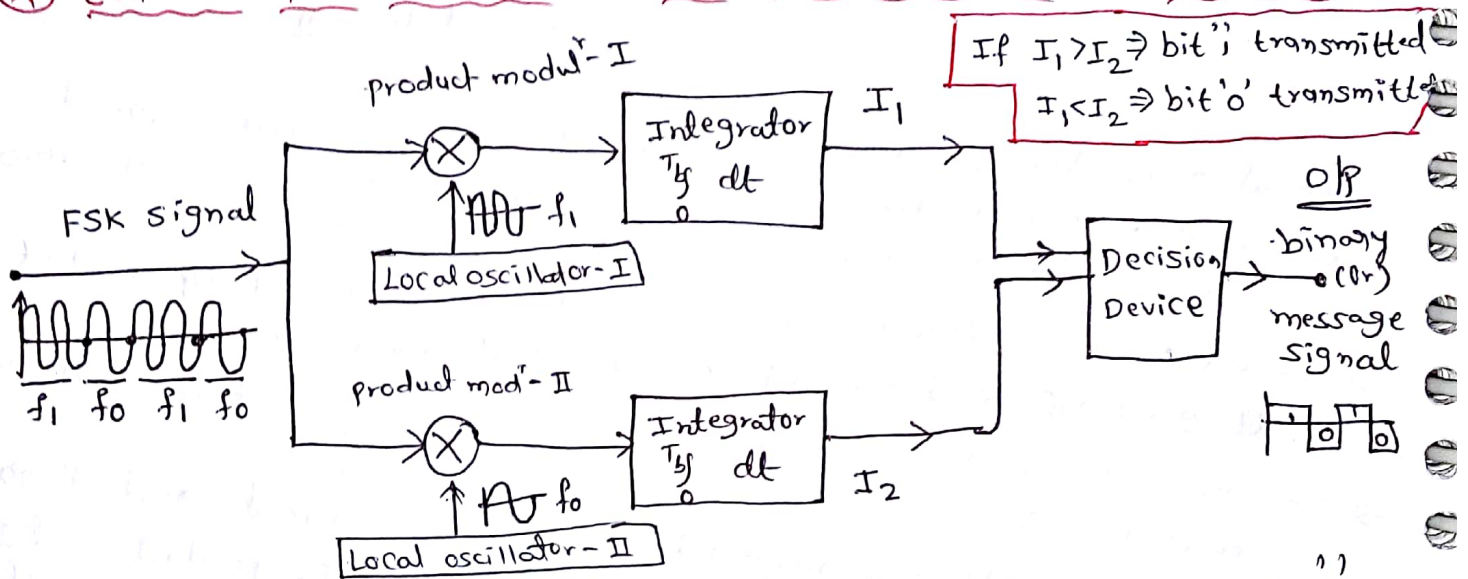
output is low frequency sgl. (i.e. 1st low freq carrier sgl = low freq carrier sgl).

↑
from inverter.

→ Now the product modulator-I output (i.e. 0) and product modulator-II output (i.e. low frequency carrier sgl f_0) are given to adder. The adder adds two sgl i.e. low frequency carrier sgl $f_0 + 0 = \text{low freq carrier sgl } (f_0)$. Hence for message bit '0' the fsk output is low frequency sgl f_0 .

→ Hence we conclude $\left\{ \begin{array}{l} \text{fsk o/p} = \text{high freq sgl for bit '1'.} \\ \text{= Low freq sgl for bit '0'.} \end{array} \right.$

* Coherent FSK detection (or) FSK demodulation (or) FSK receiver:-



→ Local oscillator-I generates high frequency carrier signal f_1 and Local oscillator-II generates low frequency carrier signal f_0 .

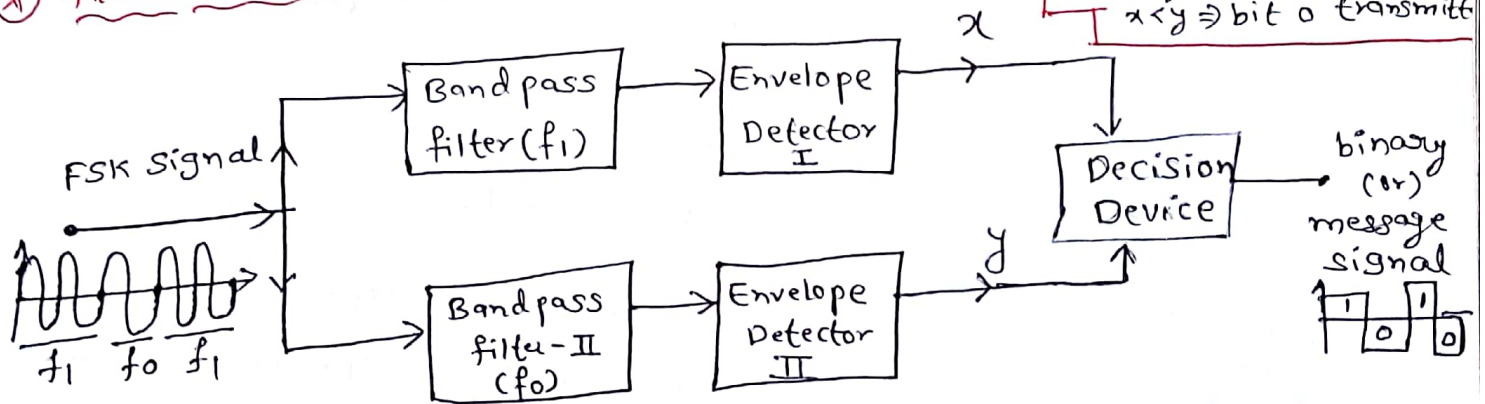
→ FSK signal and high frequency carrier signal is multiplied by product modulator-I. FSK signal and low frequency carrier signal (f_0) are multiplied at product modulator-II.

* when FSK signal is high frequency signal (f_1) then product modulator-I will be synchronized hence PM-I produces output. For the same high frequency (f_1) of FSK signal, product modulator-II doesn't synchronize. Hence PM-II produces '0' output.

* when FSK signal is low frequency signal (f_0) then product modulator-I doesn't synchronize with it. Hence PM-I output is '0'. For the same low frequency signal (f_0) of FSK signal, product modulator-II will be synchronized. Hence PM-II produces o/p.

- The output of product modulator-I and product modulator-II are given to Integrator-I and Integrator-II.
- Integrator-I integrates the output of product modulator-I for each bit time period ' T_b '.
- Integrator-II integrates the output of product modulator-II for each bit time period ' T_b '.
- The output of Integrator-I is denoted by ' I_1 ' and o/p of Integrator-II is denoted by ' I_2 '.
- The decision device decides whether to transmit bit '1' to the o/p (or) bit '0' to the o/p.
- Decision device compares I_1 and I_2 values to take a decision.
- If $I_1 > I_2$ then bit '1' is transmitted to the output. If $I_1 < I_2$, then bit '0' is transmitted to the output.
- Hence the output of coherent FSK detector is original message signal.

* Non-coherent FSK detection (or) FSK demodulation (or) FSK receiver



Block diagram of non-coherent FSK demodulator/detector

- FSK signal can be demodulated non coherently by using envelope detector.
- FSK signal (modulated signal) is applied to bandpass filters in which one bandpass filter is tuned to high frequency (f_1) and another bandpass filter is tuned to low frequency (f_0). Each bandpass filter is followed by envelop detector.
- Bandpass filter-I allows only frequency ' f_1 ' and rejects ' f_0 '.
- Band pass filter-II allows only ' f_0 ' and rejects ' f_1 ' frequency.

- when fsk signal is having high frequency (f_1) then it is passed only through band pass filter - I. The o/p of bpf-I is given to envelope detector - I. As the i/p is high frequency (f_1) to the envelope detector - I it produces high o/p (x).
- for the same high frequency (f_1) it doesn't allowed by the band pass filter - II. Hence BPF-II output is '0', Envelope detector - II output i.e 'y' also zero.
- By comparing the values 'x' and 'y' the decision device takes a decision whether to transmit bit '0' (or) bit '1' to the output. If $x > y$ then decision device takes a decision to transmit bit '1' to the output. And if $x < y$ decision device decides to transmit bit '0' to the output.
- Hence the output of non-coherent fsk detector is a original message signal.

Advantages of FSK :=

- i) Noise is Low.
- ii) power efficiency (η) is more.

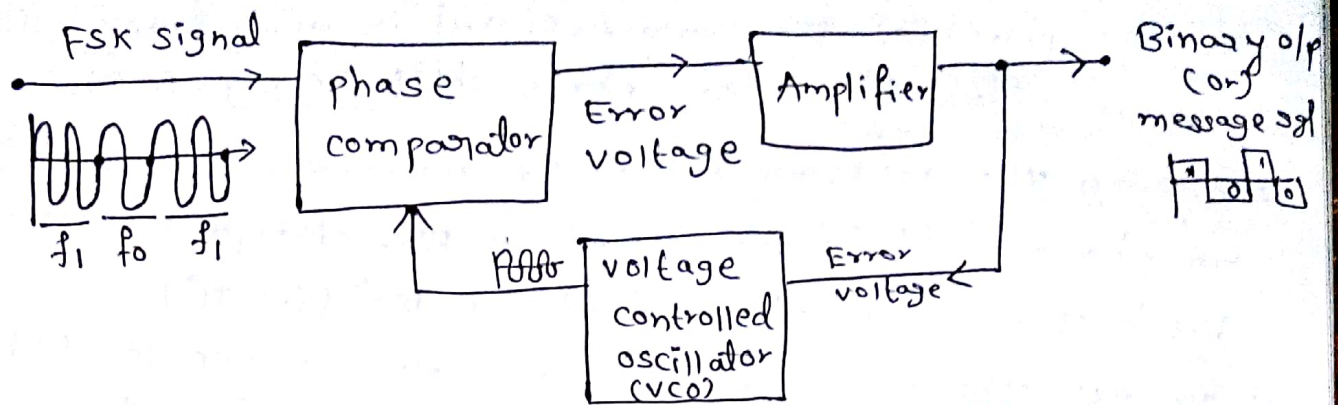
Disadvantages of FSK :=

- i) FSK requires high bandwidth
- ii) FSK generation & detection is complex.
- iii) Bit rate is low.
- iv) High cost.

Applications of FSK :=

- i) FSK used in telephone systems.
- ii) FSK used in telephone modems.
- iii) FSK used in high frequency radio transmissions.

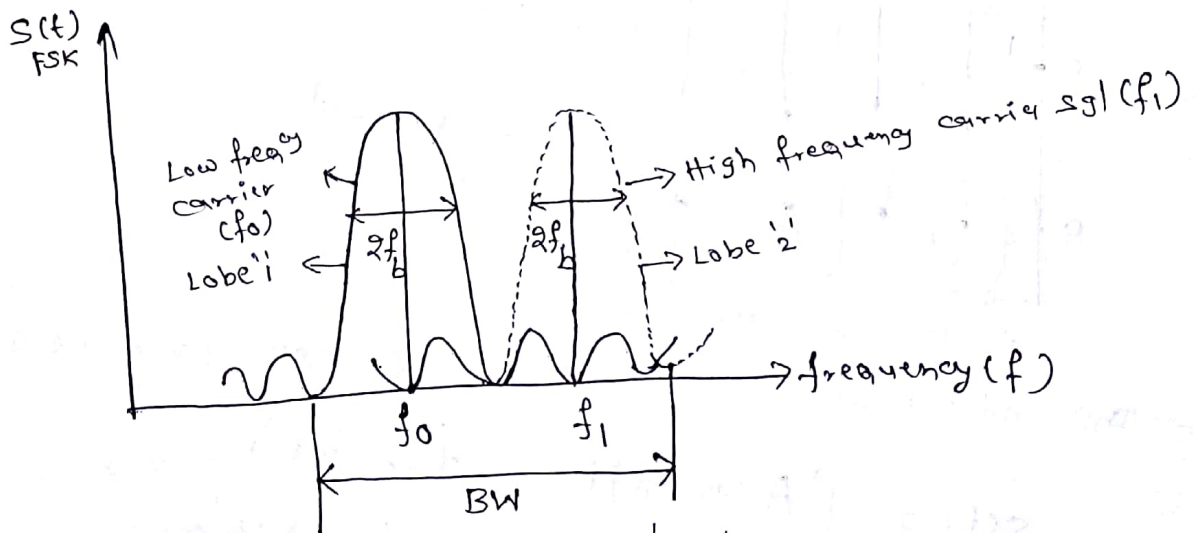
- (*) FSK detection using PLL (Phase Locked Loop) :=
- phase Locked Loops are used in demodulation (or) detection of fsk signal.
- The error voltage in PLL is proportional to the difference between phase (or) frequency of input fsk signal and VCO signal.



Block diagram of FSK detection using PLL

- The voltage controlled oscillator (VCO) generates free running frequency between f_1 and f_0 .
- The phase comparator compares the phase of FSK signal and VCO signal. The phase difference is known as error voltage.
- When FSK signal is having high frequency (f_1) then error voltage becomes +ve and o/p becomes bit '1' as long as FSK signal is having high frequency (f_1).
- When FSK signal is having low frequency (f_0) then error voltage becomes -ve and o/p becomes bit '0' as long as FSK signal is having low frequency (f_0).

* Frequency spectrum & Bandwidth of FSK :=



- The bandwidth of one lobe is $2f_b$.
- ∴ The bandwidth of FSK due to two lobes is given as,

$$(BW)_{FSK} = 2f_b \xrightarrow{\text{lobe-1}} + 2f_b \xrightarrow{\text{lobe-2}}$$

$$\boxed{(BW)_{FSK} = 4f_b} ; \text{ where } f_b = \text{one bit frequency.}$$

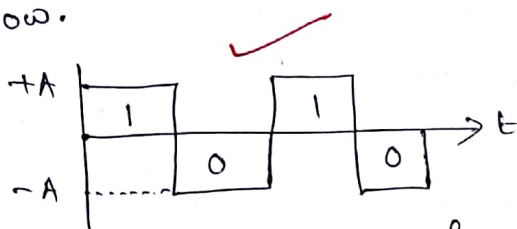
$$T_b = \text{one bit period. (time)}$$

* Phase Shift Keying (PSK) := The process of varying (or) shifting phase of the carrier signal in accordance with the message signal is called "phase shift keying (PSK)".

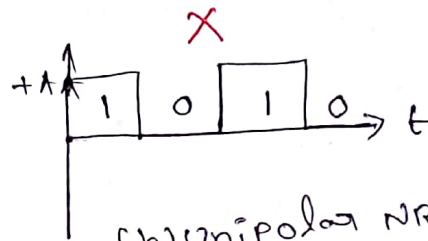
→ During the message bit '1' of message signal there is no phase shift (0° phase shift) in PSK. During the message bit '0' there is a phase shift of 180° (i.e. π).

→ In PSK the message signal is always represented by Bipolar Non Return to Zero (Bipolar NRZ) format as shown

↓
'2 polarities
below.

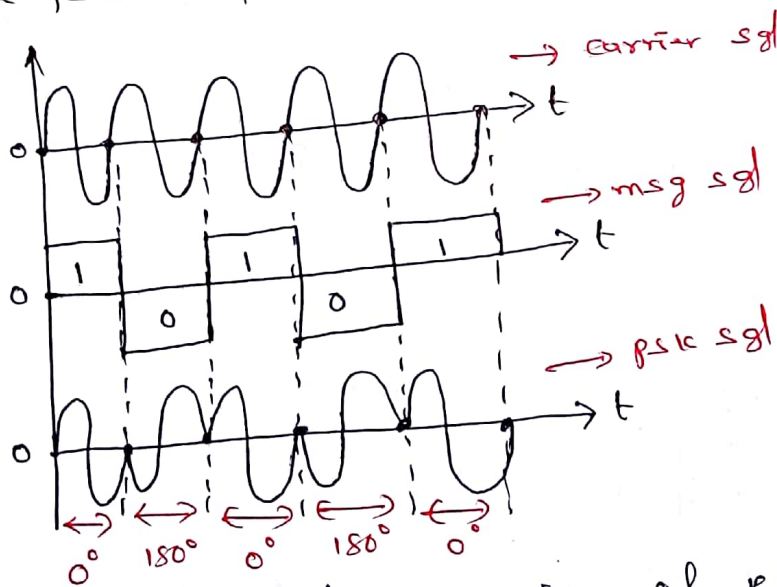


(a) Bipolar NRZ format



(b) Unipolar NRZ format.

→ The PSK waveforms are shown below,



→ The mathematical expression of PSK given below,

$$s_{psk}(t) = \begin{cases} A \cos 2\pi f t & \text{for bit '1'} \\ A \cos(2\pi f t + 180^\circ) & \text{for bit '0'} \end{cases}$$

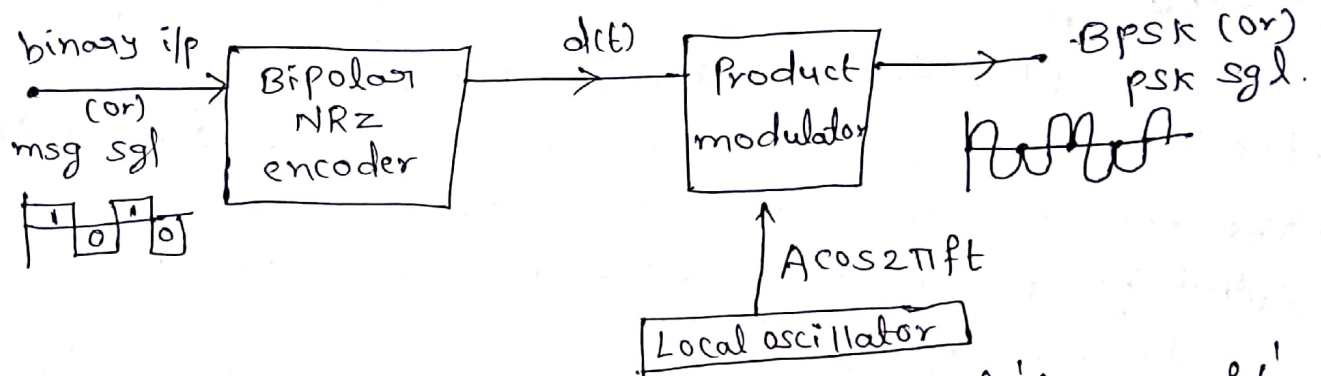
$\hookrightarrow -A \cos 2\pi f t \quad \left\{ \because \cos(180^\circ + \theta) = -\cos \theta \right\}$

$\hookrightarrow$ phase shift of 180°

* carrier is taken as cos wave.

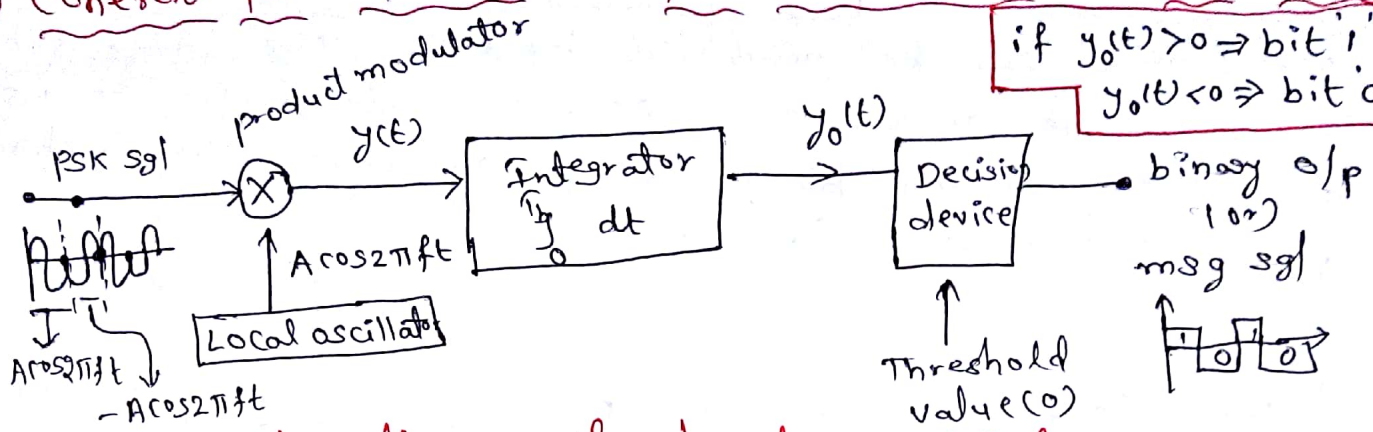
* psk generator (or) psk modulator (or) psk transmitter:-

→ psk signal can be generated by using the same scheme used in the generation of ASK. The only difference is that the incoming binary data (or) message signal should be in bipolar NRZ format.



- Local oscillator generates a carrier signal ' $A\cos 2\pi ft$ '.
- Message signal (or) binary i/p is given to bipolar NRZ encoder.
- When i/p is '0' to the bipolar NRZ encoder it gives the output as '-1'. When input is '1' to the bipolar NRZ encoder it gives the output as '1'. i.e. $d(t) = 1$.
- When message bit is '0' then o/p of bipolar NRZ encoder $d(t)$ will be '-1'. Thus o/p of balanced modulator (or) product modulator will be $-A\cos 2\pi ft$ (i.e. $-1 \times A\cos 2\pi ft = -A\cos 2\pi ft$). It indicates that phase is shifted by 180° .
- When message bit is '1' then o/p of bipolar NRZ encoder $d(t)$ will be '1'. Thus o/p of balanced modulator (or) product modulator will be $A\cos 2\pi ft$ (i.e. $1 \times A\cos 2\pi ft = A\cos 2\pi ft$). It indicates there is no phase shift.
- Thus the resultant o/p is psk signal.

* Coherent PSK detector (or) PSK demodulator (or) PSK Receiver



Block diagram of coherent PSK detector.

- Local oscillator generates a carrier signal ' $A \cos 2\pi ft$ '.
- The incoming PSK signal and carrier signal are multiplied by product modulator. The o/p of product modulator is denoted by ' $y(t)$ '.
- The o/p of PM ' $y(t)$ ' is applied to integrator. Now it performs integration over one bit period ' T_b '. The o/p of integrator is denoted by ' $y_0(t)$ '.
- The o/p of integrator ' $y_0(t)$ ' is compared with a threshold value of decision device.
- If $y_0(t) > 0$ then decision device decides to transmit bit '1' to the output. If $y_0(t) < 0$ then decision device decides to transmit bit '0' to the output.
- Case (i) :- consider first time slot of PSK signal, i.e. $s_{PSK}(t) = A \cos 2\pi ft$

The o/p of product modulator will be,

$$y(t) = \text{PSK s}_g * \text{carrier s}_g$$

$$y(t) = (A \cos 2\pi ft) * (A \cos 2\pi ft)$$

$$y(t) = A^2 \cos^2 2\pi ft$$

$$y(t) = A^2 \left(\frac{1 + \cos 4\pi ft}{2} \right) \left\{ \because \cos^2 \theta = \frac{1 + \cos 2\theta}{2} \right\}$$

Now this o/p $y(t)$ is given as i/p to the integrator. It performs integration over 0 to T_b .

The o/p of integrator $y_0(t)$ is given as,

$$y_0(t) = \int_0^{T_b} y(t) dt$$

$$y_0(t) = \int_0^{T_b} \frac{A^2}{2} (1 + \cos 4\pi f t) dt$$

$$y_0(t) = \frac{A^2}{2} \left[\int_0^{T_b} 1 \cdot dt + \int_0^{T_b} \cos 4\pi f t \cdot dt \right]$$

$$y_0(t) = \frac{A^2}{2} \left[t \Big|_0^{T_b} + \frac{\sin 4\pi f t}{4\pi f} \Big|_0^{T_b} \right]$$

$$y_0(t) = \frac{A^2}{2} \left[(T_b - 0) + \left(\frac{\sin 4\pi f T_b}{4\pi f} - 0 \right) \right]$$

→ Integrator acts as a low pass filter. hence it is removed

$$\therefore y_0(t) = \frac{A^2 T_b}{2} > 0.$$

→ As $y_0(t)$ is greater than '0', hence decision device takes a decision to transmit bit '1' to the o/p.

→ case (ii) :- consider second time slot of PSK signal,
i.e. $s_{psk}(t) = -A \cos 2\pi f t$;

The o/p of product modulator $y(t)$ will be,

$$y(t) = s_{psk} \text{ sgl} * \text{carrier sgl.}$$

$$y(t) = (-A \cos 2\pi f t) * (A \cos 2\pi f t)$$

$$y(t) = -A^2 \cos^2 2\pi f t;$$

$$y(t) = -\frac{A^2}{2} (1 + \cos 4\pi f t);$$

Now this o/p is given as i/p to integrator. The o/p of the integrator is given as,

$$y_0(t) = \int_0^{T_b} y(t) \cdot dt$$

$$y_0(t) = \int_0^{T_b} -\frac{A^2}{2} (1 + \cos 4\pi f t) dt$$

$$y_0(t) = -\frac{A^2}{2} \left[\int_0^{T_b} 1 \cdot dt + \int_0^{T_b} \cos 4\pi f t \cdot dt \right]$$

$$y_0(t) = -\frac{A^2}{2} \left[(\tau_b - 0) + \left(\frac{\sin 4\pi f \tau_b}{4\pi f} - 0 \right) \right]$$

↳ Integrator also acts as low pass filter. Hence it is removed.

$$\therefore y_0(t) = -\frac{A^2}{2} (\tau_b) < 0$$

→ As $y_0(t)$ is less than 0. decision device decides to transmit bit '0' to the output.

→ Hence ops of coherent psk detector is a msg sgl.

* Composition of ASK, FSK, PSK techniques :-

S.no	parameter	Amplitude shift keying (ASK)	Frequency shift keying (FSK)	phase shift keying (PSK)
1)	Definition	Amplitude of the carrier is varied in accordance with message signal	Frequency of the carrier is varied in accordance with message signal	phase of the carrier is varied with message sgl
2)	Variable parameter	Amplitude	Frequency	phase
3)	constant parameter	freq & phase	Amplitude & phase	Amp & freq
4)	Bandwidth	low ($2f_b$)	High ($4f_b$)	low ($2f_b$)
5)	noise	High	Low	Low
6)	S/N ratio	Low	High	High
7)	Data rate	Less	Less	High
8)	polarity of msg signal	unipolar / bipolar	Uni / bipolar	bipolar
9)	Applications	Telephone modems. optical fibres	telephone modems. telephone ex-changes.	modems, satellite commn, DTH, video broadcast.

Advantages of PSK :=

- i) Noise is low.
- ii) High data rate.
- iii) PSK generation is simple.
- iv) more efficiency (n).

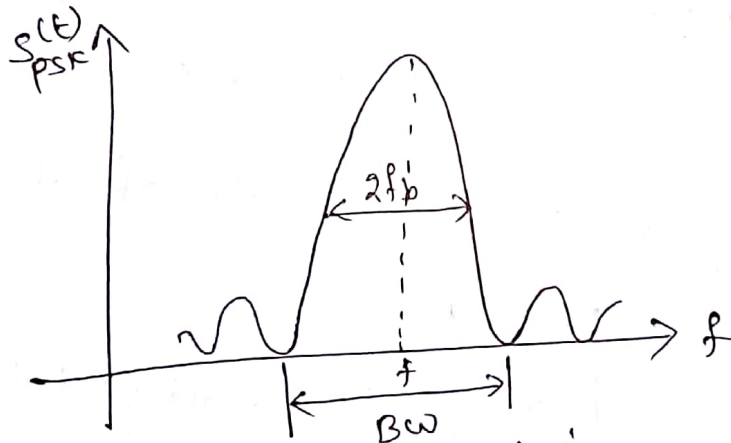
disadvantages of PSK :=

- i) PSK detection is complex
- ii) bit rate is low.

Applications of PSK :=

- i) PSK used in satellite communication.
- ii) PSK used in video broadcasting.
- iii) used in telephone modems.
- iv) used in DTH technology.

* Frequency spectrum and bandwidth of PSK (or) BPSK :=



Bandwidth of one lobe is $2f_b$.

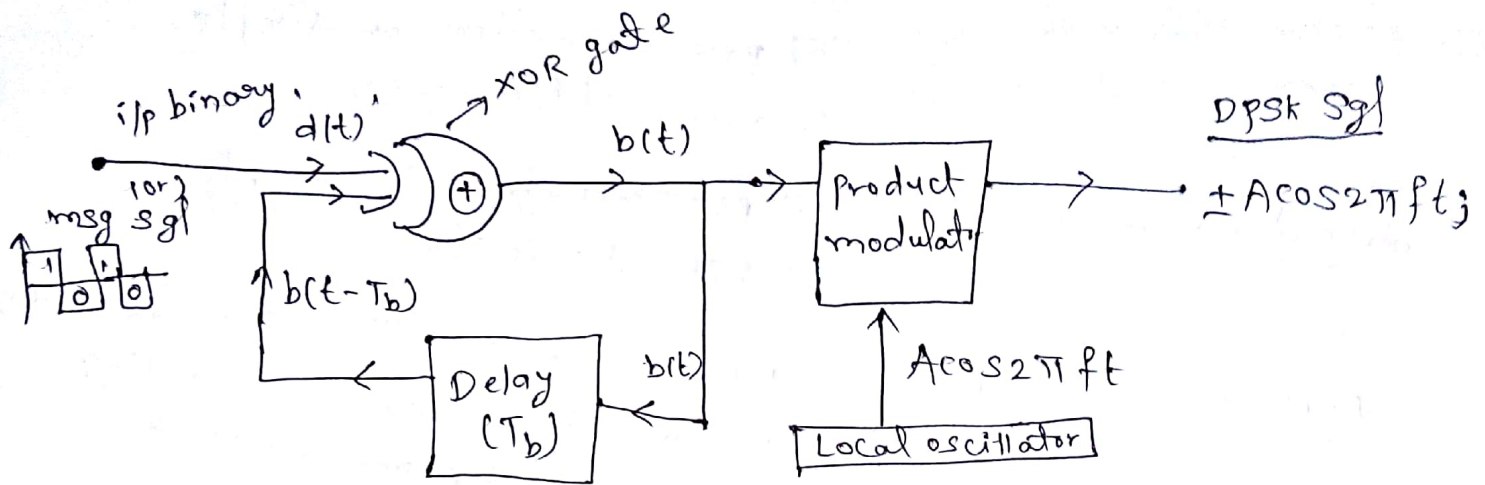
$$\therefore (Bw)_{PSK} = 2f_b \quad ; \text{ where } f_b = \text{one bit frequency.}$$

* Differential phase shift keying (DPSK) :=

→ DPSK is a modification of PSK that avoids the need of carrier synchronization in detection (or) demodulation.

→ In DPSK next bit depends on previous bit hence in the receiver (or) detection process previous received bits are used to detect present binary bit.

DPSK generator (or) DPSK transmitter (or) DPSK modulator



Block diagram of DPSK generator.

→ The input binary signal (or) message signal is denoted by $d(t)$. The output of XOR gate is $b(t)$. And $b(t-T_b)$ is the previous output delayed by one bit period T_b .

→ Depending on the values of $d(t)$ and $b(t-T_b)$ the XOR gate generates the output sequence $b(t)$.

→ Following figure shows the truth table of above XOR gate

$d(t)$	$b(t-T_b)$	$b(t)$
0	0	0
0	1	1
1	0	1
1	1	0

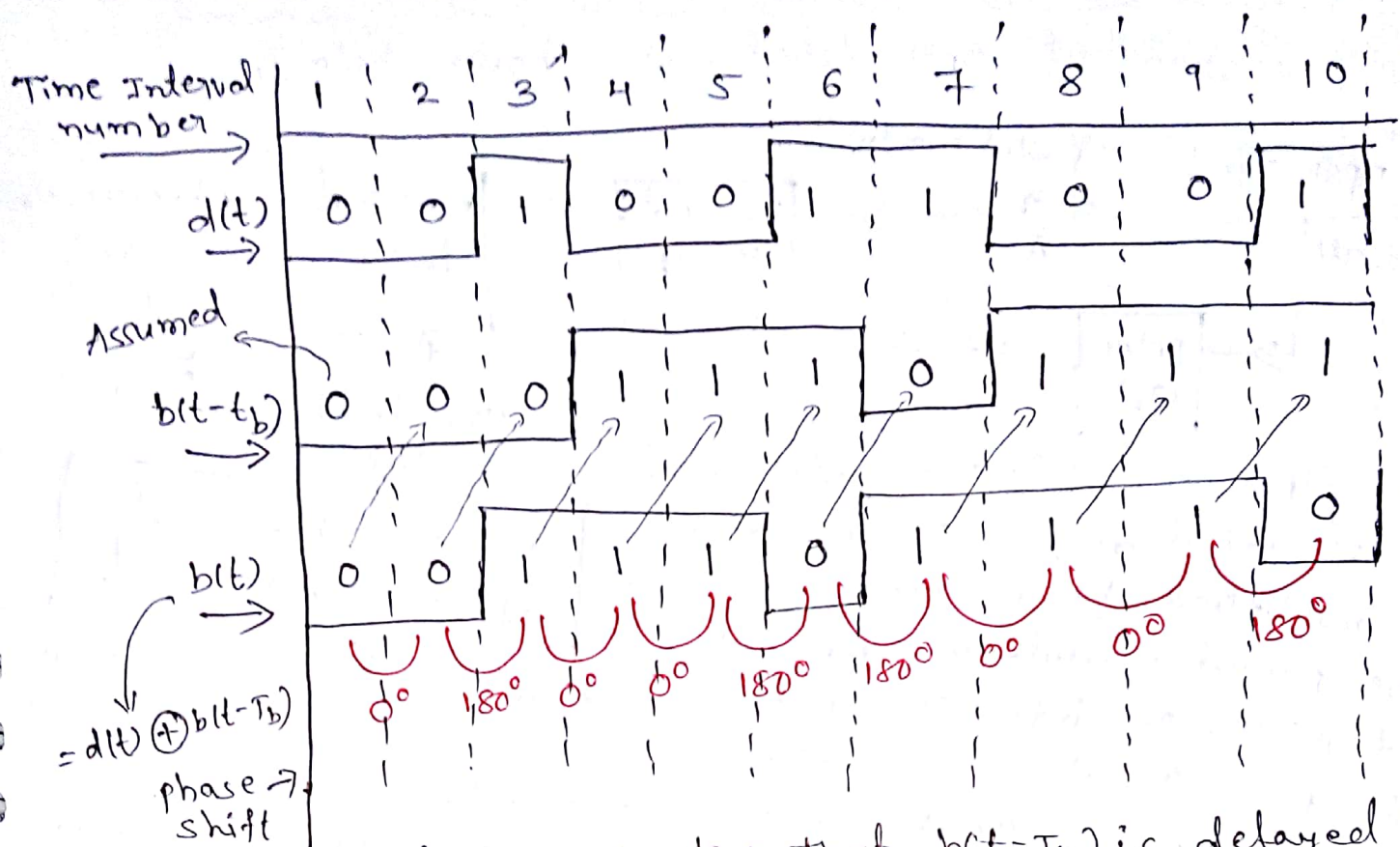
$$\Rightarrow b(t) = d(t) \oplus b(t-T_b)$$

→ Following fig. shows the waveforms of DPSK.

→ Assume input binary (or) message signal is i.e. $d(t)$ is

0 0 1 0 0 1 1 0 0 1.

→ Assume initially $b(t-T_b)$ is '0'. i.e. previous o/p is '0'.



→ From the waveform it is clear that $b(t-T_b)$ is delayed version of $b(t)$ by one bit time period T_b . and $b(t) = d(t) \oplus b(t-T_b)$

→ The value of $b(t-T_b)$ is not known initially in the interval no. 1, hence it is assumed to be zero and waveforms are drawn.

→ From the waveforms it is clear that,
 when $d(t) = 0$ then $b(t) = b(t-T_b)$ and
 when $d(t) = 1$ then $b(t) = \overline{b(t-T_b)}$.

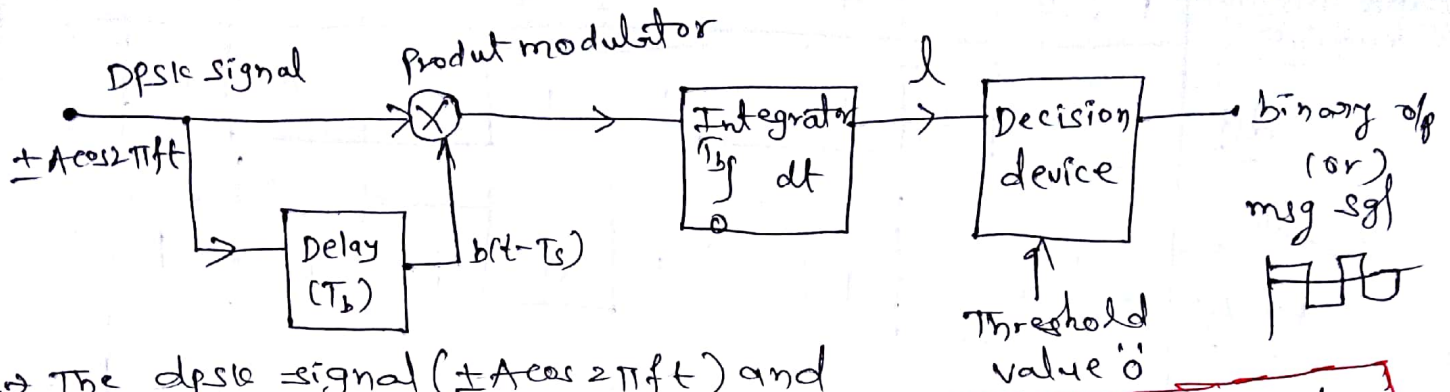
→ From the waveforms it is clear that, $b(t)$ changes its level only if $d(t) = 1$. Hence it shows that phase of the carrier is changed only if msg bit $d(t) = 1$.

$$\therefore s(t)_{\text{dpsk}} = \begin{cases} +A \cos 2\pi f t; & \text{when } d(t) = 0 \\ -A \cos 2\pi f t; & \text{when } d(t) = 1. \end{cases}$$

↳ phase shift of 180°.

* DPSK detector (or) dpsk demodulation (or) dpsk receiver :-

→ Block diagram of DPSK detector is shown below,



→ The dpsk signal ($\pm A \cos 2\pi f t$) and one bit period delayed signal $b(t-T_b)$ are multiplied by product modulator.

→ The product modulator output is given to integrator. The integrator performs the integration over one bit period T_b . Next the decision device takes a decision to transmit bit '1' (or) bit '0' to the output.

If $I > 0 \Rightarrow$ bit '0' txd
If $I < 0 \Rightarrow$ bit '1' txd.

* → When phase of the DPSK signal and phase of the one bit delayed signals are same (i.e. $++$ or $--$) then output of product modulator will be positive. Then the integrator output I becomes positive, i.e. $I > 0$. Hence decision device decides to transmit bit '0' to the output.

* → When phase of the DPSK signal and phase of the one bit delayed signals are ~~not~~ same (i.e. $+-$ (or) $-+$) the output of product modulator becomes negative. The integrator output I becomes negative, i.e. $I < 0$. Hence decision device decides to transmit bit '1' to the output.

→ Hence the output of dpsk receiver is same as message sgl.

→ Here we are not using any carrier in this demodulation.

hence it comes under non-coherent dpsk receiver.

* Bandwidth of DPSK :- In DPSK one previous bit is used to decide the phase shift of next bit. Hence the bandwidth of DPSK is equal to one bit frequency f_b .

i.e. $(BW)_{DPSK} = f_b$

f_b = one bit frequency;

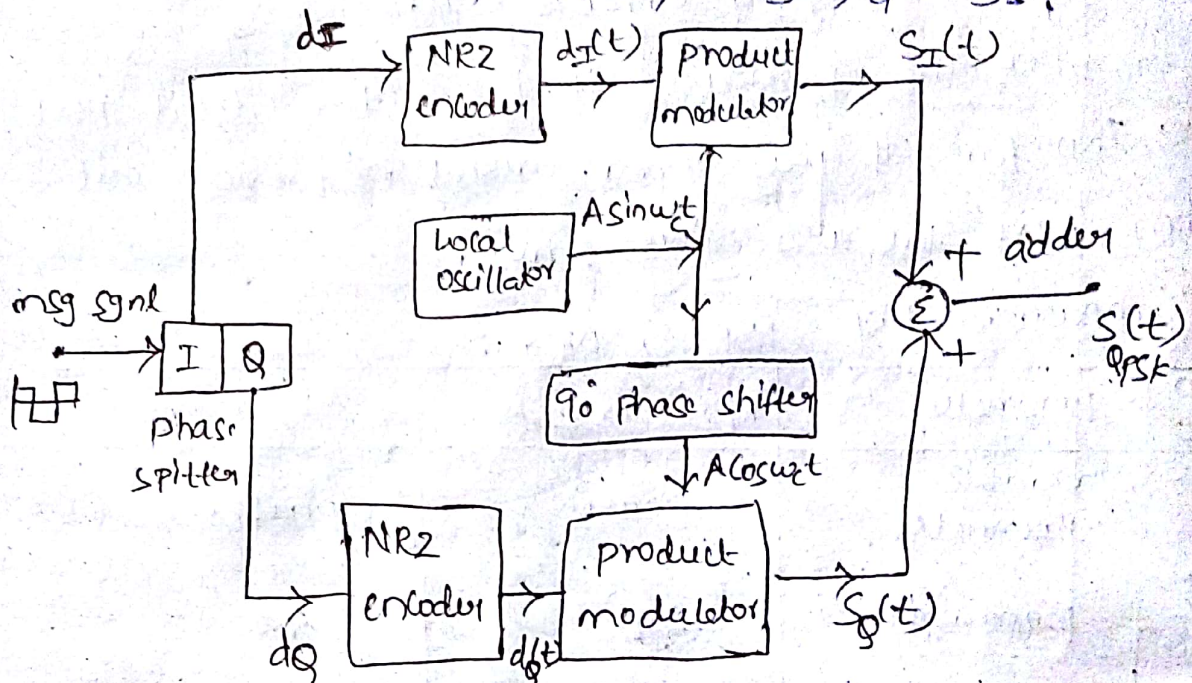
- ⊛ Advantages of dpsk := i) DPSK doesn't need carrier at its receiver. Hence complicated circuitry for detector is avoided.
ii) The required bandwidth is reduced compared to PSK.

- ⊛ Disadvantages of dpsk := i) The probability of error (or) bit error rate is high.
ii) In dpsk the previous bit is used to detect next bit. Hence if error is present in previous bit, the next bit also gives error.

⊛ comparison of psk and DPSK :=

S. no	parameter	BPSK (or) PSK	DPSK
1)	Variable parameter.	phase	phase
2)	Bandwidth	$2f_b$	only ' f_b '
3)	probability of error (P_e)	Low	High
4)	Noise effect	Low	High
5)	Detection method	coherent detection	non coherent detection.

* Quadrature of phase shift keying [QPSK]: [QPSK modulator block diagram]
 → QPSK Transmitter ² bits as ^{a symbol} like 00, 01, 10 & 11. Hence there may be 4 types of phase shifts $+45^\circ$, -45° , $+135^\circ$, & -135° .



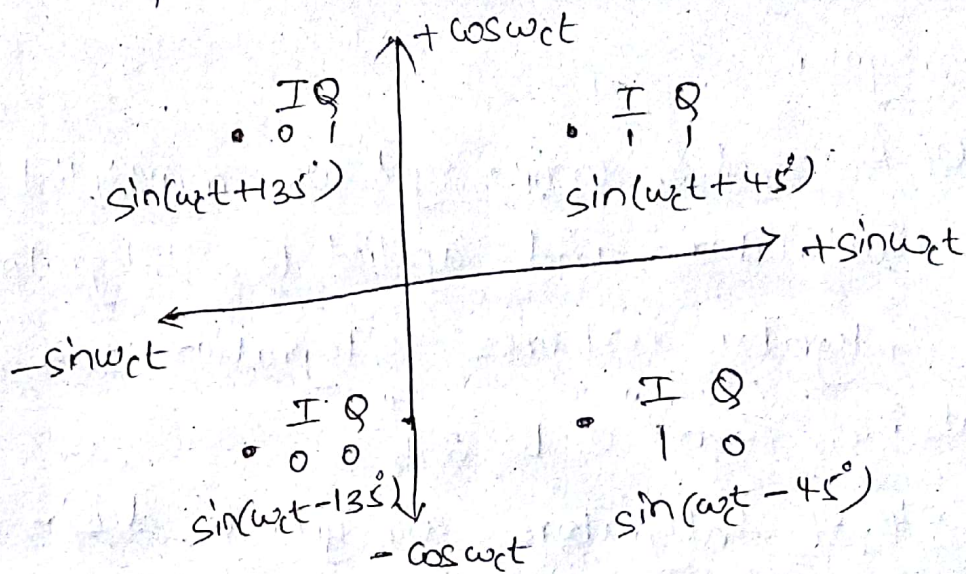
→ The phase splitter separates even number bits and odd number bits.

→ If the i/p is "0", the o/p of NRZ encoder is "-1". If the i/p is "1", then o/p of NRZ encoder is also "1".

→ The following table shows the phase shift of QPSK modulator

msg bits		phase shift of QPSK sgnl
I	Q	
0	0	-135°
0	1	$+135^\circ$
1	0	-45°
1	1	$+45^\circ$

→ The following figure shows the phasor diagram of QPSK.



* QPSK demodulator

If $I=0, Q=0 \Rightarrow -A \sin \omega_c t, -A \cos \omega_c t$

$$S(t) = -A \sin \omega_c t - A \cos \omega_c t$$

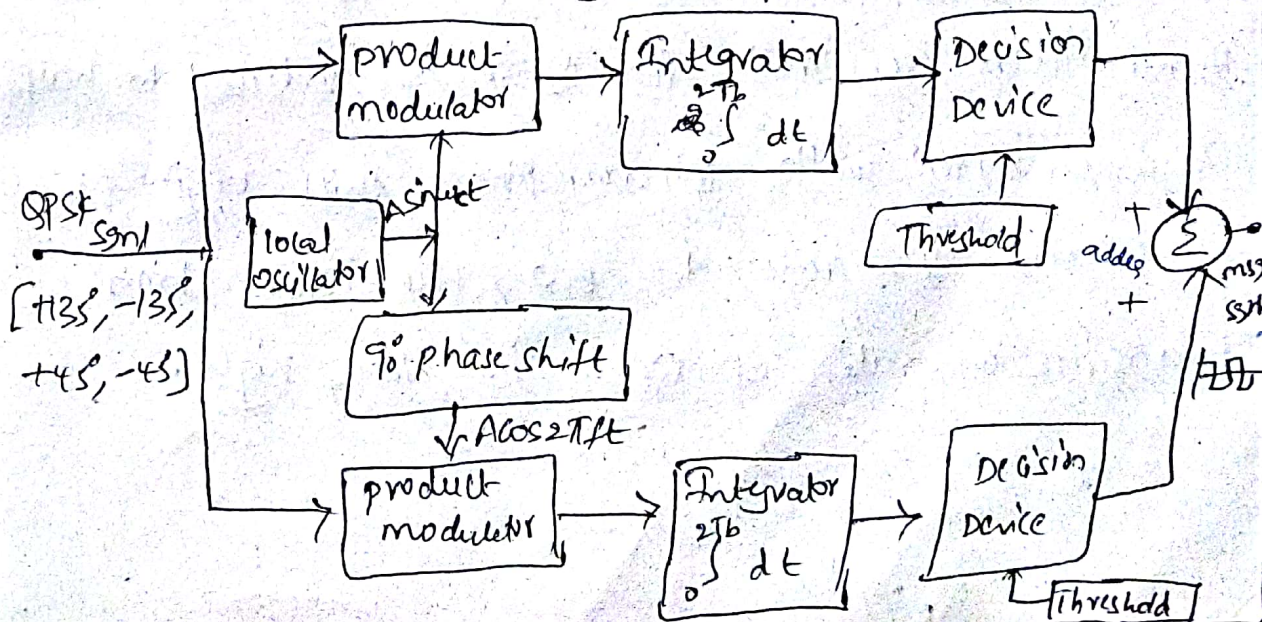
$$\begin{aligned} S_{\text{QPSK}} &= A \sin(\omega_c t + 135^\circ) \left[\because \sin \omega_c t \cos 135^\circ - \cos \omega_c t \sin 135^\circ \right] \end{aligned}$$

$$= \frac{-1}{\sqrt{2}} \sin \omega_c t - \cos \omega_c t \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} [-\sin \omega_c t - \cos \omega_c t]$$

$$= A [-\sin \omega_c t - \cos \omega_c t]$$

* QPSK demodulator Correlation detector



→ The QPSK signal is applied to product modulators where as another i/p for product modulator is carrier signal.

→ The QPSK signal & carrier signal are multiplied by product modulator and applied to Integrators.

→ The Integrator performs Integration over 2 bit intervals i.e., from "0" to $2T_b$.

→ The o/p of Integrators are given to decision device, now decision device takes a decision to transmit bit "0" [or] bit "1".

→ If o/p of Integrator is greater than "0" then bit "1" is transmitted to the o/p.

→ If Integrator o/p is less than "0" then bit decision device transmits bit "0" to the o/p.

→ The o/p of decision device -1 & decision device-2 are added at adder to form original msg signal.

* Advantages of QPSK :

→ The Bandwidth of QPSK is " f_b ". Hence the Bandwidth required for QPSK is reduced to half as compared with Binary Phase Shift Keying [BPSK].

→ Because of reduced Bw, the information transmission rate of QPSK is very high.

UNIT - V

SPREAD SPECTRUM MODULATION

Defination of spread spectrum (ss) :-

The spread modulation can be defined into two parts.

⇒ The transmitted data sequence occupies a much more bandwidth than the minimum required bandwidth.

⇒ The spectrum spreading at the transmitter and despreading at the receiver is obtained by "special code" called PN sequence code (Pseudo-Noise Sequence).

Applications of spread spectrum :-

① Anti jamming capacity for military applications :- The spread spectrum has the ability to resist the effect of intentional jamming. The anti jamming capability was used in military applications.

② Low probability of intercept :- Low probability of intercept is an application of spread spectrum in military. In this

case, the signal spectral density is kept small such that the presence of the signal is not detected easily.

- ③ Mobile communications $\div$ spread spectrum is used in mobile communications. This is because the spread spectrum signal has the ability to resist the effects of multipath fading.
- ④ Secured communications $\div$ The spread spectrum communications are 'secure' since pseudo-noise codes are used to generate spread spectrum signals, unwanted receivers cannot recognize the spread spectrum signals.
- ⑤ Distance measurements $\div$ spread spectrum communications are used in distance measurements. Broadband signals can be resolved in time more precisely than narrowband signals. spread spectrum signals are used in radars and navigation systems for distance measurement.
- ⑥ Selective calling $\div$ In this, the central station communicates with number of different receiving points.
- ⑦ CDMA communication $\div$ In this system many users communicates at the same time and through the same channel. Each user is allotted a particular code.

Techniques used in spread spectrum:

The two techniques that are used in spread spectrum is:

① Averaging systems

② Avoidance systems

① Averaging systems:

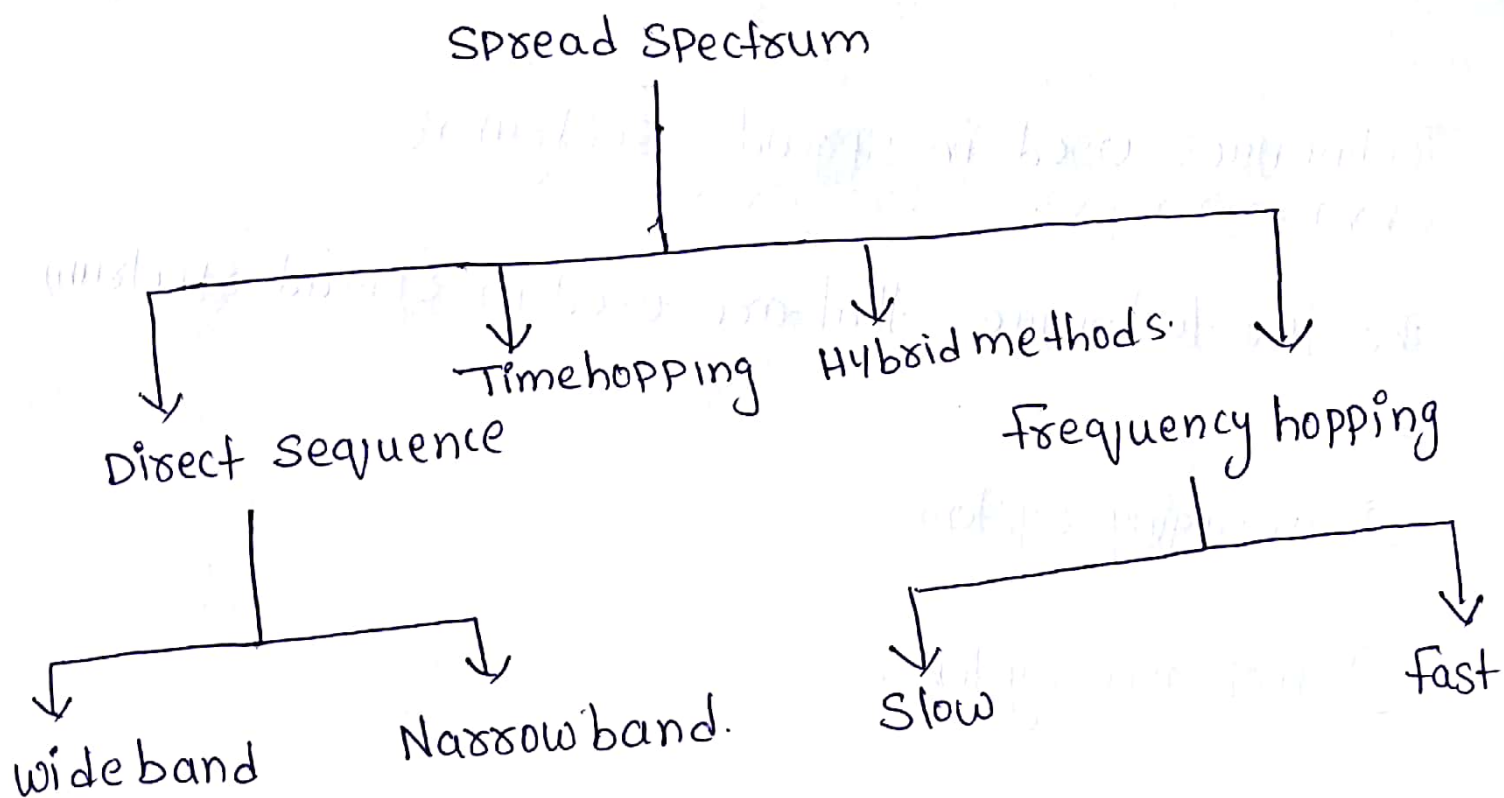
In this systems the interference is reduced by long period of time.

② Avoidance systems:

In this systems the interference is reduced by making the signal to avoid the interference a large fraction of time.

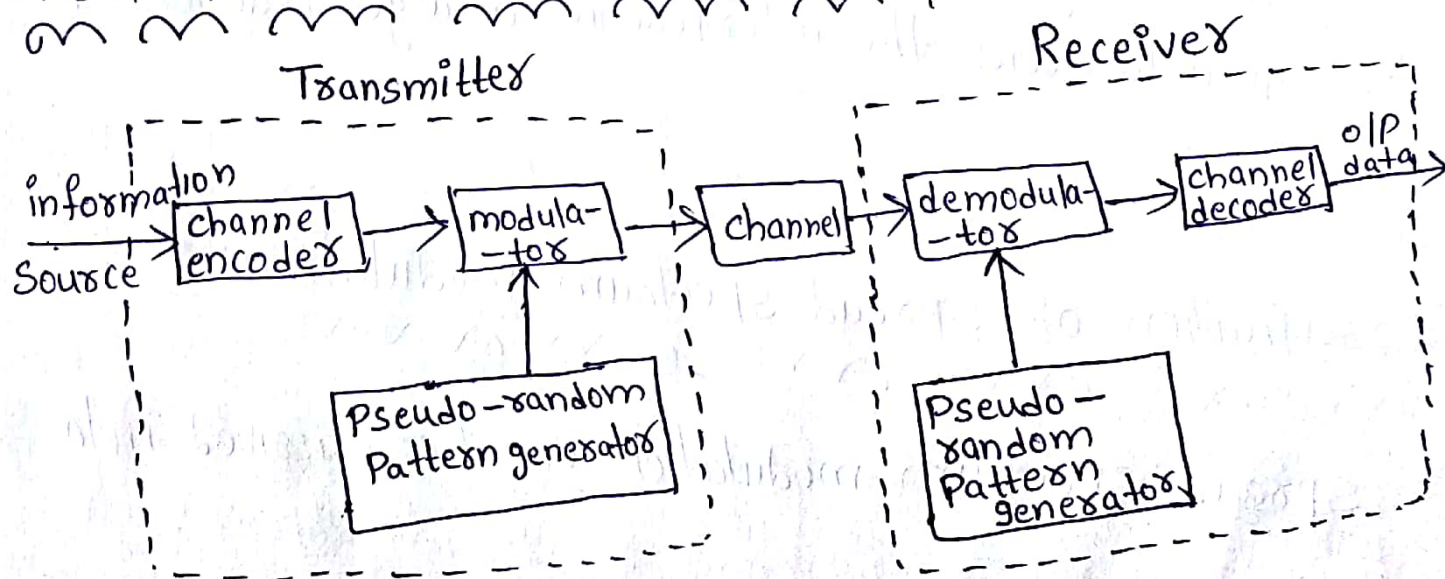
Classification of spread spectrum modulation:

Spread spectrum modulation can be classified into



From the above techniques, Direct sequence is an averaging system and remaining techniques are in avoidance system.

Model of spread spectrum digital communication system



Operation of Transmitter:

- * The binary information sequence is input to the channel encoder on transmitter side.
- * The channel encoder encodes these input sequence according to some error coding controlling techniques.
- * The coded sequence is then given to the modulator.
- * The modulator gets Pseudo-random (or) Pseudo-noise (PN) sequence from the Pseudo-random Pattern generator.
- * The PN sequence spreads the signals randomly over the wide frequency bands. The signal at the output of modulator is the spread spectrum modulated signal.
- * This signal is then transmitted over communication channel.

Operation of Receiver:

- * At the receiver the demodulator gets coded back from the spread spectrum signal.

* To recover the demodulator requires the same PN signal ~~such~~ which was used at the transmitter end.

* Hence, the pseudo-random pattern generator at the transmitter and receiver side operate in synchronization with each other.

* The channel decoder at the receiver gets the binary information sequence back.

* Thus the receiver can detect the transmitted spread spectrum signals only if it knows the PN sequence.

* For any ~~bi~~ arbitrary receiver it is difficult to know the PN sequence since it appears like noise.

Types of modulation:

⇒ The PN sequence at the modulator is used with the PSK modulator to shift the phase of this PSK signal randomly. Such technique is called direct sequence spread spectrum (DSSS) modulation. It is also known as PN noise spread spectrum modulation. In FSK signal randomly then the technique is called frequency hopping (FH) spread spectrum modulation.

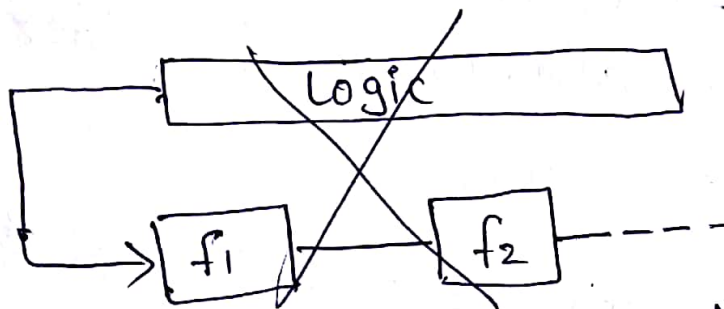
PN Sequence :-

⇒ A PN sequence is defined as a periodic binary sequence with a noise like high frequency signal. This signal is binary in nature.

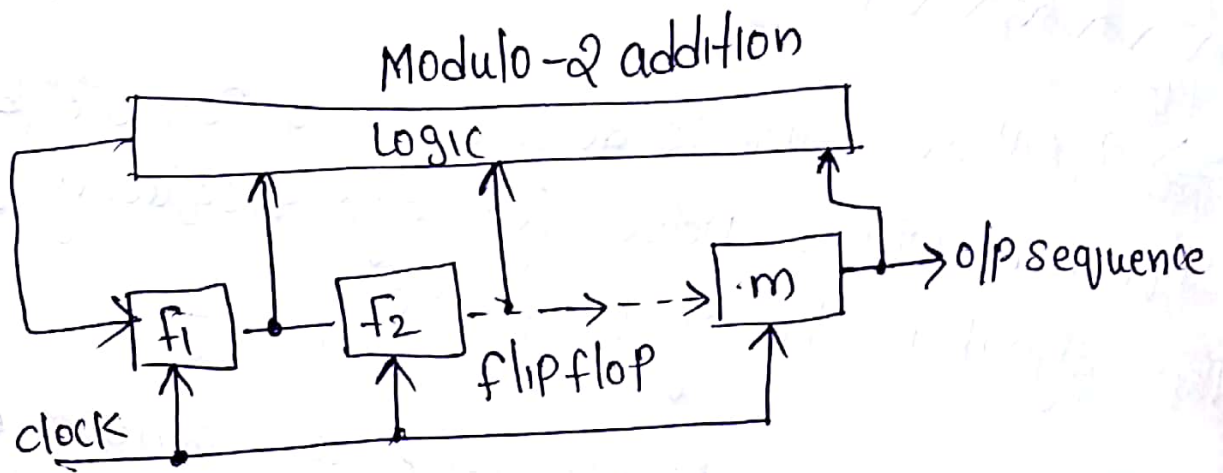
⇒ The sequence is not completely random, but it is generated by a well defined logic, this logic is used in a transmitter and receiver.

⇒ Since the sequence is generated by a well defined logic, it is rather 'pseudo' random, it is called PN Sequence.

Generation of PN Sequence :-



⇒ A PN sequence is usually generated using feedback state shift register.



⇒ A feedback shift register consists of flip flops, the flipflops in the registers are controlled by a single clock pulse.

⇒ For every clock pulse, the content of each stage is shifted by one position to another position (left to right).

⇒ An a modulo-2 operation (i.e XOR gate) takes place at the last stage of shift register and other selected o/p's from intermediate stage.

⇒ PN sequence consists of 'm' no. of flipflops or registers. length of PN sequence can be indicated by L . (or) N . i.e $L = 2^m - 1$

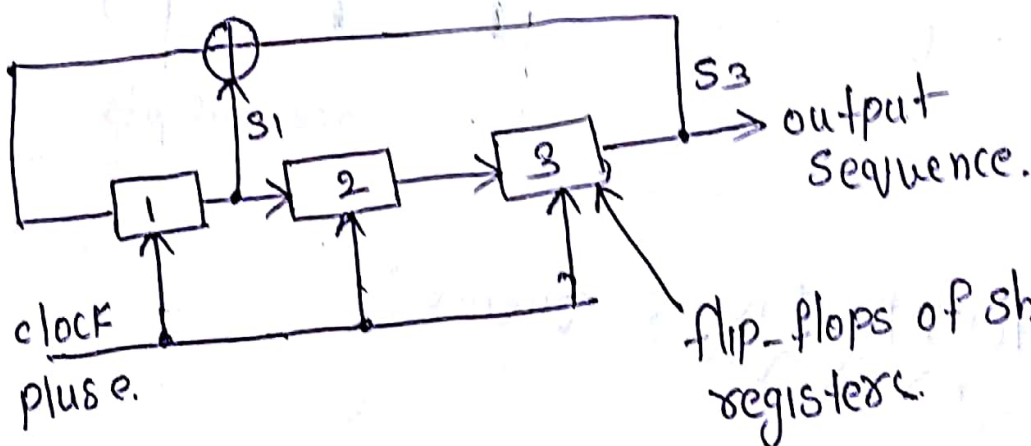
⇒ PN sequence is periodic in nature (ie it is continuous with respect to time it is repeated. Signal)

⇒ A feedback shift registers is said to be linear when the feedback logic consists of modulo-2 adders.

⇒ In such a case the '0' state is not permitted because for zero state the o/p is also '0'.

An 3 stage. PN sequence generator

Let us consider PN sequence generator.



state of shift register $S_1 \quad S_2 \quad S_3$			Modulo-2 addition $S_1 \oplus S_3$	PN-sequence. S_3
1	0	0	$1 \oplus 0 = 1$	0
↓	↓	↓	$1 \oplus 0 = 1$	0
1	1	0	$1 \oplus 1 = 0$	1
↓	↓	↓	$0 \oplus 1 = 1$	1
0	1	1	$1 \oplus 1 = 0$	0
↓	↓	↓	$0 \oplus 1 = 0$	0
0	0	1	$0 \oplus 1 = 1$	1
↓	↓	↓	$0 \oplus 1 = 1$	0
0	0	0	$0 \oplus 0 = 0$	0
↓	↓	↓	sequence repeats.	

Properties of PN Sequence :-

① Balance property :-

The number of 1s is always one more than the number of zeros in each period of

of a maximum length sequence.

② Run property

⇒ The run property is defined as the appearance of the alternate digit in a sequence, starts a new run (1's and 0's).

⇒ The length of the run is equal to length of subsequence. When the maximum length sequence is generated by feedback shift register of length m , is $2^m - 1$.

0	0	1	1	0	1	0	1
run 1	2	3	4	5	6		

③ Correlation property

The auto correlation function of maximum length sequence is periodic and it is binary valued.

chip rate $\hat{c}$

The No. of bits per second (chips per second) used in the spreading signal.

$$T_c = \frac{1}{\text{chip rate}}$$

$$T_c = \frac{1}{R_c} \text{ } \mu\text{sec. units}$$

Hence the period of o/p sequence is represented

as $T_b = N T_c$ $\mu\text{sec. units}$

Direct sequence spread spectrum with coherent BPSK:

$\Rightarrow$ Every single bit represented by multiple bits using spreading code.

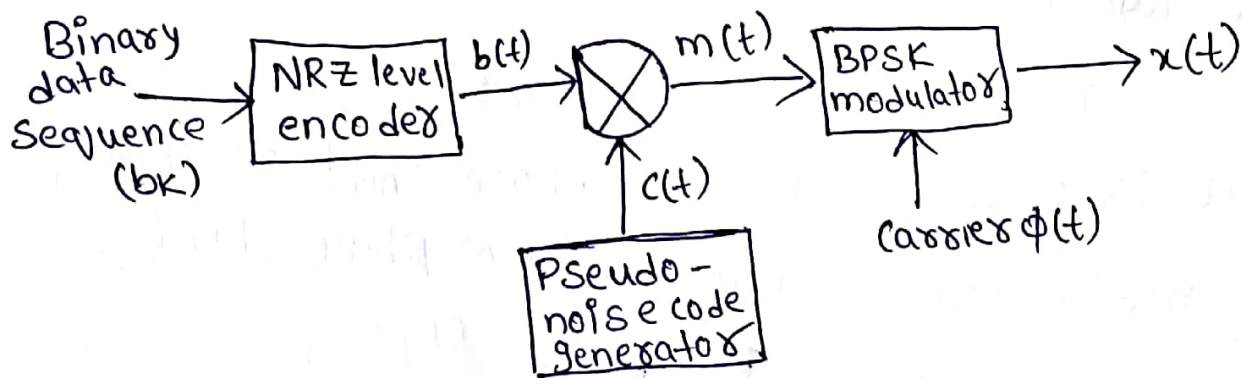
$\Rightarrow$ Spreading code spreads signal across wider frequency band.

$\Rightarrow$ In direct sequence spread spectrum, for long distance transmission (or) transmission over a satellite channel, BPSK can be used.

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Whenever a user wants to send data using this Direct Sequence Spread Spectrum technique (DSSS), each and every bit of user data is multiplied by a secret called as chipping code. This chipping code is nothing but the spreading code which is multiplied with the original message and transmitted.

DS-SS BPSK Transmitter



⇒ The binary data sequence (i.e. binary 0's and 1's) is given to NRZ level encoder. This encoder converts binary data sequence (i.e. b_k) into bipolar NRZ waveform.

⇒ The pseudo-noise sequence generator generates and encodes this sequence in bipolar NRZ signal.

⇒ The multiplexer multiplies the two signals $b(t)$ and $c(t)$. The output of multiplexer is direct sequence spread signal $m(t)$.

⇒ This signal is given as modulating signal to BPSK transmitter. The direct sequence BPSK signal is generated at the o/p i.e $x(t)$

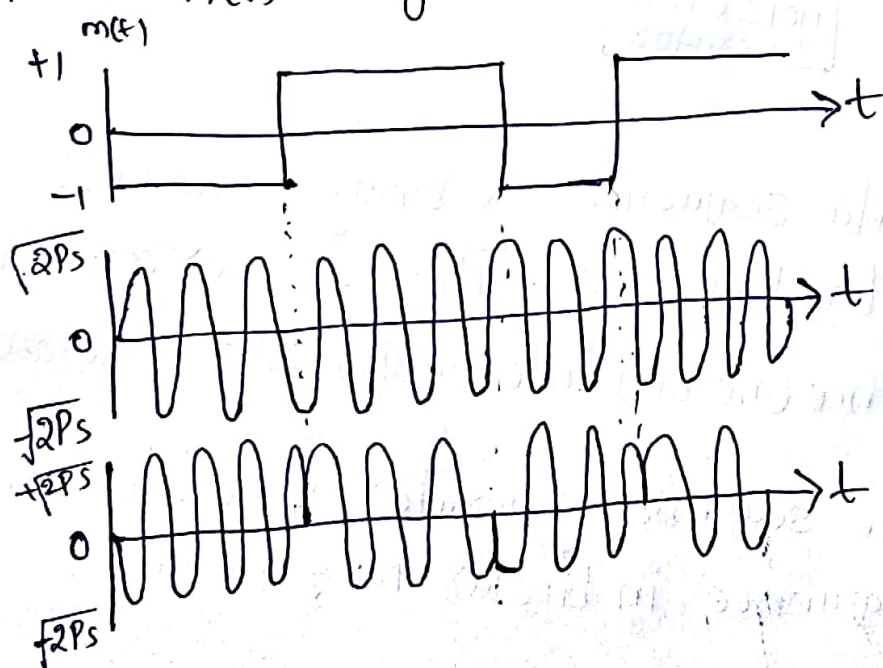
The carrier is represented as

$$\phi(t) = \sqrt{2P_s} \sin(2\pi f_c t)$$

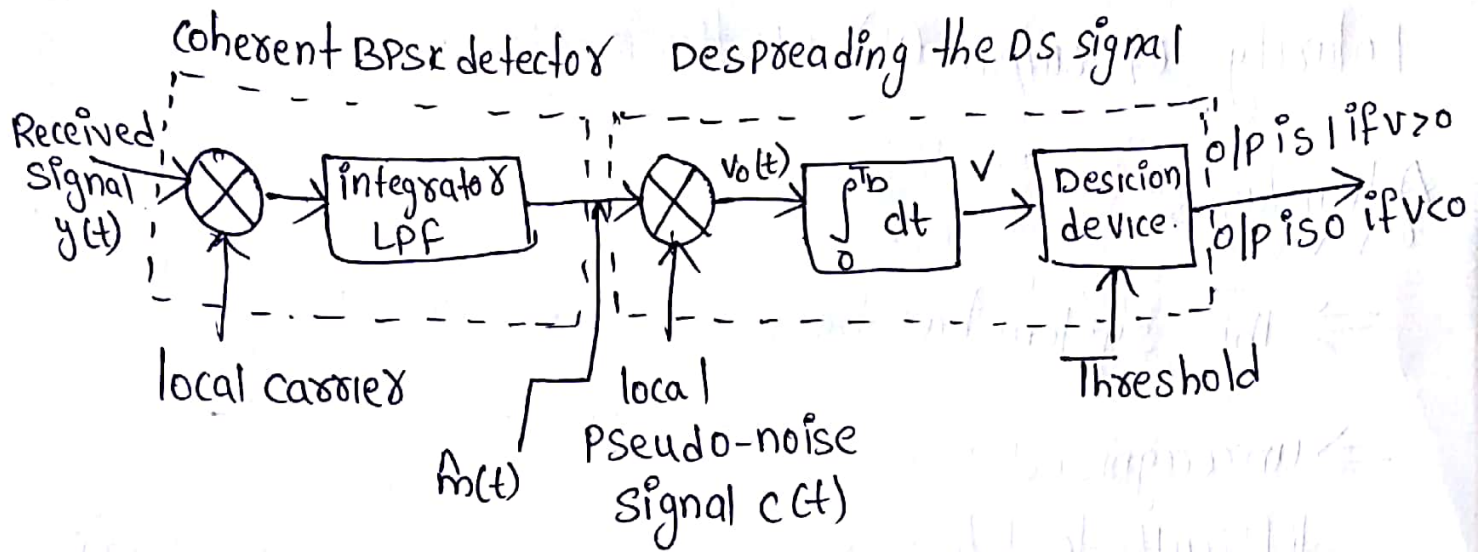
The transmitted signal is

$$x(t) = \sqrt{2P_s} m(t) \sin(2\pi f_c t)$$

When $m(t)$ is positive, there is phase shift of 0° and if the $m(t)$ is negative, there is 180° phase shift.



DS-SS BPSK Receiver



- ⇒ The received signal $y(t)$ is applied to the multiplier which is also supplied with locally generated coherent carrier
- ⇒ The op of the multiplier is then applied the lowpass filter (integrator). The bandwidth of this lowpass filter is equal to that of $m(t)$. This stage is BPSK detector.
- ⇒ The signal is applied to the second demodulator which despreads the signal. The local Pseudo-noise signal is exact of that used in the transmitter.
- ⇒ The integrator integrates the product of detected message signal and Pseudo-noise signal over one bit period ' T_b '.

⇒ The decision is then taken depending upon the Polarity of output (V) of the integrator.

Advantages:-

- ⇒ This system has best noise and antijam performance.
- ⇒ unrecognized receivers find it most difficult to detect direct sequence signals.
- ⇒ It has best discrimination against multipath signals.

disadvantages:-

- ⇒ It requires wideband channel with small phase distortion.
- ⇒ It has long acquisition time.
- ⇒ The PN generator should generate sequence at high rates.
- ⇒ This system is distance Relative.

Applications:-

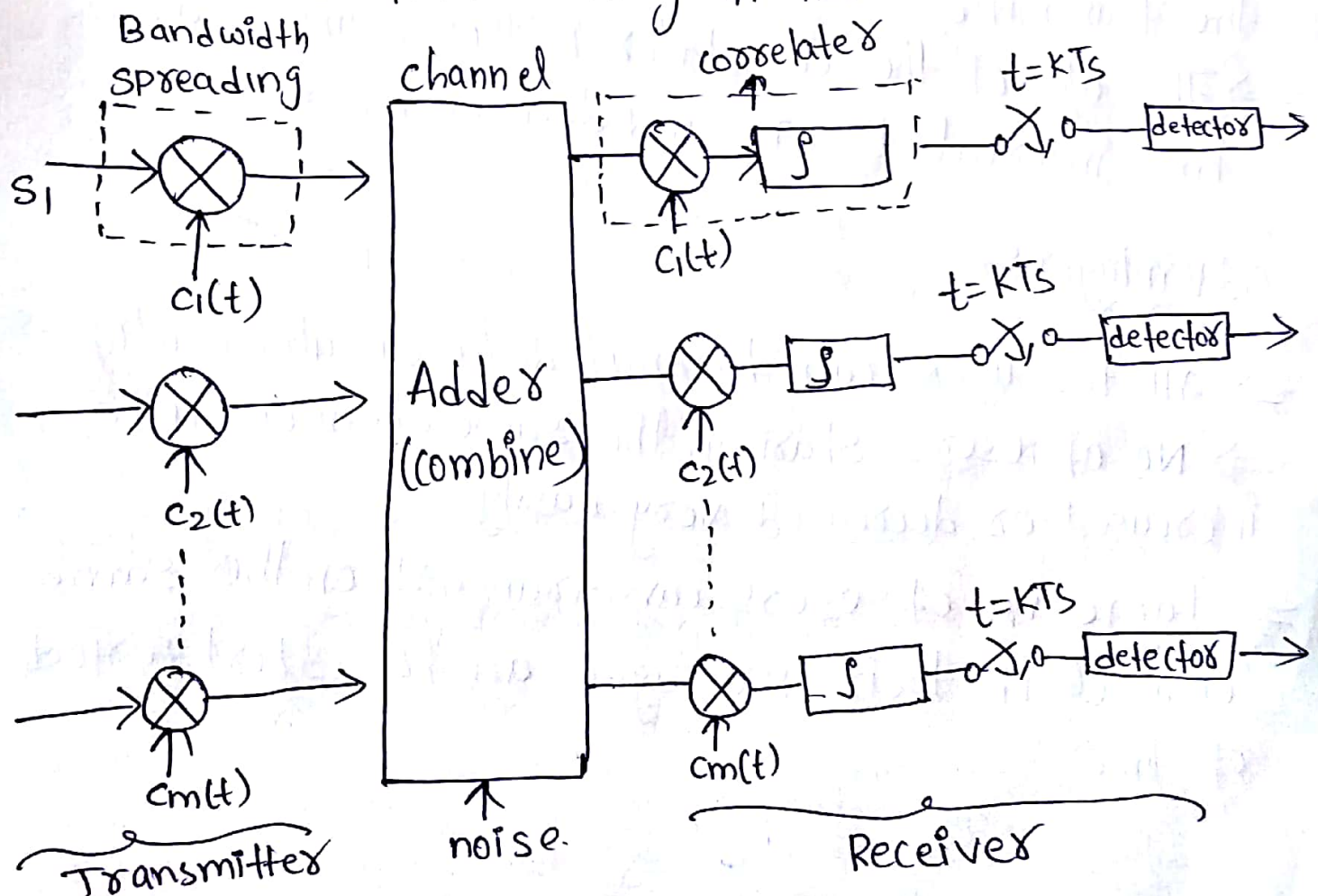
- ⇒ CDMA (code division multiple Access)
- ⇒ Ranging using DS-SS.

9

* Code division multiple Access with DS-SS (CDMA)
⇒ CDMA is also known as spread spectrum multiple Access (SSMA).

⇒ The principle of CDMA is, that the many users transmits their signals on the same channel bandwidth. Each transmitter receiver pair has a distinct PN sequence.

⇒ Thus the signals of a particular transmitter are received by its intended receiver only even if many users are transmitting at the same time.



- ⇒ The 'M' users transmit their signals $s_1, s_2, \dots, s_m$ simultaneously. Each user is supplied unique spreading sequence $c_k(t)$. Thus $c_1(t), c_2(t), \dots, c_m(t)$ are the spreading sequence for $s_1, s_2, \dots, s_m$ respectively.
- ⇒ The adder combines all the m-multiple users and transmits them over the channel simultaneously. This also adds channel noise to the transmitted signals.
- ⇒ At the receiver each signal is detected with the help of separate correlator. Each correlator is supplied with the same spreading sequence that was used at the transmitter.
- ⇒ The o/p of the correlator is then sampled at time interval $t = kT_s$ and decision made.

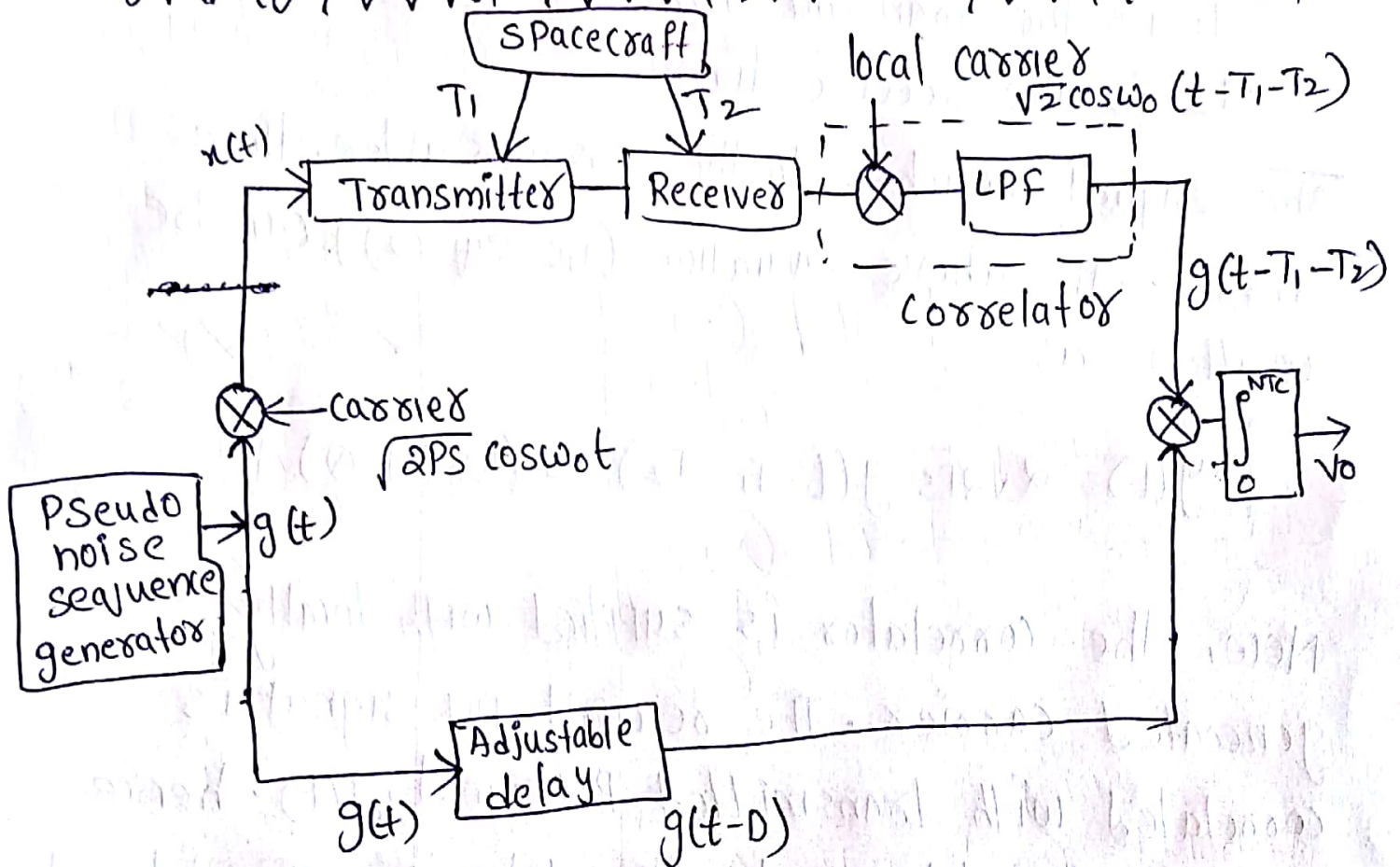
Advantages:-

- ⇒ All the users can transmit data simultaneously.
- ⇒ No. of users sharing the same channel can be increased or decreased very easily.
- ⇒ Large no. of users can transmit on the same channel if their messages are for short period of time.

Disadvantages:-

- ⇒ The signals from the other users appears as additive interference.
- ⇒ The PN sequence used for different users must be mutually orthogonal to avoid interference from each other.

* Ranging using direct sequence spread spectrum:-



The PN sequence generator generates the PN signal $g(t)$. It modulates the carrier $\sqrt{2P_s} \cos \omega_0 t$. we get

$$x(t) = \sqrt{2P_s} g(t) \cos \omega_0 t. \quad \text{--- (1)}$$

The signal is transmitted towards the spacecraft. The reflected signal from the spacecraft is

$$y(t) = \alpha x(t - T_1 - T_2) \quad \text{--- (2)}$$

Here α is the attenuation of the signal.

T_1 is the transmit time

T_2 is the receive time.

The signal reaches to the receiver after, time $T_1 + T_2$. The above equation (i.e eq. (2)) can be written as.

$$y(t) = \alpha \sqrt{2P_s} g(t - T_1 - T_2) \cos(\omega_0 t + \theta)$$

Now, the correlator is supplied with locally generated carrier. The received PN signal is correlated with transmitted PN signal, $g(t)$. hence PN signal is delayed by the delay 'D'. This delayed PN signal is received PN signal are multiplied

and integrated. The multiplier and integrator at o/p sequence performs correlation. The o/p of integrator is given as.

$$U_0 = R(D)$$

$$U_0 = \int_0^{NT_c} g(t - T_1 - T_2) g(t - D) dt$$

Here $R(D)$ indicates correlation.

N is the length of PN sequence.

Processing gain (PG)

It is defined as the ratio of the bandwidth of spread message signal to the bandwidth of unspread data signal

$$PG = \frac{\text{BW spreaded signal}}{\text{BW unspreaded signal}} = \frac{1/T_c}{1/T_b}$$

$$PG = \frac{1/T_c}{1/T_b}$$

$$PG = \frac{T_b}{T_c}$$

Frequency Hop Spread Spectrum (FH-SS)

The frequency hopping means to transmit the data bits in different frequency slots. The total bandwidth of the o/p signal is equal to sum of all these frequency slots are "hops".

⇒ Thus in the frequency hop spread spectrum the carrier frequency hops randomly from one frequency to another. M-ary FSK is used along with frequency hop spread spectrum.

⇒ There are two types of FH-SS, they are named as

- * Slow frequency hopping.

- * Fast frequency hopping.

⇒ Hop rate :- It is denoted as R_h . The rate of change of frequency "hops" is called hop rate.

⇒ Symbol rate (r_s) :- The rate at which k-bit symbols of data input sequence are generated is called symbol rate.

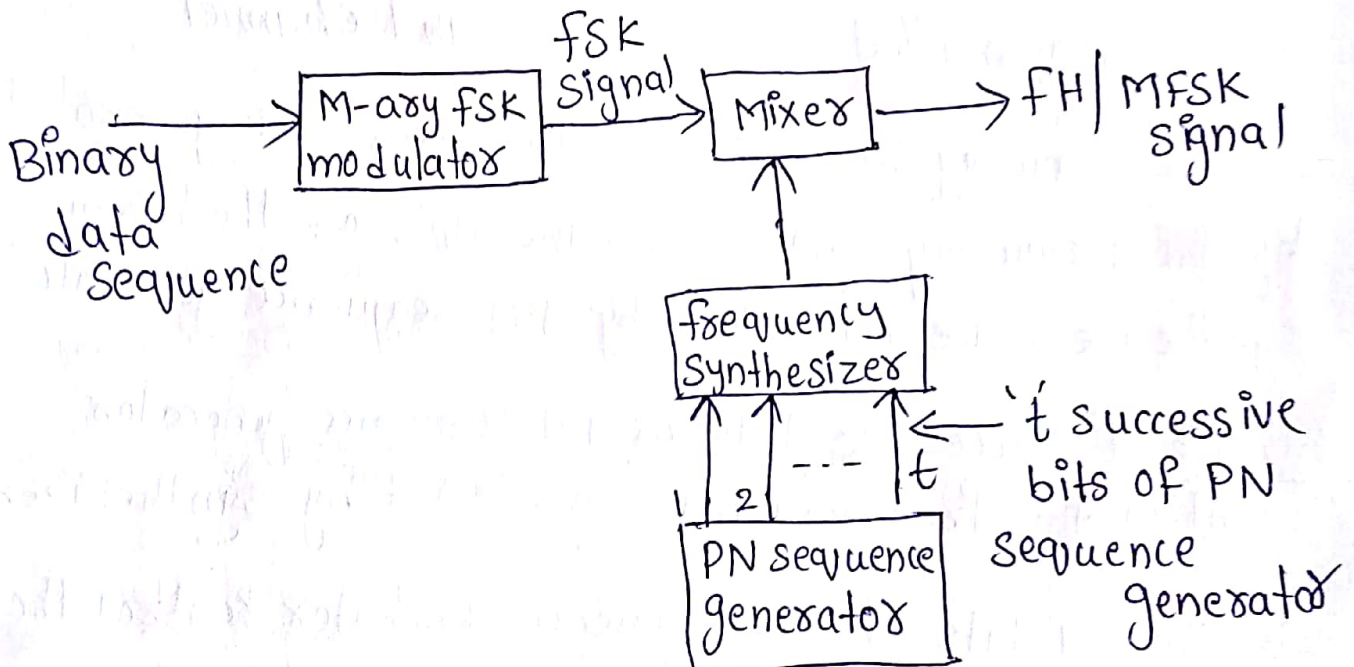
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Slow frequency hopping :-

When several symbols of data are transmitted in one frequency hop (slot), then it is called Slow frequency hopping.

one frequency hop = several symbol transmitted.

Transmitter :- (FH/MFSK)



⇒ The i/p binary data sequence is applied to the M-ary fsk. This modulator o/p is the particular frequency depending upon the i/p symbol.

⇒ The o/p of fsk modulator is then applied to a mixer. The other i/p to the mixer is particular frequency from frequency synthesizer.

⇒ The o/p of frequency synthesizer at particular instant is the frequency slot. This hop or slot is mixed with the fsk signal.

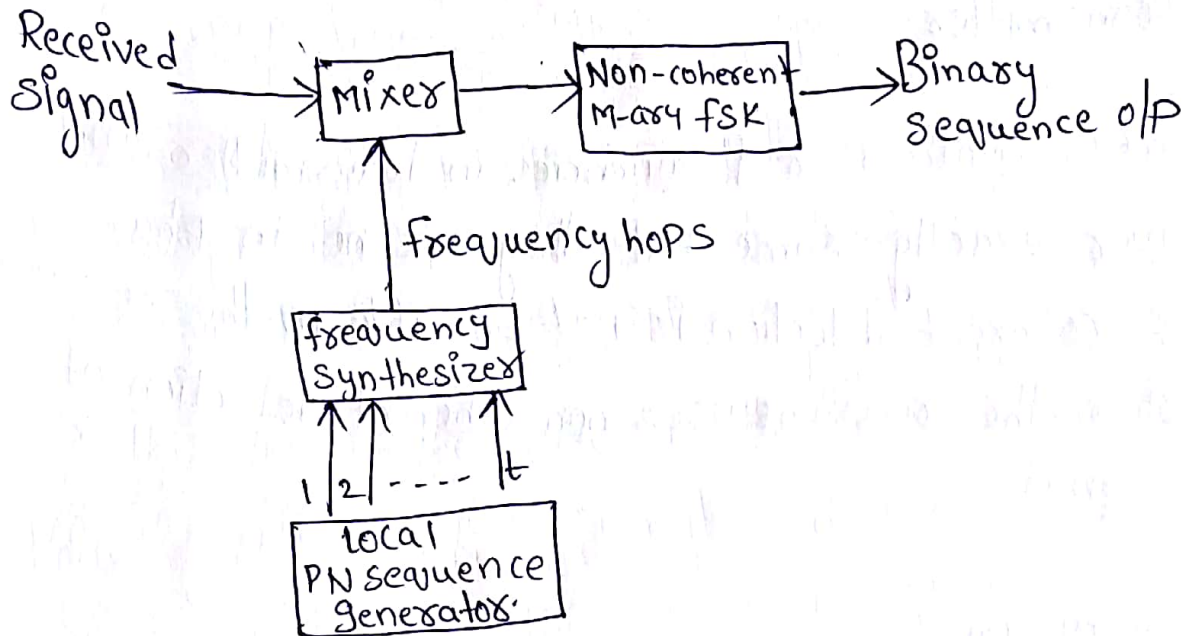
⇒ The o/p of the mixer is the 'sum' frequency component of fsk signal and frequency hop from synthesizer. This signal is FH/MFSK signal and is transmitted over the wideband channel.

⇒ The frequency hops given to the mixer are generated by the frequency synthesizer. The i/p's of the frequency synthesizer are controlled by PN sequence generator.

⇒ The 't' successive bits of PN sequence generator control the frequency hops generated by synthesizer.

⇒ Since 't' bits of PN sequence generator control the frequency hops, there will be distinct '2^t' frequency hops generated. Therefore the total bandwidth of the o/p signal will be equal to sum of individual frequency hops. This bandwidth goes in range of 'GHz'.

Receiver of FH/MFSK



⇒ The received FH/MFSK signal is applied to the mixer. The o/p of frequency synthesizer is also given to the mixer. The sum and difference frequencies are generated by mixer.

⇒ only difference frequencies are allowed to pass out of the mixer. Those difference frequencies are exactly the M-ary FSK signal.

⇒ These signals are given to the non-coherent M-ary FSK detector. The detector detects particular symbol transmitted.

⇒ The frequency synthesizer doesn't generate the inphase (or phase locked) frequencies with that of synthesizer at transmitter.

⇒ That is synthesizer frequencies in transmitter and receiver exactly same but they are not in phase. Hence coherent detection is not possible in the receiver. Therefore the receiver uses non-coherent detection of FSK signal.

Processing Gain:

$$PG = \frac{2^t \cancel{f_s}}{\cancel{f_s}}$$

$$PG = 2^t$$

Fast frequency hopping:

When several frequency hops take place to transmit one symbol, then it is called fast frequency hopping.

Several frequency hops ⇒ one symbol transmitted

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Comparison b/w slow and fast frequency hopping:

Slow frequency hopping	Fast frequency hopping
⇒ Multiple symbols are transmitted in one frequency hop ⇒ Symbol rate is equal to chip rate ⇒ Hop rate is lower than symbol rate ⇒ one or more symbols are transmitted over the same carrier frequency ⇒ This signal can be detected by jammer if carrier frequency in one hop is known.	⇒ multiple hops are taken to transmit one symbol. ⇒ Hop rate is equal to chip rate. ⇒ Hop rate is higher than symbol rate. ⇒ one symbol is transmitted over multiple carriers in different hops ⇒ This signal is difficult to detect one symbol is transmitted on multiple carrier frequencies

Comparison b/w DS-SS and FH-SS :-

FH-SS	DS-SS / CDMA
→ Multiple frequencies are used.	→ Single frequency is used.
→ Frequency reuse is allowed.	→ Frequency reuse is not allowed.
→ Sender need not wait	→ Sender has to wait if the spectrum is busy.
→ Power strength of the signal is high.	→ Power strength of the signal is low.
→ Stronger and Penetrates through the obstacles	→ It is weaker compared to FH-SS
→ It is never affected by interference.	→ It can be effected by interference.
→ It is cheaper	→ It is expensive
→ This is the commonly used technique	→ This technique is not frequently used

Advantages of FH-SS:

- ⇒ These systems bandwidth are very high.
- ⇒ The distance effect is less.
- ⇒ They can be programmed to avoid some portions of the spectrum.
- ⇒ They have relatively short acquisition time.

Disadvantages:

- ⇒ Those systems need complex frequency synthesizers.
- ⇒ They are not useful for range and range-rate measurement.
- ⇒ They need error correction

Applications:

- ⇒ In mobile communication, the signal reach to the receiver from different paths.
- ⇒ There is one direct path and many indirect paths due to reflections from near by objects.
- ⇒ The signal due to indirect path interfere with the received signal in amplitude as well as phase, it is called multipath fading.

Synchronization in spread spectrum systems:-

⇒ spread spectrum systems are essentially synchronous.

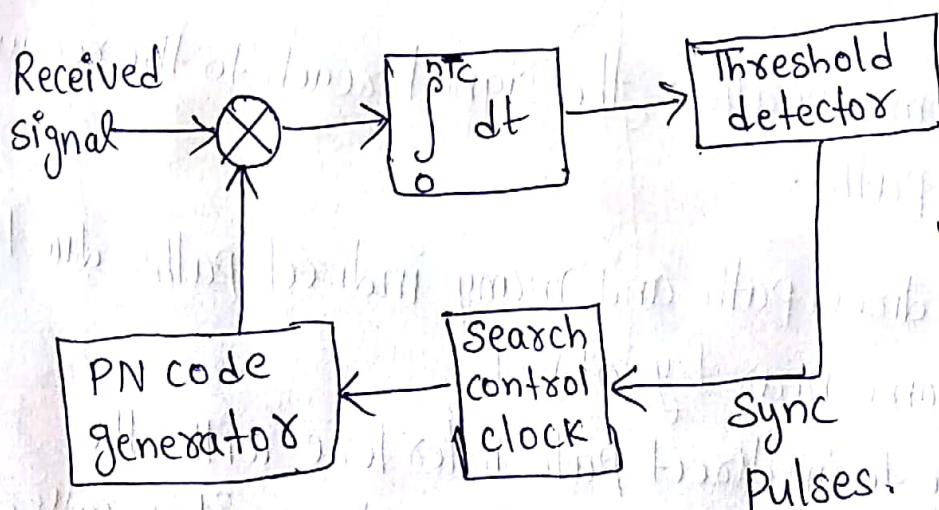
The pseudo noise sequence generated at the receiver and the transmitter must be same and locked to each other so that the ~~sig~~ transmitted signal can be extracted.

⇒ The synchronization of the spread spectrum systems can be considered in two parts.

→ Acquisition is also called as "coarse synchronization".

→ Tracking is also called as "fine synchronization".

Acquisition of DS-signal using serial search:-



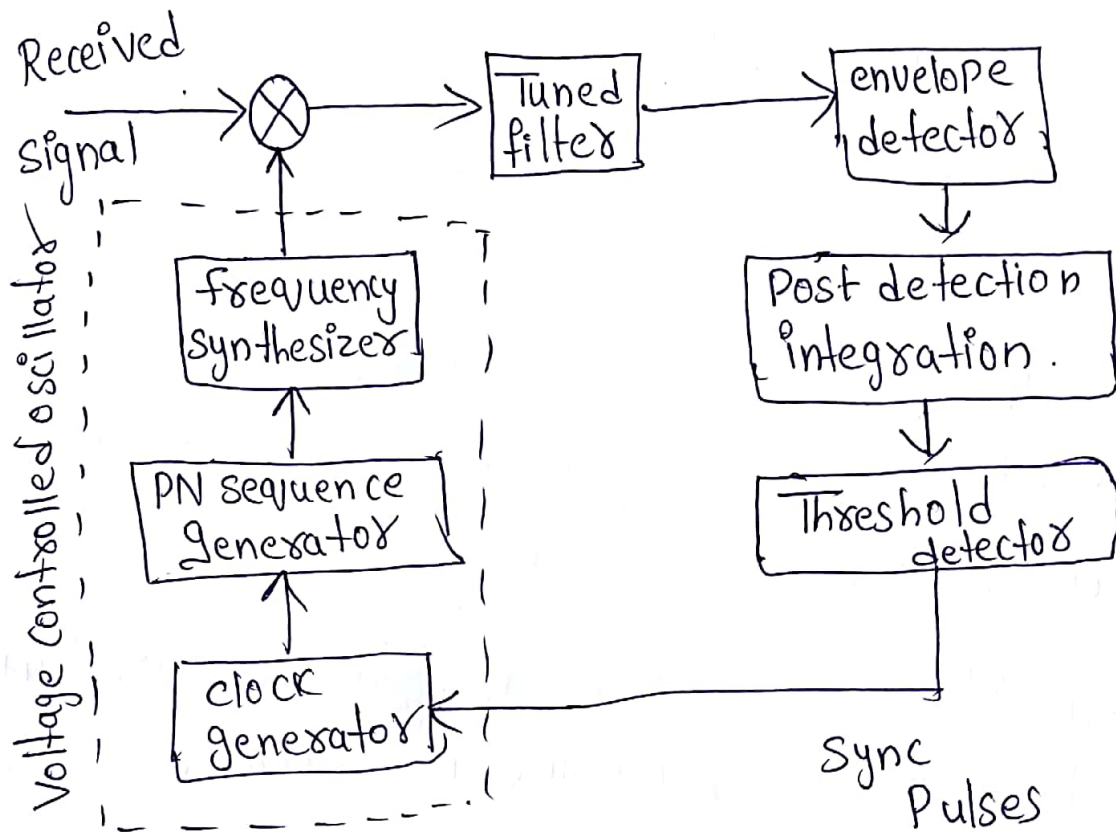
It is also known as "A sliding correlator".

- ⇒ The received signal is correlated with the generated PN sequence. This cross-correlation is performed over the time interval of NT_c
- ⇒ The o/p of the correlator is compared with a threshold. If it exceeds the threshold, then the required signal is obtained.
- ⇒ If the threshold is not exceeded, then the PN generator o/p is advanced by half chip duration $(\frac{1}{2}T_c)$ and the correlation is performed.
- ⇒ The o/p of the correlator is again compared with the threshold and the procedure is repeated.

Acquisition of FH signal using serial search:-

- ⇒ The VCO consists of frequency synthesizer, PN generator and clock generator. The received signal is correlated with the o/p of VCO.
- ⇒ The tuned filter passes only the intermediate frequency f_0 .

⇒ The envelope detector generates the o/p which is compared with the threshold voltage.



⇒ When the i/p frequency and frequency of VCO are same, then o/p of threshold detector is high and the clock generator starts running continuously. Then the signal is said to have acquired and tracking starts.

Advantages of spread spectrum:-

- * cross talk elimination.
- * Better op with data integrity.
- * Reduced effect of multipath fading.
- * Better security
- * Reduction in noise.
- * Hard to detect.
- * Difficult to jam the signals.
- * Not easy to decode/demodulate.
- * Longer operative distance.
- * ~~co-exist~~ co-existence with other systems.