UNIT-I

Microwave Transmission Lines

Introduction of Microwaves:

Microwaves are electromagnetic waves whose frequencies range from 1GHz to 1000GHz. Microwaves (μw's) are so called since they are defined interms of their wavelength in the sense that micro refers to tinyness - tinyness referring to the wavelength and the period of a cycle of a cm wave. In other words, the wavelength (λ) of cm waves at microwave frequencies are very short.

Microwave is a signal that has a wavelength of 1 foot or less i.e., $\lambda \leq 30.5$ cms. This converts to a frequency of 984 MHz, approximately =1GHz.

Applications of Microwave:-

→ Microwaves are used to satisfy many functions in our modern society.

Ex: oven for cooking things.

→ In Radar systems we are using Microwaves. They are used to detect aircraft, guide supersonic missiles, To control the flight traffic.

→ In satellite communication we are using.

→ In Telephone wireless network.

→ In T.V. communication.

· In some commercial and industrial purpose also us[e]

I) Drying machines – textile, food and paper industry for drying clothes, potato chips, printed matter etc.

II) Food processing industry – precooling / cooking

III) Rubber industry, plastics, chemical; forest product industries.

IV) breaking rock, breaking up concrete, breaking up c[oal] seems,

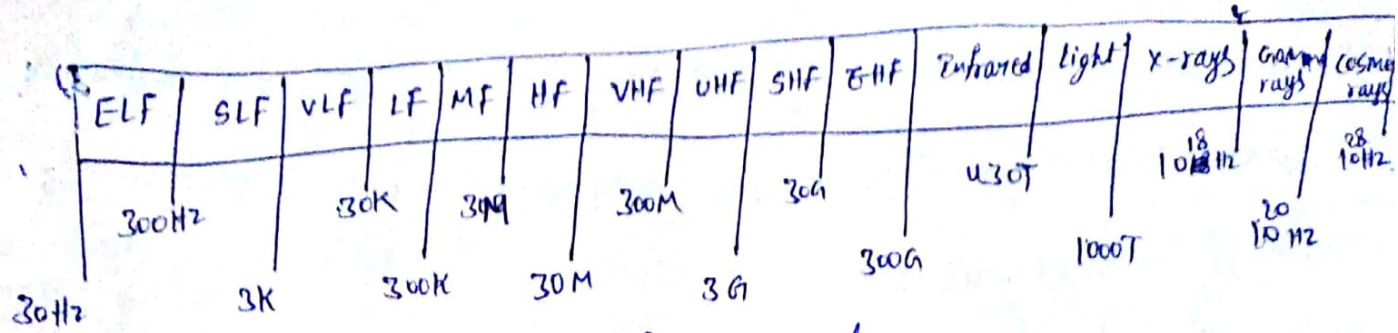
→ In space communication (earth to space; space to ea[rth]) we are using,

Microwave frequency Bands :–

microwave band							millimeter	sub millimeter
L	S	C	X	Ku	K	Ka		

1GHz, 2G, 4G, 8G, 12G, 18G, 27G, 40G, 0.300THz

Generally microwaves includes the whole super high frequ[ency] band (SHF band) having freq range from 8GHz to 30GH[z]



Electromagnetic freq spectrum.

Advantages of Microwaves:-

There are some advantages of Microwaves over low freqs.

1. Increased bandwidth availability.

Microwaves have large b.ws compared to the common bands namely MW, SW & UHF waves. The advantage of large b.w is that the freq range of information channels will be a small percentage of the carrier freq & more information can be transmitted in MW freq ranges.

2. Improved directive properties:-

As freq increases, directivity increases and beamwidth decreases. Hence the beam width of radiation θ is propotional to λ/D. At low freq bands, the size of the antenna becomes very large if it is required to get sharp beams of radiation..

3. Fading Effect & Reliability:

Due to the variation in transmission medium, the fading effect is more severe at low freq bands. The mw communication is more reliable, because of less fading effect at high freq due to line of sight propagation.

4. Power requirements:

At μw, the transmitter/ receiver power requirements are very low compared to that at low freq bands.

5. Transparency property of μw:

μw freq band ranging from 300 MHz - 10 GHz are capable of freely propagating through the ionized layers surrounding the earth, as well as through the atmosphere. Hence the study of μw radiation from the sun & stars is possible due to the presence of such a transparent window in a μw band.

Disadvantages:

→ At μw freq circuit design is complex.

→ Measurements are difficult at μw freq.

Rectangular waveguide:-

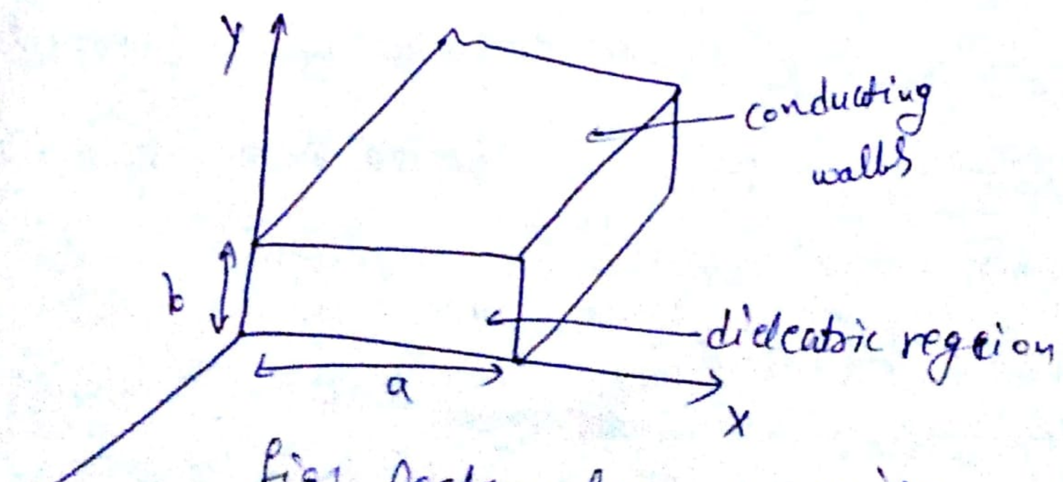


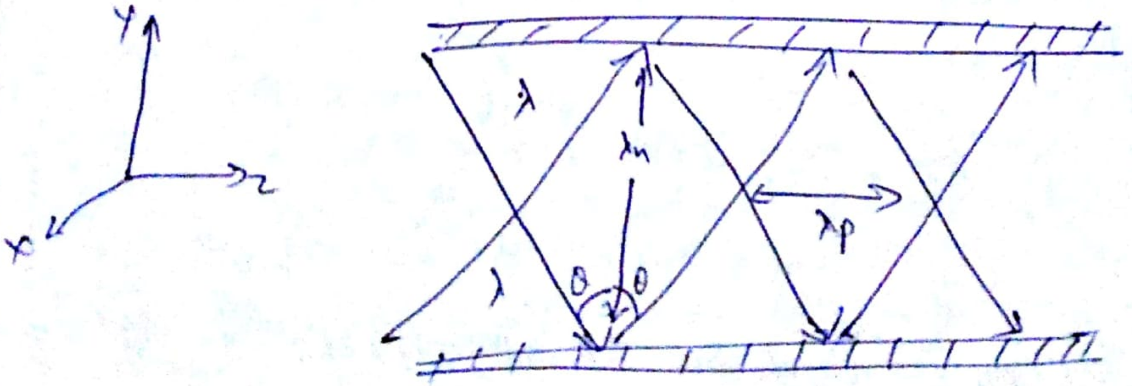
fig: Rectangular waveguide

· The conducting walls are made of usually brass or aluminium & the dielectric medium is usually air. The electromagnetic wave to be transmitted travel longitudinally in z- direction.

→ The wave is reflected b/w the walls, because of multiple reflections no. of distinct field configurations can exist in waveguides each of these field configuration is called a MODE.

→ The waveguide structure is a single conductor, therefore a d.c. voltage can't be applied in the usual manner across the waveguide, it appears across whatever separates the waveguide from ground. The basic property of a rectangular waveguide is its ability to carry high freq s/n but not low freq.

The field polarization field components are E_x, E_y, H_x, H_y, H_z



Fig! wave reflection in waveguide

λ is the wavelength.

θ be the angle of the wave

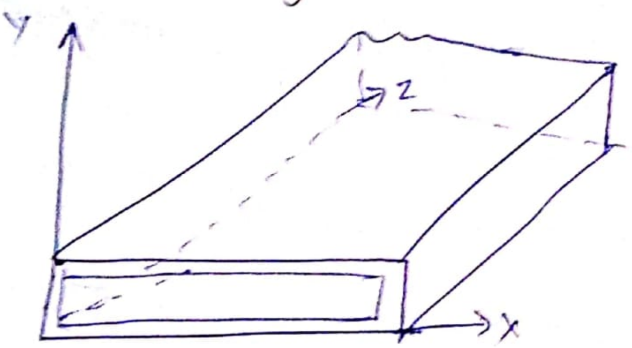
λ_n is the direction normal to the reflecting plane.

λ_p is the parallel to the reflecting plane.

$$\lambda_n = \frac{\lambda}{\cos\theta}$$

$$\lambda_p = \frac{\lambda}{\sin\theta}.$$

Wave eqn in Rectangular co-ordinates:-



The electric & magnetic wave eqns in freq domain are given by,

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 H = \gamma^2 H.$$

where γ is vector wave eqn

$$\gamma = \alpha + j\beta$$

The E & H components satisfies the helmholtz eqn given as,

$$\nabla^2 \psi = \gamma^2 \psi$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \gamma^2 \psi$$

$$\psi = X_{(x)} Y_{(y)} Z_{(z)}.$$

solving the eqn will give the helmholtz eqn in rectangular co-ordinates is

$$\psi = [A \sin k_x x + B \cos k_x x][C \sin k_y y + D \cos k_y y][E \sin k_z z + F \cos k_z z]$$

Field eq^n:-

maxwell eq^{n}s $\nabla \times E = -j\omega\mu H_s$ — (1)

$\nabla \times H = j\omega\epsilon E_s$ — (2)

$$\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z} = -j\omega\mu H_{xs} \quad \text{— (3)}$$

$$\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} = j\omega\varepsilon E_{xs} \quad \text{— (4)}$$

$$\frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x} = -j\omega\mu H_{ys} \quad \text{— (5)}$$

$$\frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x} = j\omega\varepsilon E_{ys} \quad \text{— (6)}$$

$$\frac{\partial E_{xs}}{\partial y} - \frac{\partial E_{ys}}{\partial x} = -j\omega\mu H_{zs} \quad \text{— (7)}$$

$$\frac{\partial H_{xs}}{\partial y} - \frac{\partial H_{ys}}{\partial x} = j\omega\varepsilon E_{zs} \quad \text{— (8)}$$

From (4) & (5)

$$\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} = j\varepsilon\omega E_{xs}$$

$$\frac{\partial H_{zs}}{\partial y} - \frac{\partial}{\partial z}\left[\frac{-1}{j\omega\mu}\left(\frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x}\right)\right] = j\omega\varepsilon E_{xs}$$

$$\frac{\partial H_{zs}}{\partial y} + \frac{1}{j\omega\mu}\left[\frac{\partial}{\partial z}\left(\frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x}\right)\right] = j\omega\varepsilon E_{xs}$$

$$\frac{\partial H_{zs}}{\partial y} + \frac{1}{j\omega\mu}\frac{\partial^2}{\partial z^2}E_{xs} - \frac{\partial}{\partial z\,\partial x}\frac{E_{zs}}{j\omega\mu} = j\omega\varepsilon E_{xs}$$

Assume $E_{xs} = e^{-\gamma z}$

$$\frac{\partial}{\partial z}E_{xs} = -\gamma \cdot e^{-\gamma z}$$

$$\frac{\partial^2}{\partial z^2}\cdot E_{xs} = -\gamma \cdot -\gamma \cdot e^{-\gamma z}$$

$$= \gamma^2 \cdot E_{xs}.$$

$E_{zs} = e^{-\gamma z}$

$$\frac{\partial}{\partial z}E_{zs} = -\gamma \cdot e^{-\gamma z}$$

$$= -\gamma \cdot E_{zs}.$$

$$\frac{\partial H_{zs}}{\partial y} + \frac{1}{j\omega\mu}\gamma^2 E_{xs} + \frac{\partial E_{zs}}{\partial x}\cdot\frac{\gamma}{j\omega\mu} = j\omega\varepsilon E_{xs}$$

$$\frac{\partial H_{zs}}{\partial y} + \frac{\partial E_{zs}}{\partial x}\cdot\frac{\gamma}{j\omega\mu} = j\omega\varepsilon E_{ys} - \frac{\gamma^2}{j\omega\mu}E_{xs}$$

$$= \frac{E_{xs}}{j\omega\mu}\left[j\omega\varepsilon\cdot j\omega\mu - \gamma^2\right]$$

$$= \frac{E_{xs}}{j\omega\mu}\left[-\omega^2\mu\varepsilon - \gamma^2\right]$$

$$= \frac{-E_{xs}}{j\omega\mu}\left[\omega^2\mu\varepsilon + \gamma^2\right]$$

$$\boxed{\gamma^2 + \omega^2\mu\varepsilon = h^2}$$

$$\frac{\partial H_{zs}}{\partial y} + \frac{\partial E_{zs}}{\partial x}\cdot\frac{\gamma}{j\omega\mu} = \frac{-E_{xs}}{j\omega\mu}\cdot h^2$$

$$-\frac{\partial H_{zs}}{\partial y} - \frac{\partial E_{zs}}{\partial x}\cdot\frac{\gamma}{j\omega\mu} = \frac{E_{xs}}{j\omega\mu}\cdot h^2$$

$$E_{xs} = \frac{-j\omega\mu}{h^2}\cdot\frac{\partial H_{zs}}{\partial y} - \frac{\cancel{j\omega\mu}}{h^2}\cdot\frac{\gamma}{\cancel{j\omega\mu}}\cdot\frac{\partial E_{zs}}{\partial x}$$

$$E_{xs} = \frac{-j\omega\mu}{h^2}\cdot\frac{\partial H_{zs}}{\partial y} - \frac{\gamma}{h^2}\cdot\frac{\partial E_{zs}}{\partial x} \quad \text{— (A)}$$

$$E_{ys} = \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_{zs}}{\partial x} - \frac{\gamma}{h^2}\cdot\frac{\partial E_{zs}}{\partial y} \quad \text{— (B)}$$

$$H_{xs} = \frac{j\omega\varepsilon}{h^2}\cdot\frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2}\cdot\frac{\partial H_{zs}}{\partial x} \quad \text{— (C)}$$

$$H_{ys} = \frac{-j\omega\varepsilon}{h^2}\cdot\frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2}\cdot\frac{\partial H_{zs}}{\partial y} \quad \text{— (D)}$$

Modes of propagation:-

There are two types of modes the waveguide can support:

1. TM mode (Transverse magnetic)
2. TE Mode (Transverse Electric)

TM mode: In TM mode magnetic lines are entirely transverse to the direction of propagation of electromagnetic wave. The electric field has a component in that direction. $H_z = 0,\ E_z \neq 0$.

TE mode: In TE mode electric field lines are entirely transverse to the direction of the propagation of electromagnetic wave. The magnetic field has a component in that direction. $E_z = 0,\ H_z \neq 0$.

Depending on the values of m & n, any mode is denoted by TE_{mn} or TM_{mn} mode.

Field eqn for TM mode:-

In transverse magnetic mode, the direction of propagation is in the positive z direction.

The transverse magnetic component exist and magnetic component in the direction of propagation (z) does not exist, so $H_z = 0$.

The helmholtz eqn for electric field in the rectangular co-ordinates can be written as-

$$E_{zs} = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y)\, e^{-\gamma z} \quad (1)$$

Boundary conditions:-

$E_{zs} = 0$ at $y = 0$ — ② (all along bottom wall)

$E_{zs} = 0$ at $y = b$ — ③ (Top wall)

$E_{zs} = 0$ at $x = 0$ — ④ (left side wall)

$E_{zs} = 0$ at $x = a$ — ⑤ (Right side wall)

$E_{zs} = A_3 A_1 \cos k_x x \cdot \cos k_y y + A_1 A_4 \cos k_x x \cdot \sin k_y y + A_2 A_3 \sin k_x x \cdot \cos k_y y +$

$A_2 A_4 \sin k_x x \cdot \sin k_y y) e^{-\gamma z}$ — ⑥

From eqn ② & ④

$0 = A_1 A_3 \cos 0 \cdot \cos 0.$

$0 = A_1 A_3.$

$A_1 = 0, A_3 = 0.$ — ⑦

sub ⑦ in ⑥

$E_{zs} = A_2 A_4 \sin k_x x \cdot \sin k_y y \cdot e^{-\gamma z}$

$E_{zs} = E_0 \sin k_x x, \sin k_y y \cdot e^{-\gamma z}$

using ③ & ⑤ eqn.

$0 = E_0 \sin k_x a \cdot \sin k_y b e^{-\gamma z}.$

$\sin k_x a = 0$	$\sin k_y b = 0.$
$\sin k_x a = \sin m\pi$	$\sin k_y b = \sin n\pi$
$k_x a = m\pi$	$k_y \cdot b = n\pi$
$k_x = \frac{m\pi}{a}$	$k_y = \frac{n\pi}{b}.$

where $m = 1, 2, \ldots$

$n = 1, 2 \ldots$

a = wide wall

b = narrow wall.

$E_{zs} = E_0 \sin\left(\frac{m\pi}{a}\right) x \cdot \sin\left(\frac{n\pi}{b}\right) \cdot y \cdot e^{-\gamma z}$ — ⑧

Sub (8) in (A), (B), (C), (D) $\leftarrow$ [$\because H_z = 0$]

$$E_{xs} = \frac{-\gamma}{h^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_{ys} = \frac{-\gamma}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y\, e^{-\gamma z}$$

$$H_{xs} = \frac{j\omega\varepsilon}{h^2}\left(\frac{n\pi}{b}\right) \cdot E_0 \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y\, e^{-\gamma z}$$

$$H_{ys} = \frac{j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) \cdot E_0 \cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right) \cdot y \cdot e^{-\gamma z}.$$

Derivation for propagation constant (γ) :-

$$h^2 = \gamma^2 + \omega^2 \mu\varepsilon$$

$$\Rightarrow k^2 = \omega^2 \mu\varepsilon$$

$$h^2 = \gamma^2 + k^2$$

Assume $h^2 = k_x^2 + k_y^2$

$$k^2 = -\gamma^2 + k_x^2 + k_y^2$$

$$k^2 = -\gamma^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma^2 = -k^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = \sqrt{-k^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \quad \text{— (1)}$$

Now there can be 3 conditions for propagation constant γ.

Condition 1: $\omega^2\mu\varepsilon = k_c^2$, $\gamma = 0$ in eqn (1)

$$\boxed{k = \omega\sqrt{\mu\varepsilon}}.$$

$$0 = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}$$

$$k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$(\omega\sqrt{\mu\varepsilon})^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega^2 \mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\omega^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

condition 2:

$$\omega^2 \mu\varepsilon > k^2$$

$$\omega^2 \mu\varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

condition 3:

$$\omega^2 \mu\varepsilon < k^2$$

$$\omega^2 \mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2.$$

cut-off freq (fc):-

$$f_c = \frac{\omega_c}{2\pi}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$u' = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$f_c = \frac{u'}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\lambda_c = \frac{u'}{f_c}.$$

$$\lambda_c = \frac{u'}{\frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$\cancel{\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\gamma^2 = (j\beta)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu\varepsilon \quad \text{—— (1)}$$

At $f = f_c$, $\omega = \omega_c$, $\gamma = 0$

$$0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2 \mu\varepsilon$$

$$\omega_c^2 \mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad \text{—— (2)}$$

sub (2) in (1)

$$\gamma^2 = (j\beta)^2 = \omega_c^2 \mu\varepsilon - \omega^2 \mu\varepsilon$$

$$\gamma^2 = \beta^2 = \omega^2 \mu\varepsilon - \omega_c^2 \mu\varepsilon$$

$$\beta = \sqrt{\mu\varepsilon}\sqrt{(\omega^2 - \omega_c^2)}$$

$$= \sqrt{\mu\varepsilon} \cdot \sqrt{(2\pi f)^2 - (2\pi f_c)^2}$$

$$= 2\pi f \sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\boxed{\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\beta = \beta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

phase velocity $u_p = \frac{\omega}{\beta}$.

η_{TM} intrinsic impedance :-

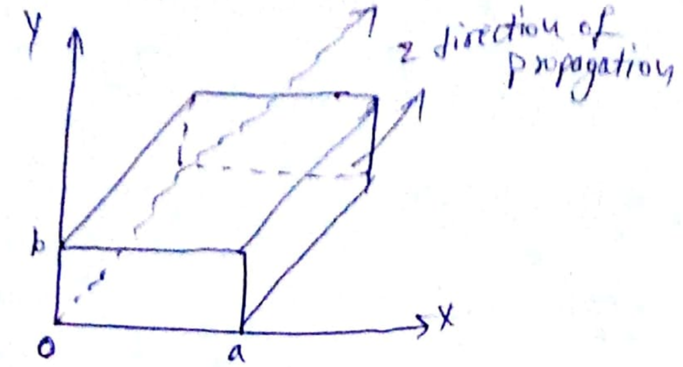
$$\eta_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x}.$$

$$\eta_{TM} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Field eq's for TE mode :-

$$H_{zs} = (A\cos k_x x + B\sin k_x x)(C\cos k_y y + D\sin k_y \cdot y)\, e^{-\gamma z} \quad \text{—— (a)}$$

$E_z = 0$ but we have components along x and y direction.



$E_x = 0$ all along bottom & top walls of the waveguide.

$E_y = 0$ all along left and right walls of the waveguide.

<u>Boundary conditions:-</u>

$E_x = 0$ at $y = 0$, $x \to 0$ to a (bottom wall) — ①

$E_x = 0$ at $y = b$, $x \to 0$ to a (top wall) — ②

$E_y = 0$ at $x = 0$, $y \to 0$ to b (left side wall) — ③

$E_y = 0$ at $x = a$, $y \to 0$ to b (right side of wall) — ④

∴ sub ① eqn in Ⓐ eqn.

Let us write E_x in terms of H_z.

$$E_{xs} = \frac{-j\omega\mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial y} - \frac{\gamma}{h^2} \cdot \frac{\partial E_{zs}}{\partial x} \quad [\because \text{eqn Ⓐ}]$$

since $E_z = 0$, the 2nd term $= 0$.

$$E_{xs} = \frac{-j\omega\mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial y}.$$

$$E_x = \frac{-j\omega\mu}{h^2} \left[A\cos K_x \cdot x + B\sin K_x \cdot x\right] \cdot \left[-C\sin K_y \cdot y \cdot K_y + D\cos K_y \cdot y \cdot K_y\right] e^{-\gamma z}.$$

sub ① eq^n in ⓐ eq^n the

$$0 = \frac{-j\omega\mu}{h^2}\left[A\cos K_x\cdot x + B\sin K_x\cdot x\right]\left[0 + DK_y\right]\bar{e}^{\gamma z}$$

$$0 = \frac{-j\omega\mu}{h^2}\left[A\cos K_x\cdot x + B\sin K_x\cdot x\right]\left[DK_y\right].$$

since $A\cos K_x\cdot x + B\sin K_x\cdot x \neq 0,\ K_y \neq 0$

so $D = 0$.

sub $D=0$ in eq^n ⓐ.

$$H_{zs} = (A\cos K_x x + B\sin K_x\cdot x)(C\cos K_y y)\,\bar{e}^{\gamma z} \quad \text{——— (S)}$$

3rd boundary condition:

sub 3rd eq^n in eq^n (B).

$$E_{ys} = \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x} - \frac{\gamma}{h^2}\cdot\frac{\partial E_{zs}}{\partial y} \quad \text{——— (B)}.$$

$$E_{ys} = \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_{zs}}{\partial x} - 0. \qquad [\because E_z = 0].$$

sub the eq^n (S) in above eq^n.

$$E_{ys} = \frac{j\omega\mu}{h^2}\cdot\frac{\partial}{\partial x}\left[A\cos K_x\cdot x + B\sin K_y\cdot x\right]\cdot C\cos K_y\cdot y\cdot\bar{e}^{\gamma z}$$

$$E_{ys} = \frac{j\omega\mu}{h^2}\cdot\left[-A\sin K_x\cdot x\,K_x + B\cos K_x\cdot x\,K_x\right]\cdot C\cos K_y\cdot y\,\bar{e}^{\gamma z}.$$

sub 3rd eq^n in above eq^n

$$0 = \frac{j\omega\mu}{h^2}\left[BK_x\right]C\cos K_y\cdot y\,\bar{e}^{\gamma z}$$

since, $\cos K_y\cdot y \neq 0,\ C \neq 0,\ K_x \neq 0$.

$B = 0$,

sub value of B in eq^n (5)

$$H_{zs} = A \cos K_x x \, C \cdot \cos K_y \cdot y \cdot e^{-\gamma z} \quad \text{——— (6)}$$

2^nd Boundary condition:

$E_x = 0$ at $y = b$.

From eq^n (A).

$$E_{xs} = \frac{-j\omega\mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial y} - \frac{\gamma}{h^2} \cdot \frac{\partial E_{zs}}{\partial x} \quad \text{—— (A)}$$

sub (6) eq^n in (A) eq^n

$$= \frac{-j\omega\mu}{h^2} \cdot \frac{\partial}{\partial y}\left[A \cos K_x \cdot x \cdot C \cdot \cos K_y \cdot y \cdot e^{-\gamma z}\right] - \frac{\gamma}{h^2} \cdot 0$$

$[\because E_z = 0]$.

$$= \frac{-j\omega\mu}{h^2} \cdot AC \cdot \cos K_x \cdot x\,(-\sin K_y \cdot y) \cdot K_y \cdot e^{-\gamma z}$$

$$E_{xs} = \frac{j\omega\mu}{h^2} \cdot AC \cdot \cos K_x \cdot x \cdot \sin K_y \cdot y \cdot K_y \cdot e^{-\gamma z}$$

sub eq^n (2) in above eq^n.

$$0 = \frac{j\omega\mu}{h^2} \cdot AC \cdot \cos K_x \cdot x \cdot \sin K_y \cdot b \cdot K_y \cdot e^{-\gamma z}$$

$$0 = \frac{j\omega\mu}{h^2} \cdot AC \cos K_x x \cdot \sin K_y b \cdot K_y .$$

$\cos K_x \cdot x \neq 0$, $A \neq 0$, $C \neq 0$.

$\sin K_y \cdot b = 0$

$b \cdot \cancel{\sin} K_y = \cancel{\sin}\, n\pi$

$b \cdot K_y = n\pi$

$$\boxed{K_y = \frac{n\pi}{b}}$$

From 4th boundary condition:

$E_y = 0$ at $x = a$.

$$E_{ys} = \frac{-\gamma}{h^2} \cdot \frac{\partial E_{zs}}{\partial y} + \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial x}$$

$$E_{ys} = \frac{j\omega\mu}{h^2} \cdot \frac{\partial}{\partial x}\left[A \cos K_x \cdot x \cdot C \cdot \cos K_y \cdot y \, e^{-\gamma z}\right]$$

($\because E_z = 0$ & from eqn ⑥).

$$E_{ys} = \frac{j\omega\mu}{h^2} AC \left(-\sin K_x \cdot x \, K_x\right) \cdot \cos K_y \cdot y \, e^{-\gamma z}.$$

$$0 = \frac{-j\omega\mu}{h^2} \cdot AC \sin K_x \cdot a \cdot K_x \cdot \cos K_y \cdot y \cdot e^{-\gamma z}$$

$\cos K_y \neq 0$, $A; C \neq 0$.

$\sin K_x \cdot a = 0$.

$\cancel{\sin} K_x \cdot a = \cancel{\sin}(m\pi)$

$K_x \cdot a = m\pi$

$$\boxed{K_x = \frac{m\pi}{a}}$$

sub K_x, K_y values in eqn ⑥

$$H_{zs} = AC \cos\left(\frac{m\pi}{a}\right) x \cdot \cos\left(\frac{n\pi}{b}\right) \cdot y \cdot e^{-\gamma z}.$$

$$H_{zs} = H_0 \cos\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)\cdot y \cdot e^{-\gamma z} \quad \text{—— (7)}$$

sub eqn (7) in eqn (A), (B), (C), (D) & $E_z = 0$.

$$E_{xs} = \frac{-j\omega\mu}{h^2} \cdot H_0 \cdot \cos\left(\frac{m\pi}{a}\right) \cdot \frac{n\pi}{b} \cdot \sin\left(\frac{n\pi}{b}\right) \cdot y \, e^{-\gamma z}.$$

$$E_{ys} = \frac{-j\omega\mu}{h^2} H_0 \cdot \frac{m\pi}{a} \cdot \sin\left(\frac{m\pi}{a}\right) x \cdot \cos\left(\frac{n\pi}{b}\right) y \, e^{-\gamma z}.$$

$$H_{xs} = \frac{\gamma}{h^2} \cdot H_0 \frac{m\pi}{a} \cdot \sin\left(\frac{m\pi}{a}\right) x \cdot \cos\left(\frac{n\pi}{b}\right) y \cdot e^{-\gamma z}$$

$$H_{ys} = \frac{\gamma}{h^2} \cdot H_0 \cos\left(\frac{m\pi}{a}\right) x \cdot \left(\frac{n\pi}{b}\right) \cdot \sin\left(\frac{n\pi}{b}\right) y \cdot e^{-\gamma z}.$$

TE Modes in Rectangular waveguides:-

TE_{mn} is the general mode and the specific modes are given by various values of m and n as discussed below.

1) <u>TE_{00} mode</u>: For m=0 & n=0 i.e., the no. of half wave variations on wide dimension & narrow dimension are zero, therefore all the field components vanish inside the waveguide, therefore this mode can't exist.

2) <u>TE_{10} mode</u>: for m=1, n=0, there is only one-half wave variation of electric field along the wide dimension & there is no electric field variation along the narrow dimension. Therefore TE_{10} mode can exist & it is the simplest mode.

<u>TE_{01} mode</u>: For m=0 & n=1,

$E_y = 0$, $H_x = 0$, E_x & H_y exist.

TE_{11} mode: for $m=1, n=1$.

TE_{11} mode exist.

Modes of TM wave in Rectangular waveguides:-

TM_{00} mode: For $m=0, n=0$.

All the electric & magnetic field components E_x, E_y, H_x, H_y vanishes hence TM_{00} mode can't exist.

TM_{01} mode: For $m=0, n=1$.

All the field components vanishes hence it does not exist.

TM_{10} mode: For $m=1, n=0$.

Again all field components vanishes so this mode also not valid.

TM_{11} mode: For $m=1, n=1$.

In this mode all field components exist.

Dominant Mode:-

→ The mode for both TE, TM which offers highest cut-off wavelength (λ_0) or lowest cut-off freq (f_0) in a particular waveguide is called as dominant mode.

→ For TE_{mn} mode, TE_{10} is the dominant mode & for TM_{mn} mode, TM_{11} is the dominant mode.

→ Dominant mode is almost a low loss, distortion less transmission while other modes contains harmonic distortion & losses. Therefore TE_{10} & TM_{11} modes are used for all practical electro magnetic transmission.

Dominant mode is that mode for which the cut off wave-length (λ_c) assumes a maximum value.

we know that $\lambda_{cmn} = \dfrac{2ab}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$

For TM_{11} mode:

$m=1, n=1.$

$$\lambda_{c11} = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}}$$

$$\lambda_{c11} = \frac{2}{\sqrt{\frac{b^2+a^2}{a^2b^2}}} = \frac{2ab}{\sqrt{a^2+b^2}}$$

For TE_{10} Mode:-

$m=1, n=0.$

$$\lambda_{c10} = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2 + 0}}$$

$$\lambda_{c10} = \frac{2}{\frac{1}{a}} = 2a.$$

Degenerate Mode:-

→ The higher order modes having the same cut-off freq are referred as degenerate modes.

→ In rectangular waveguide, TE_{mn} & TM_{mn} modes (both $m \neq 0, n \neq 0$) are always degenerate. In square wave guide ($a=b$), modes TE_{pq}, TE_{qp}, TM_{pq}, TM_{qp} modes are degenerate.

characteristic eq^n of TM_{mn} modes:-

I. cut-off frequency:-

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

ϵ = permitivity

μ = permeability

Phase-constant:-

$$\beta g = \omega\sqrt{\mu\epsilon} \sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

ω = Angular freq

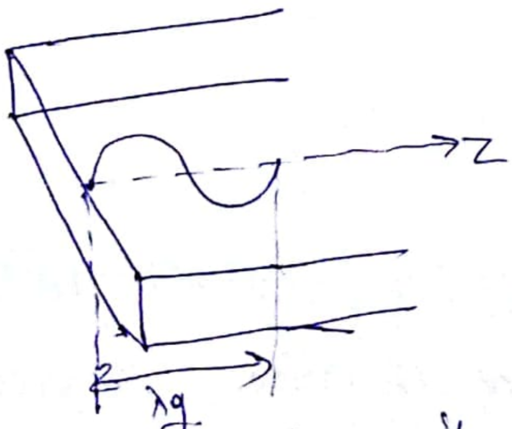
f_c = cut-off freq.

f = freq of propagation

Guide wavelength:-

$$\cancel{\lambda g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}}$$

The distance travelled by the wave with a phase shift of 2π radians is called guide wavelength (λg).



$$\lambda g = \frac{2\pi}{\beta}$$

where λg = guide wavelength

β = phase constant

The relation ship b/w guide wavelength λg & cut off wavelength λ_c & free space wavelength λ is

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2}$$

$$\frac{1}{\lambda_g^2} = \frac{\lambda_c^2 - \lambda^2}{\lambda^2 \lambda_c^2}$$

$$\lambda_g^2 = \frac{\lambda^2 \lambda_c^2}{\lambda_c^2 - \lambda^2}.$$

$$\lambda_g = \frac{\lambda_c \lambda}{\sqrt{\lambda_c^2 - \lambda^2}} = \frac{\lambda_c \cdot \lambda}{\lambda_c \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

Phase velocity:-(v_p)

The velocity with which a wave changes its phase in a direction parallel to walls of the waveguide is called phase velocity.

$$v_p = f \cdot \lambda_g = \frac{2\pi f}{2\pi} \cdot \lambda_g = \frac{\omega}{\beta}$$

(or)

$$v_p = \lambda_g \cdot f.$$

$$= f \cdot \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} = \frac{c}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

where v_p = phase velocity

c = velocity of light

λ_c = cut-off wavelength

λ = free space "

λ_g = Guide " .

Group velocity (v_g):

The velocity of the group of waves in the direction parallel to the conducting surface is called group velocity.

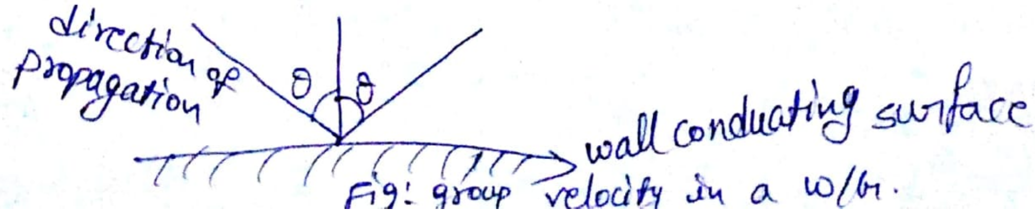


Fig: group velocity in a w/G.

$$v_g \cdot v_p = c^2$$

$$v_g = \frac{c^2}{v_p} = \frac{c^2}{\frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

$$v_g = c \cdot \sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$$

<u>wave impedance:-</u>

$$\eta_{TM} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$H_z = 0.$

$$Z_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x}.$$

$$= \frac{-\frac{j\omega\mu}{h^2} \cdot \frac{\partial H_z}{\partial y} - \frac{\gamma}{h^2} \cdot \frac{\partial E_z}{\partial x}}{-\frac{j\omega\varepsilon}{h^2} \cdot \frac{\partial E_z}{\partial x} - \frac{\gamma}{h^2} \cdot \frac{\partial H_z}{\partial y}}.$$

$$= \frac{-\frac{\gamma}{h^2} \cdot \frac{\partial E_z}{\partial x}}{-\frac{j\omega\varepsilon}{h^2} \cdot \frac{\partial E_z}{\partial x}} = \frac{\gamma}{j\omega\varepsilon}$$

$$= \frac{\cancel{j}\beta}{\cancel{j}\omega\varepsilon} = \frac{\cancel{\omega}\sqrt{\mu\varepsilon} \cdot \sqrt{1-\left(\frac{f_c}{f}\right)^2}}{\cancel{\omega}\varepsilon}$$

$$Z_{TM} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

propagation constant (γ):

$$\gamma = j\beta$$
$$= j\,\omega\sqrt{\mu\varepsilon}\,\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

Characteristic eq^n of TE_{mn} modes :-

cut-off wave number:-

$$K_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$K_c = \omega_c \sqrt{\mu\varepsilon}$$

where a, b are in meters.

cut-off freq: (f_c)

$$f_c = \frac{1}{2\sqrt{\mu\varepsilon}} \cdot \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Phase constant :- (β)

$$\beta_g = \omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

f_c = cut-off freq.

f = operating freq.

Phase velocity :- (V_p) :-

$$V_p = \frac{\omega}{\beta_g} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$V_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

characteristic impedance :-

In TE mode $E_z = 0$.

wave impedance is defined as the ratio of the strength of electric field in one transverse direction to the strength of magnetic field in other transverse direction.

$$Z_{TE} = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$

$$= \frac{-\frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial y} - \frac{\gamma}{h^2}\cdot\frac{\partial E_z}{\partial x}}{-\frac{j\omega\mu}{h^2}\cdot\frac{\partial E_z}{\partial x} - \frac{\gamma}{h^2}\cdot\frac{\partial H_z}{\partial y}},$$

$$= \frac{-\frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2}\cdot\frac{\partial H_z}{\partial y}} = \frac{j\omega\mu}{\gamma}.$$

$$= \frac{j\omega\mu}{j\beta} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$Z_{TE} = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

→ A 6 GHz signal is to be propagated in the dominate mode in a rectangular waveguide. If its group velocity is to be 80% of the free space velocity of light what must be the breadth of wave guide? what impedance will it offer to this signal if its correctly matched.

$F = 6\,GHz.$

it is dominant mode so $m=1, n=0$.

$V_g = 80\%\, c$

$V_g = \frac{80}{100} \times c = 0.8 \times c$

breadth $a = ?$

$Z = ?$

$V_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$c \times 0.8 = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$$0.8 = \sqrt{1 - \left(\frac{f_c}{6\times10^9}\right)^2}$$

$$(0.8)^2 = 1 - \left(\frac{f_c}{6\times10^9}\right)^2$$

$$1-(0.8)^2 = \left(\frac{f_c}{6\times10^9}\right)^2$$

$$f_c = 3.6\,GHz.$$

$$F_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$3.6\times10^9 = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2}$$

$$3.6\times10^9 = \frac{3\times10^{10}}{2}\times\frac{1}{a}.$$

$$a = \frac{3\times10}{2\times3.6} = \frac{30}{7.2} = 4.166\,cm.$$

$$Z = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$Z = \frac{\sqrt{\frac{4\pi\times10^{-7}}{8.854\times10^{-12}}}}{\sqrt{1-\left(\frac{3.6\times10^9}{6\times10^9}\right)^2}} = 471.23\,\Omega$$

Eg: A Rectangular waveguide has a cross-section of 1.5cm × 0.8 cm, $\sigma=0$, $\mu=\mu_0$ & $\varepsilon=4\varepsilon_0$. The magnetic field component is given as $H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi}{b}\right)y \cdot \sin(\pi\times10^{11}t - \beta z)$ A/m.

Determine
- i) The mode of operation.
- ii) The cut-off freq
- iii) The phase constant.
- iv) propagation constant.
- v) wave impedance.

Given data :- $a = 1.5\ cm$

$b = 0.8\ cm$

$\sigma = 0$

$\mu = \mu_0$

$\varepsilon = 4\varepsilon_0$

$\varepsilon = 4 \times 8.854 \times 10^{-12}\ F/M$

$\mu_0 = 4\pi \times 10^{-7}\ H/m$

$H_x = 2\sin\left(\frac{\pi x}{a}\right) \cdot \cos\left(\frac{3\pi y}{b}\right) \cdot \sin(\pi \times 10^{11} t - \beta z)$

$m = 1,\ n = 3$

i) $TE_{mn} = TE_{13}$

$TM_{mn} = TM_{13}$

ii) $f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

$= \frac{1}{2\sqrt{4\times10^{-7} \times 4 \times 8.854 \times 10^{-12}}} \sqrt{\left(\frac{1}{1.5}\right)^2 - \left(\frac{3}{0.8}\right)^2}$

$f_c = 28.55\ GHz$

iii) $\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

$\omega = \pi \times 10^{11}$

$2\pi f = \pi \times 10^{11}$

$f = 10^{11} \times 0.5 = 50\ GHz$

$\beta = \pi \times 10^{11} \sqrt{1 - \left(\frac{28.55 \times 10^9}{50 \times 10^9}\right)^2}\ \sqrt{\mu\varepsilon}$

$\beta = 1720.57\ rad/m$

iv) Propagation constant $\gamma = j\beta$

$\gamma = j\,1720.57$

v) impedance $Z_{TM_{13}} = \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

$= \sqrt{\frac{4\pi \times 10^{-7}}{4 \times 8.854 \times 10^{-12}}} \sqrt{1 - \left(\frac{28.55 \times 10^9}{50 \times 10^9}\right)^2}$

$$Z_{TM_{13}} = 154.6\,\Omega$$

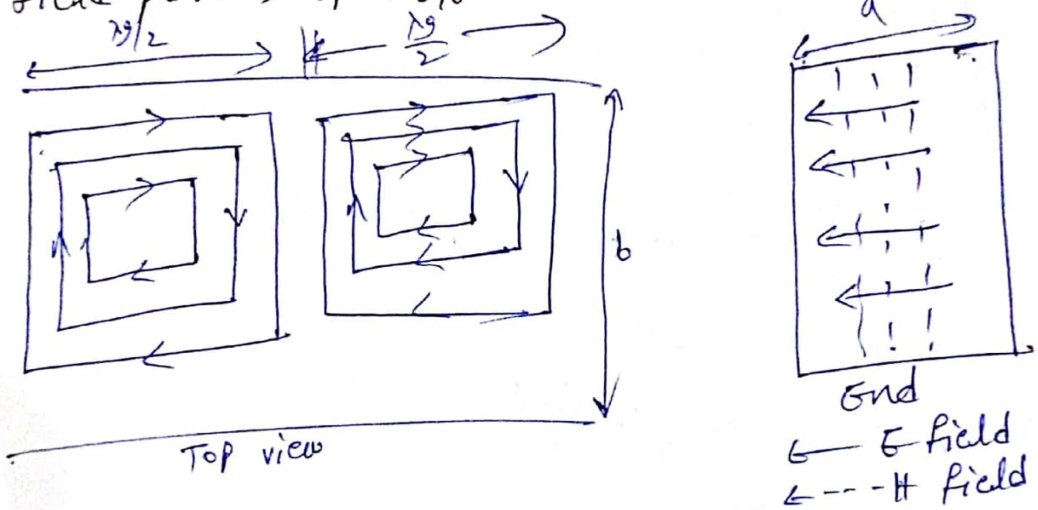
$$Z_{TE_{13}} = \frac{\sqrt{\frac{\mu}{\epsilon}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{\sqrt{\frac{4\pi\times10^{-7}}{4\pi\times8.854\times10^{-12}}}}{\sqrt{1-\left(\frac{28.55\times10^{9}}{50\times10^{9}}\right)^2}}$$

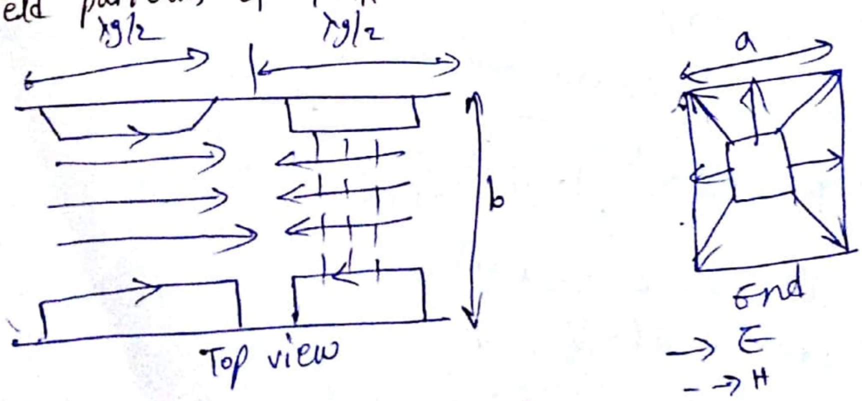
$$Z_{TE_{13}} = 229.4\,\Omega.$$

Dominant mode :-

Field patterns of TE_{10} mode :-



Field patterns of TM_{11} mode :-



Power Transmission in waveguide :-

The power transmitted through a waveguide and the power loss in the guide walls can be calculated by means of complex pointing theorem. we assume that the waveguide is terminated in such a way that there is no reflection from the receiving end.

The power transmitted P_{tr}, through a waveguide is

$$P_{tr} = \int P \cdot ds$$

$$P = E \times H^*$$

$$P_{tr} = \int E \times H^* \cdot ds$$

The real average power transmitted is

$$P = \frac{1}{2} \int E \times H^* \cdot ds$$

$$Z_g = \frac{E_x}{H_y}$$

For TM mode

$$P = \frac{1}{2} \int Z_g \cdot H_y \times H \cdot ds$$

$$P = \frac{Z_g}{2} \int |H|^2 \cdot ds$$

For TE mode

$$P = \frac{1}{2} \int \frac{1}{Z_g} \cdot E_x \times E \cdot ds$$

$$P = \frac{1}{2Z_g} \cdot \int |E|^2 ds$$

where $|E|^2 = |E_x|^2 + |E_y|^2$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

$$P_{tr} = \frac{c}{2} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \cdot \int (H_x^2 + H_y^2) \cdot ds$$

$$P_{tr} = \frac{1}{2c} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \cdot \int (E_x^2 + E_y^2) ds$$

power losses in waveguides–

1) power loss in dielectric filling.
2) power loss in waveguide walls.
3) Misaligned waveguide sections.

power loss in dielectric filling :-

When the guide is filled with a low loss dielectric the attenuation constant α is

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \quad \text{--- (1)}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \quad \text{--- (2)}$$

sub (2) in (1)

$$\alpha = \frac{\sigma \eta}{2}$$

The attenuation in waveguide α_g for TE_{mn} & TM_{mn} mode is given by,

For TE_{mn} mode,

$$\alpha_g = \frac{\sigma}{2} \cdot \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\alpha_g = \frac{\sigma}{2}\frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\alpha}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$$

For TM_{mn} mode,

$$\alpha_g = \frac{\sigma\eta}{2}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\alpha_g = \alpha \cdot \sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$$

power loss in waveguide walls :-

In waveguide the wave is propagated by reflections from walls. The tangential component of electric field & normal component of magnetic field develops losses in the walls. Due to this the average power in the waveguide is dissipated. The attenuation in waveguide α_g is

$$\alpha_g = \frac{P_L}{2P_{tr}}$$

Scanned by CamScanner

P_L = power loss per unit length

P_{tr} = power transmitted through the waveguide.

Misaligned waveguide sections:-

When the waveguide sections are joined and if the joint is not appear proper misaligned, there will be some loss due to reflection.

Impossibility of TEM mode:-

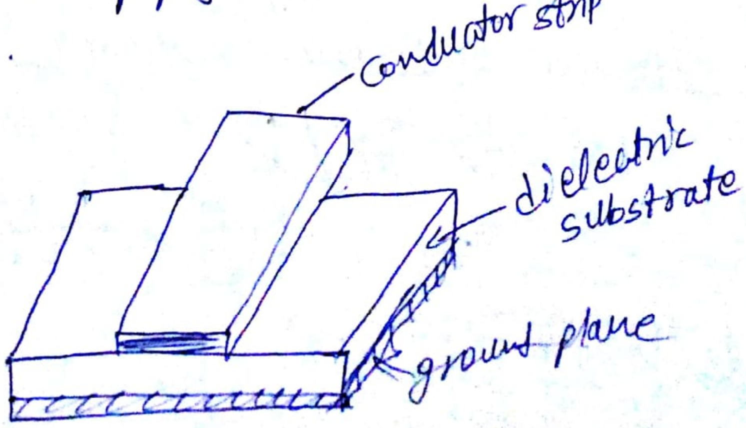
The TEM (Transverse Electromagnetic wave) will not propagate in a waveguide because certain boundary conditions apply. while the wave in the waveguide propagates through the air or inert gas dielectric manner similar to free space propagati The process is bounded by the walls of the waveguide which implies that certain conditions must be met.

The boundary conditions for waveguides are as follows.

1. The electric field must be orthogonal to the conductor in order to exist at the surface of that conductor.
2. The magnetic field must not be orthogonal to the surface of the waveguide.

In order to satisfy these boundary conditions the waveguide gives rise to two types of propagation modes, one is TE mode, another one TM mode.

Microstrip Lines:-



→ Microstrip line has very simple geometry. Microstrip line consists of a conductor & a ground plane separated by dielectric material. Microstrip is an unsymmetrical strip line. The upper ground plane is not present in microstrip, hence it is called as open strip line.

→ microstrip line construction has following advantages.

1) Better inter connection features.
2) Easier fabrication.

→ Microstrip circuits are fabricated using printed circuits techniques. Three commonly used dielectric materials are alumina, quartz and duroid.

→ It is used to avoid radiations from fringing fields.

→ when the s/n is poor to avoid that weakness for increasing the s/n strength we are using microstrip.

→ since the upper plane is not existing the electric field lines remain partially in the air & in dielectric substrate. Hence microstrip line does not support pure TEM mode for propagation.

→ Because of open structure the microstrip line radiates electromagnetic energy.

Effective dielectric constant :-

For $\frac{w}{h} \leq 1$:-

$$\epsilon_{eff} = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2}\left[\left(1+\frac{12h}{w}\right)^{-\frac{1}{2}} + 0.04\left(1-\frac{w}{h}\right)^2\right]$$

for $\frac{w}{h} >> 1$:-

Scanned by CamScanner

$$\varepsilon_{eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1+\frac{12h}{\omega}\right)^{-\frac{1}{2}}$$

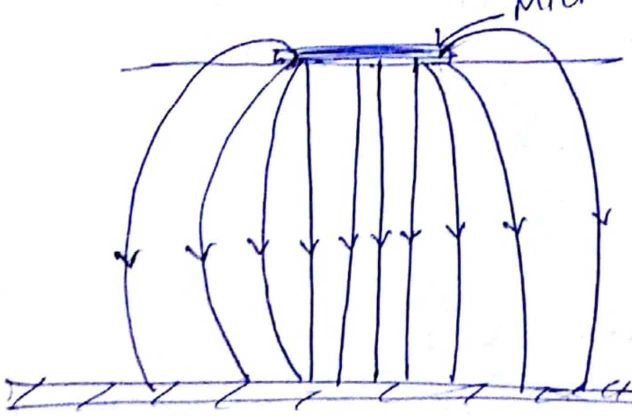
where ε_r is relative permittivity.

ω is width of strip line.

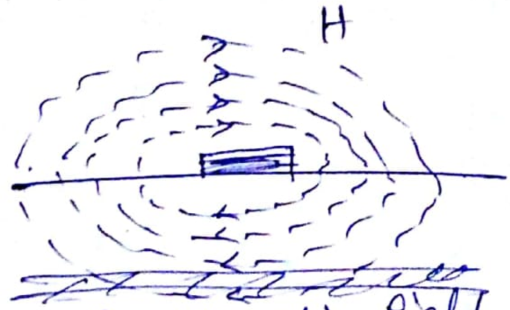
characteristic Impedance (Z_0):-

The characteristic impedance (Z_0) of microstrip line is a function of the strip line width (ω), thickness(t) and ground plane separation (h)

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left[\frac{8h}{\omega} + \frac{\omega}{4h}\right]\Omega \quad \text{if } \frac{\omega}{h} \le 1.$$



(a) electric field.



(b) Magnetic field

Fig: Quasi TEM Mode of microstrip line.

characteristic impedance of microstrip line:-

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{re}}} \ln\left(\frac{4h}{d}\right); \quad h > 2d.$$

ε_{re} = effective relative dielectric constant.

$$\varepsilon_{re} = 0.475\,\varepsilon_r + 0.67$$

$$d = 0.67\omega\,(0.8 + t/\omega)$$

$$Z_0 = \frac{60}{\sqrt{0.475\varepsilon_r + 0.67}} \ln\left(\frac{4h}{0.67\omega(0.8+t/\omega)}\right)$$

$$= \frac{60}{\sqrt{0.475}\left(\sqrt{\varepsilon_r + \frac{0.67}{0.475}}\right)} \ln\left(\frac{4h}{0.67(0.8\omega+t)}\right)$$

$$= \frac{87.05}{\sqrt{\varepsilon_r + 1.41}} \ln\left(\frac{5.97h}{0.8w + 8}\right)$$

Q factor of a microstrip line:-

attenuation constant α_c

conductor Q factor Q_c

The quality factor of a microstrip is very high. The value of Q of a microstrip is limited by the radiation losses of the substrate and dielectric constant.

$$Q_c = \frac{27.3}{\alpha_c} \quad \text{--- (1)}$$

$$\alpha_c = \frac{8.686\, R_s}{Z_0 w} \ \mathrm{dB/cm} \quad \text{--- (2)}$$

$$R_s = \pi \sqrt{\frac{f\mu}{\sigma}}$$

$$Z_0 = \frac{h}{w} \cdot \frac{120\pi}{\sqrt{\varepsilon_r}}$$

$$\alpha_c = \frac{8.686 \times \pi \sqrt{\frac{f\mu}{\sigma}}}{\frac{h}{\cancel{w}} \cdot \frac{120\pi}{\sqrt{\varepsilon_r}} \cdot \cancel{w}}$$

$$= \frac{8.686 \sqrt{\frac{f\mu\varepsilon}{\sigma}}}{120h}$$

$$\alpha_c = \frac{0.128}{h} \sqrt{\frac{f\mu\varepsilon_r}{\sigma}}$$

$$Q_c = \frac{27.3}{\frac{0.128}{h}\sqrt{\frac{f\mu\varepsilon_r}{\sigma}}}$$

$$Q_c = \frac{212.78\, h \sqrt{\sigma}}{\sqrt{f\mu\varepsilon_r}}$$

Q_c is measured in dB/λ_g.

$$\lambda_g = \frac{30}{f\sqrt{\varepsilon_r}}$$

$$Q_c = \frac{\frac{212.78\,h\sqrt{f}}{\sqrt{f\mu}\,\varepsilon_r}}{\frac{30}{f\sqrt{\varepsilon_r}}}$$

$$= \frac{212.78\,h\sqrt{f}\; f\sqrt{\varepsilon_r}}{30\sqrt{f\mu}\,\varepsilon_r}$$

$$= \frac{7.092\sqrt{\varepsilon_r f}\cdot h}{\sqrt{\mu}}$$

$\mu = 4\pi \times 10^{-7}$ H/m.

$Q_c = 0.63\,h\sqrt{\varepsilon_r f}$. GHz

$$Q_d = \frac{27.3}{\alpha_d}$$

$$\alpha_d = 27.3\left(\frac{q\varepsilon_r}{\varepsilon_{re}}\right)\frac{\tan\theta}{\lambda_g}$$

$$Q_d = \frac{27.3}{27.3\left(\frac{q\varepsilon_r}{\varepsilon_{re}}\right)\cdot\frac{\tan\theta}{\lambda_g}}$$

$$= \frac{1}{\frac{\varepsilon_{re}}{\varepsilon_{re}}\cdot\frac{\tan\theta}{\frac{\lambda_0}{\sqrt{\varepsilon_{re}}}}} \qquad \left[\because q\varepsilon_r = \varepsilon_{re}\right]$$

$$= \frac{\lambda_0}{\sqrt{\varepsilon_{re}}\cdot\tan\theta} \qquad \left[\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_{re}}}\right]$$

$$Q_d \sim \frac{1}{\tan\theta}$$

Losses in Microstrip lines:-

1. Dielectric losses 2) ohmic losses 3) Radiation losses.

Dielectric losses:-

The power dissipated by a dielectric. The losses resulting from the heating effect on the dielectric material b/w conductors.

dielectric attenuation constant

$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}.$$

σ = conductivity of dielectric.

commonly used dielectric materials are alumina, quartz.

Ohmic losses:-

ohmic losses are due to the non-perfect conductors.

we know that every material has its own resistance & when electric current flows through that material, due to resistance 'I^2R' losses will occur. And we know that unit of resistance is ohm. That's why it is called as ohmic losses.

The conducting attenuation constant.

$$\alpha_c = \frac{8.86\,R_s}{Z_0 W}\ \mathrm{dB/cm}.$$

R_s = surface skin resistance

Z_0 = characteristic impedance

W = width of microstrip

Radiation losses:-

Radiation loss depends on the substrate thickness & dielectric constant.

Radiated power loss is given as

$$P_{rad} = 240\,\pi^2\left(\frac{h}{\lambda_0}\right)\cdot\frac{F}{Z_0}\cdot P_t.$$

h = distance b/w ground plane & microstrip.

F = radiation factor

Z_0 = characteristic impedance

λ_0 = free space wavelength.

Ex: A certain Microstrip line has the following parameters
$\varepsilon_r = 5.23$, $h = 7$ miles, $t = 2.8$ miles, $w = 10$ miles calculate
I) effective dielectric constant
II) diameter of the wire over the ground.
III) characteristic impedance IV) wavelength at freq 2GHz.

I) $\varepsilon_{re} = 0.475\,\varepsilon_r + 0.67$

$= 0.475(5.23) + 0.67$

$\varepsilon_{re} = 3.154.$

II) $d = 0.67\,w(0.8 + t/w)$

$= 0.67 \times 10\left(0.8 + \frac{2.8}{10}\right)$

$d = 7.78$ miles

III) $z_0 = \frac{60}{\sqrt{\varepsilon_{re}}} \ln\left(\frac{4h}{d}\right)$

$= \frac{60}{\sqrt{3.154}} \ln\left(\frac{4 \times 7}{7.78}\right)$

$z_0 = 45.7\,\Omega$

IV) $\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_{re}}}$

$\lambda_0 = \frac{c}{f} = \frac{3 \times 10^8}{2 \times 10^9} = 0.15$

$\lambda = \frac{0.15}{\sqrt{3.154}} = 0.084\,m.$

1

II UNIT

Cavity Resonators

Introduction:

A cavity Resonator is a metallic enclosure formed by shorting two ends of waveguide. Cavity Resonator confines the electro magnetic energy. [so cavity Resonator is the limits of the electro magnetic energy.]

The stored electric & magnetic field components inside the cavity determines the equivalent inductance & capacitance.

within the cavity TE_{mnp} & TM_{mnp} modes are possible.

A very high value of Q can be calculated by using Resonator.

A cavity resonator can take any size or shape, which is depends on different factors such as -

→ Resonant freq required to be produced by cavity.

→ Mechanical considerations.

→ Desired value of quality factor (Q).

→ Required value of shunt conductance.

In microwave applications the commonly used cavity resonators are.

1. Rectangular cavity Resonator.
2. circular cavity "
3. Re-entrant " ".

Rectangular cavity resonator:-

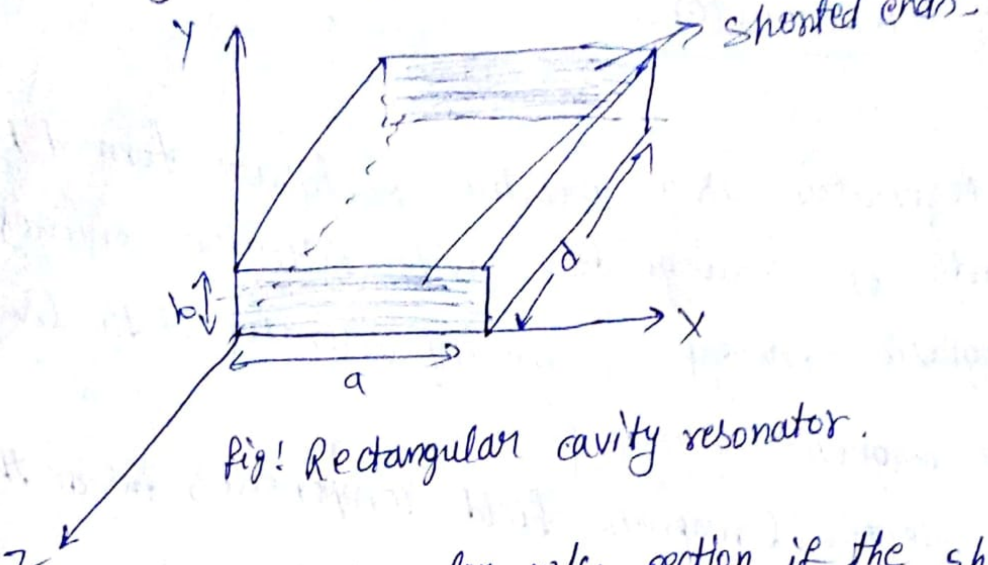


fig: Rectangular cavity resonator.

In a Rectangular w/G section if the short circuit is placed at two ends. The resultant configuration is called a rectangular cavity resonator in which the signal bounces back & forth b/w the opposite walls.

Resonant freq:-

If M = No. of half wave periodicity in the x - direction.

n = No. of " " " y - " .

P = " " z - "

$h^2 = \gamma^2 + \omega^2\mu\epsilon$

$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$\gamma = j\beta$

$\gamma^2 = -\beta^2$, $\beta = \frac{p\pi}{d}$ → [∴ $P = 1, 2, \ldots \infty$) d = length of resonator]

$\gamma^2 + \omega^2\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$.

$\omega^2\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \gamma^2$

$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta^2$

$$\omega^2 \mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

here $\omega = 2\pi f_r$.

$$(2\pi f_r)^2 \mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

$$f_r^2 = \frac{1}{4\pi^2\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]$$

$$f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}} \cdot \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

$$f_r = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

<u>Q-factor of cavity Resonator</u>:-

Q-factor is the measure of the freq selectivity of a circuit, Q is defined as

$$Q = 2\pi \times \frac{\text{max energy stored during a cycle}}{\text{Average energy dissipated per cycle}}.$$

$$Q = \omega_r \cdot \frac{W_s}{P_L}.$$

W_s = Energy stored in cavity

ω_r = Resonant freq.

P_L = Average power loss in cavity.

The average power loss can be divided in to 3 parts.

i) power loss in walls (P_w)

ii) power loss in dielectric (P_d)

iii) power loss due to loading of coupled device (P_L coupling)

$$P_L = P_{Lw} + P_{Ld} + P_{L\,coup}$$

For loaded circuit

$$Q_L = \omega_r \cdot \frac{W_s}{P_{Lw} + P_{Ld} + P_{Lc}}$$

$$\frac{1}{Q_L} = \frac{P_{lw}}{P_{lgws}} + \frac{P_{ld}}{\omega_r w_s} + \frac{P_{lc}}{\omega_r \cdot \omega_s}$$

$$\frac{1}{Q_L} = \frac{1}{Q_w} + \frac{1}{Q_d} + \frac{1}{Q_c}.$$

where Q_w = Q - factor of wall

Q_d = " dielectric

Q_c = " coupling

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_c}$$

Q_0 is unloaded factor.

$Q_d >> Q_w$,

$$\frac{1}{Q_w} >> \frac{1}{Q_d}$$

$$\frac{1}{Q_0} \approx \frac{1}{Q_w}$$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_c}$$

$$\frac{1}{Q_L} = \frac{Q_c + Q_0}{Q_0 Q_c}$$

$$Q_L = \frac{Q_0}{1+K}.$$

here $\frac{Q_c}{Q_c + Q_0} = \frac{1}{1+K}$.

where K = coupling coefficient.

There can be 3 values of coupling coefficients. :-

I critical coupling:

When resonator and generator are matched, then $K = 1$.

$$Q_L = \frac{Q_0}{1+1} = \frac{Q_0}{2}$$

II. over coupling: $K > 1$

Here cavity terminals are at max voltage, and the impedance at maxi voltage is standing wave ratio (ρ) i.e $K = \rho$

$$Q_L = \frac{Q_0}{1+\rho}$$

Scanned by CamScanner

III. under coupling : $K < 1$

- Here cavity terminals are of mini voltage & the impedance is equal to reciprocal of SWR $\left(\frac{1}{\rho}\right)$.

$$K = \frac{1}{\rho}.$$

$$Q_L = \frac{\rho}{\rho+1} \cdot Q_0.$$

The relationship of coupling coefficient K and SWR is

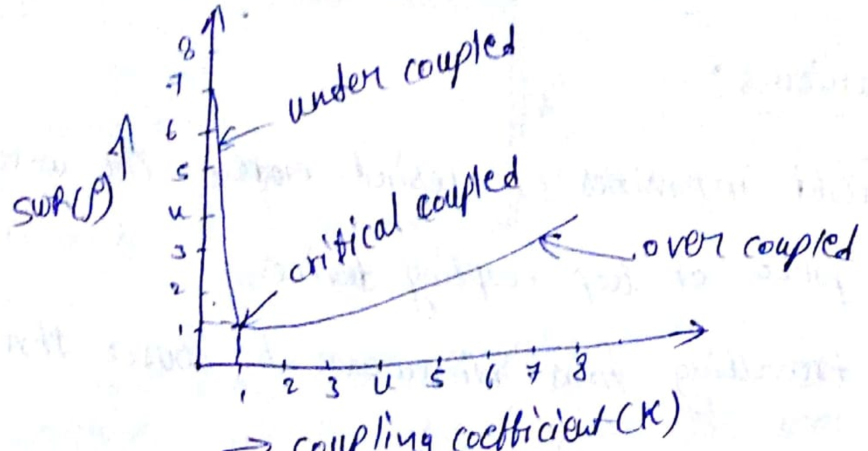


fig: coupling coefficient Vs SWR.

Ex: A metal box is 3x4x5 cm in dimensions filled with air calculate the resonante freq of the cavity for TE_{102}.

Given $a = 3$, $b = 4$, $d = 5$ cm.

$$a = 3 \times 10^{-2}\,m$$
$$b = 4 \times 10^{-2}\,m$$
$$d = 5 \times 10^{-2}\,m.$$

$$m = 1$$
$$n = 0$$
$$p = 2.$$

$$f_r = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

$$= \frac{3 \times 10^8}{2} \cdot \sqrt{\left(\frac{1}{3 \times 10^{-2}}\right)^2 + 0 + \left(\frac{2}{5 \times 10^{-2}}\right)^2}$$

$$f_r = 7.8\,GHz.$$

Applications of cavity Resonators:-

They can be used as tuned circuits in UHF tubes, klystron amplifier/oscillators, cavity magnetron, in duplexers of radars, cavity wavemeters in measurement of freq etc.

Wave guide components and Applications:-

coupling mechanisms:-

To produce field intensities of desired mode in the waveguide we are using probe or loop coupling device.

The w/G are travelling from microwave s/u source through a co-axial cable.

Probe coupling:-

The probe is placed at $\lambda/4$ distance from w/G short at the center of broader dimension of w/G.

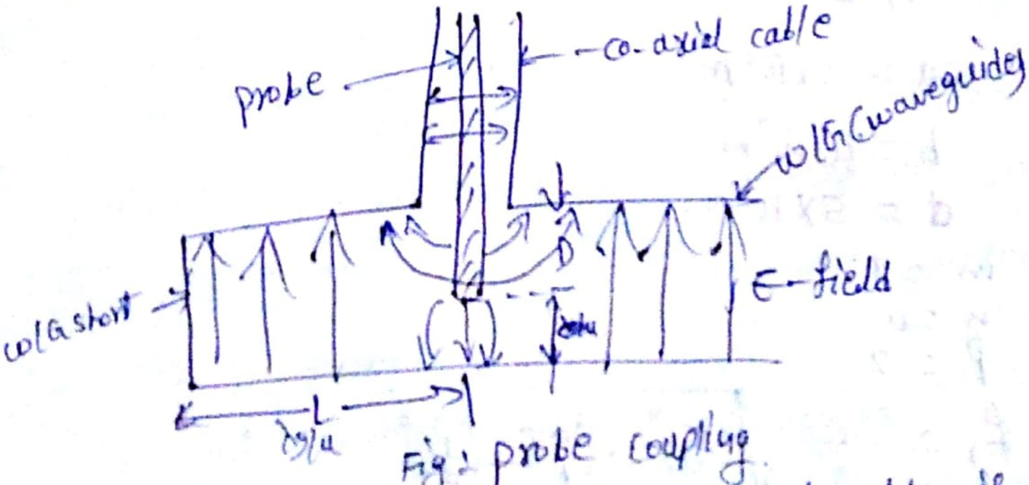


Fig: probe coupling

The inner conductor of the co-axial cable is projected inside the waveguide. This conductor acts as an electric dipole. The dipole is oriented so as to exite the electric field intensity of the mode and the coupling loop so as to generate the magnetic field intensity of the desired mode, The TE_{10} mode with the probe at the centre from the broad wall or perpendicular to the max E-field.

Loop coupling :-

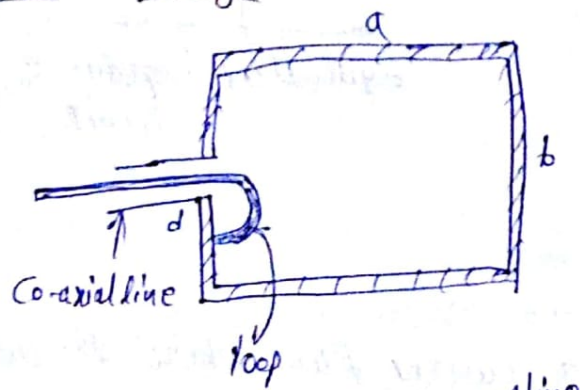


Fig : loop coupling

A curved conductor that connects the ends of a co-axial cable.

→ In-order to excite a particular mode, the wave guide must be properly coupled to an external source. Different coupling methods are used in microwave filters & wave meters.

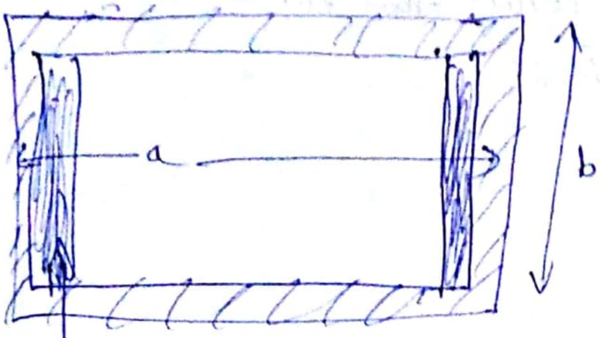
→ The loop size is very small & the current in the loop can be considered to be constant. The conduction current in the loop produces a magnetic field. The loop is capable of exciting any mode. The plane of the loop is placed perpendicular to the magnetic flux lines.

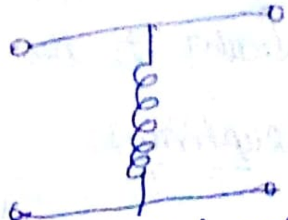
wave guide ~~Aperture~~ / Irises (or) windows :-

Irises are fixed or adjustable projections from the walls of waveguide. Irises are also known as windows. Irises are used for impedance matching purposes. In any w/G system, when there is a mismatch there will be reflections.

There 3 types of Irises:

Inductive Irises:-





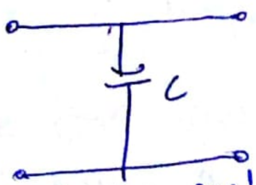
equivalent impedance circuit.

Projections from side walls of guide.

fig: Inductive iris

An ip inductive iris allow a current flow where the current not flowing in waveguide. The irises are placed where the magnetic field is strong (or where the electric field is week) since the plane of polarisation of electric field is parallel to the plane of iris. Energy storage of magnetic field takes place & there is an increase in inductance at that point of the waveguide.

capasitive irises:-



equivalent capacitive ckt.

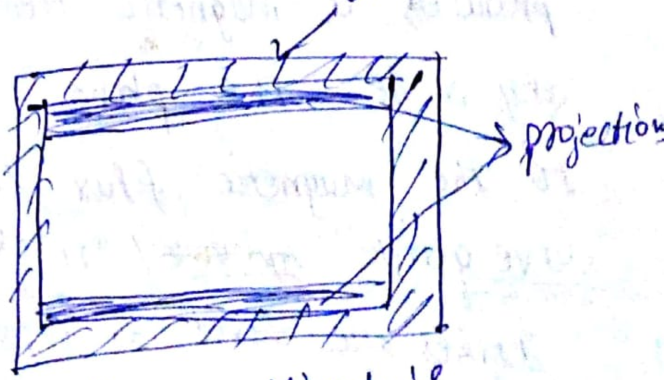
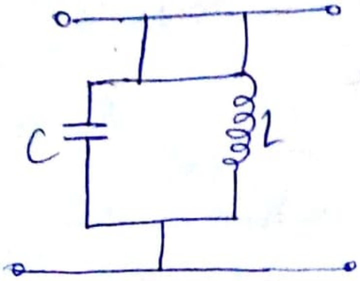


fig: capacitive iris

In capacitive iris the potential exitsted b/w the top and bottom walls of the waveguide now exist b/w surfaces which are closer & therefore the capacitance has increased at that point. The capacitive iris is placed in a position where the electric field is strong.

Resonant iris:-



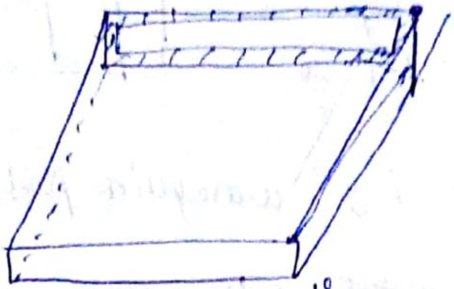


Fig: Resonant iris.

The inductive & capacitance are connecting parallel to the walls of waveguide. The impedance of iris is very high for dominant mode & the shunting effect is also negligible. There fore resonant window acts as a bandpass filter.

posts and Tuning screws!-

When a metallic cylindrical post is introduced in to the broader side of W/G., it produces the lumped reactance same as in iris. If the post is inserting into the W/G. with the distance of less than $\lambda g/4$, it behaves capacitively.

The capacitive susceptence increases with depth is equal to $\lambda g/4$, The post acts as a series resonant circuit. If it is > $\frac{\lambda g}{4}$, the post behaves ~~the depth is equal to~~ $\cancel{\frac{\lambda g}{2}}$ inductively. The Inductive susceptance vs penetration (d) characteristics is shown in fig.

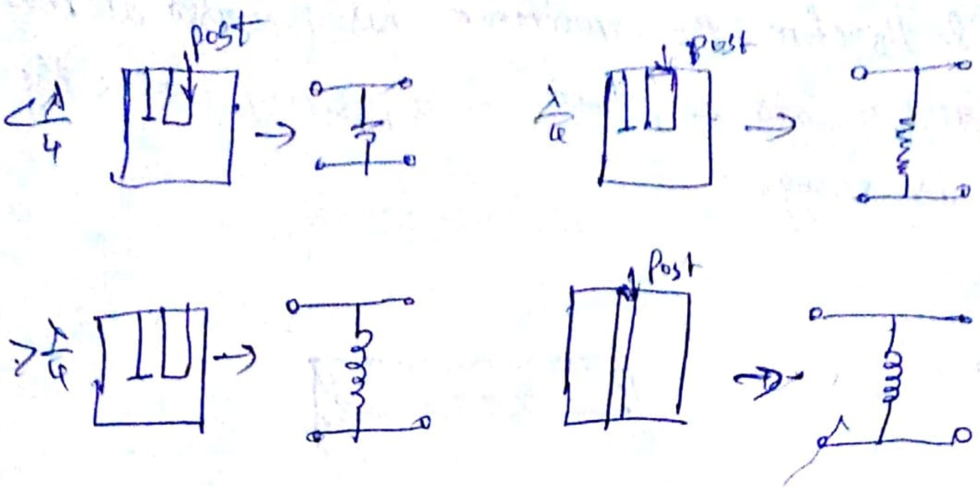
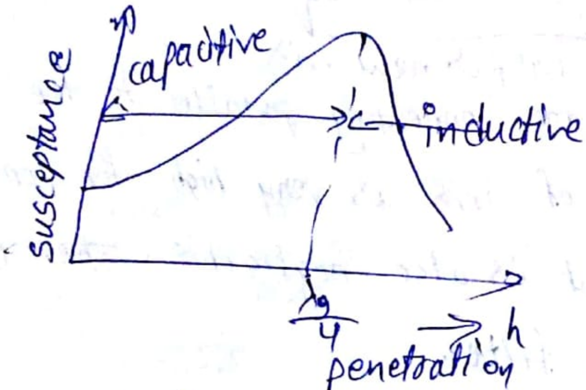
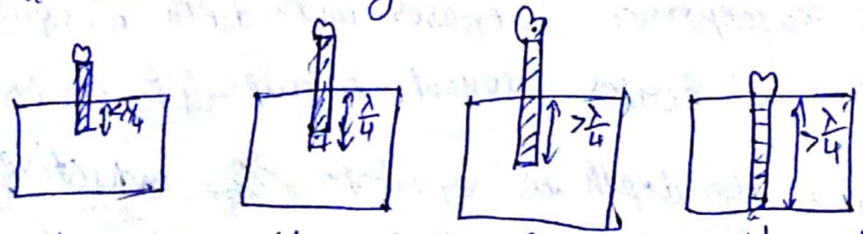


Fig: waveguide posts



The amount of susceptance decreases as the diameter of the post is reduced. If the post is made thicker, effective Q will be lowered & can act as a bandpass filter similar to an iris.

The big advantage of the post over an iris is known as a screw or slug.



Depending upon the depth of penetration, the tuning screw may introduce inductive or capacitive susceptance.

6

Waveguide Discontinuities :-

Any interruption (disturbances) in the waveguide is defined as a waveguide discontinuties.

Different forms in waveguide discontinuities:

1) change in waveguide height.
2) " " width
3) Dielectric discontinuity.
4) Inductive septums (part).
5) capacitive septums.
6) Resonant iris.

change in w/G height :-

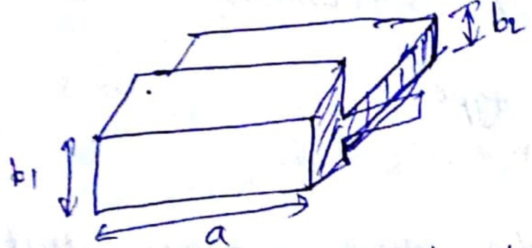
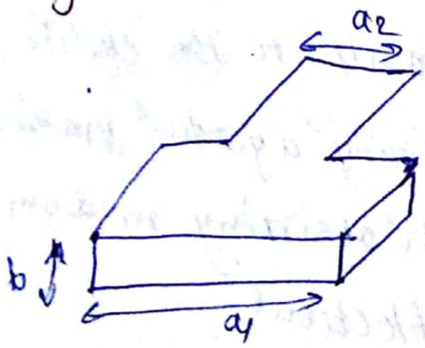


Fig:- change in w/G height.

The change in w/G causes a distortion of TE_{10} electric field near step discontinuity. This condition is analogous to the TEM mode distortion caused by change in the diameter of the co-axial line.

change in w/G width :-



This situation can be represented by shunt inductance since only higher-order TE modes are generated by the discontinuity.

Dielectric discontinuity :-

The discontinuity b/w two planar dielectric waveguides is examined on the basis of an integral-equation formulation. The aperture field is approximated by a finite set of discrete modes, characteristic of the region $z<0$. Results are presented for the launching efficiency & reflection at the junction b/w a solid-state heterojunction laser & an optical waveguide.

Microwave Attenuators :-

Attenuators are used for measuring power gain or loss in dBs, for providing isolation b/w instruments, for reducing power input to a particular stage.

μw attenuators control the flow of μw power either by reflecting it or absorbing it. Attenuators can be classified as fixed or variable type.

Fixed Attenuator :-

Fixed attenuator absorbs all the energy entering into it, we call it as a waveguide terminator. It consists of a dissipative element called pad. which is placed in a waveguide. The pad is connecting parallel to the electric field. The pad is taperd for providing a gradual transition from the waveguide medium to absorbing medium of pad. This also reduces the reflections.

Fig - shows such a fixed attenuator where a dielectric slab (pad) consisting of glass slab coated with carbon film. The amount of power that a fixed attenuator can absorb depends on

8.

7

8) strength of dielectric field.

II) Location of dielectric slab within a guide

III) Area of slab

IV) freq of operation

V) slab material used for power absorption.

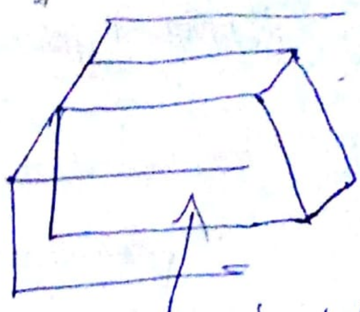


Fig: Fixed attenuator

Variable Attenuator:-

variable attenuator provides continuous or stepwise variable attenuation. For rectangular waveguides. these attenuators can be flap type or vane type for circular waveguides rotary type is used.

Flap type Attenuator:-

The flap type attenuator consists of a resistive card inserted longitudinally into center of the guide. The amount of attenuation introduced is controlled by the depth of insertion of absorbing plate inside the waveguide. For this a knob & gear assembly is used. The knob can be calibrated suitably. The maximum attenuation will be occured when the resistive card extends all the way across the guide.

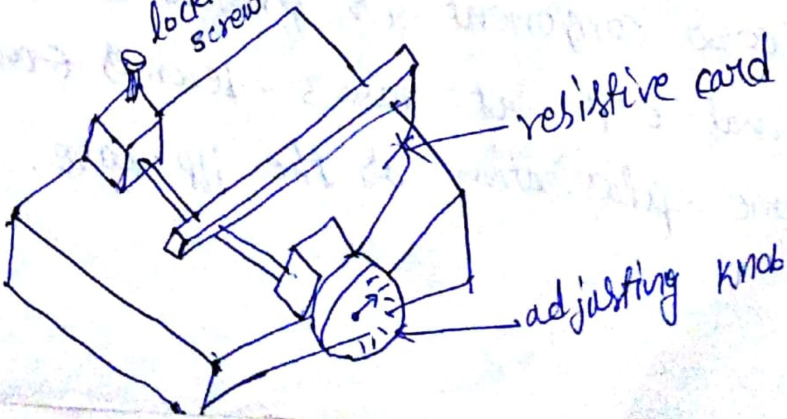


Fig: variable attenuator

Scanned by CamScanner

Rotary type attenuator:-

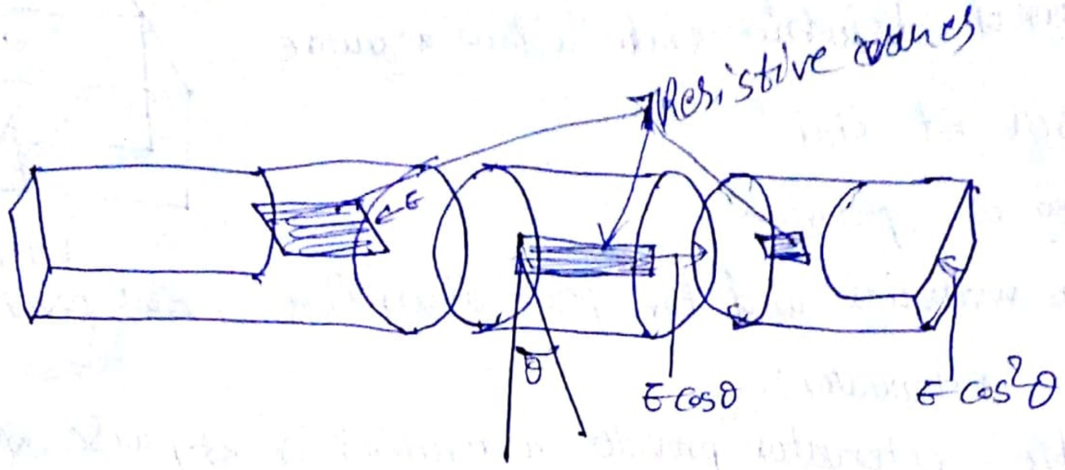


Fig: Rotary wave attenuator.

It consists of three vanes. The central vane rotating type placed in the central section of a circular w/G arrangement tapered at both ends. The other two vanes are in the rectangular section. when all the three vanes are aligned their planes are at 90° to the direction of electric field. Hence there is no attenuation. vane 1 prevents any horizontal polarization & hence electric field at the o/p of vane 1 is vertically polarized. The center vane 2 is rotating type & if it rotated by an angle θ, the $E\sin\theta$ component is attenuated and $E\cos\theta$ component is present at the o/p of vane 2 and final o/p at vane 3 becomes $E\cos^2\theta$, which has the same polarisation as the i/p wave.

Waveguide μw Junctions! –

At a certain position in a waveguide system, many a times it becomes necessary to split all or part of the μw energy into particular directions. This is achieved by microwave junctions. These are combined to form coupler units that direct the energy as required. Alternately the same junction may be used to combine two or more signals. In general, a μw junction is an interconnection of two or more μw components as shown in fig 1.

This junction has four ports similar to low freq two-port networks. Fig 2 shows a μw source at port① & μw loads at ports ②, ③ & ④.

The μw junction is analogous to a traffic junction where a no. of roads meet on which vehicles enter and leave the traffic junction. In a similar manner, when i/p from μw source is applied at port① a part of it comes out of port② another part out of port③ some part out of port④ & the remaining part may come out of port① itself due to mismatch b/w port① & microwave junction.

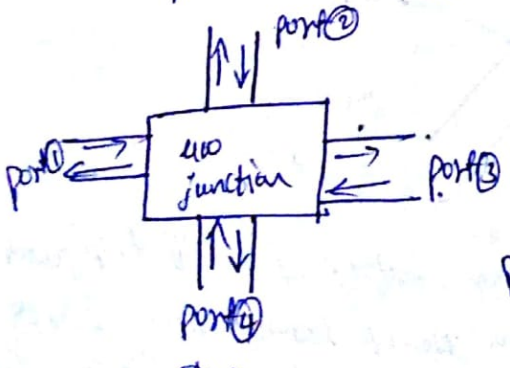


Fig 1.

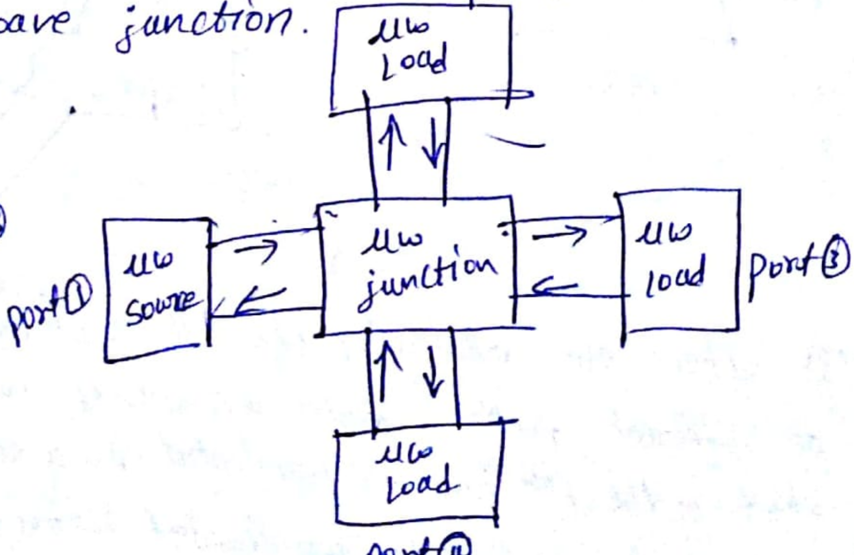


Fig.2.

Scattering parameters:-

low freq circuits can be described by two port n/w & their parameters such as Z, Y, H, ABCD etc. As per n/w theory. Here n/w parameters relate the total voltages & total currents as shown in fig 3.

In a similar manner u/w freq, we talk of travelling waves with associated powers instead of voltages & currents & the u/w junction can be defined by what are called as S-parameters or scattering parameters. In fig 2 for an i/p at one port, we have four o/p's. similarly if we apply i/p's to all the ports, we will have 16 combinations, which are represented in a matrix form & that matrix is called as scattering matrix. It is a square matrix which gives all the combinations of power relationships b/w various i/p and o/p ports of a u/w junction. The elements of this matrix are called scattering parameters

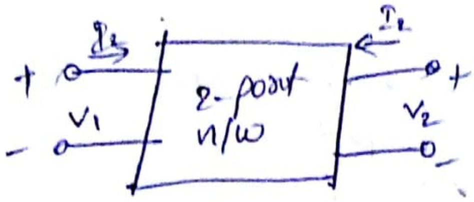


Fig 3.

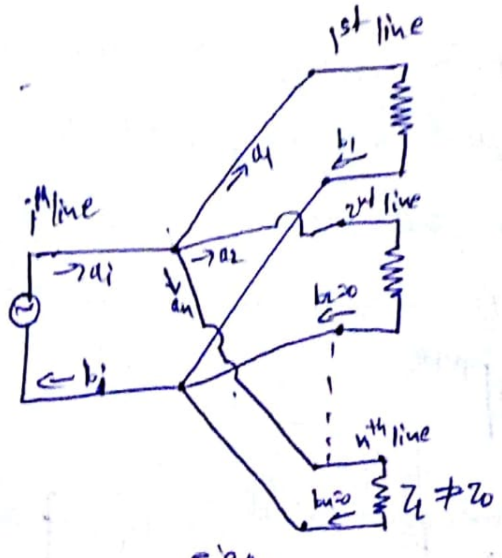


Fig 4.

To obtain the relationship b/w the scattering matrix & the i/p / o/p powers at different ports, consider a junction of 'n' no. of transmission lines where in the ith line is terminated in a source as shown in fig 4.

let the first line be terminated in an impedance other than the characteristic impedance (i.e., $Z_L \neq Z_0$) & all the remaining lines in an impedance equal to Z_0 (i.e., $Z_L = Z_0$).

If a_i be the incident wave at the junction due to a source at the i^{th} line, then it divides itself among $(n-1)$ no. of lines as $a_1, a_2 - a_n$ as shown in fig 4. There will be no reflections from 2^{nd} to n^{th} line & the incident waves are absorbed since their impedances are equal to characteristic impedance (Z_0). But, there is a mismatch at the 1^{st} line & hence there will be a reflected wave b_1 going back into the junction.

b_1 is related to a_1 by,

$b_1 =$ (reflection coefficient) $a_1 = S_{i1} \cdot a_1$

where, S_{i1} = reflection coefficient of 1^{st} line.

a_1 = reflection from 1^{st} line &

i = source connected at i^{th} line.

$b_i = S_{i1} \cdot a_1$. $\quad [\because b_2 = b_3 = \cdots = b_n = 0]$.

Let all the $(n-1)$ lines be terminated in an impedance other than Z_0. Then, there will be reflections in to the junction from every line and hence the total contribution to the outward travelling wave in the i^{th} line is given by

$$b_i = S_{i1} \cdot a_1 + S_{i2} \cdot a_2 + S_{i3} \cdot a_3 + \cdots + S_{in} \cdot a_n.$$

$i = 1$ to n since i can be any line from 1 to n.

$$b_1 = S_{11} a_1 + S_{12} a_2 + \cdots S_{1n} a_n$$
$$b_2 = S_{21} a_1 + S_{22} a_2 + \cdots S_{2n} a_n$$
$$\cdots$$
$$\cdots$$
$$b_n = S_{n1} a_1 + S_{n2} a_2 + \cdots S_{nn} a_n$$

Scanned by CamScanner

In matrix form,

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

column matrix [b] corresponding to Reflected waves or o/p

scattering column matrix [S] of order n×n

Matrix [a] corresponding to incident waves or i/p.

$$[b] = [S][a]$$

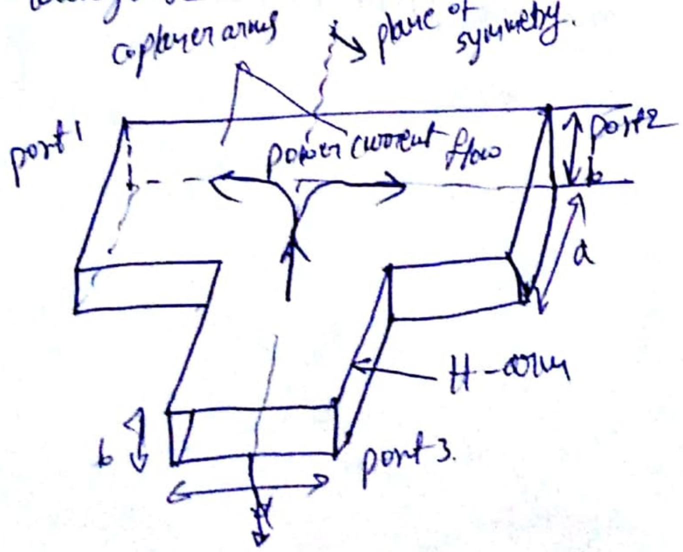
Microwave T-Junctions:-

A T-junction is an intersection of three waveguides in the form of english alphabet 'T'. There are several types of Tee junctions.

1. H-plane Tee junction
2. E-plane "
3. E-H " "
4. Magic T " "

H-plane Tee junction:-

A H-plane Tee junction is formed by cutting a rectangular slot along the width of a main waveguide & attaching another waveguide. the side arm called the H-arm.



The port ① & port ② of the main waveguide are called collinear ports and port ③ is the H-arm or side arm. H-plane Tee is so called because the axis of the side arm is possible parallel to the planes of the main transmission line. As all three arms of H-plane Tee lie in the plane of magnetic field, the magnetic field divides itself into the arms.

The properties of a H-plane Tee can be completely defined by its [S] matrix. The order of scattering matrix is 3x3 since there are three possible i/p's & 3 possible o/p's.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Now we determine the S-parameters S_{ij}, $i \to 1,2,3$, $j \to 1,2,3$ by applying the properties of [S].

1. Because of plane of symmetry of the junction scattering coefficients S_{13} & S_{23} must be equal

$$S_{13} = S_{23}.$$

2- From symmetry property $S_{ij} = S_{ji}$

$$S_{12} = S_{21},\ S_{23} = S_{32} = S_{13},\ S_{13} = S_{31}.$$

3- Since port is perfectly matched to the junction

$$S_{33} = 0.$$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \quad \text{——} (1)$$

4- From unitary ~~real~~ property

$$[S][S]^* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_1C_1: $S_{11} S_{11}^* + S_{12} S_{12}^* + S_{13} \cdot S_{13}^* = 1.$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1. \quad \text{—— (2)}$$

R_2C_2: $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$ —— (3)

R_3C_3: $|S_{13}|^2 + |S_{13}|^2 = 1.$ —— (4)

R_3C_1: $S_{13} S_{11}^* + S_{13} \cdot S_{12}^* = 0.$ —— (5)

from eqn (4)

$$2|S_{13}|^2 = 1$$

$$S_{13} = \frac{1}{\sqrt{2}}. \quad \text{—— (6)}$$

comparing eqn (2), (3). $|S_{11}|^2 = |S_{22}|^2$

$$S_{11} = S_{22} \quad \text{—— (7)}$$

from eqn (5) $\because S_{13} \neq 0$, $(S_{11}^* + S_{12}^*) = 0.$

$$S_{11} = -S_{12} \text{ or } S_{12} = -S_{11} \quad \text{—— (8)}$$

using these in eqn (2).

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1.$$

$$2|S_{11}|^2 = \frac{1}{2} \Rightarrow S_{11} = \frac{1}{2}.$$

from eqn (7) & (8)

$$S_{12} = -\frac{1}{2}$$

$$S_{22} = \frac{1}{2}.$$

$$[S] = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

[b] = [S] [a].

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$b_1 = \frac{1}{2}a_1 + \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3$$

$$b_2 = -\frac{1}{2}a_1 - \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 + \frac{1}{2}a_2$$

case I: $a_3 \neq 0,\ a_1 = a_2 = 0,$

~~case II:~~

$$b_1 = \frac{a_3}{\sqrt{2}},\ b_2 = \frac{a_3}{\sqrt{2}}\ \&\ b_3 = 0.$$

case II: $a_1 = a_2 = a,\ a_3 = 0$

$$b_1 = \frac{a_3}{\sqrt{2}} = 0.$$

$$b_2 = \frac{a_3}{\sqrt{2}} = 0.$$

$$b_3 = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}}$$

E plane Tee:

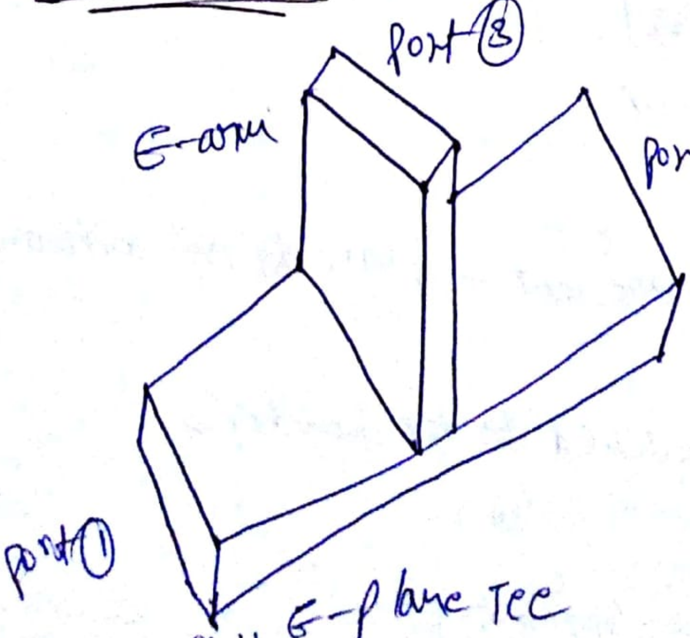


fig1: E-plane Tee

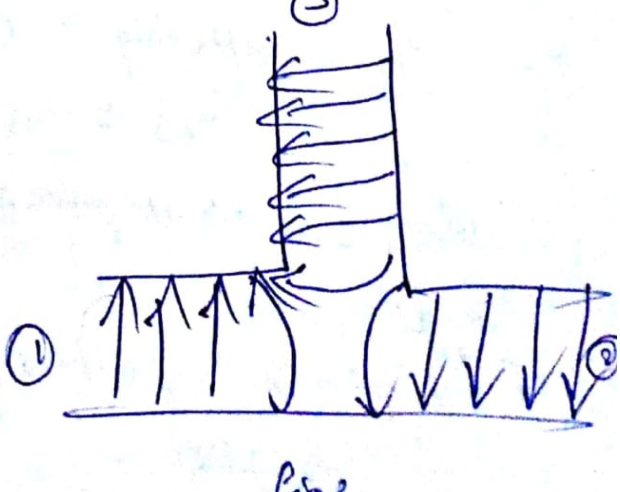


fig2.

A Rectangular slot is cut along the broader dimension of a long waveguide & a side arm is attached as shown in fig 1. ports ① & ② are the collinear arms & port ③ is the E-arm.

When TE_{10} mode is made to propagate into port ③, the two o/p's at port ① & ② will have a phase shift of 180° as shown in fig 2. since the electric field lines change their direction when they come out of port ① & ②, it is called a E-plane Tee. E-plane Tee is a voltage or series junction symmetrical about the central arm. Hence any signal that is to be split or any two signal that are to be combined will be fed from E arm. The power out of port ③ is propotional to the difference b/w Instantaneous powers entering from port ① & ②.

1. $[S]$ is a 3x3 matrix since there are 3 ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

2. The scattering co-efficient

$$S_{23} = -S_{13}$$

since outputs at ports ① & ② are out of phase by 180° with an i/p at port ③.

3. If port ③ is perfectly matched to the junction

$$S_{33} = 0.$$

4. From symmetric property $S_{ij} = S_{ji}$

$$S_{12} = S_{21},\ S_{13} = S_{31},\ S_{23} = S_{32}$$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \quad \text{—— (1)}$$

5. From unitary property, $[S]\cdot[S]^* = [I]$

i.e.,
$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_1C_1 : $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1.$ —— (2)

R_2C_2 : $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1.$ —— (3)

R_3C_3 ! $|S_{13}|^2 + |S_{13}|^2 = 1$ —— (4)

R_3C_1 : $S_{13}\cdot S_{11}^* + S_{13}\cdot S_{12}^* = 0.$ —— (5)

equating (2) & (3)

$S_{11} = S_{22}$

from eqn (4) $S_{13} = \frac{1}{\sqrt{2}}$

from eqn (5) $S_{13}(S_{11}^* - S_{12}^*) = 0$ or $S_{11} = S_{12} = S_{22}$.

Using these values in eqn (2)

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} = 1.$$

$$2|S_{11}|^2 = \frac{1}{2} \quad \text{or} \quad S_{11} = \frac{1}{2}.$$

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix}.$$

$$[b] = [S][a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$b_1 = \frac{1}{2}a_1 + \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3$$

$$b_2 = \frac{1}{2}a_1 + \frac{1}{2}a_2 - \frac{1}{\sqrt{2}}a_3$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 - \frac{1}{\sqrt{2}}a_2$$

case I: $a_1 = a_2 = 0$, $a_3 \neq 0$.

$b_1 = \frac{1}{\sqrt{2}}a_3$; $b_2 = \frac{-1}{\sqrt{2}}a_3$; $b_3 = 0$.

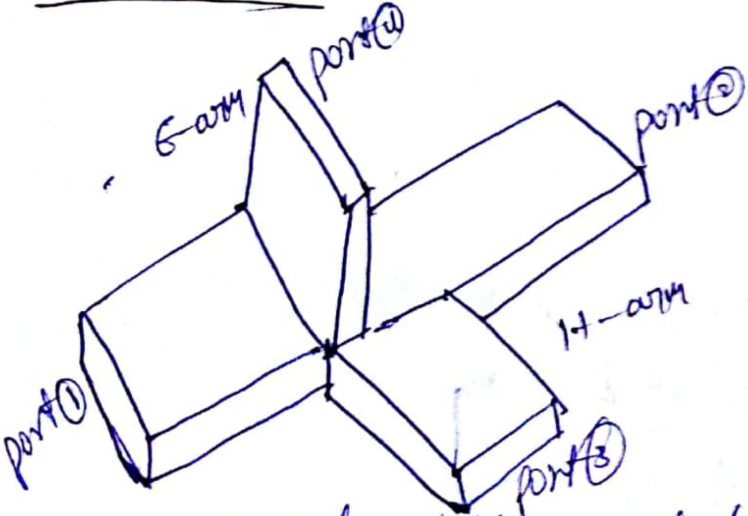
case 2: $a_1 = a_2 = a$, $a_3 = 0$.

$b_1 = \frac{a}{2} + \frac{a}{2}$; $b_2 = \frac{a}{2} + \frac{a}{2}$; $b_3 = \frac{1}{\sqrt{2}}a - \frac{1}{\sqrt{2}}a = 0$.

case 3: $a_1 \neq 0$, $a_2 = 0$, $a_3 = 0$.

$b_1 = \frac{a_1}{2}$, $b_2 = \frac{a_1}{2}$, $b_3 = \frac{-a_1}{\sqrt{2}}$

E-H plane Tee:-



Here rectangular slots are cut both along the width & breath of a long waveguide and side arms are attached as shown in fig above. port ① & ② are colinear arms, port ③ is the H-arm & port ④ is the E arm.

1. [S] is a 4x4 matrix since there are 4 ports

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

2. Because of H-plane Tee $S_{23} = S_{13}$

3. Because of E " $S_{24} = -S_{14}$.

4. Because of geometry of the junction an i/p at port ③ cannot come out of port ④ since they are isolated ports & vice versa.

$S_{34} = S_{43} = 0$.

5. From symmetric property, $S_{ij} = S_{ji}$

$S_{12} = S_{21}$, $S_{13} = S_{31}$, $S_{23} = S_{32}$

$S_{34} = S_{43}$, $S_{24} = S_{42}$, $S_{41} = S_{14}$.

6. If ports ③ & ④ are perfectly matched to the junction.

$S_{33} = S_{44} = 0$.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad \text{——} ①$$

7. From unitary property, $[S][S^*] = [I]$

i.e., $$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

R_1C_1 : $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$ —— ②

R_2C_2 : $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$. —— ③

R_3C_3 : $|S_{13}|^2 + |S_{13}|^2 = 1$ —— ④

R_4C_4 : $|S_{14}|^2 + |S_{14}|^2 = 1$. —— ⑤

eqn ④, ⑤ $S_{13} = \frac{1}{\sqrt{2}}$ —— ⑥

$S_{14} = \frac{1}{\sqrt{2}}$ —— ⑦

comparing ① & ③.

$S_{11} = S_{22}$

using these values from eqn ⑥, ⑦ in ④

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1.$$

$$|S_{11}|^2 + |S_{12}|^2 = 0.$$

$$S_{11} = S_{12} = 0.$$

from eqn ③. $S_{22} = 0$.

This means ports ① & ② are also perfectly matched to the junction. Hence in any four port junction, if any two ports are perfectly matched to the junction, then the remaining ports are automatically matched to the junction. such a junction where in all the four ports are perfectly matched to the junction is called a magic Tee.

$$\begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} = [S]$$

$$[b] = [S][a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

E-H plane (Hybrid or Magic) Tee :-

Here rectangular slots are cut both along the width and breadth of a long waveguide and side arms are attached as shown in fig below. ports ① & ② are collinear arms, port ③ is the H-arm, and port ④ is the E-arm.

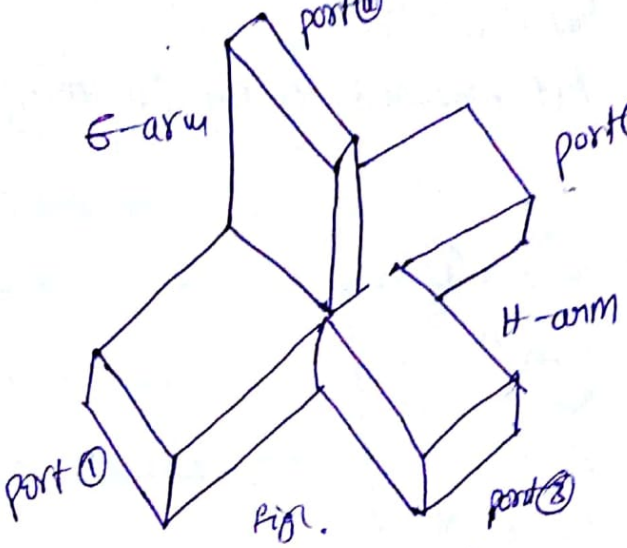


fig1.

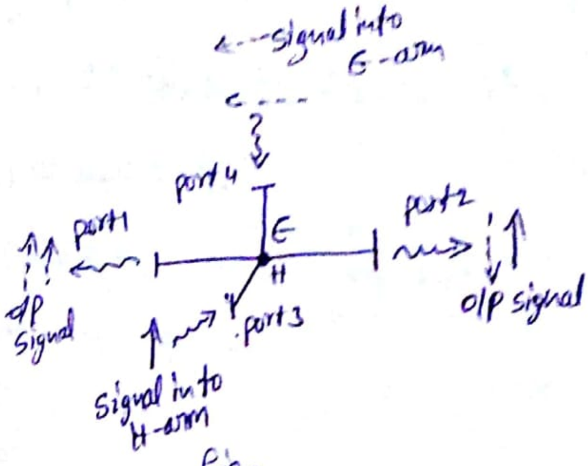


fig2

Such a device became necessary because of the difficulty of obtaining a completely matched three port Tee junction. This four port hybrid Tee junction combines the power dividing properties of both H-plane Tee and E-plane Tee as shown in fig2 ~~below~~.

1.· $[S]$ is a 4×4 matrix since there are 4 ports

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \text{———} (1)$$

Because of H-plane Tee section

$$S_{23} = S_{13} \quad \text{———} (2)$$

Because of E-plane Tee section

$$S_{24} = -S_{14} \quad \text{———} (3)$$

Because of geometry of the junction an i/p at port ③ cannot come out of port ④ since they are isolated ports and vice versa.

$$S_{34} = S_{43} = 0 \quad \text{——— (4)}$$

From symmetric property, $S_{ij} = S_{ji}$

$$S_{12} = S_{21};\ S_{13} = S_{31},\ S_{23} = S_{32}$$

$$S_{34} = S_{43},\ S_{24} = S_{42},\ S_{41} = S_{14}.$$

If ports ③ & ④ are perfectly matched to the junction

$$S_{33} = S_{44} = 0.$$

Sub the above properties ~~from~~ in ①

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad \text{——— (5)}$$

From unitary property, $[S][S]^{*} = [I]$

i.e., $$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1C_1 = |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1. \quad \text{——— (6)}$$

$$R_2C_2 : |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1. \quad \text{——— (7)}$$

$$R_3C_3 : |S_{13}|^2 + |S_{13}|^2 = 1 \quad \text{——— (8)}$$

$$R_4C_4 : |S_{14}|^2 + |S_{14}|^2 = 1 \quad \text{——— (9)}$$

from eqⁿ ⑧, ⑨

$$S_{13} = \frac{1}{\sqrt{2}} \quad \text{——— (10)}$$

$$S_{14} = \frac{1}{\sqrt{2}} \quad \text{——— (11)}$$

Comparing eqn ⑥, ⑦ we get

$$S_{11} = S_{22}$$

Using these values from eqn ⑩, ⑪ we get

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$$S_{11} = S_{12} = 0$$

i.e.,

$$S_{22} = 0$$

From eqn ⑦

This means ports ① & ② are also perfectly matched to the junction.

Hence in any four port junction, if any two ports are perfectly matched to the junction, then the remaining two ports are automatically matched to the junction. Such a junction where in all the four ports are perfectly matched to the junction is called a Magic Tee.

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

we know that $[b] = [S][a]$ ~~for~~

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$b_1 = \frac{1}{\sqrt{2}}(a_3 + a_4), \quad b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2)$$

$$b_2 = \frac{1}{\sqrt{2}}(a_3 - a_4), \quad b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2)$$

case II: $a_3 \neq 0,\ a_1 = a_2 = a_4 = 0$

$$b_1 = \frac{a_3}{\sqrt{2}},\ b_2 = \frac{a_3}{\sqrt{2}},\ b_3 = b_4 = 0$$

This is the property of H-plane Tee.

case II: $a_4 \neq 0,\ a_1 = a_2 = a_3 = 0$

$$b_1 = \frac{a_4}{\sqrt{2}},\ b_2 = \frac{-a_4}{\sqrt{2}},\ b_3 = b_4 = 0$$

This is the property of E-plane Tee.

case 3: $a_1 \neq 0,\ a_2 = a_3 = a_4 = 0$

$$b_1 = 0,\ b_2 = 0,\ b_3 = \frac{a_1}{\sqrt{2}},\ b_4 = \frac{a_1}{\sqrt{2}}$$

i.e., when power is fed into port①, nothing comes out of port② even though they are collinear ports. Hence ports ① & ② are called isolated ports. Similarly an i/p at port ② cannot comes out of port ①. Similarly E & H ports are isolated ports.

case 4: $a_3 = a_4,\ a_1 = a_2 = 0$.

$$b_1 = \frac{1}{\sqrt{2}}(2a_3),\ b_2 = 0,\ b_3 = b_4 = 0.$$

case 5: $a_1 = a_2,\ a_3 = a_4 = 0.$

$$b_1 = 0 = b_2 = b_4,\ b_3 = \frac{1}{\sqrt{2}}(2a_1)$$

Directional couplers:-

Directional couplers are flanged, built in waveguide assemblies which can sample a small amount of uw power for measurement purposes. They can be designed to measure incident and/or reflected powers, SWR values, provide a signal path to a receiver or perform other desirable operations. They can be unidirectional or bi-directional powers. In its most common form, the directional coupler is a four port waveguide junction consisting of a primary main waveguide and a secondary auxiliary waveguide as shown in fig 1.

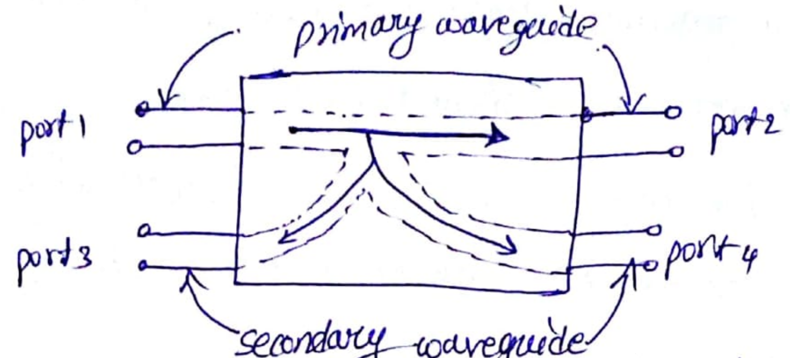


fig 1. schematic of a directional coupler

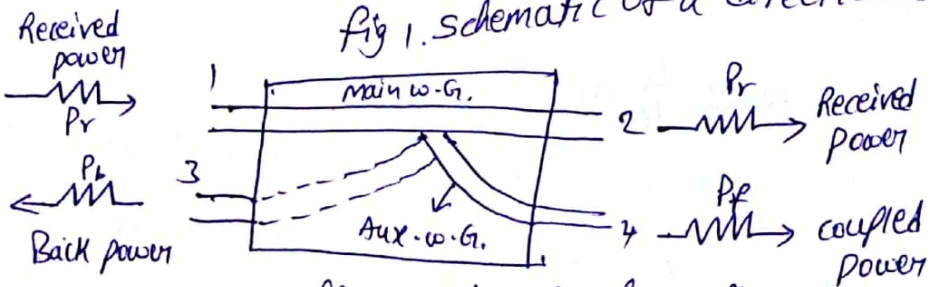


fig 2: Directional coupler indicating powers.

with matched terminations at all its ports, the properties of an ideal directional coupler can be summerized as follows.

1. A portion of power travelling from port ① to port ② is coupled to port ④ but not to port ③.
2. A portion of power travelling from port ② to port ① is coupled to port ③ but not to port ④.
3. A portion of power incident on port ③ is coupled to port ② but not to port ④ & a portion of the power incident on port ④ is coupled to port ① but not to port ②. Also ports ① & ③ are decoupled as are ports ② & ④.

A small portion of i/p power at port ① is coupled to port ④ so that measurement of this small power is possible. Ideally no power should come out of port③.

P_i = incident power at port①.

P_r = received " port②

P_f = Forward coupled power at port ④.

P_b = back power at port ③.

The performance of a directional coupler is usually defined in terms of two parameters which are defined as follows.

<u>Coupling factor C</u>:- The coupling factor of a directional coupler (D.C). is defined as the ratio of the incident power 'P_i' to the forward power 'P_f' measured in dB.

$$C = 10\log_{10}\frac{P_i}{P_f}\ dB.$$

<u>Directivity D</u>: The directivity of a D.C. is defined as the ratio of forward power P_f to the back power 'P_b' expressed in dB.

$$D = 10\log_{10}\frac{P_f}{P_b}\ dB.$$

For a typical D.C.

$$C = 20dB,\ D = 60dB.$$

$$C = 20 = 10\log\frac{P_i}{P_f}$$

$$\frac{P_i}{P_f} = 10^2 = 100$$

$$P_f = \frac{P_i}{100}$$

$$D = 60 = 10\log\frac{P_f}{P_b}.$$

$$\frac{P_f}{P_b} = 10^6.$$

$$P_b = \frac{P_f}{10^6} = \frac{P_i}{10^8}.$$

since P_b is very small, $(\frac{1}{10^8})P_i$, the power coming out of port③ can be neglected.

Isolation:- Another parameter called Isolation is sometimes defined to describe the directive properties of a directional coupler. It is defined as the ratio of the incident power P_i to the back power P_b expressed in dB.

$$I = 10\log_{10}\frac{P_i}{P_b}\ dB.$$

It may be noted that isolation in dB equals coupling factor plus directivity.

Two-hole Directional coupler:-

The principle of operation of a two-hole directional coupler is shown in fig. It consists of two guides the main & the auxiliary with two tiny holes common b/w them as shown. The two holes are at a distance of $\lambda g/4$ where λg is the guide wavelength.

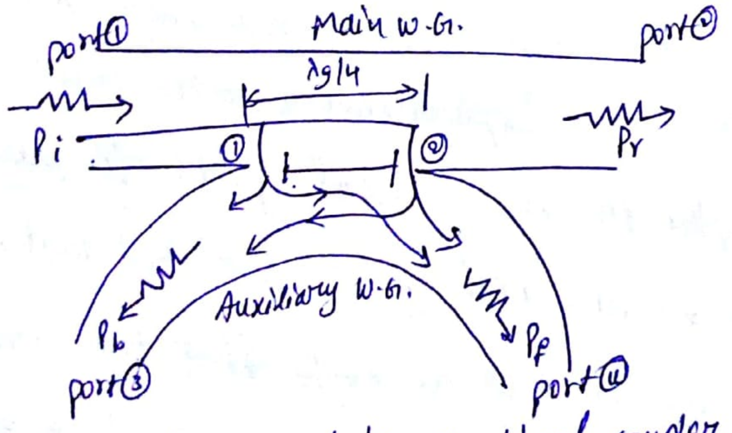


Fig: Two hole directional coupler.

The two leakages out of holes ① & ② both in phase at the position of 2nd hole and hence they add up contributing to P_f. But the two leakages are out of phase by 180° at the position of the 1st hole and therefore they cancel each other making $P_b = 0$. The magnitude of the power coming out of 2 holes depends upon the dimension of the two holes. of the two holes depends upon the dimension of the two holes. made '0' (since the since the distance b/w holes is $\lambda g/4$, P_b is made '0' (since the incident power will have to travel a distance of ($\lambda_g/4 + \lambda_g/4$) when it comes back from hole ② resulting in 180° phase shift. Compared to incident power leakage through hole ① entering port ③.

Bethe or single-hole coupler:-

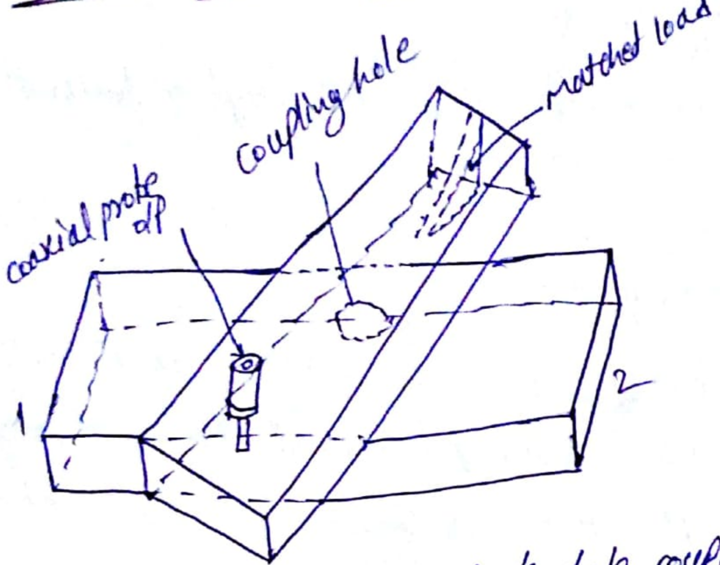


Fig: Bethe or single hole coupler.

A single-hole directional coupler is shown in fig. Here the directivity is improved as the Bethe coupler relies on a single hole for coupling process rather than the separation b/w two holes. The power entering port① is coupled to the co-axial probe o/p and the power entering port② in absorbed by the matched load. The auxiliary guide is placed at such an angle that the magnitude of the magnetically excited wave a made equal to that of the electrically excited wave for improved directivity. In this coupler, the waves in the auxiliary guide are generated through a single hole which includes both electric and magnetic fields. Because of the phase relationships involved in the coupling process, the signals generated by the two types of coupling cancel in the forward direction & reinforce in the reverse direction.

Scattering Matrix of a Directional coupler:-

We use the properties of the directional coupler to arrive at the S matrix.

1. Directional coupler is a four port n/w. Hence [S] is a 4×4 matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

In a directional coupler all four ports are perfectly matched to the junction. Hence the diagonal elements are zero.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0.$$

From symmetric property, $S_{ij} = S_{ji}$

$$S_{23} = S_{32}, \quad S_{13} = S_{31}, \quad S_{24} = S_{42}$$

$$S_{34} = S_{43}, \quad S_{41} = S_{14}.$$

Ideally back power is zero ($P_b = 0$) i.e., There is no coupling b/w ports ① & ③.

$$S_{13} = S_{31} = 0.$$

Also there is no coupling b/w port ② & ④

$$S_{24} = S_{42} = 0$$

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

since $[S][S^*] = I$, we get

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1 : $|S_{12}|^2 + |S_{14}|^2 = 1$ —— ①

R_2C_2 : $|S_{12}|^2 + |S_{23}|^2 = 1$ —— ②

R_3C_3 : $|S_{23}|^2 + |S_{34}|^2 = 1$ —— ③

R_1C_3 : $S_{12} S_{23}^* + S_{14}^* S_{34}^* = 0$. —— ④

comparing eqn ① & ②

$S_{14} = S_{23}$

comparing eqn ② & ③

$$|S_{12}|^2 + \cancel{|S_{23}|^2} = \cancel{|S_{23}|^2} + |S_{34}|^2$$

$S_{12} = S_{34}$.

let us assume that S_{12} is real & positive = 'P'.

$S_{12} = S_{34} = P = S_{34}^*$ —— ⑤

from eqn ④ & ⑤

$$P S_{23}^* + S_{23} \cdot P = 0$$

$$P (S_{23} + S_{23}^*) = 0.$$

$$P \neq 0, \; S_{23} + S_{23}^* = 0.$$

$$S_{23} = jy.$$

$$S_{23}^* = -jy$$

i.e., S_{23} must be imaginary.

let $S_{23} = jq = S_{14}$.

$S_{12} = S_{34} = P$.

$S_{23} = S_{14} = jq$.

$$[S] = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix}$$

Ferrite Devices :-

Ferrites are non-metallic materials with resistivities (ρ) nearly 10^{14} times greater than metals and with dielectric constants (ϵ_r) around 10-15 & relative permiabilities of the order of 1000. They have magnetic properties similar to those of ferrous metals. They are oxide based compounds having general composition of the form $MeO.Fe_2O_3$ i.e., a Mixture of a metallic & ferric oxide where MeO represents any divalent metallic oxide such as MnO, ZnO, CdO, NiO or a mixture of these. They are obtained by firing powdered oxides of materials at 1100°C or more & pressing them into different shapes. This processing gives them the added characteristics of ceramic insulators so that they can be used at uw frequencies.

Ferrites have atoms with large no. of spinning electrons resulting in strong magnetic properties. These magnetic properties are due to the magnetic dipole moment associated with the electron spin.

Faraday Rotation in Ferrites :-

Consider an infinite lossless medium. A static field B_0 is applied along the z-direction. A plane TEM wave that is linearly polarised along the x-axis at t=0 is made to propagate through the ferrite in the z-direction. The plane of polarisation of this wave will rotate with distance, a phenomenon known as Faraday Rotation.

Any linearly polarised wave can be regarded as the vector sum of two counter rotating circularly polarised wave. The ferrite materials offers different characteristics to these waves, with the result that the phase change for one wave is larger than the other wave resulting in rotation 'θ' of the linearly polarised wave, at $z=l$.

It is observed that a rotation of 100 degrees or more per cm of ferrite length is typical for ferrites at a freq of 10 GHz.

If the direction of propagation is reversed, the plane of polarisation continues to rotate in the same direction i.e., $z=l$ to $z=0$, the wave will arrive back at $z=0$ polarised at an angle 2θ relative to x-axis.

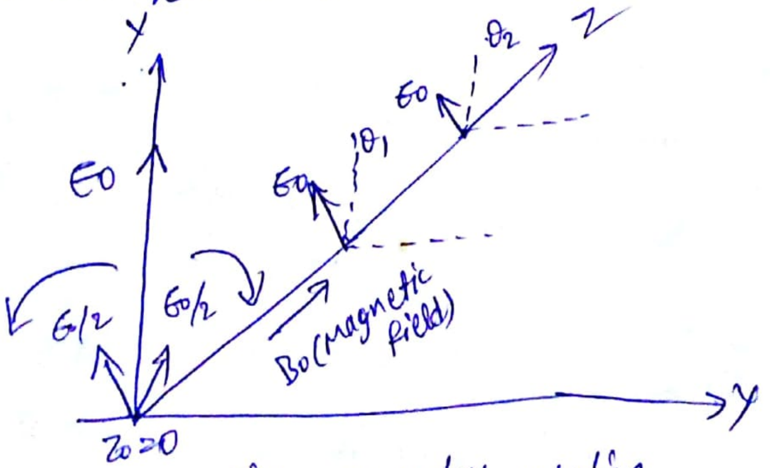


Fig: Faraday rotation

In fact, the angle of rotation 'θ' is given by

$$\theta = \frac{l}{2}(\beta_+ \sim \beta_-)$$

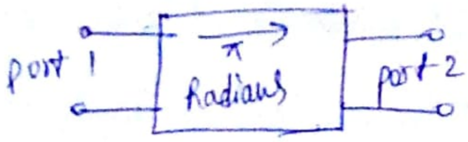
where, l = length of the ferrite rod

β_+ = phase shift for the right circularly polarised (component in clock wise direction) wave

β_- = phase shift for the left circularly polarised wave.

Microwave Devices which make use of Faraday rotation:-

a) Gyrator:-



It is a two port device that has a relative phase difference of 180° for transmission from port ① port ② and 'no' phase shift for transmission from port ② to port ① as shown in above fig.
The construction of a gyrator is as shown in fig 2 below. It consists of a piece of circular waveguide carrying the dominant TE_{11} mode with transitions to a standard rectangular waveguide with dominant mode (TE_{10}) at both ends. A thin circular ferrite rod tapered at both ends is located inside the circular waveguide supported by polyfoam and the waveguide is surrounded by a permanent magnet which generates dc magnetic field for proper operation of ferrite. To the i/p end a 90° twisted rectangular waveguide is connected as shown. The ferrite rod is tapered at both ends to reduce the attenuation & also for smooth rotation of polarized wave.

<u>operation</u>:- when a wave enters port ① its plane of polarisation rotates by 90° because of the twist in the waveguide. It again undergoes Faraday Rotation through 90° because of ferrite rod and the wave which comes out of port ② will have a phase shift of 180° compared to the wave entering port ①.

Scanned by CamScanner

But when the same wave enters port ②, it undergoes faraday rotation through 90° in the same anticlock wise direction. Because of the twist, this wave gets rotated back by 90° comes out of port ① with 0° phase shift as shown in fig 2. Hence a wave at port ① undergoes a phase shift of π radians but a wave fed from port ② does not change its phase in a gyrator.

Isolator :-

An isolator is a 2 port device which provides very small amount of attenuation for transmission from port ① to port ② but provides maximum attenuation for transmission from port ② to port ①. This requirement is very much desirable when we want to match a source with a variable load.

~~In most~~ .

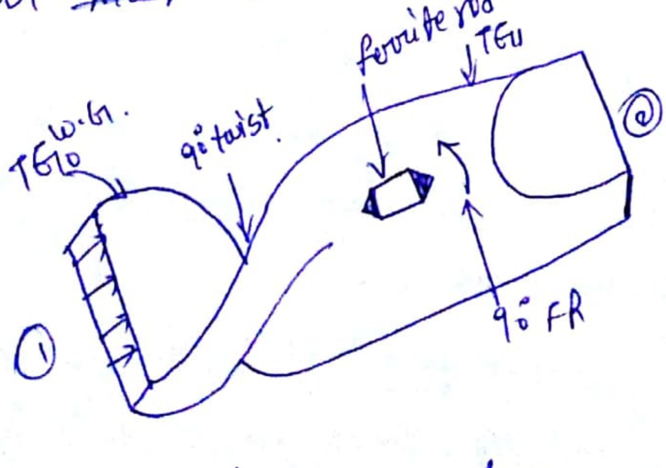
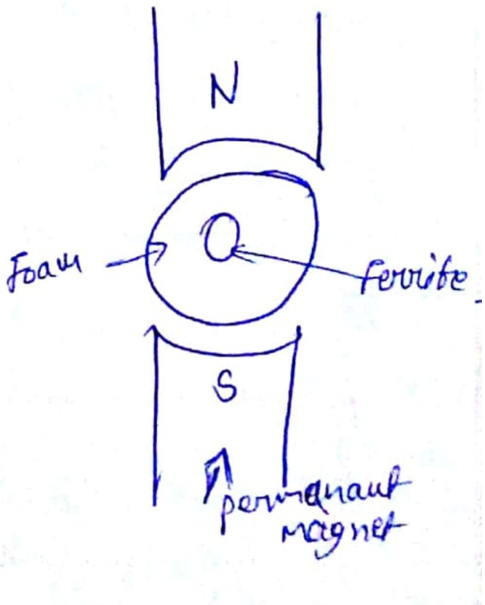


Fig2: Gyrator.



When isolator is inserted b/w generator & load, the generator is coupled to the load with zero attenuation & reflections if any from the load side are completely absorbed by the isolator without affecting the generator o/p. Hence the generator appears to be matched for all loads in the presence of isolator so that there is no change in freq and o/p power due to variation in load. This is shown in fig 3.

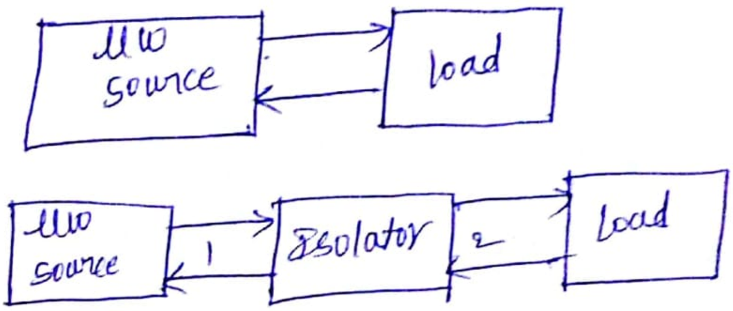


Fig 3.

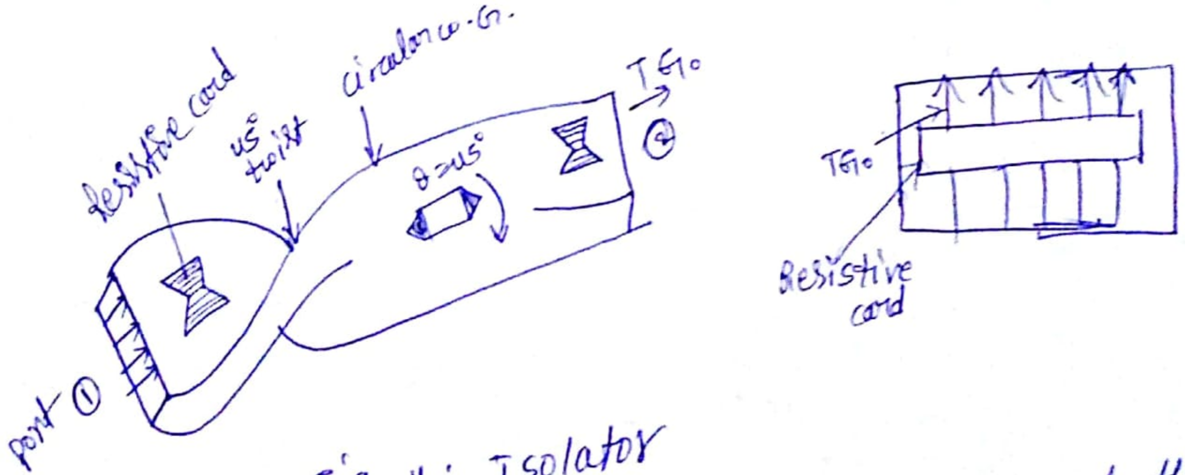


Fig 4: Isolator

<u>operation</u>:- A TE_{10} wave passing from port ① through the resistive card and is not attenuated. After coming out of the card, the wave gets shifted by 45° because of the twist in anticlockwise direction & then by another 45° in clockwise direction because of the ferrite rod and hence comes out of port ② with the same polarisation

as at port① without any attenuation.

But a TE_{10} wave fed from port② gets a pass from the resistive card placed near port② since the plane of polarisation of the wave is perpendicular to the plane of resistive card. Then the wave gets rotated by 45° due to Faraday rotation in clock wise direction & further gets rotated by 45° in clockwise direction due to the twist in the waveguide. Now the plane of polarization of the wave will be parallel with that of the resistive card and hence the wave will be completely absorbed by the resistive card and the o/p at port① will be zero. This power is dissipated in the card as heat. In practice 20 to 30 dB isolation is obtained for transmission from port② to port①.

Circulator:-

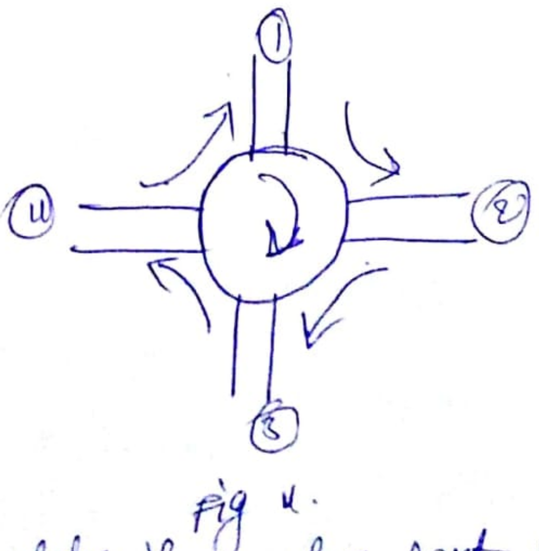
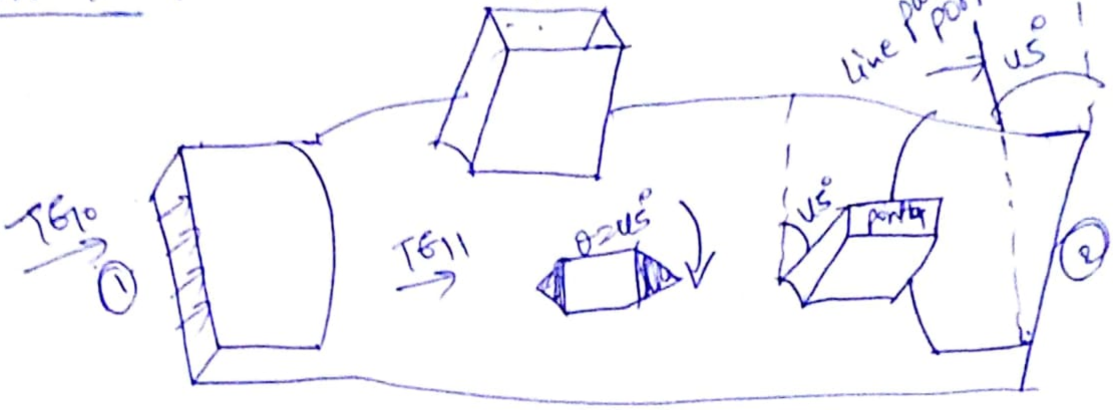


Fig 4.

A circulator is a four port microwave device which has a peculiar property that each terminal is connected only to the next clock wise terminal i.e., port① is connected port① only not to port ③ & ④ and port ② is connected only to port ③ etc. This is shown in fig 4. Although there is no restriction on the no. of ports, four ports are most commonly used. They are useful in parametric amplifiers, tunnel diode, amplifiers & duplexer in radars.

<u>Construction:</u>



Four port circulator.

A four port faraday rotation circulator is shown in above fig. The power entering port① is TE_{10} mode and is converted to TE_{11} mode because of gradual rectangular to circular transition. This power passes port③ unaffected since the electric field is not significantly cut & is rotated through 45° due to the ferrite, passes port④ unaffected and finally emerges out of port②, power from port② will have plane of polarization already tilted by 45° with respect to port①. This power passes port④ unaffected because again the electric field is not significantly cut. This wave gets rotated by another 45° due to ferrite rod in the clockwise direction. This power whose plane of polarization is tilted through 90° finds port ③ suitably aligned & emerges out of it. similarly port ③ is coupled only to port ④ & port ④ to port ①.

III UNIT

Microwave Tubes and Helix TWTs

Microwave Tubes:-

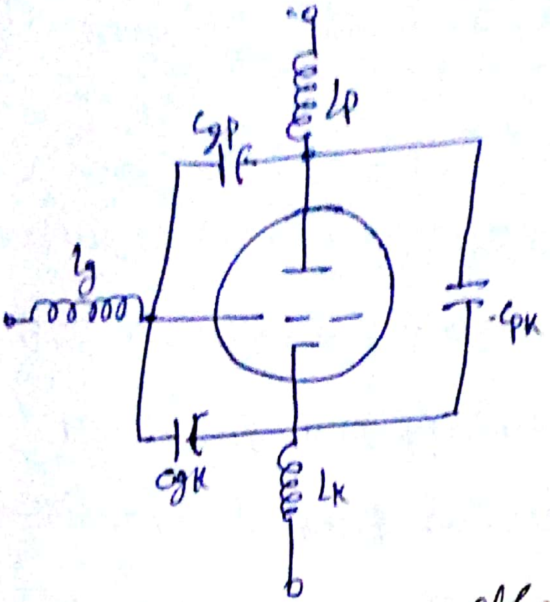
At μw frequencies, the size of electronic devices required for generation of μw energy becomes smaller & smaller. This results in lesser power handling capability and increased noise levels. Electronic devices such as tubes & transistors will be required even at μw frequencies. The tubes can provide higher o/p powers while transistors are smaller have lesser noise; better reliability with reduced o/p power levels. Conventional triodes, tetrodes and pentodes are useful only at low μw frequencies. special tubes would ~~to~~ be required even at UHF freq as conventional tubes have certain limitations at μw frequencies.

Limitations of conventional Tubes:-

Conventional ~~tubes~~ devices cannot be used for freq >100 MHz because of the following effects.

1. Interelectrode capacitance effect:-

As freq increases, the reactance $X_C = 1/2\pi fC$, decreases and the o/p voltage decreases due to shunting effect. Because at higher freq X_C becomes almost a short. C_{gp}, C_{gk} and C_{pk} are the I.E.C's which come into effect and are shown in below fig.



The effect of IEC can be minimised by reducing the IEC's C_{gk}, C_{pk} & C_{gp}. These can be reduced by decreasing the area of the electrodes i.e., by using smaller electrodes or by increasing the distance b/w electrodes.

2) Lead Inductance effect:-

As freq increases, the reactance $X_L = 2\pi f L$, increases and hence the voltages appearing at the active electrodes are less than the voltages at the base pins. This results in reduced gain for the tube amplifier. L_k, L_p, L_g are the lead inductances that limit the performance of the tube and are show in fig above.

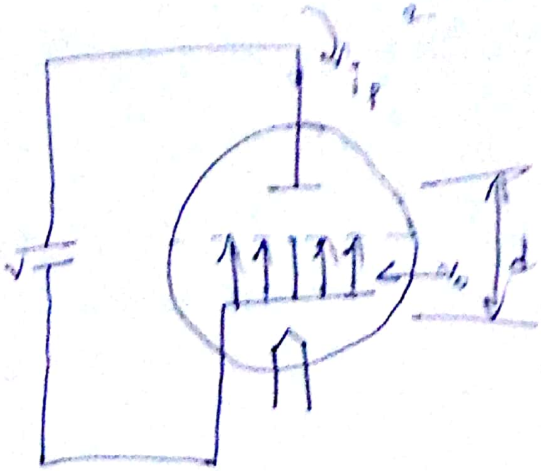
The effect of LI can be minimised by decreasing L, since L is propotional to reactance. L can be decreased by using larger sized short leads without base pins ie, by increasing 'A' and decreasing 'l'. This however reduces the power handling capatlity.

3) Transit Time effect:-

Transit time is the time taken for the electron to travel from cathode to anode as shown in below fig

i.e., transit time $= T = \frac{d}{v_0}$

where d = distance b/w anode and cathode.



V_0 = velocity of electrons

Static energy of electrons = eV

Kinetic energy = $\frac{1}{2} m V_0^2$

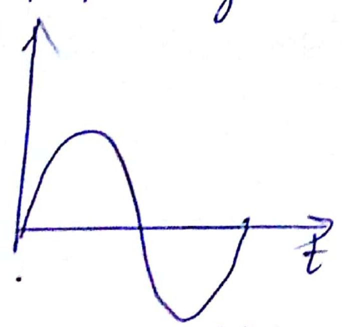
Static energy = kinetic energy

$$eV = \frac{1}{2} m V_0^2$$

$$V_0 = \sqrt{\frac{2eV}{m}}$$

$$T = \frac{d}{\sqrt{\frac{2eV}{m}}}$$

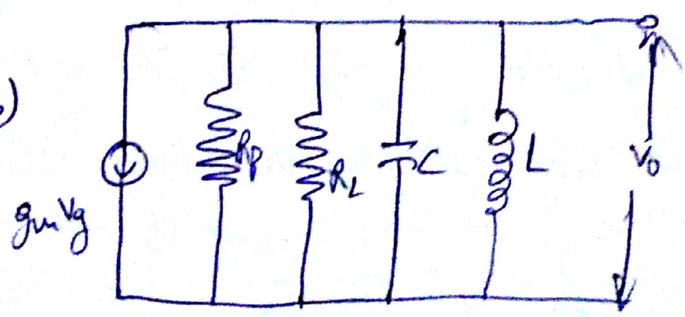
At low freq, transit time is negligible compared to the period of the signal as show in below.



Gain Bandwidth limitation:-

maximum gain is achieved when the tuned circuit is at resonance. Referring to the equivalent circuit in fig below. we have by transfer function,

$$\text{Gain } G = \frac{V_0(s)}{V_i(s)} = Z_0(s)$$



equivalent circuit.

Applying Laplace transform to the parallel circuit & replacing R_L and R_P by $R = \frac{1}{R_1} + \frac{1}{R_P}$,

$$\frac{1}{Z_0(s)} = Y_0(s) = Cs + \frac{1}{Ls} + \frac{1}{R} = \frac{s^2LCR + Ls + R}{RLs}$$

$$= \frac{s/L}{s^2 + \frac{s}{CR} + \frac{1}{LC}}$$

The characteristic eqn is given by the denominator $s^2 + \frac{1}{CR}s + \frac{1}{LC}$

The roots of the quadratic eqn give the extreme frequencies, ω_1 & ω_2 for calculating the bandwidth.

$$\omega_1 = -\frac{G}{2C} - \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}$$

$$\omega_2 = -\frac{G}{2C} + \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}$$

where $G = \frac{1}{R}$.

$$B.\omega = \omega_2 - \omega_1 = \frac{G}{C} \text{ for } \left(\frac{G}{2C}\right)^2 >> \frac{1}{LC}.$$

The max gain at resonance is $A_{max} = g_m/G$.

$\therefore$ Gain bandwidth product $= A_{max} \cdot BW = \frac{g_m}{G} \times \frac{G}{C} = \frac{g_m}{C}$.

The gain b.w product is thus independant of freq. As g_m and C are fixed for a particular tube or circuit, higher gain can be achieved at the cost of bandwidth only. In MW circuit this restriction / limitation can be overcome by use of

1) re-entrant cavities
2) Slow wave tubes.

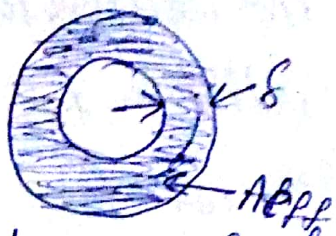
Effect due to RF Losses:-

Skin effect losses (or conductor or I^2R losses):-

These losses come into play at higher freq at which the current has the tendency to confine itself to a smaller cross section of the conductor towards its outer surface

$$\delta = \text{skin depth} = \sqrt{2/\omega\mu\sigma}$$

$$\delta \propto \frac{1}{\sqrt{\omega}} \ \& \ \delta \propto A_{eff}$$



where A_{eff} is the effective area over which current flows.

$$A_{eff} \propto 1/\sqrt{f}.$$

$$R = \frac{\rho l}{A_{eff}} = \frac{\rho l}{1/\sqrt{f}} = \rho l\sqrt{f}.$$

i.e., As f increases R increases

Hence losses will increase at higher freq. These losses can be reduced by increasing the size of conductors

Dielectric losses:-

This occurs in various types of insulating materials used in the device i.e., spacers, glass envelope silicon or plastic encapsulations etc. The losses in any of these material is in general given by

$$P = \pi f \cdot V^2 \varepsilon_r \tan\delta$$

ε_r = relative permittivity of the dielectric

δ = loss angle of the dielectric.

As f increases the power loss increases. The remedy for this is to eliminate the tube bases to reduce the surface area of glass.

Radiation losses:-

Whenever the dimensions of the wire approaches the wavelength, it will emit radiation that is, radiation losses increase with increase in freq. The remedy for this is use power shielding of the tubes and its circuitry.

Microwave Tubes:-

As already stated μw tubes are constructed so as to overcome the limitations of conventional and UHF tubes. They differ from them in that they make use of the transit time effect rather than fight it. In fact, large transit time is required for their operation. The basic principle of operation of the μw tubes involves transfer of power from a source of dc voltage to a source of dc voltage by means of current density modulated electron beam.

At μw freq the size of electronic devices required for generation of μw energy becomes smaller.

This results in lesser power handling capability & increased noise levels. Electronic devices such as tubes & transistors will be required even at μw freq. The tubes can provide higher o/p powers while transistors are smaller have lesser noise, better reliability.

μw tubes 2 types

1) O-type (linear beam or original type)

2) M type (magnetron, cross field)

Klystrons:

A klystron is a vacuum tube that can be used either as a generator or as an amplifier of power at μw frequencies. This was invented by Russel H. Varian at Stanford University in 1939 in association with his brother S.F. Varian. We study a two cavity klystron amplifier, multicavity klystron, two cavity klystron oscillators and reflex klystron.

Two cavity klystron amplifier:-

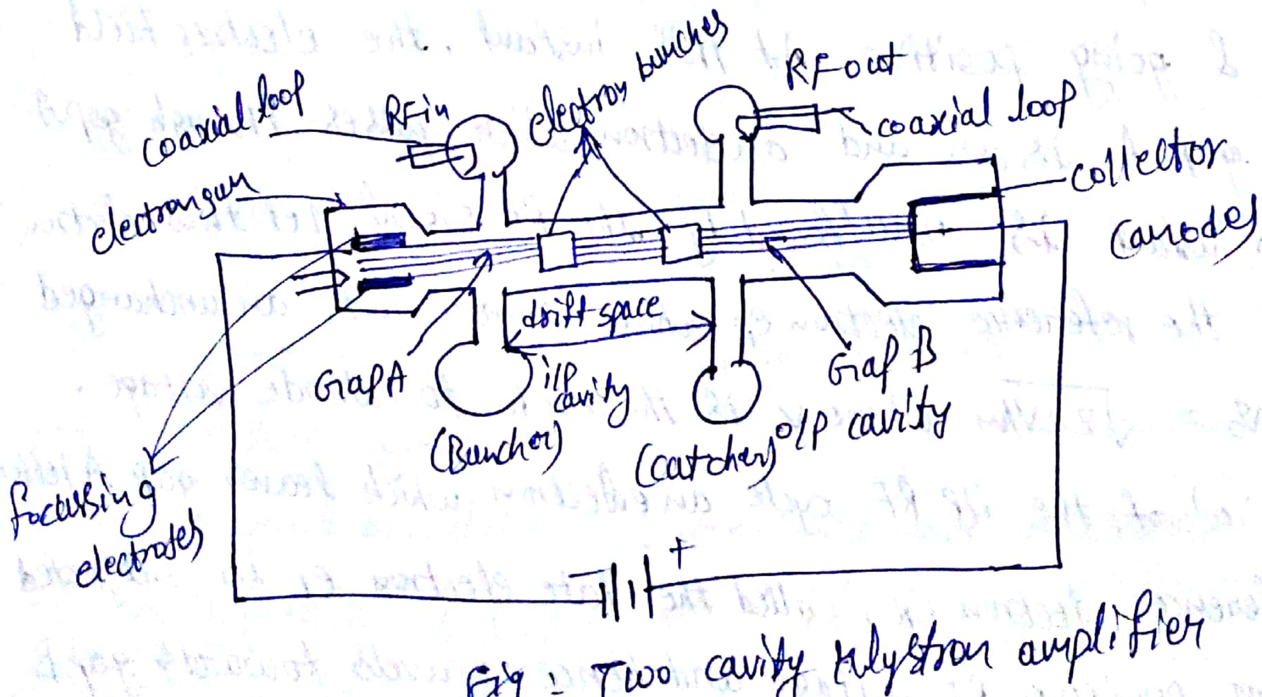
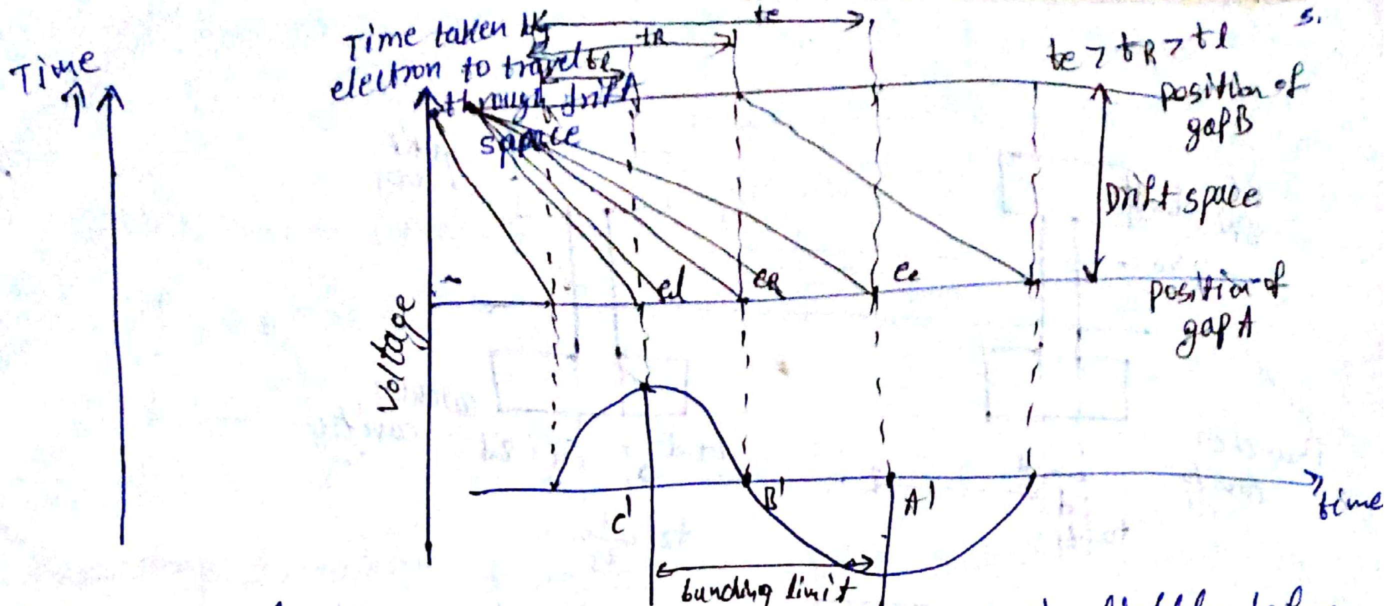


Fig: Two cavity klystron amplifier

A two cavity klystron amplifier is shown in above fig. which is basically a velocity modulated tube. Here a high velocity electron beam is formed focussed and sent down along a glass tube through an i/p cavity a field free drift space and an o/p cavity to a collector electrode or anode. The anode is kept at a positive potential with respect to cathode. The electron beam passes through a gap 'A' consisting of two grids of the buncher cavity separated by a very small distance and two other grids of the catcher cavity with a small gap 'B'. The i/p and o/p are taken from the tube via resonant cavities with the aid of coupling loops.

<u>Operation</u>: The RF signal to be amplified is used for exciting the i/p buncher cavity thereby developing an alternating voltage of signal freq across the gap A. Let us now consider the effect of this gap voltage on the electron beam passing through gap A. The situation is best explained by means of an Applegate diagram shown in fig below. At point 'B' on the i/p RF cycle, the alternating voltage is zero & going positive. At this instant, the electric field across gap A is zero and an electron which passes through gap A at this instant is unaffected by the RF signal. Let this electron be called the reference electron e_R which travels with an unchanged velocity $v_0 = \sqrt{2eV/m}$ where V is the anode to cathode voltage.

At point 'c' of the i/p RF cycle an electron which leaves gap A later than reference electron e_R, called the late electron e_l is subjected to maximum positive RF voltage and hence travels towards gap B with an increased velocity $(v > v_0)$ and this electron tries to overtake the reference electron e_R.



similarly an early electron e_e that passes the gap 'A' slightly before the reference electron e_R is subjected to a maxi negative field. Hence this early electron is decelerated and travels with a reduced velocity v_0. This electron e_e falls back & reference electron e_R catches up with the early electron.

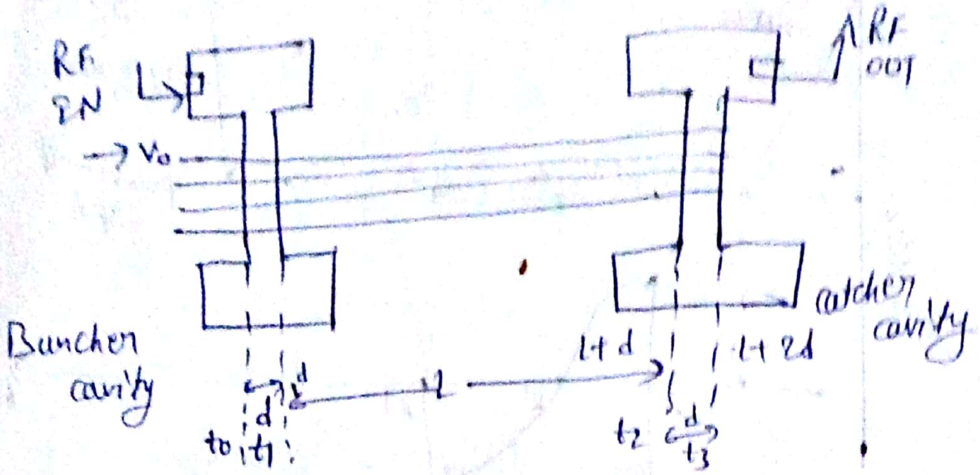
Applications :-

1. As power o/p tubes.
   a) In UHF TV transmitters.
   b) In troposphere scatter transmitters.
   c) satellite communication ground stations.
   d) Radars transmitters.
2. As power oscillator (5-50GHz) if used as a klystron oscillator.

Mathematical Analysis of a Klystron Amplifier :-

Let the dc voltage b/w cathode and anode be V_0 and v_0 be the velocity of the electron, L be the drift space length and the RF i/p signal to be amplified by the klystron be V_s.

Then,
$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593\times10^6\sqrt{V_0}\ m/sec \quad —(1)$$



$$V_0 = \sqrt{\frac{2eV}{m}} = 0.593\times10^{6}\sqrt{V_0}\ \text{m/sec}$$

where e is charge of electron, m is mass of electron.

The average microwave voltage in the buncher gap

$$V_S = V_1 \sin\omega t.$$

where V_1 is amplitude of the signal. & $V_1 << V_0$ is assumed.

The average transit time through the buncher gap distance d is

$$\tau = \frac{d}{V_0} = t_1 - t_0$$

The average gap transit angle is given by

$$\theta_g = \omega\tau = \omega(t_1-t_0) = \frac{\omega d}{V_0}.$$

The energy of the electron at the time of leaving buncher cavity is

$$\frac{1}{2}mv_1^2 = e(V + V_1\sin\omega t_1)$$

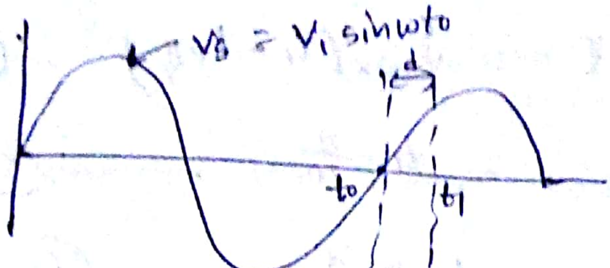
$$v_1^2 = \frac{2e(V+V_1\sin\omega t_1)}{m}$$

$$v_1 = \sqrt{\frac{2e}{m}(V+V_1\sin\omega t_1)}$$

$$v_1 = \sqrt{\frac{2e}{m}\cdot V\left(1+\frac{V_1}{V}\sin\omega t_1\right)}$$

$$= \sqrt{\frac{2eV}{m}} \cdot \sqrt{1 + \frac{V_1}{V} \sin\omega t_1}$$

$$v_1 = v_0 \left(1 + \frac{V_1}{V} \sin\omega t_1\right)^{\frac{1}{2}}$$



expanding binomially & neglecting higher powers of $\sin\omega t$, we get

$$v_1 = v_0 \cdot \left(1 + \frac{V_1}{2V} \sin\omega t_1\right)$$

Bunching process of the electron beam :-

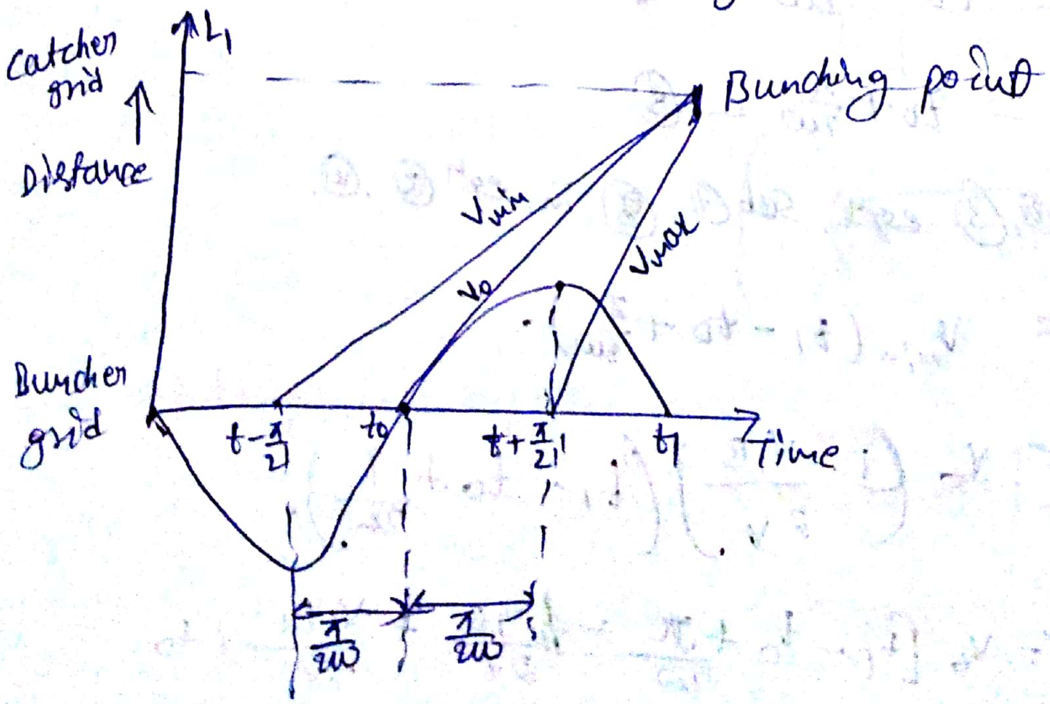
Maximum velocity occurs at $+\frac{\pi}{2}$ so that

$$v_{1\,max} = v_0 \left(1 + \frac{V_1}{2V_0}\right) \quad \text{—— (1)}$$

minimum velocity at $-\frac{\pi}{2}$, so that

$$v_{min} = v_0 \left(1 - \frac{V_1}{2V_0}\right) \quad \text{—— (2)}$$

If the distance in the drift space at which the bunching occurs from the buncher grid at time t_1 is L_1,



$L_1 = v_0(t_1 - t_0)$

The distance L_1 at $t_{-\frac{\pi}{2}} = V_{min}(t_1 - t_{-\frac{\pi}{2}})$ — (3)

The distance L_1 at $t_{+\frac{\pi}{2}} = V_{max}(t_1 - t_{+\frac{\pi}{2}})$ — (4)

$$t_{-\frac{\pi}{2}} = t_0 - \frac{\pi}{2\omega} \quad \text{—— (5)}$$

$$t_{+\frac{\pi}{2}} = t_0 + \frac{\pi}{2\omega} \quad \text{—— (6)}$$

$$L_1 \text{ at } t_{-\frac{\pi}{2}} = v_0\left(1 - \frac{V_1}{2V_0}\right)\left(t_1 - t_0 + \frac{\pi}{2\omega}\right)$$

$$L_1 \text{ at } t_{+\frac{\pi}{2}} = v_0\left(1 + \frac{V_1}{2V_0}\right)\left(t_1 - t_0 - \frac{\pi}{2\omega}\right)$$

For minimum value

$$\cancel{L_1 = v_0(t_1 - t_0) + v_0\left[\frac{\pi}{2\omega} - \frac{V_1}{2V}(t_1 - t_0)\right.}$$

$$L_1 = v_0\left[t_1 - t_0 + \frac{\pi}{2\omega} - \frac{V_1}{2V_0}t_1 + \frac{V_1}{2V_0}t_0 - \frac{V_1}{2V_0}\frac{\pi}{2\omega}\right]$$

$$= v_0(t_1 - t_0) + v_0\left[\frac{\pi}{2\omega} - \frac{V_1}{2V_0}(t_1 - t_0) - \frac{V_1 \pi}{2V_0\, 2\omega}\right]$$

where $t_{-\frac{\pi}{2}} = t_0 - \frac{\pi}{2\omega}$ — (7)

$$t_{+\frac{\pi}{2}} = t_0 + \frac{\pi}{2\omega} \quad \text{— (8)}$$

~~sub (7), (8) in (1), (8) eqn~~, sub (7), (8) in eqn (3), (4).

$$L_1 \text{ at } t_{-\frac{\pi}{2}} = V_{min}\left(t_1 - t_0 + \frac{\pi}{2\omega}\right).$$

$$= V_0\left(1 - \frac{V_1 B}{2V}\right)\left(t_1 - t_0 + \frac{T}{2\omega}\right).$$

$$L_1 \text{ at } t - \frac{\pi}{2} = V_0\left[t_1 - t_0 + \frac{\pi}{2\omega} - \frac{t_1 V_1 B}{2V} + \frac{V_1 B_1}{2V} t_0\right.$$

Output Power (P_{out}).

(7)

At the catcher cavity

RF voltage $= V_2 \sin \omega t_2$.

Energy given by the electron to the bunch.

$$= (-e) V_2 \sin \omega t_2 = -eV_2 \sin \omega t_2.$$

The average energy given to the RF field in a cycle.

$$P_{av} = \frac{1}{2\pi} \int_{\omega t_1}^{\omega t_2} (-eV_2 \sin \omega t_2)\, d\omega t_1.$$

In the field free space between cavities, the transit time for velocity modulated ~~mod~~ electron is given by

$$T = t_2 - t_1 = \frac{L}{u_x} = \frac{L}{\left[V_0\left[1+\frac{V_1}{V_0}\right] \sin \omega t_1\right]^{1/2}}.$$

$$= \frac{L}{V_0}\left[1 - \frac{V_1}{2V_0} \sin \omega t_1\right].$$

Multiplying by ω,

$$\omega T = \omega (t_2 - t_1) = \frac{\omega L}{V_0}\left[1 - \frac{V_1}{2V_0} \sin \omega t_1\right].$$

$$= v_0\left[t_1 - t_0\right] + v_0\left[\frac{-V_1 B}{2v}(t_1 - t_0] + \frac{\pi}{2\omega} - \frac{V_1 B}{2v}\cdot\frac{\pi}{2\omega}\right].$$

$$\text{Liat } t\frac{\pi}{2} = v_0\left[t_1 - t_0 - \frac{\pi}{2\omega} + t_1\frac{V_1 B}{2v} - t_0\frac{V_1 B}{2v} - \frac{\pi}{2\omega}\frac{V_1 B}{2v}.\right.$$

$$= v_0\left[t_1 - t_0\right] + v_0\left[\frac{-\pi}{2\omega} + \frac{V_1 B}{2v}(t_1 - t_0) - \frac{\pi}{2\omega}\frac{V_1 B}{2v}\right]$$

without RF voltage V_1 in buncher cavity where N is the No. of electron transit cycles in drift space.

$$\omega t_2 = \omega t_1 + \theta_0\left[1 - \frac{V_1}{2v}\sin\omega t_1\right].$$

$$P_{av} = \frac{-eV_2}{2\pi}\int_0^{2\pi}\sin\left[\omega t_1 + \theta_0\left[1 - \frac{V_1}{2v}\sin\omega t_1\right]\right]d\omega t_1$$

$$P_{av} = \frac{-eV_2}{2\pi}\int_0^{2\pi}\sin\left[\omega t_1 + \theta_0 - \theta_0\frac{V_1}{2v}\sin\omega t_1\right)d\omega t_1$$

$$= 2\pi J_1(x)\sin\theta.$$

$$P_{av} = \frac{-eV_2}{2\pi}2\pi J_1(x)\sin\theta_0.$$

$$P_{av} = -eV_2 J_1(x)\sin\theta_0.$$

~~Pav~~. Here x is bunching parameter

In the above equation, $\frac{L}{v_0} = T_0$, the transmit time without RF voltage V_1 in buncher cavity and ⑧

$$\frac{\omega L}{v_0} = \omega T_0 = \theta_0 = 2\pi N$$ is the transit angle without RF Voltage V_1 in buncher cavity (or it is due to dc voltage V_0) and N is the number of electron transit cycles in drift space.

The bunching parameter X of a klystron is defined by the equation,

$$X = \frac{V_1}{2V_0}\theta_0.$$

Which is a dimension less quantity and proportional to input power.

Eq. 8.29 can be written using Eq. 8.31.

$$P_{av} = \frac{-eV_2}{2\pi}\int_0^{2\pi} \sin\left[\omega t_1 + T\right] d\omega t_1.$$

i.e., $$P_{av} = \frac{-eV_2}{2\pi}\int_0^{2\pi} \sin\left[\omega t_1 + \theta_0\left[1 - \frac{V_1}{2V_0}\sin\omega t_1\right]\right] d\omega t_1.$$

This is Bessel function & its solution is given by,

$$P_{av} = -eV_2 J_1(X)\sin\theta_0.$$

where, $J_1(X)$ = Bessel function of the first order for the argument X.

for N electron transit cycles,

Energy transferred $= NP_{av} = -Ne\, V_2 J_1(X) \sin\theta_0$;

$Ne = I_0$, the o/p current.

Energy transferred $= -I_0 V_2 J_1(X) \sin\theta_0$

max value of $J_1(X) = 0.58$ for $X = 1.84$ (from bessel fun tables)

For max energy transfer,

$$P_{max} = -I_0 V_2 (0.58) \sin\theta_0.$$

$$\sin\theta_0 = -1, \theta_0 = 2n\pi - \frac{\pi}{2}.$$

The o/p power, $P_{out} = P_{max} = 0.58 I_0 V_2$. —①

Input power (P_{in}):-

The i/p power is basically the dc i/p given by

$$P_{in} = I_0 V_0. \quad \text{—②}$$

The efficiency (η) therefore is given by eq ①, ②.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{0.58 I_0 V_2}{I_0 V_0} = 0.58 \frac{V_2}{V_0}.$$

multicavity klystron:-

Gains of about 10 to 20 dB are typical with two-cavity tubes. A higher overall gain can be acheived by connecting several two cavity tubes in cascade, feeding the o/p of each of the tubes to the i/p of the succeeding one. ~~Ba~~

Here, each of the intermediate cavities act as a buncher with the passing electron beam inducating an enhanced RF voltage than the previous cavity. with four cavities, power gains of around 50dB can be easily acheived. The cavities can all be tuned to the same freq for narrow band operation. Bandwidth can be improved by stagger tuning of cavities upto about 80MHz of course with reduction in gain. This stagger tuning is employed in UHF klystrons for TV transmitter o/p tubes & in satellite earth station transmitters as power amplifiers at 6GHz.

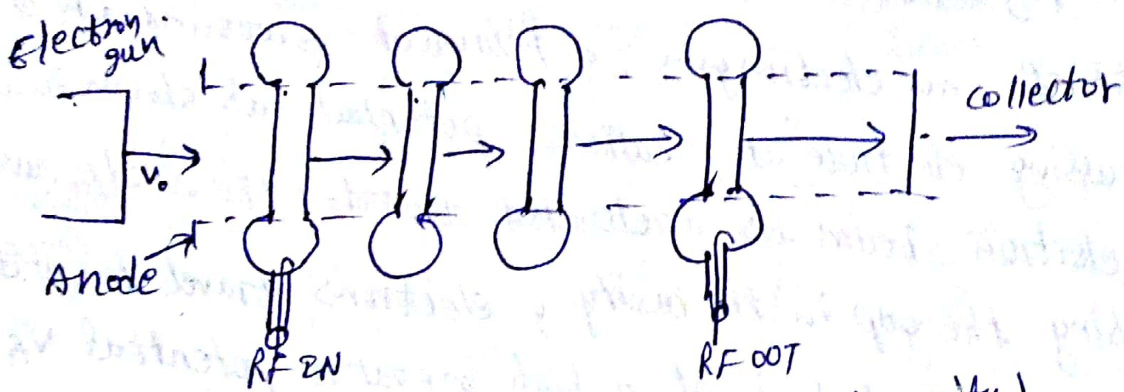


Fig: Multicavity klystron (4 cavity)

Reflex Klystron:-

The reflex klystron is a single cavity variable freq microwave generator of low power and low efficiency.

Applications:-

1. in radar receivers
2. local oscillator in μw receivers.
3. signal source in μw generator of variable freq.
4. portable μw links and
5. pump oscillator in parametric amplifier.

Construction!

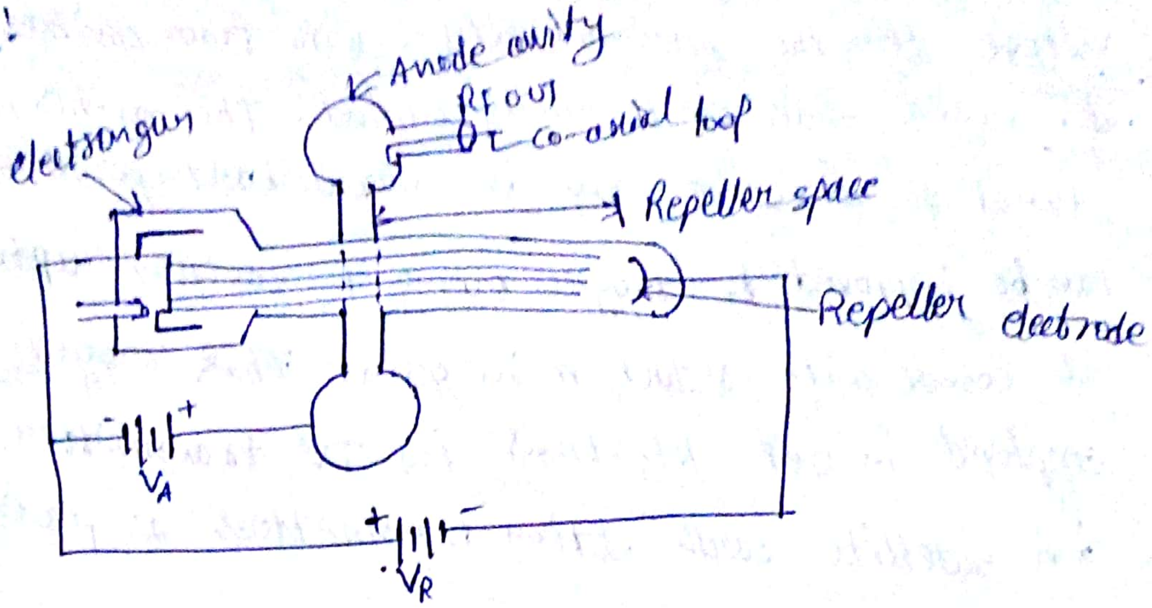


Fig ! Reflex klystron.

It consists of an electrongun, a filament surrounded by cathode and a focussing electrode at cathode potential as shown in above fig. The electron beam is accelerated towards the anode cavity. After passing the gap in the cavity, electrons travel towards a repeller electrode which is at a high negative potential V_R. The electrons never reach the repeller because of the negative field and are returned back towards the gap. Under suitable conditions, the electrons give more energy to the gap than they took from the gap on their forward journey & oscillations are sustained.

Operation!- It is assumed that the oscillations are setup in the tube initially due to noise or switching transients and these oscillations are sustained by device operation. This can be explained again by applegate diagram, shown in fig below. The RF voltage that is produced across the gap by the cavity oscillations act on the electron beam to cause velocity modulation. 'e_R' is the reference electron that passes through the gap when the gap voltage is '0' and going negative. Electron 'e_R' is unaffected by the gap voltage. This moves towards the

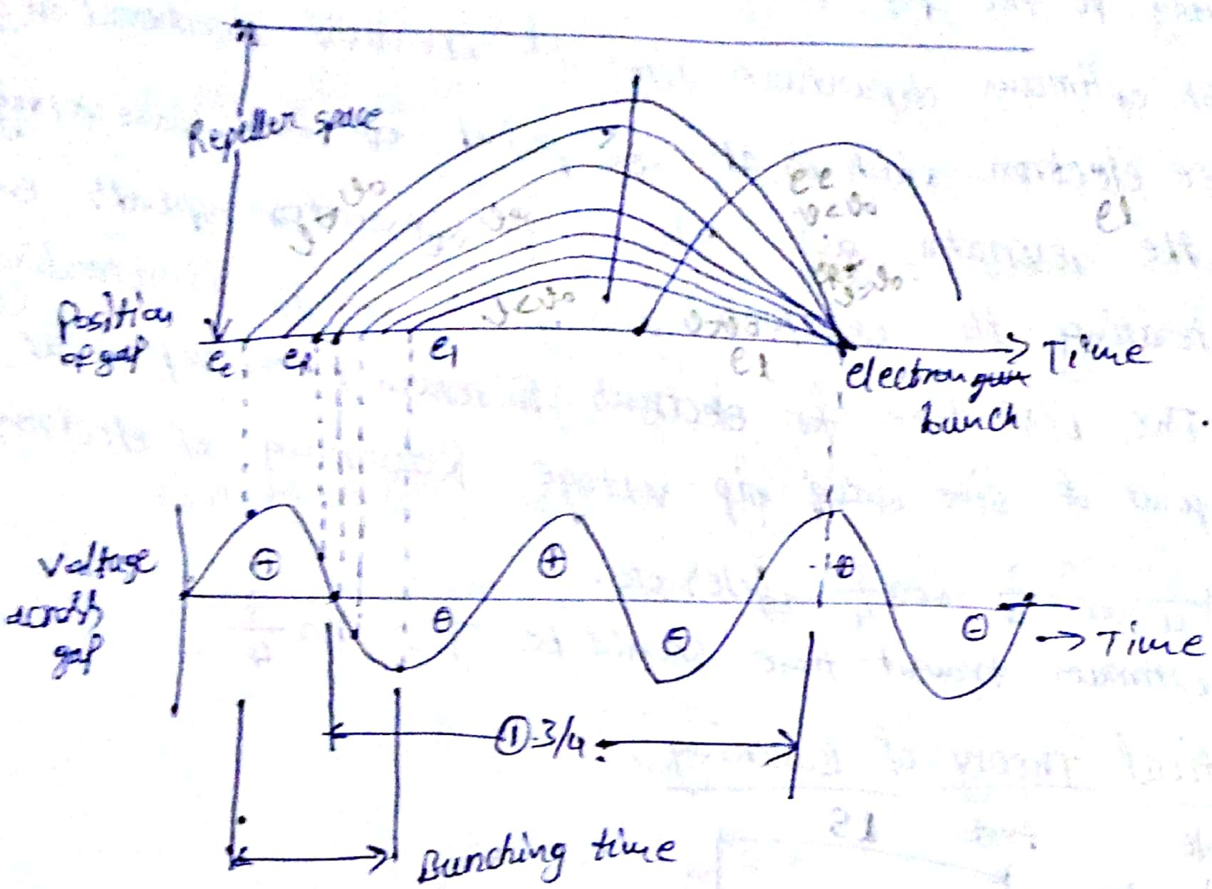


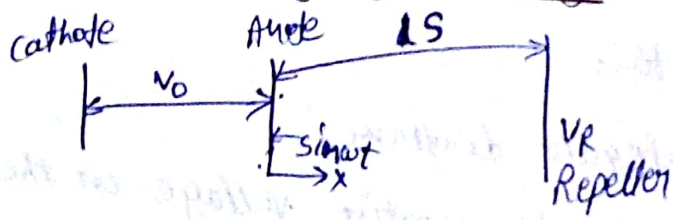
Fig : Applegate diagram

repeller and gets reflected by the negative voltage on the repeller. It returns and passes through the gap for a second time. The early electron 'e_e' that passes through the gap before the reference electron 'e_R' experiences a max positive voltage across the gap & this electron is accelerated. It moves with greater velocity & penetrates deep into repeller space is more. Hence e_e & e_R appear at the gap for the second time at the same instant. The late electron 'e_l' that passes the gap later than reference electron 'e_R' experiences a max negative voltage & moves with a retarding velocity. The return time is shorter as the penetration into repeller space is less and catches up with e_R and e_e electrons forming a bunch. Bunches occur once per cycle centred around the reference electron 'e_R' and these bunches transfer max energy to the gap to get sustained oscillations.

The most optimum departure time is obviously centered on the reference electron which is at 180° point of sine wave voltage across the resonator gap. The cavity resonator spends energy in accelerating the electrons and gains energy in retarding them. The best time for electrons to return to the gap is at the 90° point of sine wave gap voltage. Returning of electrons after $1\frac{3}{4}$ or $2\frac{3}{4}$ or $3\frac{3}{4}$ cycles etc.

The optimum transit time should be $T = n + \frac{3}{4}$.

Mathematical Theory of Bunching :-



V_0 = electron gun anode voltage

$V_1 \sin\omega t$ = RF voltage at cavity gap

V_R = Repeller voltage with respect to cathode.

s = distance b/w cavity gap & repeller electrode.

v_0 = velocity of electron in gun.

v_1 = velocity due to RF voltage in addition to the electron accelerating voltage V_0.

t_0 = time for electron entering cavity gap at $x=0$.

t_1 = time for same electron leaving cavity gap at $x=d$.

t_2 = time for same electron returned by retarding field at $x=d$.

Scanned by CamScanner

$$v_0 = \sqrt{\frac{2eV_0}{m}} \quad \left(\text{since } \frac{1}{2} m v_0^2 = eV_0\right)$$

$$v_1 = v_0 \left(1 + \frac{V_1}{V_0} \sin \omega t\right)^{\frac{1}{2}}$$

(Voltage now is $V_0 + V_1 \sin\omega t$, $V_1 << V_0$)

Voltage b/w repeller and anode

$$= V_R - (V_0 + V_1 \sin\omega t) \simeq V_R - V_0.$$

Retarding electrostatic field b/w repeller & anode is given by

$$E = -\left(\frac{V_R - V_0}{S}\right)$$

force on electron $= -eE = +e\left(\frac{V_R - V_0}{S}\right)$

force on electrons = mass x acceleration $= \frac{m d^2x}{dt^2}$

$$-eE = \frac{m d^2x}{dt^2}.$$

$$+e\left(\frac{V_R - V_0}{S}\right) = m \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{e}{mS}(V_R - V_0)$$

integrating above eqn

$$\frac{dx}{dt} = \frac{e}{mS}(V_R - V_0)t + C. \quad (1)$$

$\frac{dx}{dt} = v_1$, At $t = t_1$

$$v_1 = \frac{e}{mS}(V_R - V_0)t_1 + C. \quad (1)$$

$$C = v_1 - \frac{e}{mS}(V_R - V_0)t_1$$

sub 'C' in eqn (1)

$$\frac{dx}{dt} = \frac{e}{ms}(V_R - V_0)t + v_1 - \frac{e}{ms}(V_R - V_0)t_1$$

$$\frac{dx}{dt} = \frac{e}{ms}(V_R - V_0)(t - t_1) + v_1$$

integrating above eqn

$$x = \frac{e}{ms}(V_R - V_0) \cdot \int_{t_1}^{t} (t - t_1)\, dt + v_1 \int_{t_1}^{t} dt.$$

$$= \frac{e}{2ms}(V_R - V_0)(t - t_1)^2 + v_1(\cancel{t} - t_1) + C_1$$

At $t = t_1$. distance travelled by electron $x = d = C_1$.

$$x = \frac{e}{2ms}(V_R - V_0)(t - t_1)^2 + v_1(t - t_1) + d$$

$$\cancel{d} = \frac{e}{2ms}(V_R - V_0)(t - t_1)^2 + v_1(t - t_1) + \cancel{d}$$

$$0 = \frac{e}{2ms}(V_R - V_0)(t - t_1)^2 + v_1(t - t_1)$$

At $t = t_2$, $x = d$.

$$0 = \frac{+e}{2ms}(V_R - V_0) \cdot (t_2 - t_1)^2 + v_1(t_2 - t_1)$$

$$\frac{e}{2ms}(V_R - V_0)(t_2 - t_1)^2 = -v_1(t_2 - t_1)$$

$$\frac{e}{2ms}(V_R - V_0)^{(t_2 - t_1)} = -v_1$$

round trip transit time

$$t_2 - t_1 = \frac{-v_1 \, 2ms}{(V_R - V_0)\, e}$$

The transit angle 'ωt' is defined as transit angle at time 't'.

$$\omega(t_2-t_1) = \frac{-2ms v_1 \omega}{e(V_R-V_0)}$$

$$\omega t_2 - \omega t_1 = \frac{-2ms v_1 \omega}{e(V_R-V_0)}$$

$$\omega t_2 = \omega t_1 - \frac{2ms v_1 \omega}{e(V_R-V_0)} \quad \text{—} (3)$$

sub v_1 in eq^n (3)

$$v_1 = v_0\left(1+\frac{V_1}{V_0}\sin\omega t\right)^{\frac{1}{2}}$$

$$\omega t_2 = \omega t_1 + \frac{2ms\omega}{e(V_R-V_0)} \cdot v_0\left(1+\frac{V_1}{V_0}\sin\omega t\right)^{\frac{1}{2}}$$

let $$\theta_g = -\frac{2ms\omega v_0}{(V_R-V_0)e} \quad \text{—} (4)$$

$$\omega t_2 = \omega t_1 + \theta_g \cdot \left(1+\frac{V_1}{2V_0}\sin\omega t\right)^{\frac{1}{2}} \quad [\because \text{since } (V_1 << V_0)]$$

Relation b/w. Repeller voltage & Accelerating voltage:-

$$\omega t_2 = \omega t_1 + \theta_g\left(1+\frac{V_1}{2V_0}\sin\omega t\right)$$

when $V_1 << V_0$

$$\omega t_2 = \omega t_1 + \theta_g.$$

For max transfer of energy, the modes are $1\frac{3}{4}$ cycles apart

$$2\pi\left(n-\frac{1}{4}\right) \text{ where } n-\frac{1}{4} = 3/4,\ 1\tfrac{3}{4} \text{ etc.}$$

$[n=1,2,\ldots]$

$$2\pi n - \frac{2\pi}{4} \Rightarrow 2\pi n - \frac{\pi}{2}$$

$$\theta_g = 2\pi n - \frac{\pi}{2}$$

But $\theta_g = \dfrac{-2ms\,\omega v_0}{(V_R - V_0)e}$

$$v_0 = \frac{-\theta_g (V_R - V_0)e}{2ms\omega}$$

From mass & voltage relation ship of electron

$$\frac{1}{2} m v_0^2 = eV_0$$

$$V_0 = \frac{m v_0^2}{2e}$$

$$= \frac{m}{2e} \cdot \frac{\theta_g^2 (V_R - V_0)^2 e^2}{4 m^2 s^2 \omega^2}$$

$$= \frac{\theta_g^2 (V_R - V_0)^2 e}{8 m s^2 \omega^2}$$

$$V_0 = \frac{\left(2\pi n - \frac{\pi}{2}\right)^2 (V_R - V_0)^2 e}{8 m s^2 \omega^2}$$

$$\frac{V_0}{(V_R - V_0)^2} = \frac{e}{8 m s^2 \omega^2} \left(2\pi n - \frac{\pi}{2}\right)^2$$

Mathematical expression for change in freq due to repeller voltage:

$$(V_R - V_0)^2 = \frac{8 m s^2 V_0 \omega^2}{e \left(2\pi n - \frac{\pi}{2}\right)^2} \quad \text{(1)}$$

Differentiating V_R with respect to ω.

$$\cancel{2}(V_R - V_0) \cdot \frac{dV_R}{d\omega} = \frac{\cancel{2} \times 8 m s^2 V_0}{\left(2\pi n - \frac{\pi}{2}\right)^2 \cdot e} \cdot \omega$$

$$\frac{dV_R}{d\omega} = \frac{8 m s^2 V_0\, \omega}{\left(2\pi n - \frac{\pi}{2}\right)^2 e} \cdot \frac{1}{V_R - V_0} \quad \text{(2)}$$

Scanned by CamScanner

sub (1) in (2)

$$\frac{dV_R}{d\omega} = \frac{8V_0 m s^2 \omega}{e\left(2\pi n - \frac{\pi}{2}\right)^2 \cdot \sqrt{\frac{V_0 \cdot 8 m s^2 \omega^2}{e\left(2\pi n - \frac{\pi}{2}\right)^2}}}$$

$$= \frac{8V_0 m s^2 \omega \sqrt{e}\left(2\pi n - \frac{\pi}{2}\right)}{e\left(2\pi n - \frac{\pi}{2}\right)^2 \cdot \sqrt{V_0 \cdot 8m} \cdot s\omega}$$

$$= \sqrt{\frac{8V_0 m}{e}} \frac{s}{\left(2\pi n - \frac{\pi}{2}\right)}$$

$$\frac{dV_R}{2\pi\, df} = \frac{s}{\left(2\pi n - \frac{\pi}{2}\right)} \sqrt{\frac{8mV_0}{e}}$$

$$\frac{dV_R}{df} = \frac{2\pi s}{\left(2\pi n - \frac{\pi}{2}\right)} \sqrt{\frac{8mV_0}{e}}$$

Efficiency (η) :-

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{a.c}}{P_{d.c}}$$

$P_{d.c} = V_0 I_0$.

$\therefore P_{a.c} = I_0 \cdot V_2 J_1(X) \sin\theta_g$.

$P_{a.c} = I_0 V_1 (J_1(X)) \cdot [\sin\theta_g = 1]$

But Bunching factor

$$X = \frac{V_1 \theta_g}{2V_0}$$

$$V_1 = \frac{2X V_0}{\theta_g}$$

$$V_1 = \frac{2X V_0}{2\pi n - \frac{\pi}{2}}$$

$$P_{a.c} = I_0 \left[\frac{2X V_0}{2\pi n - \frac{\pi}{2}} \right] \cdot J_1(X)$$

$$\eta = \frac{\dfrac{I_0\, 2V_0 \cdot J_1(X) \cdot X}{2\pi n - \frac{\pi}{2}}}{V_0 I_0}.$$

$$\eta = \frac{2 J_1(X) \cdot X}{2\pi n - \frac{\pi}{2}}$$

sub $X = 2.408$, $J_1(X) = 0.52$.

$n = 2$.

$\eta = 22.8\%$

power o/p in terms of repeller voltage V_R :-

$$P_{out} = \frac{2V_0 I_0 X J_1(X)}{2\pi n - \frac{\pi}{2}}.$$

$$2\pi n - \frac{\pi}{2} = \theta_g = \frac{2mS\omega}{e(V_R - V_0)} \cdot v_0.$$

The negative sign is not taken as electron bunch travels in the reverse direction, -X.

$$P_{out} = \frac{2V_0 I_0 X J_1(X)}{2mS \cdot \omega v_0} X (V_R - V_0) e.$$

eliminating v_0 $\quad P_{out} = \frac{2V_0 I_0 X \cdot J_1(X)}{\omega S} \times (V_R - V_0) \sqrt{\frac{e}{2mV_0}}$

For max value of X', $J_1(X) = 1.25$, we get

$$P_{max} = \frac{1.25 V_0 I_0 (V_R - V_0)}{\omega S} \times \sqrt{\frac{e}{2mV_0}}$$

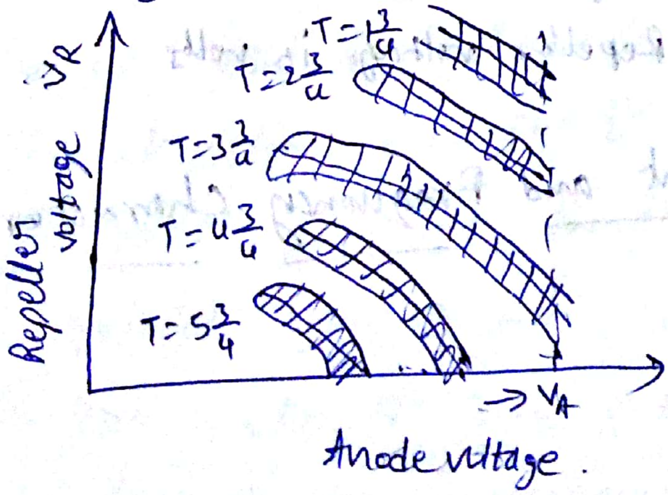
performance characteristics of a reflex klystron :-

The performance of a reflex klystron depends on two types of characteristics. They are,

1. voltage characteristics :-

The required specified combinations of anode and repeller voltages are used to produce the oscillations and that will give a favourable transit time i.e., $T = (n + \frac{3}{4})$.

The possible oscillation combinations and optimum combinations are shown by shaded areas and heavy lines respectively in fig below. For different values of 'n' i.e, n=1, 2, 3, --- the reflex klystron oscillator is operated in different modes. The mode with larger o/p power is not advantageous but it should have higher voltages as shown in fig 1. This leads to the possibilities of lower efficiencies & insulation problems. Hence, modes corresponding to n=2 or 3 are most widely used.



2) Power output and Frequency characteristics

Figure (2) shows the frequency characteristics of the mode curves. The frequency of oscillation is decided by the frequency of resonance of the cavity. This frequency is slightly changed by the variations in repeller voltage. This will describe the electronic tuning of reflex klystron and is also illustrated in figure (2).

Thus, the reflex klystron can be used as,

1. Voltage tuned oscillator
2. Frequency modulated oscillator.

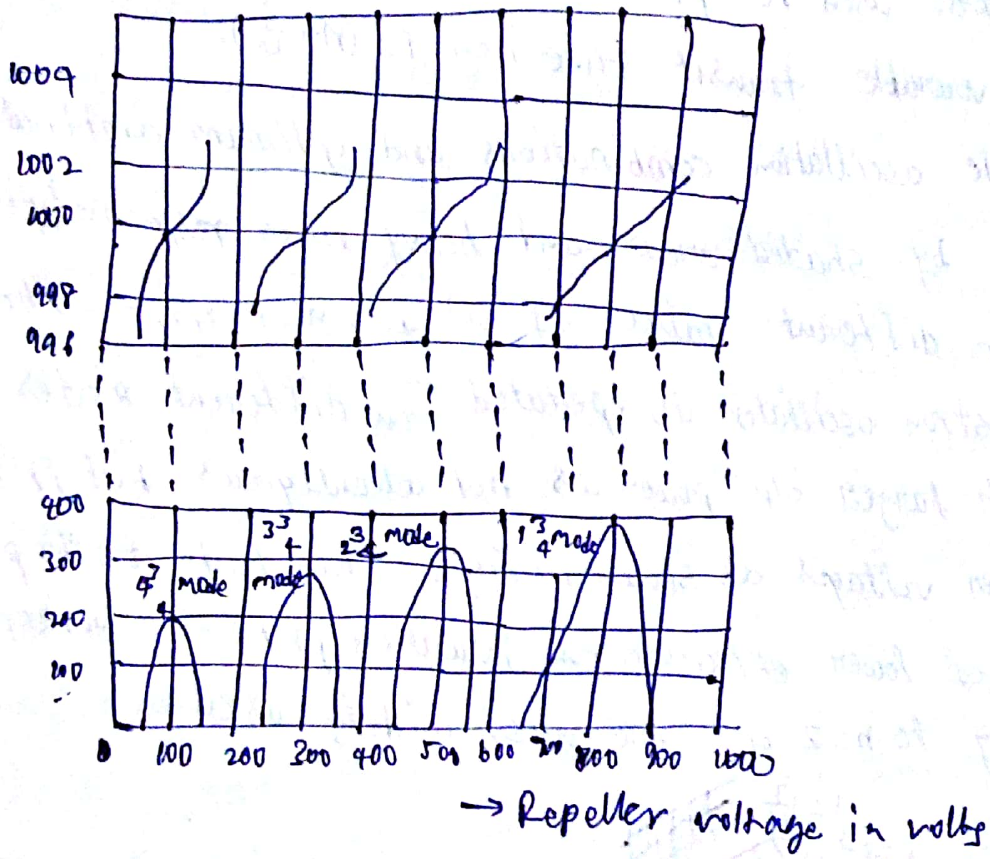


Figure (2): Power output and Frequency characteristics of Reflex

Travelling wave Tube (TWT):-

The travelling wave tube is a new type of tube which has displayed considerable promise as a broad band amplifier, proposed by pierce and others in 1946. These amplifiers are different from klystron amplifiers.

Klystrons are essentially narrow band devices as they utilise cavity resonators to velocity modulate the electron beam over a narrow gap whereas TWT's are broad band devices in which there are no cavity resonators. The interaction space in a TWT is extended and the electron beam exchanges energy with the RF wave over the full length of the tube.

The TWT is a high gain, low noise, broad band mw amplifier.

-> Its operation is based on the interaction b/w the electron beam and RF field and both are travelling in the same direction with nearly equal to the velocity of light.

-> The RF field propagates with a velocity equal to light, v_c.

-> The interaction b/w the RF field & the moving electron beam will takes place when the RF field is retarded (slow). RF field is retarded by using slow wave structure (i.e., helix)

-> In TWT; when RF wave & the electron beam are moving with similar velocities, a continuous interaction b/w the "input wave" and the "electron beam" takes place resulting in bunches and these bunches grow as the beam moves further in the helix. In this way, the i/p is amplified.

construction:- It consists of

Electron gun: It produces narrow constant velocity electron beam.

Magnet: It is used to prevent the beam from spreading & to guide it through the center of the helix.

Attenuator: It is used to isolate i/p and o/p waves.

Helix: It is a loosely wound thin conduction helical wire which acts as a slow wave structure. RF i/p is applied at one end of the helix and o/p is taken at other end of the helix as shown in fig below.

Collector: It is made positive potential and as a result, beam is attracted to the collector and acquires a high velocity.

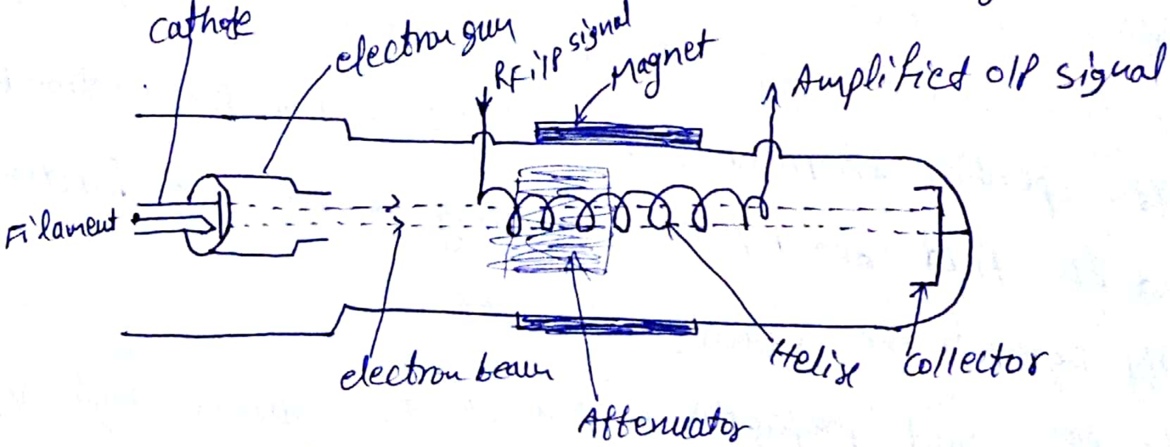


Fig: TWT structure

Operation:- The electron gun produces a electron beam is travel through the center of the helix without touching the helix.

→ When the applied RF signal propagates around the turns of the helix, it produces an electric field at the center of the helix. The electron beam & RF field both are travelling in the same direction with nearly velocity of light.

→ The RF field due to the RF signal travel with a velocity of light multiplied by the ratio of helix pitch to helix circumference ($2\pi r$).

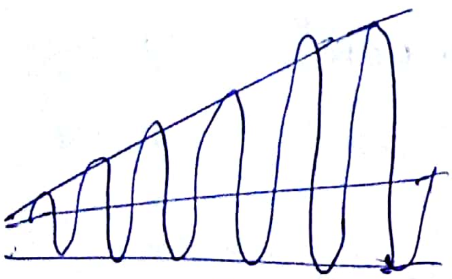
The interaction takes place b/w the electron beam and RF field when they are travelling through the helix and the electron beam delivers the energy to the RF wave on the helix.
Thus, the RF signal wave grows and amplified o/p is obtained at the o/p of the TWT.

The phase velocity (V_p) is given by,

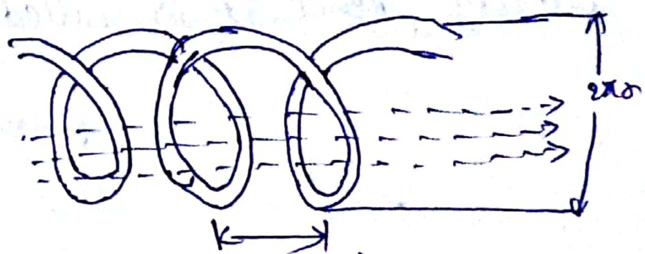
$$V_p = V_c \times \frac{\text{Helix pitch}}{\text{Helix circumference}}$$

$$V_p = V_c \times \frac{\text{Helix pitch}}{2\pi r}$$

where, V_p = Phase velocity
V_c = velocity of light
r = Radius of the helix.



a) Amplified o/p of TWT



b) pitch Diagram of helix.

Advantages:-

1. low noise.
2. High bandwidth.
3. High Gain.
4. low noise figure about 6 dB.
5. No need of resonant cavities

Applications:

1. low noise RF amplifier in broad band u/w receivers.

2. Repeater amplifier in wideband communication links and co-axial cables (long distance telephones).

3. Used as o/p tube in communication satellites.

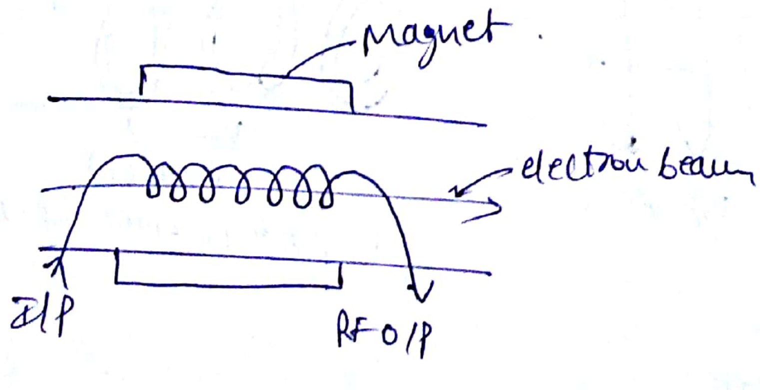
4. Used in Airborne and ship borne pulsed high power Radars.

Slow wave structure:-

It is a device in which an electron beam is velocity modulated without a resonant cavity.

1. The principle involved in this structure is the interaction of electron beam continuously with a weak electric field which is travelling along the beam.

2. The wave interacts with the electron beam continuously if their velocities are equal. The wave velocity is less than that of light velocity, hence it is called "slow wave".



slow wave structure

velocity modulation: velocity of electron varies in accordance with the RF i/p voltage is called "velocity modulation" of the electron beam.

Merits & Demerits of Different slow wave structures:-

a) Helical - line

Merits: → Helix structure provides sufficient bandwidth required for broad band travelling wave tubes.

→ It features good chromatic dispersion.

→ The fluctuations of its o/p parameters in broad band ~~travelling wave tubes~~ range are very small.

→ Helical - wave s/s provides wideband characteristics.

→ suitable for low temperature experiments.

Demerits:-

→ low energy & power handling capability.

→ Efficiency is less.

→ Gain is reduced by the internal attenuator.

→ It suffers from manufacturing problems at high freq.

→ sufficient amount of heat dissipation is not provided

b) Folded Back line:-

Merits: folded-back transmission line is suitable in

→ designing Backward wave oscillators (BWO)

→ It supports the characteristics of helix structure.

Demerits:

→ Bandwidth is limited.

c) Zig-zag line:-

Merits:

→ The structure forms inductive part of slow wave structure which provides synchronization with electron beam.

→ It performs shaping of dispersion characteristics.

→ Insertion loss is reduced.

Demerits:

→ weight is more at very high freq.

→ cost is more.

d) Interdigital line:-

Merits:

→ The dispersion is negative, due to which design of amplifier and BWO is made easy.

→ Heat dissipation is good.

Demerits:-

→ Requires high temperature for operation.

→ The structure suffers from fabrication problems.

e) corrugated waveguide:-

merits:

→ coupling efficiency is high.

→ It is suitable for high freq applications.

→ propagation loss is very less.

Demerits:

→ Narrow bandwidth is provided.

→ weight is more.

→ It has high cross polarization.

→ Various difficulties are encountered at the time of manufacture.

A TWT operates under following parameters beam voltage $V_0 = 3KV$, beam current $I_0 = 20mA$, characteristic impedance of helix $Z_0 = 10$, circuit length $N_1 = 50$ and freq $f = 10GHz$. Determine, i) Gain parameter. ii) o/p power gain in dB.

Given that,

- for a TWT,

Beam voltage, $V_0 = 3\,kV$,

Beam current, $I_0 = 30\,mA$.

characteristic impedance, $Z_0 = 10\,\Omega$

circuit length, $N = 50$.

Freq, $f = 10\,GHz$.

i) Gain parameter

The expression for gain parameter of a helix TWT is given by,

$$C = \left[\frac{I_0 Z_0}{4 V_0}\right]^{\frac{1}{3}} = \left[\frac{20\times10^{-3}\times10}{4\times3\times10^{3}}\right]^{1/3}$$

$$C = 2.55\times10^{-2}$$

ii) O/P power gain

The o/p power gain of helix TWT is given by,

$$A_p = -9.54 + 47.3\,NC$$

$$= -9.54 + 47.3\times50\times2.55\times10^{-2}$$

$$A_p = 50.77\ dB$$

iii) The four propagation constants, $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are given by,

$$\gamma_1 = -\beta_e C\frac{\sqrt{3}}{2} + j\beta_e\left(1+\frac{C}{2}\right)$$

where,

$$\beta_e = \frac{\omega}{V_0} = \left(\frac{2\pi\times10^{10}}{0.593\times10^{6}\cdot\sqrt{3\times10^{3}}}\right)$$

$$\beta_e = 1.93\times10^{3}\ rad/m$$

$$\gamma_1 = -1.93\times10^{3}\times2.55\times10^{-2}\times0.87 + j1.93\times10^{3}\left[1+\frac{2.55\times10^{-2}}{2}\right]$$

$$\gamma_1 = -42.82 + j1954.61.$$

$$\gamma_2 = \beta_e \cdot c \frac{\sqrt{3}}{2} + j\beta_e \cdot (1 + \frac{c}{2})$$

$$= 1.93 \times 10^3 \times 2.55 \times 10^{-2} \times 0.87 + j1.93 \times 10^3 \left(1 + \frac{2.55 \times 10^{-2}}{2}\right)$$

$$\gamma_2 = 42.82 + j1954.61$$

$$\gamma_3 = j\beta_e (1 - c) = j1.93 \times 10^3 (1 - 2.55 \times 10^{-2})$$

$$\gamma_3 = j1880.78$$

$$\gamma_4 = -j\beta_e \cdot (1 - c^3/4)$$

$$= -j\,1.93 \times 10^3 \left(1 + \frac{(2.55 \times 10^{-2})^3}{4}\right)$$

$$\gamma_4 = -j1930$$

Scanned by CamScanner

UNIT-IV

M-Type Tubes And microwave solid state Devices

Introduction:-

The tubes discussed earlier are linear beam tubes generally called O tubes or original type. The other type of µw tubes are cross field tubes in which the electric & magnetic fields are perpendicular to each other. The principal tube in this type called the M-type is the magnetron.

The magnetron was invented by Hull in 1921 & an improved high power magnetron was developed by Randall & Boot around 1939. Magnetrons provide microwave oscillations of very high peak power.

There are three types of magnetrons.

1. Negative Resistance type.
2. cyclotron frequency type.
3. Travelling wave or cavity type.

Negative resistance magnetron make use of negative resistance b/w two anode segments but have low efficiency & are useful only at low frequencies (<500MHz)

cyclotron freq Magnetrons depend upon synchronism b/w an alternating component of electric and periodic oscillations of electrons in a direction parallel to this field. These are useful only for frequencies greater than 100MHz.

cavity Magnetrons depend upon the interaction of electrons with a rotating electro-magnetic field of constant angular velocity. These provide oscillations of very high peak power and hence are very useful in radar applications.

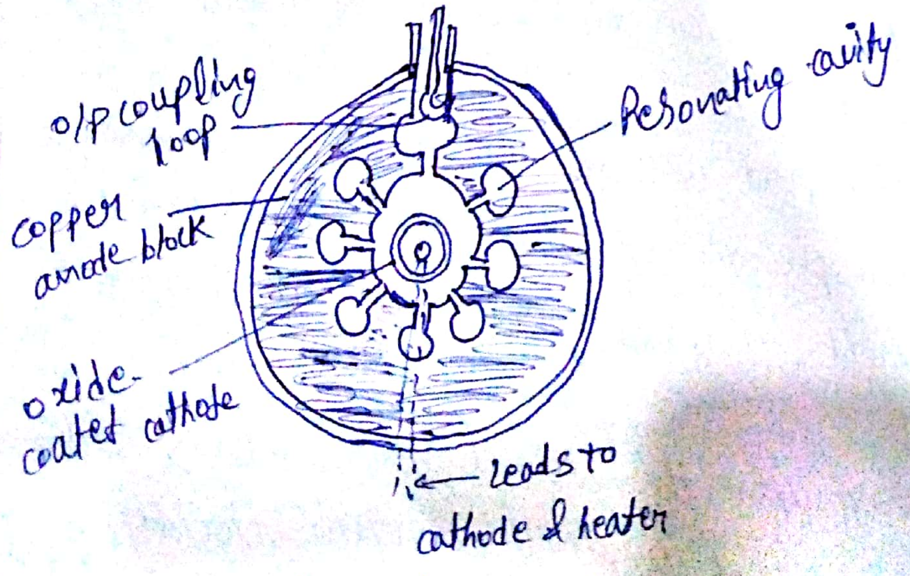
Cross field effect in magnetron:-

Magnetron is a self excited high power μw oscillator in which it is used to interact magnetic & electric fields of a cavity to provide oscillations.

M-types or cross field devices are those devices whose electric field b/w anode and cathode is radial. since magnetic field is perpendicular to plane of radial electric field, magnetron is also reffered as cross field device.

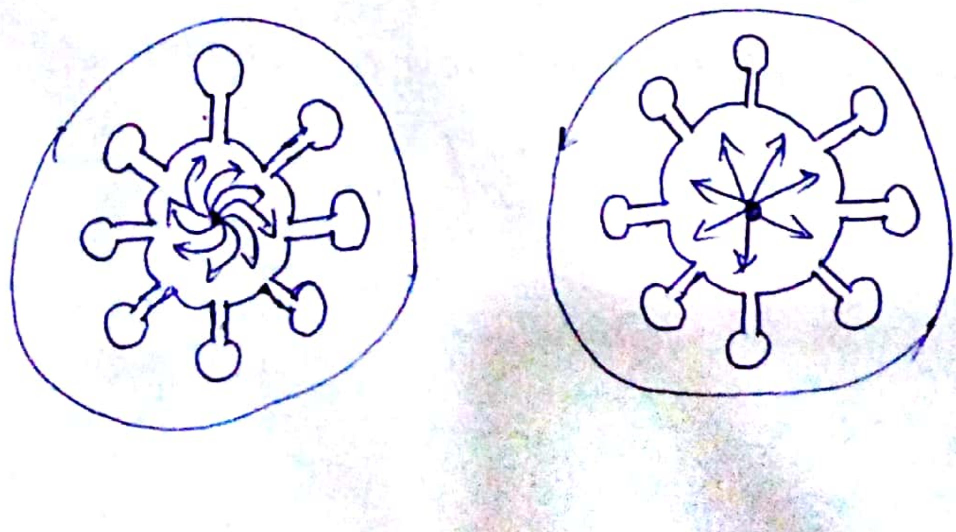
cross-field effect: If the orientation of electric and magnetic fields are perpendicular to each other the motion of electrons depend on electric or magnetic fields respectively. This field is called as cross field. Hence, the magnetic field exerts no force on electrons when the direction of electric and magnetic fields are same or opposite.

Cylindrical magnetron:-



It consists of a C-shaped permanent magnet containing cylindrical anode block of equally spaced | resonant cavity. The

cavities control the o/p freq as their diameter is adjusted @ only one half of the wavelength of required freq.
A copper anode block at the centre contains an oxide coated cathode and a filament. Filament leads support cathode & filament.
The permanent magnet provides magnetic field parallel to the axes cathode axis where as interaction chamber provides interaction b/w electric & magnetic fields for exerting force on electrons.
The other end of magnetron which is coupled to coaxial cable or waveguides provides the required o/p.
The principle of operation of magnetron depends on electron motion based on electric & magnetic fields.
In interaction chamber, when a magnetic field is applied, the motion of electrons become curved & starts flowing into circular path. The magnetic field & anode voltage are responsible for the movement of electrons in circular loop.
this circular movement of electrons causes cavities to excite & convert them into oscillation. The electron movement with & without applied magnetic field is as shown in fig 2.



Hull cut-off equation for cylindrical magnetron:-

critical Magnetic field:-

In a linear magnetron, the electrons travel in the straight path a from cathode to the anode at $B=0$. The electrons travel in a slightly covered path i.e., b, (when the magnetic field strength is slightly increased). For an applied magnetic field B_c, then electron return back to cathode just grazing the surface of the anode i.e., path c is shown in fig 1. This is called a critical magnetic field B_c. When $B>B_c$, the electrons return back to the cathode quickly without reaching the anode as they experience high rotational force.

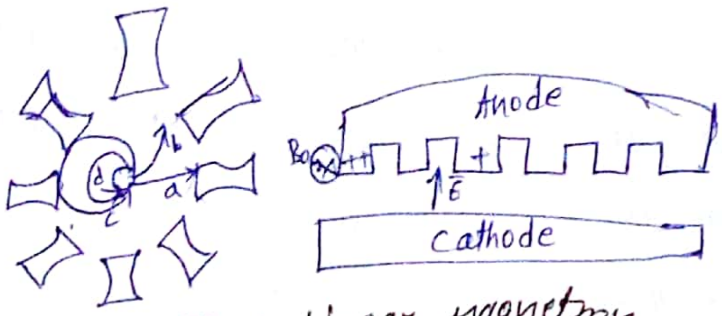


Fig 1: linear magnetron

Expression for cut-off magnetic field in linear magnetron:-

In the crossed electric & magnetic fields, the differential equations for motion of electrons can be written as,

$$\frac{d^2x}{dt^2} = -\frac{e}{m}\left[E_x + B_z\frac{dy}{dt}\right] \quad \text{—(1)}$$

$$\frac{d^2y}{dt^2} = \frac{e}{m} B_z \cdot \frac{dy}{dt}. \quad \text{—(2)}$$

$$\frac{d^2z}{dt^2} = 0. \quad \text{—(3)}$$

Hull cut-off magnetic field density (B_{oc}):-

It is the magnetic field strength for a given voltage V_0, which causes the electrons to just graze the anode surface & return towards cathode. It is denoted by B_{oc}.

Force acting on the electron is $F = Bev$.

In the direction of ϕ, the force component F_ϕ is given by

$$F_\phi = Bev_\rho.$$

where v_ρ = velocity in the direction of the radial distance ρ.

Torque in ϕ direction is

$$T_\phi = \rho F_\phi = \rho . Bev_\rho \quad \text{——} ①$$

Angular momentum = angular velocity × moment of inertia.

$$L = \omega \times I$$

$$= \frac{d\phi}{dt} \times m\rho^2$$

Time rate of angular momentum $= \frac{d}{dt}\left(\frac{d\phi}{dt} \times m\rho^2\right)$ —— ②

which gives the Torque in ϕ direction.

equating ① & ②

$$\frac{d}{dt}\left(\frac{d\phi}{dt} \times m\rho^2\right) = \rho . Bev_\rho.$$

$$\frac{d}{dt}\left(\frac{d\phi}{dt} \times \rho^2\right) = \frac{\rho Be v_\rho}{m}$$

where $\boxed{\omega_c = \frac{eB}{m}}$ [$\because \omega_c$: angular freq]

$$\frac{d}{dt}\left(\frac{d\phi}{dt} \times \rho^2\right) = \rho . v_\rho . \omega_c \quad \text{——} ③$$

here $v_\rho = \frac{d\rho}{dt}$

$$\rho . v_\rho = \rho . \frac{d\rho}{dt} \quad \text{— (4)}$$

sub (4) in (3)

$$\frac{d}{dt}\left(\frac{d\phi}{dt} \times \rho^2\right) = \rho . \frac{d\rho}{dt} . \omega_c .$$

$$\frac{d\phi}{dt} \rho^2 = \int \rho . \frac{d\rho}{dt} . \omega_c .$$

$$\frac{d\phi}{dt} . \rho^2 = \omega_c . \frac{\rho^2}{2} + c . \quad \text{— (5)}$$

For $\rho = a$ & $\frac{d\phi}{dt} = 0$ (radius of cathode)

$$0 = \omega_c \frac{a^2}{2} + c .$$

$$c = -\omega_c \frac{a^2}{2}$$

sub c value in (5)

$$\frac{d\phi}{dt} . r^2 = \omega_c \frac{\rho^2}{2} - \omega_c \frac{a^2}{2}$$

$$\rho^2 . \frac{d\phi}{dt} = \frac{\omega_c}{2}(\rho^2 - a^2)$$

$$\frac{d\phi}{dt} = \frac{\omega_c}{2}\left(1 - \frac{a^2}{\rho^2}\right)$$

At $\rho = b$.

$$\frac{d\phi}{dt} = \frac{\omega_c}{2}\left(1 - \frac{a^2}{b^2}\right)$$

Electron velocity is given as,

$$b^2\left(\frac{d\phi}{dt}\right)^2 = \frac{2e}{m} . V_0 .$$

$$b^2\left[\frac{\omega_c}{2}\left(1 - \frac{a^2}{b^2}\right)\right]^2 = \frac{2e}{m} V_0 .$$

sub $\omega_c = \frac{eB_{0c}}{m}$

$$b^2\left[\frac{eB_{oc}}{2m}\left(1-\frac{a^2}{b^2}\right)\right]^2 = \frac{2e}{m}V_0$$

$$\left(\frac{eB_{oc}}{2m}\right)^2 = \frac{\frac{2e}{m}V_0}{\left(1-\frac{a^2}{b^2}\right)^2 b^2}$$

$$\frac{eB_{oc}}{2m} = \frac{\left(\frac{2e}{m}V_0\right)^{\frac{1}{2}}}{\left(1-\frac{a^2}{b^2}\right)b}$$

$$B_{oc} = \frac{\left(\frac{2e}{m}V_0\right)^{\frac{1}{2}} \cdot 2m}{b\left(1-\frac{a^2}{b^2}\right) e}$$

$$B_{oc} = \frac{\left(8V_0\frac{m}{e}\right)^{\frac{1}{2}}}{b\left(1-\frac{a^2}{b^2}\right)}$$

The above eq^n is called hull cut-off magnetic eq^n.

cut-off voltage:-

$$V_{oc} = \frac{e}{8m} B_0^2 b^2 \left(1-\frac{a^2}{b^2}\right)^2$$

Hartree anode voltage eq^n for magnetron:-

In π-mode of operation, Hartree condition speifies an anode voltage that is required to synchronize the electron velocity with RF wave phase velocity.

It is defined as,

$$V_{on} = \frac{\omega}{\beta}\left(B_o d - \frac{m}{2e}\frac{\omega}{\beta}\right)$$

Fig shows the arrangement used to derive the Hartree resonance condition.

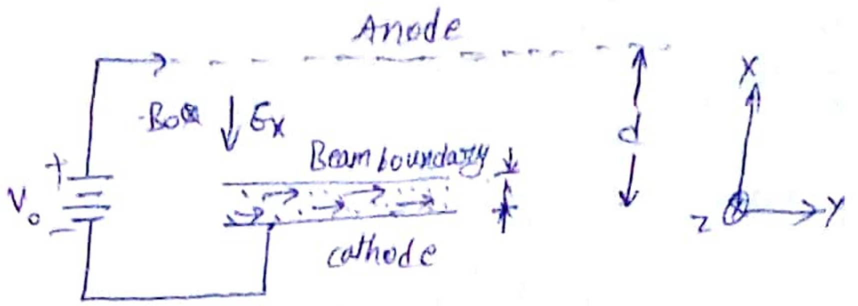


Figure.

The velocity of electron is assumed to be in +ve Y-direction and is given by,

$$V_y = -\frac{E_x}{B_0} = \frac{1}{B_0} \cdot \frac{dv}{dx} \quad \text{—①}$$

where, E_x = Electric field intensity in x-direction

$B_0 = B_z$ = The magnetic flux density in the +ve z-direction

V = potential applied.

But, from the principle of energy conservation

Kinetic energy of electron = potential electron energy

$$\frac{1}{2} m v_y^2 = eV$$

$$V_y^2 = \frac{2eV}{m} \quad \text{—②}$$

from eq^n ① & ②

$$\left(\left(\frac{dv}{dx}\right) \cdot \frac{1}{B_0}\right)^2 = \frac{2eV}{m}$$

$$\left(\frac{dv}{dx}\right)^2 = \frac{2eV}{m} \cdot B_0^2 \quad \text{—③}$$

$$\frac{dv}{dx} = \left(\frac{2eV}{m}\right)^{\frac{1}{2}} B_0$$

$$dx = \frac{1}{\left(\frac{2eV}{m}\right)^{\frac{1}{2}} B_0} \, dv$$

$$dx = \left(\frac{m}{2eV}\right)^{\frac{1}{2}} \frac{dv}{B_0}$$

Integrating on both sides we get,

$$\int dx = \int \left(\frac{m}{2eV}\right)^{\frac{1}{2}} \frac{1}{B_0} \cdot dV.$$

$$\left(\frac{m}{2eB_0^2}\right)^{\frac{1}{2}} \int \frac{1}{\sqrt{V}}\, dV = \int dx.$$

$$\left(\frac{m}{2eB_0^2}\right)^{\frac{1}{2}} \cdot \frac{V^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = x + c.$$

$$\left(\frac{m}{2eB_0^2}\right)^{\frac{1}{2}} \cdot \frac{V^{\frac{1}{2}}}{\frac{1}{2}} = x + c.$$

$$\left(\frac{m}{2eB_0^2}\right)^{\frac{1}{2}} \cdot 2 \cdot \sqrt{V} = x + c.$$

$$\left(\left(\frac{m}{2eB_0^2}\right)^{\frac{1}{2}}\right)^2 \left(2\sqrt{V}\right)^2 = x^2$$

$$\frac{m \cdot \cancel{4}^{2} V}{\cancel{2} e B_0^2} = x^2$$

$$V = \frac{x^2 e B_0^2}{2m} \quad \text{—— (4)}$$

since, the constant of integration is eliminated for $V=0$ at $x=0$. Then, the expression for potential & electric field at the hub surface are given by,

$$V(h) = \frac{e}{2m} B_0^2 \cdot h^2 \quad \text{—— (5)}$$

and $$E_x = -\frac{dV}{dx} = \frac{-e}{m} \cdot B_0^2 h. \quad \text{—— (6)}$$

Then the anode potential is given by,

$$V_0 = -\int_0^d E_x\, dx.$$

$$= -\int_0^h E_x\,dx - \int_h^d E_x\,dx$$

$$= V(h) + \frac{e}{m} B_0^2\, h(d-h)$$

sub eqn ⑤ in above eqn, we get,

$$V_0 = \frac{e}{2m} B_0^2 h^2 + \frac{e}{m} B_0^2 h(d-h)$$

$$V_0 = \frac{e}{m} B_0^2 h\left[d - \frac{h}{2}\right] \quad \text{(7)}$$

The velocity of electron at the hub surface is obtained from eqn ① & ⑥ as,

$$V_y(h) = \frac{e}{m} B_0 h \quad \text{(8)}$$

The phase velocity of the slow wave structure is equal to the above velocity of electron i.e,

$$\frac{\omega}{\beta} = \frac{e}{m} B_0 h \quad \text{(9)}$$

Then sub eqn ⑨ in eqn ⑦, the final anode potential for π-mode is given by,

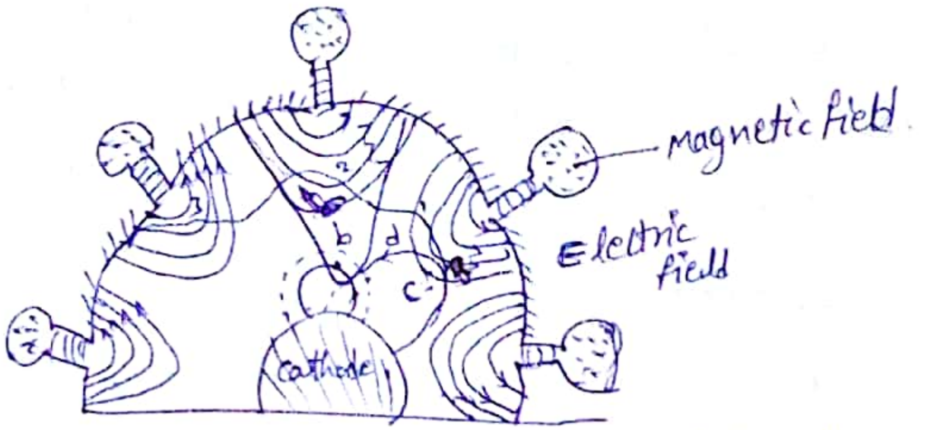
$$V_{0h} = \frac{\omega B_0 d}{\beta} - \frac{m}{2e}\frac{\omega^2}{\beta^2}$$

$$V_{0h} = \frac{\omega}{\beta}\left(B_0 d - \frac{m}{2e}\cdot\frac{\omega}{\beta}\right) \quad \text{(10)}$$

The above eqn is called Hartree response condition which is the function of the magnetic flux density, B_0 and the spacing b/w cathode and anode, d.

π- mode of magnetron :-

Bunching is caused due to the sustained interaction b/w the RF
wave & the electron beam travelling with equal velocities.
noise transients present in the magnetron introduces a RF field
in its cavity, which gives continuous oscillations in the operation les
of the drive. The oscillations produced are stable only if the
adjoining anode poles haves a phase-shift of $\frac{n\pi}{4}$ or $2n\pi$. (us)
$\varphi_v = \frac{2\pi n}{N}$ [$n = \frac{N}{2}$ then $\varphi_v = \pi$.
For instance, if 'n' is considered as 4, then we get,
[$\because n = 0, \pm 1, \pm 2 \dots \pm(\frac{N}{2}-1), \pm(\frac{N}{2})$]
$\frac{n\pi}{4} = \frac{4\pi}{4} = \pi$ - mode of operation as shown in below fig.



where a, b, c & d represents four electrons.
It can be seen that anodes are situated at a phase-shift of
π - radians. The dotted lines indicate electron paths without RF
fields i.e., in static field where as the electron paths in the
presence of RF field is represented by the durable lines.
a) The electron 'a' travels along distance from cathode to
anode with its speed being slowed down. It also gives its
energy to the oscillations. Hence, these electrons to the RF
field are known as favoured electrons & are accountable

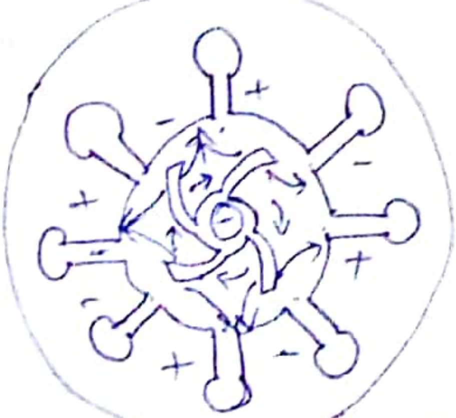
for bunching effect.

II) The electron 'b', in contrast to electron 'a', grasps energy from the oscillations, which leads to an increase in its velocity. Because of its high speed, electron 'b' comes to the cathode quickly & is not responsible for bunching. Hence it is known as unfavoured electron. This in-turn affects the system adversly by heating it.

III) The electron 'c', is released after a bit delay to get an appropriate position. It travels with a speed similar to that of electron 'a'. This electron contributes to the Bunching of electron 'a'.

IV) The electron 'd' is also slowed down to catch the electron 'a'. so this electron is also responsible for Bunching.

Hence, the electrons that are responsible for bunching are a, c and d. These are restricted to the spokes (e-clouds) only. It is known as phase focussing effect which is related to a bunch of electrons (a, c & d with 'a' as reference electron), as shown in fig 2.



Electron cloud motion & electric fields in resonant magnetron.

(7). $V_{Ht} = \frac{2\pi f B}{N} (b^2 - a^2)$.   7

separation of π-mode:- (strapping)

strapping is a method to separate π-mode from other modes. In this strapping technique for π-mode, each strap is at same potential and no π-mode current flows in strap. The straps offer capacitance & causes the freq separation b/w π-mode and the other modes as shown in fig 1.

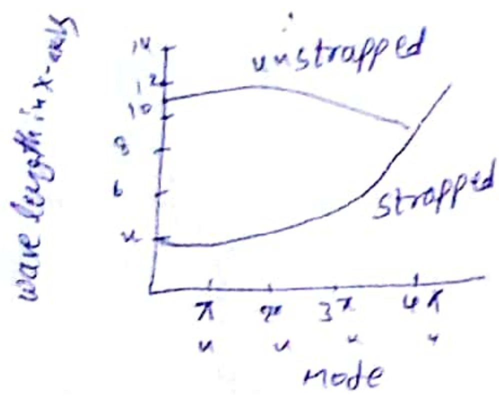


fig 1.

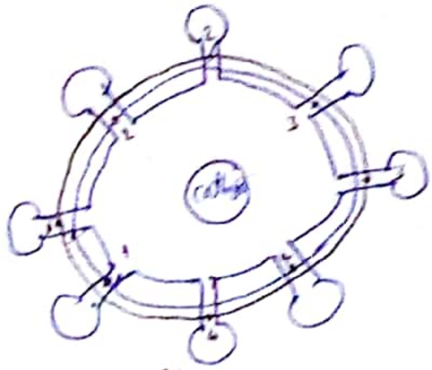


fig 2

strapping consists of two metallic rings of heavy gauge with one ring connected to the even number anode and the other to the odd number anode cavities poles as shown in fig 2.

strapping increases the freq separation b/w the cavities & also avoids mode jumping.

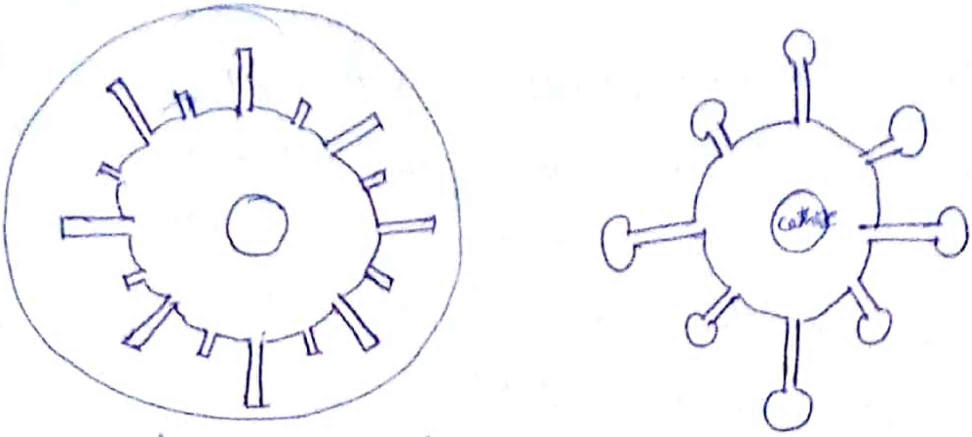
Disadvantages of strapping:-

→ causes power loss in the conducting rings.

→ It produces stray effects.

→ As the no. of cavities increase (16 or 32), strapping has no effect on mode jumping.

Rising sun magnetron does not require strapping. It gets even anode cavities which give a wide separation of resonant freq & hence provide high efficiency with low copper losses as shown in fig 3.



Rising sun Magnetron

Fig 3.

Mode jumping in magnetron:-

In magnetron cavities, the alternating RF magnetic flux lines are parallel to the cathode axis where as alternating RF electric fields are across the slot and fringe to the space b/w anode & cathode. N resonant or nodes exist for N no. of resonant coupled cavities.

For a closed slow wave structure, total phase shift must be integral multiple of 2π oscillations.

The phase shift b/w two adjacent cavities is given by,

$$\phi_n = \frac{2\pi n}{N}, \quad n = 0, \pm 1, \pm 2, \pm 3 \ldots$$

For opposite phase in successive cavities i.e. $\phi = \pi$ $(n = \frac{N}{2})$.

Excitation is max in the cavities.

When the change in the operating conditions of magnetron occur, its oscillations tend to jump from one mode to other, causing sudden changes in power o/p & oscillation freq. This condition is called "Mode jumping".

Ex: In a circular magnetron, a = 0.10 m, b = 0.40 m, B = 1.0 mT, V_0 = KV. Find the Hulls cut-off voltage & cut off magnetic flux density.

Ans: Given For a circular magnetron,

$a = 0.10\,m$

$b = 0.40\,m$

$B = 1.0\,mT$

$V_0 = 5\,KV$

Hull cut off voltage, $V_0 = ?$

cut off magnetic flux density, $B_0 = ?$

$$V_0 = \frac{eB^2b^2}{8m}\left[1 - \frac{a^2}{b^2}\right]^2$$

$$V_0 = \frac{1.759 \times 10^{11} \times (1 \times 10^{-3})(0.40)^2}{8}\left[1 - \frac{(0.10)^2}{(0.40)^2}\right]^2$$

$= 3092$

$V_0 \simeq 3\,KV$

$$B_0 = \frac{\sqrt{8 V_0 m/e}}{b\left(1 - \frac{a^2}{b^2}\right)} \quad \left[\because e = 1.6 \times 10^{-19},\ m = 9.1 \times 10^{-31}\right]$$

$$= \frac{\sqrt{8 \times 3 \times 10^3}}{\sqrt{1.759 \times 10^{11}}} \times \frac{1}{0.4\left[1 - \frac{(0.1)^2}{(0.4)^2}\right]}$$

$B_0 = 0.984\ wb/m^2$

microwave solid state devices:-

uw solid state devices are those which are used for low power microwave applications.

Scanned by CamScanner

These are of small size, light weight with high reliability.

μw solid state devices are classified into two types. They are,

1) Transferred - electron devices.

2) Avalanche transit-time devices.

1. Transferred Electron Devices:-

In electron devices, bulk effect is observed when the electron drift velocity decreases with increasing electric field above threshold value. This effect is due to negative conductance effect at μw frequencies. Hence, as the electric field increases, the mobility of electron shift from high to low. This phenomena is called as transferred-electron mechanism & the device exhibiting such phenomena is turned as transferred-electron device.

Ex: Gunn diode.

2. Avalanche Transit-time Devices:

When a reverse biased voltage increases beyond the junction voltage in a p-n-junction diode, breakdown occurs & current flows even for slight increase in voltage. This breakdown is due to avalanche multiplication of holes & electron of the junction in the space charge region. Hence p-n diode exhibits negative resistance characteristics in avalanche condition. The devices which exhibit such

such phenomena to generate microwave power to amplify the microwave signals are referred as avalanche transit-time devices.

In simple the junction devices which give the negative resistance by suitable combination of impact avalanche breakdown & charge carrier transit time effects is refered as Avalanche transit time devices.

Ex: IMPATT diodes, TRAPATT diode BARITT diodes.

<u>Applications:-</u>

The applications of solid state microwave devices are,

1. They are used in radio transmitters, such as CW doppler radar
2. They are used in broadband linear amplifiers.
3. They are used as pump sources in parametric amplifiers.
4. They find application in transponders.
5. They are used in both the combinational & sequential logic circuits
6. They are used in low receivers.

<u>IMPATT diode</u>:- (Impact Ionization Avalanche Transit Time devices)

IMPATT stands for impact ionization avalanche transit-time

The IMPATT diode is fabricated from silicon carbide material due to its high break down fields it operates in the freq-range of 3 GHz to 100 GHz. As, this diode contains high power, It is employed in high freq electronics & microwave devices.

The family of IMPATT diode involves several distinct junctions and metal semiconductors devices. Initially, IMPATT oscillation was produced from a silicon p-n junction diode mounted in a μω cavity, in which, the diode is biased into reverse avalanche breakdown region.

operation:

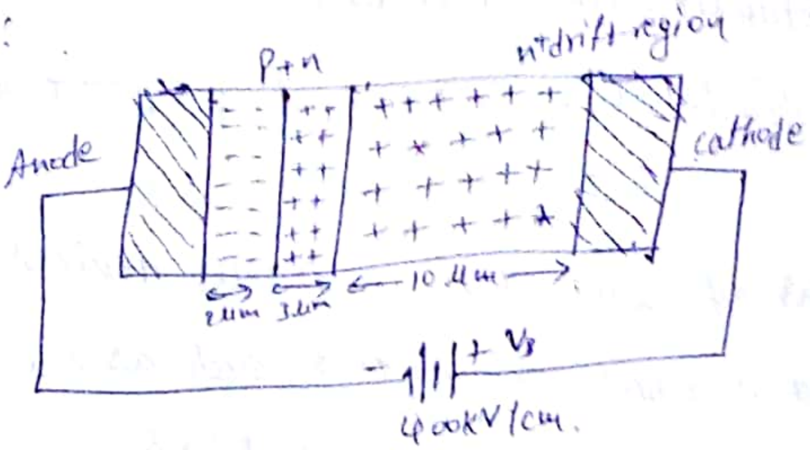


fig: IMPATT diode.

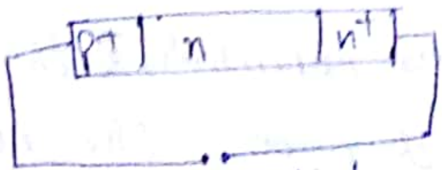
When the DC bias voltage V_B is applied to the diode, the holes in the n^+ drift region moves into high resistivity region called drift zone and from there electrons generated in avalanche region move towards the anode. Assume an AC voltage of large magnitude that is superimposed on the DC bias, in such a way that diode goes into avalanche breakdown during positive half cycle of the AC voltage. When $t=0$, the AC voltage is also zero and only little amount of breakdown current passes through the diode. When the t value increases, the voltage also increases and crosses the breakdown voltage level.

As a result secondary electron-holes pairs are formed through impact ionization. The concentration of electron hole pair grows exponentially with I_s till the field in an avalanche region remains above the breakdown field. when the field goes below the breakdown field during negative half cycle of AC voltage the concentration of electron-hole pairs decreases exponentially. Then generated holes in avalanche region dissappear in the p^+ region & are collected at the cathode. where, in i-zone, injected electrons move towards the n^+ region.

Advantage:- It has high power capability, Thus, it is used in wide applications like, low power radar systems, alarms etc.

Disadvantage:- It generates high level of phase noise due to the statistical nature of the avalanche process.

TRAPATT Diodes:- (Trapped plasma Avalanche Triggered Transit device) in which TRAPATT diodes are derived from IMPATT diode in which doping level b/w the junction & anode changes gradually. silicon or gallium arsenide is used for fabricating TRAPATT diodes.



Reverse bias

The current density will be high as the p-n junction is reverse biased beyond the breakdown region, this decreases the electric field in space charge region & increases carrier transit time. At this instant, the operating freq is limited to less than 10 GHz. The efficiency of this diode is very high.

The characteristics of TRAPATT diode are shown in fig 2

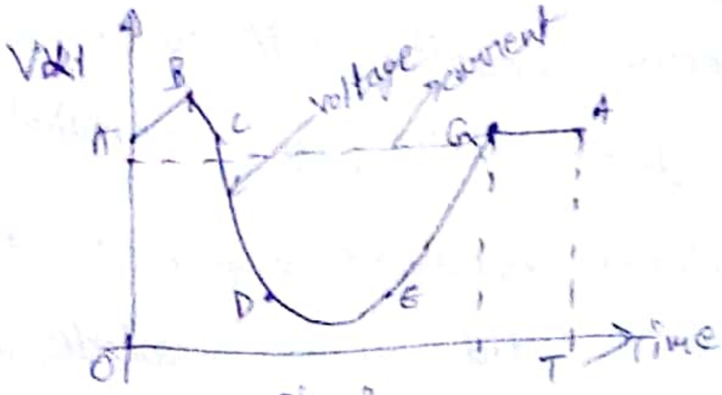


Fig 2.

The electric field is uniform at the point 'A' and its magnitude is large. The diode charges at point 'A' because of charge carriers, when a certain field is reached the electric field is further decreased to point D. Now, by removing the plasma & residual charge, the electric field is increased to point F.

Now from point F the electric field again increases to point G due to the charging of capacitor. At point G the voltage remains constant as the diode current drops to zero. The drift through this diode is much slower than through a comparable IMPATT diode because the electrons & holes drift at velocities determined by the low field mobilities, & the transit time of the carriers becomes much longer. The normal freq operating freq of a TRAPATT diode is 3-50 GHz.

because of slow drift time, the carrier transit time is increased This causes the operating freq of this diode to be limited to below 10 GHz.

BARITT Diodes:- (Barrier Injected Transit Time device):

BARITT diodes are the latest addition to the family of active mw diodes. They have long drift regions similar to those of IMPATT diodes. The carrier traversing the drift region of BARITT diodes however are generated by minority carrier injection from forward biased junctions instead of being extracted from the plasma of an avalanche region.

Such diodes are much less noisy than IMPATTs. with noise figures as low as 15 dB. The major disadvantage of BARITT being narrow band width and power o/p limitation.

The minority injection is provided by punch througth of the intermediate region. This process is different in the sense that it has lower noise than impact ionization responsible for current injuction in an IMPATT diode.

The negative resistance in a BARITT device is obtained on account of the drift of the injected holes to the collector end of the diode, made of p type material.

Fig below shows the constructional details of a typical BARITT device. It consists of an emitter, base intermediate drift or depletion region and collector.

An essential requirement for the BARITT device is therefore that the intermediate drift region be entirely depleted to cause punch through to the emitter base junction without causing available breakdown of the base collector junction.

For a m-n-m BARITT diode Ps-si schottky barrier contacts metals with n-type si wafer in b/w, a rapid increase in current with applied voltage is due to the thermionic hole injection into the semiconductor as shown in fig 2.

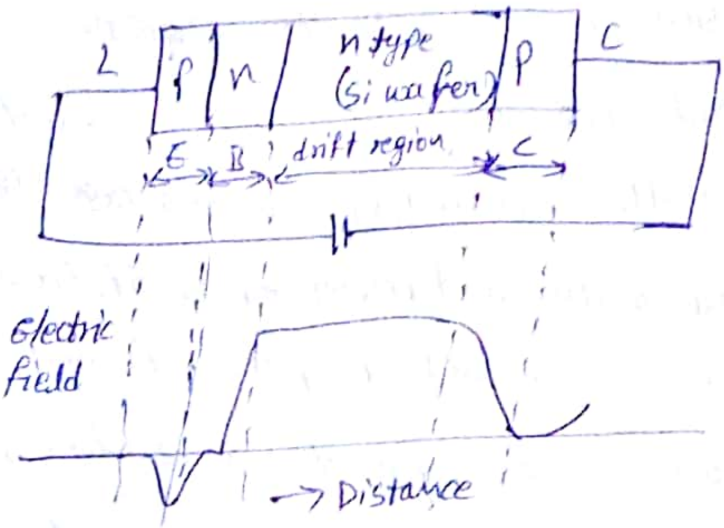


Fig 1: construction of BARITT diode

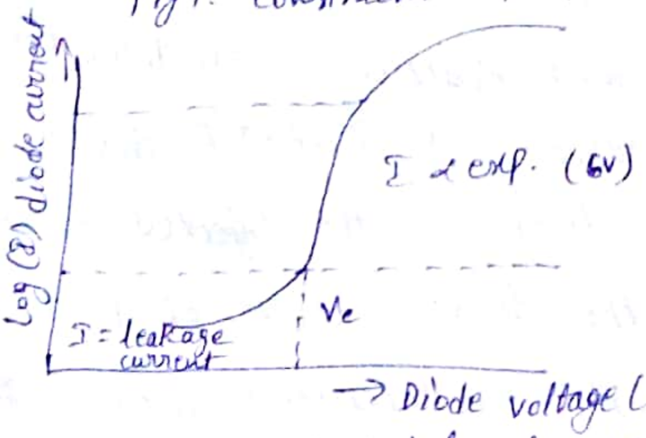


Fig 2: characteristics of BARITT diode.

Transferred Electron Devices (TED's):-

As we have seen before, the common characteristics of all active two terminal devices is their negative resistance. The real part of their impedance is negative over a range of frequencies. In a positive resistance the current through the resistance and the voltage across it are in phase, the voltage drops across positive resistance is +ve. & a power of I^2R is dissipated in the resistor, In a negative resistance the current & voltage are out of phase by 180°, the voltage drop across it is -ve & a power of $(-I^2R)$ is generated by the power supply associated with the -ve resistance.

TED's are fabricated from compound semiconductors such as GaAs, InP or CdTe as against Ge and Si of transistors.

Construction of GUNN Diode :-

Gunn diodes can be constructed by using single piece of n-type silicon as shown in fig 1.

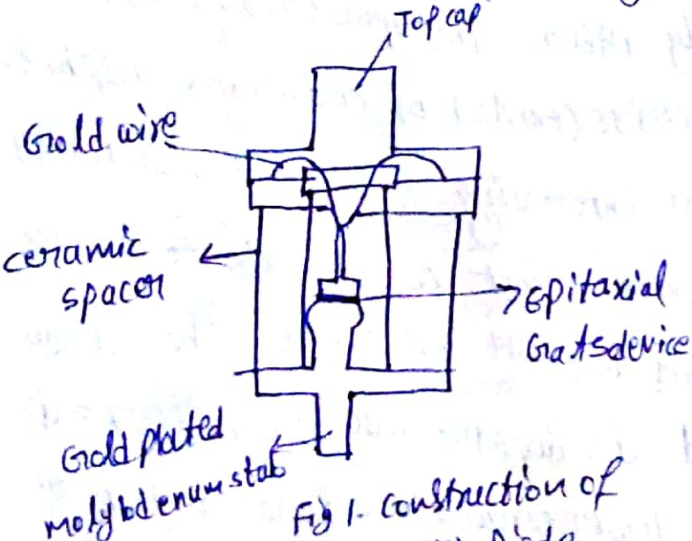


Fig 1. Construction of GUNN Diode

From the fig 1. it can be observed that the top & bottom areas of the GUNN diode are doped heavily to produce N^+ material. This material forms a high conductivity for making connections to the device. The device is placed on conducting base and a wire connection is made in order to work as a heat sink for generated heat. The gold connection deposited on the top surface of the device is used to make the connection with the other terminal. The gold is necessarily used in the gunn diode construction as it provides high conductivity and relative stability.

The middle area of the device called as "active region" is also doped heavily, in order to represent all voltages deposited across this device.

Gunn diode is only made up of n-type material hence, It does not have any junction. This it is not a true diode.

· Construction of GUNN diode using RWH theory :-

The principle of operation of Gunn diode depends on the properties of -ve resistance of bulk semiconductor materials.

consider the energy band diagram of GaAs as shown in fig 1.

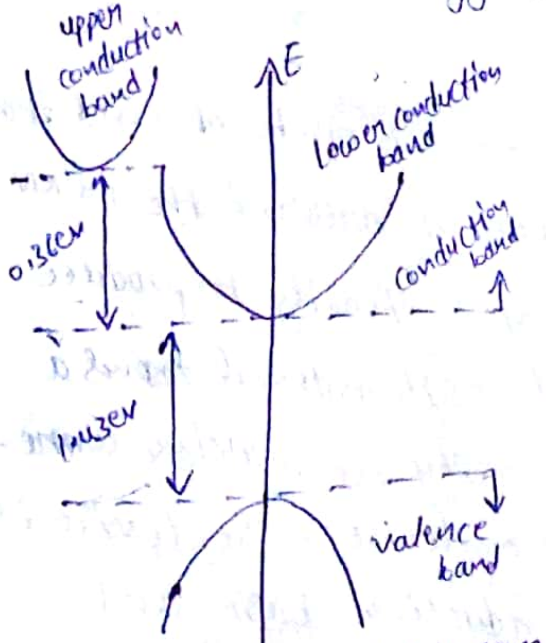


Fig: Energy Band Diagram of GaAs.

The Gunn effect was discovered by J.B.Gunn in the early 1960s. The materials such as gallium Arsenide (GaAs) or cadmium sulphide exhibit an interesting effect of conductivity known as Gunn effect. consider the following energy band diagram of GaAs. The conduction band is divided into two valley bands known as lower conduction band and upper conduction band which are seperated by an energy Gap of 0.36 eV.

under normal condition the electrons are in valence band. When an electric field is applied the electrons are transfered to the lower conduction band where the mobility of electrons increases with respect to the increased value of electric field which also increases current. This is called as +ve resistance characteristics. As the voltage increases further at a certain critical value the electrons are transferred to the upper conduction band where mobility of electrons is less as compared to the mobility of electrons at lower conduction band. The mobility further decreases with increased voltage which also decreases the current. This is called as -ve resistance characteristics as shown in below fig. Hence the Gunn effect is defined as "the effect exhibit by some materials such as GaAs and cds, after an electric field in the material reaches a threshold level, the mobility of electrons decreases as the electric field is increased, thereby producing negative resistance which results in reducing current.

Modes of operation:-

There are four different modes of operation that contribute to the μw oscillations in a Gun diode they are,

1. GUNN or Transit-time Mode:-

When a voltage is applied to a Gunn diode i.e., n^+nn^+ GaAs crystal is above the threshold level, the electrons present in low energy level are transferred to high energy level. At cathode, these electrons of high energy level accumulate to form an electric field dipole domain. For a constant applied voltage, electric field across domain increases above the average field. The resultant electric field decreases below threshold level & avoids the formation of further domains. Since the low mobility electrons drift with reduced velocity and conduction band electrons with constant velocity, the domain also decreases.

At the end of contact, the current increases which result in high field domain formation as shown in fig 2.

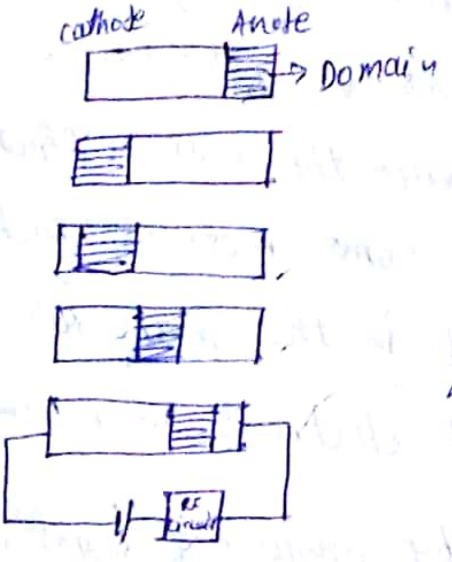


Fig 2:

Each domain produces a current pulse which result in a signal at low impedance RF circuit for μw frequencies. As the high field domain is quenched before reaching the anode, the transit time gets decreased with increased frequency.

LSA Mode:

LSA stands for "Limited space charge Accumulation". In this mode, the gunn diode works as a resonant circuit as shown in fig 3.

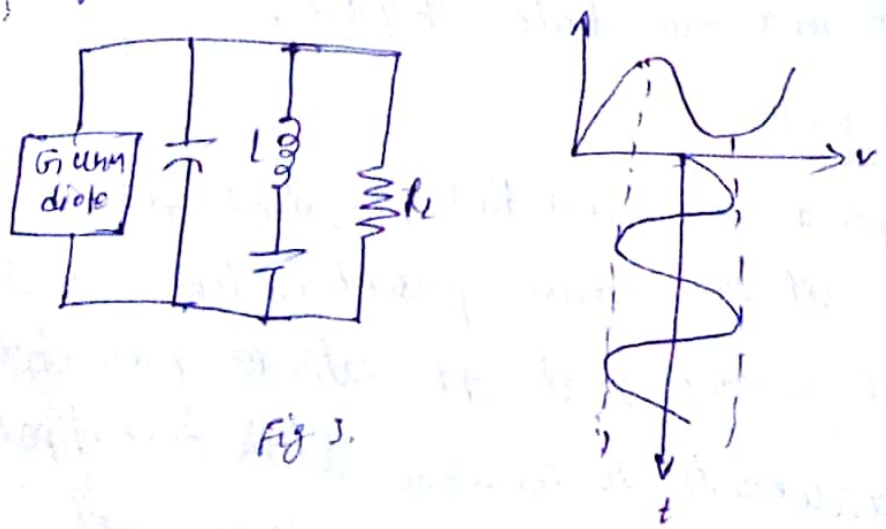


Fig 3.

The circuit is turned to operate as a -ve resistance oscillator with no formation of dipole domains. When dc voltage exceeds the threshold voltage and R_L adjusted to 20% higher than max resistance value, oscillations are developed. These oscillations become constant when the average negative resistance of diode equals load Resistance R_L.

Quenched Domain mode:-

In this mode, the dipole domain gets quenched before it reaches the anode by -ve swing of oscillation. Though the Gunn diode appears like operating in Gunn diode mode, this mode can be achieved when the resonant circuit is tuned above the transit time mode.

Delayed Mode:

In this mode, the new dipole domains get delayed until oscillation voltage exceeds the threshold value. Here, the resonator circuit is tuned below the gunn mode such the dipole domains reach the anode in time.

Fig 4 shows the diagramatic representation of operation GUNN diode in different modes.

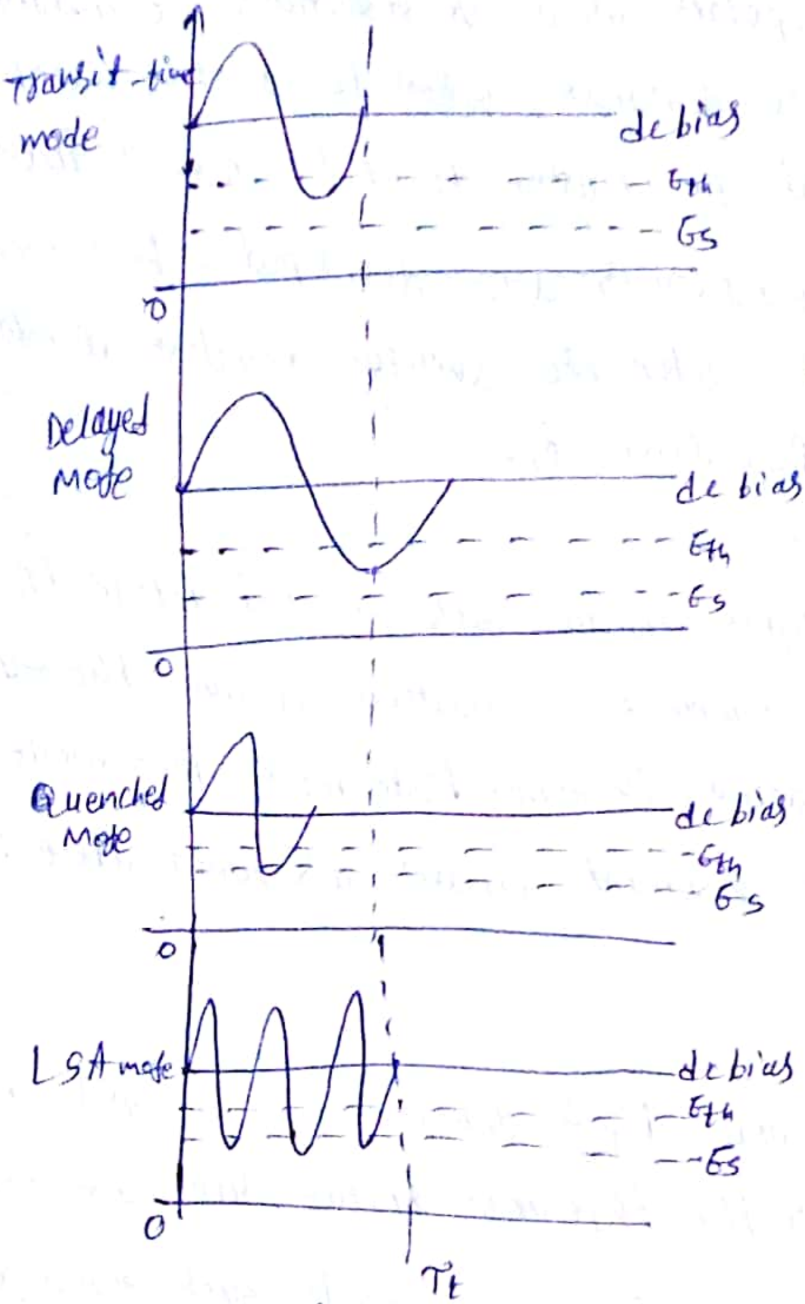


Fig 4.

Scanned by CamScanner

V UNIT

Microwave Measurements

Introduction:

At low freq, it is convenient to measure voltage & current and use them to calculate power. However at μw frequencies they are difficult to measure and since they vary with position in a transmission line, are of little value in determining power. Therefore at μw freq, it is more desirable and simpler to measure power directly.

At low freq, circuits use lumped elements which can be identified and measured. At μw freq circuit elements are distributed and as such it is usually not important to know what element make up a line.

For power measurements, it is usually sufficient to know the ratio of two powers rather than exact i/p or o/p powers. The following parameters can be measured at μw frequencies: 1) Frequency 2) power 3) Attenuation 4) voltage standing wave Ratio 5) phase 6) Impedance 7) Insertion loss 8) Dielectric constant 9) Noise factor.

Microwave Bench set-up:-

The general set up for measurement of any parameter in microwaves normally done by a μw bench.

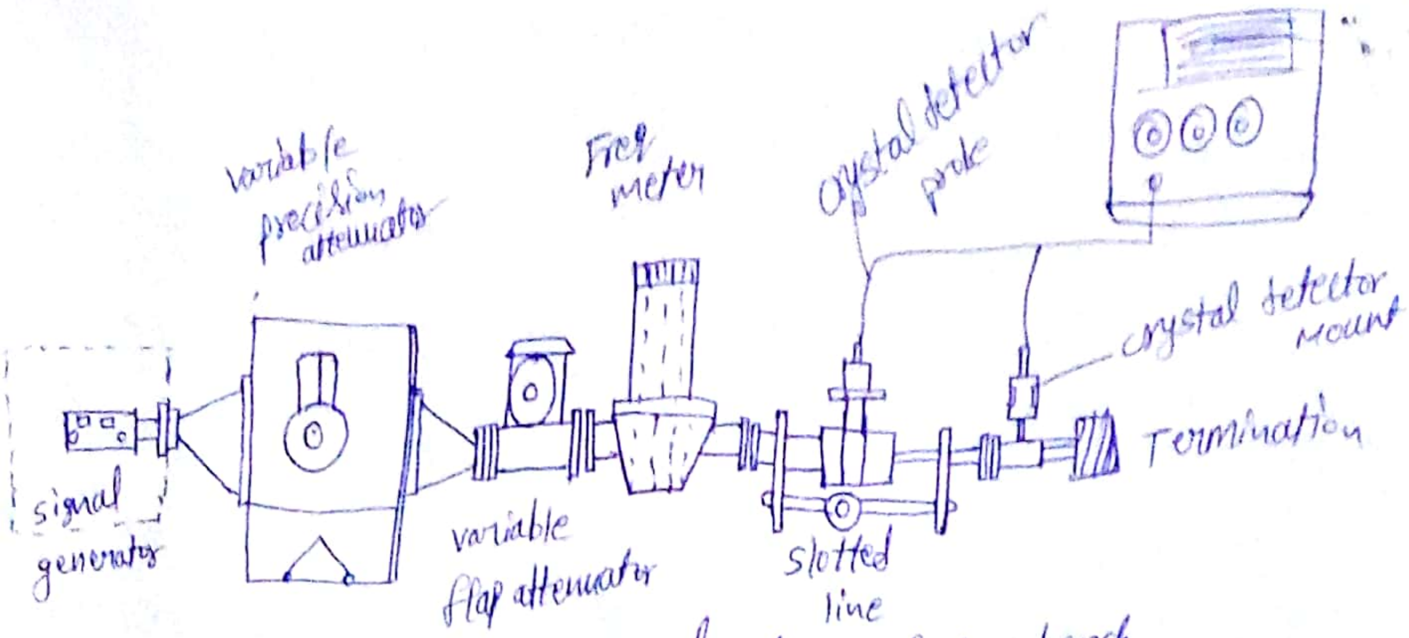


Fig: General set-up of uw bench

Signal Generator:

The signal generator is a uw source whose o/p is of the order of milliwatts. It could be a Gunn diode oscillator or a reflex klystron tube. It can provide either a continuous wave (cw) or square wave modulated at an audio rate which is normally 1KHz

Variable precision attenuator :-

The precision attenuator can provide 0 to 50 dB attenuation above insertion loss.

Variable flap attenuator :-

The variable flap attenuator is also used in addition, whose calibration can be checked against readings of the precision attenuator.

Frequency Meter:- It is used for direct reading of freq that consists of single cylindrical cavity which can be adjusted to resonance and is slot coupled to the waveguide.

The readings of freq is directly taken from the freq meter.

∴ Slotted line:- Slotted line is used for measuring standing wave ratio. It consist of slotted section of transmission line a travelling probe carriage.

Standing wave Ratio:- It is defined as the ratio of voltage max to voltage minimum.

$$SWR = \frac{V_{max}}{V_{min}}.$$

Termination:- It is used to produce the standing wave pattern.

Crystal detector:- The crystal detector is inserted in the E-probe of the slotted line is used to detect the modulated signal.

SWR indicator:- Finally direct readings of standing wave ratio is given by a sensitive tuned voltmeter called SWR indicator.

precautions:-

→ components should be connect titly for avoiding power leakage.

→ Air cooling is required for refler klystron oscillator,

→ Microwave power should not be measured directly as it effects the vision (harmful to eyes).

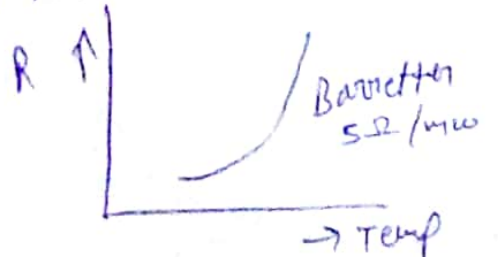
Measurement of power:-

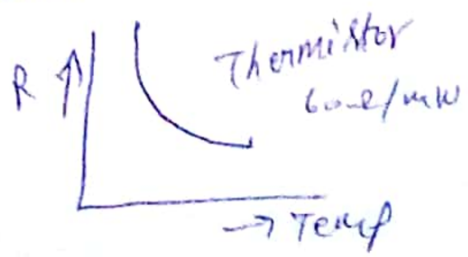
To measure the MW power we are using 3 techniques:

1) Measurement of low power (0.01 mW – 10 mW)
   – Bolometer technique.
2) Measurement of medium power (10 mW – 10 W)
   – calorimetric technique
3) Measurement of high MW power (>10 W)
   – calorimetric watt meter.

1) Measurement of low MW power:-

Devices such as bolometers & thermocouples whose resistance chan-
ges with the applied power are capable of measuring low MW powers
Bolometers are most widely used among these.
Bolometer is a simple temperature sensitive device whose resistan
ce varies with temperature. These are of two types viz Barretters
and Thermistors. Barretters have positive temperature coefficient
and their resistance increases with increase in temperature
as shown in fig 1. It basically consist of a short length of
fine platinum wire mounted in a cartridge, like an ordinary fuse. It is
very delicate device. Thermistors have negative temperature coefficient
of resistance & their resistance decreases with increase in temperature.





A bolometer such as crystal diode is a square law device and it produces a current that is popotional to the applied power. Bolometer is mounted inside the waveguide as shown in fig below

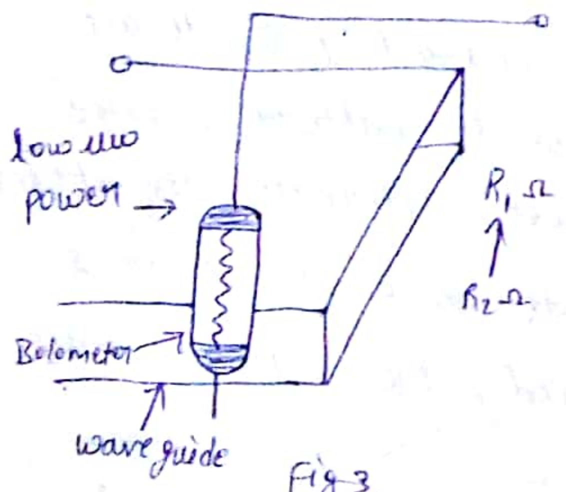


Fig 3.

where the bolometer itself is used as a load, with the operation resistance as $R_1 \Omega$. Now the low µw power which is to be measured is applied. Some power is obsorbed in the bolometer load and dissipated as heat and the resistance changes to R_2. This change in resistance $(R_1 \sim R_2)$ is propotional to the µw power which can be measured using a bridge. In the balanced bolometer bridge technique, the bolometer itself is made to be one of the arms of the bridge as shown in fig below.

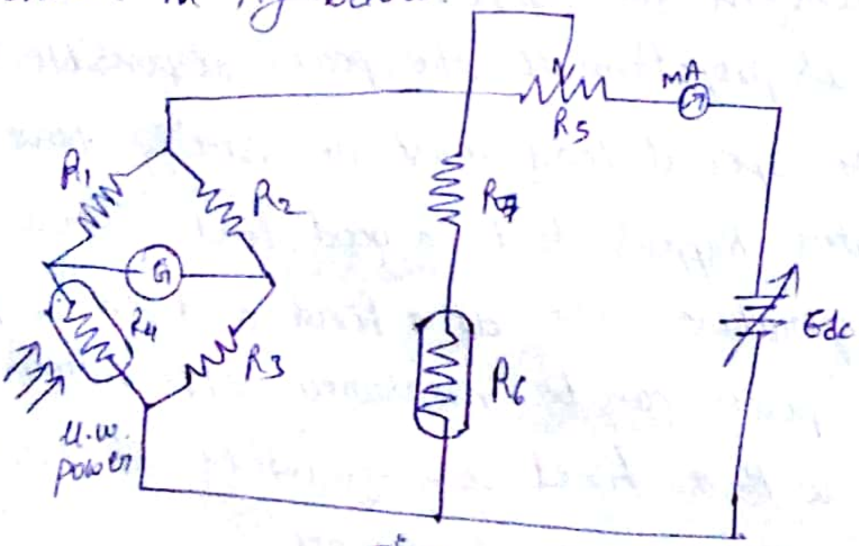


Fig 3.

Initially, the bridge is balanced by adjusting R_5, which varies the dc power applied to the bridge and the bolometer element is brought to a predetermined operating resistance before μw power is applied. let the voltage of the battery be E_1 at balance. The μw power is now applied and this power gets dissipated in the bolometer. The bolometer heats up and it changes its resistance. Therefore the bridge becomes unbalanced. The applied dc power is changed to E_2 to get back the balance and this change in dc battery voltage will be propotional to the μw power. Alternately the detector 'G' can be directly calibrated in terms of μw power so that when the bridge is unbalanced, the detector reads the μw power directly.

Measurement of medium power or high power:

medium power as already stated in the range of 10mw to 10w. such powers can be measured by calorimetric techniques. The principle is very simple wherein the temperature rise of a special load monitored which is propotional to the power responsible for the rise ~~as show~~. The special load must necessarily have high specific heat. water happens to be a good load. Knowing mass specific heat & temperature rise at a fixed and known rate of fluid flow, the power can be measured. Alternately rate of temperature rise with a fixed ~~fix~~ quantity of fluid also can be adopted for measurement of power.

In this method the power is measured based on mass, specific heat, temperature rise values at a constant fluid flow rate.

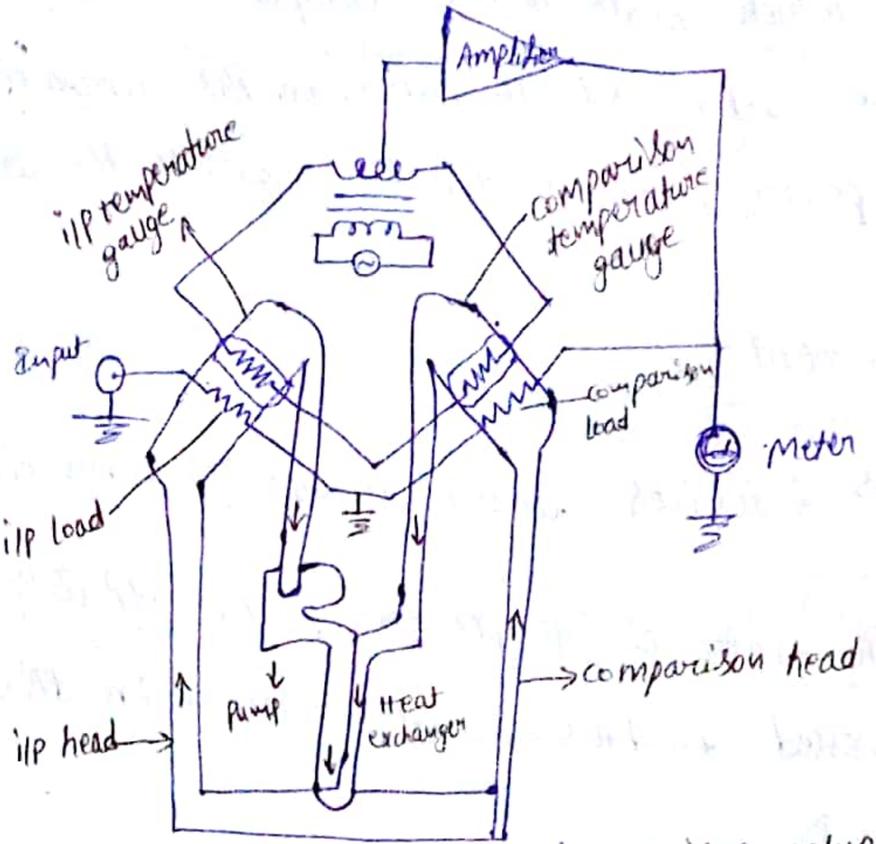


Fig : self Balancing Bridge setup

The setup consists of,

1. Two identical temperature gauges
2. An indicating meter
3. Two load resistors.

The two identical gauges along with the two load resistors are placed in the two arms

The o/p arm is referred to as comparison head, the resistor load as comparison load and the gauge as comparison temperature gauge.

similarly the i/p side arm, resistor load & guage are called as i/p head, i/p load & input temperature gauge respectively.

<u>operation</u>:- When an unknown i/p ~~its~~ power is applied by a µw source, heat is generated in the i/p load resistor which raises the temperature of the gauge. This unbalances the bridge. The resulting signal is amplified and applied to the

Scanned by CamScanner

comparison load resistor which heats up the comparison temperature gauge & the bridge is rebalanced The meter on the comparison side measures this power which is directly equal to the unknown i/p power.

Attenuation measurement :-

→ Microwave components & devices almost always provides some degree of attenuation.

→ Attenuation is the ratio of i/p power to the o/p power and is normally expressed in dB's i.e. attenuation in dB's equal to $10 \log_{10}\left(\frac{P_{in}}{P_{out}}\right)$

$$A = 10 \log_{10}\left(\frac{P_{in}}{P_{out}}\right)$$

where P_{in} – input power

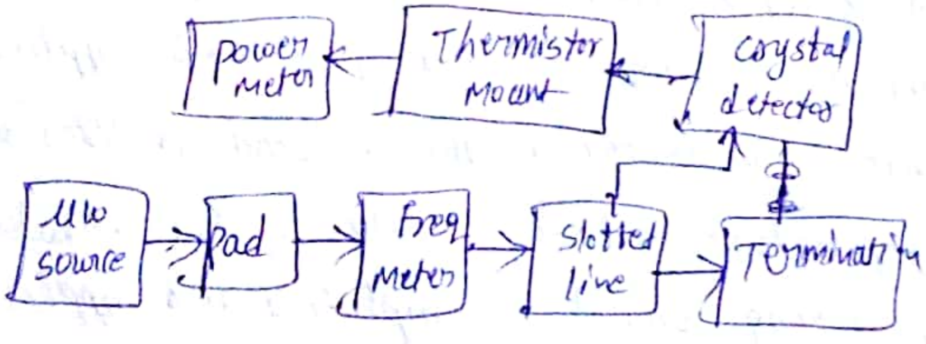
P_{out} – o/p power.

The amount of attenuation can be measured in 2 methods.

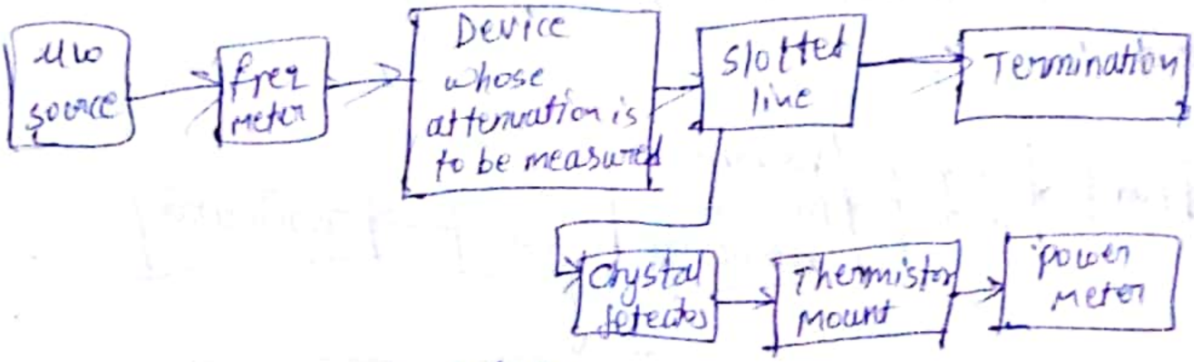
i) power ratio method or direct method.

ii) RF substitution method.

power ratio method :-



∴ In this method, i/p & o/p power are measured with or without the device whose attenuation is to be measured. In setup the device whose attenuation is to be measured is added in b/w the freq meter & the slotted line. The ratio of power measured in each setup, i.e., P_1/P_2 gives the attenuation in decibels.



setup 2

RF substitution method:-

The commonly used method for the measurement of attenuation is RF substitution method. This method is particularly suitable for the networks with large attenuation & low i/p powers because the attenuation is measured at a single power position. Thus, the results obtained are accurate compared to the power ratio method. Fig 1. shows an arrangement used for the measurement of o/p power, P by including a network whose attenuation has to be measured.

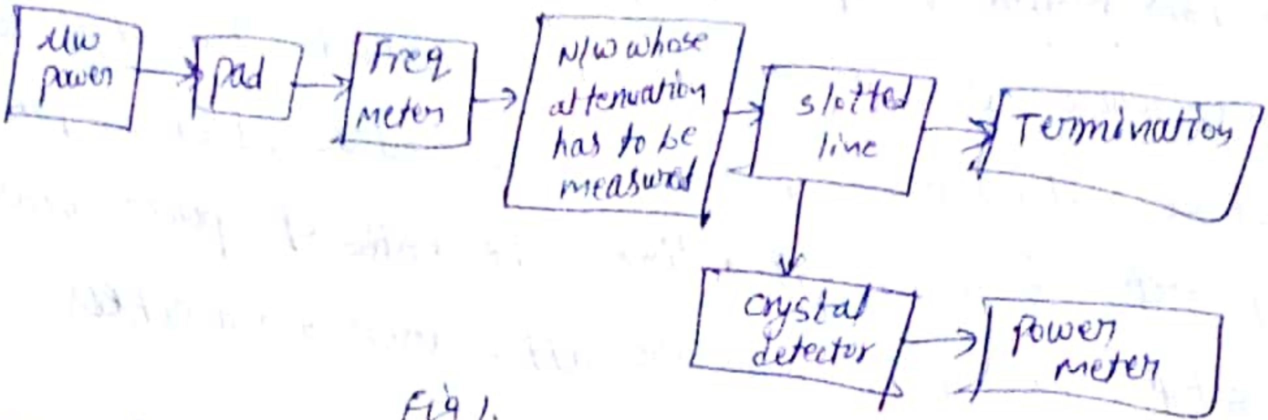


Fig 1.

In fig 2, the network is replaced by a precision calibrated attenuator.

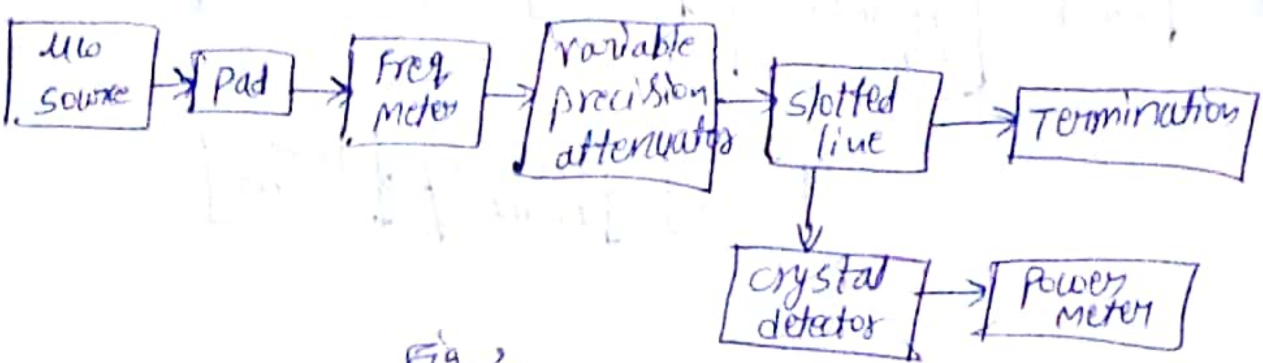


Fig 2

The precision attenuator is adjusted to obtain the same power P as measured in arrangement of fig1. In this case the attenuation of the n/w is measured by directly reading value from precision attenuator.

VSWR :-

A VSWR meter is a high gain, high Q, low noise amplifier which is tuned normally to a fixed freq (1KHz) of a modulated uw signal. The detected uw signal at the slotted waveguide will be the i/p to the VSWR meter and the amplified o/p is measured through crystal detector so the o/p is proportional to the square of the i/p voltage at the measured position.

6

The VSWR can be known directly by V_{max}/V_{min}. If V_{min} is adjusted to unity, then V_{max} is equal to VSWR. with the ~~following feature~~

Any mismatched load leads to reflected waves resulting in standing waves along the length of the line.

The ratio of max to mini voltages gives the VSWR.

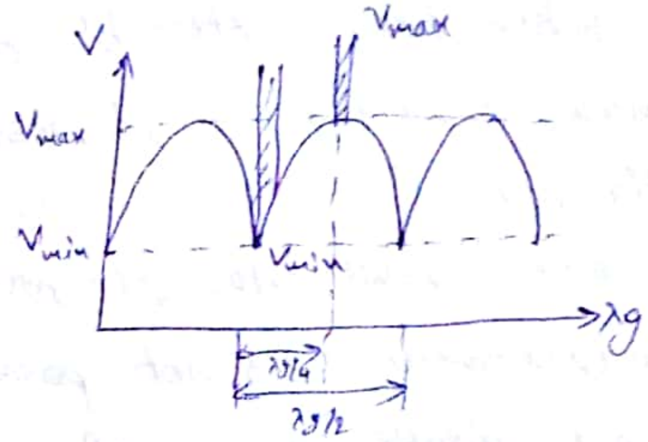
$$S = \frac{V_{max}}{V_{min}} = \frac{1+\rho}{1-\rho}$$

'S' varies 1 to ∞

ρ varies 0 to ∞

ρ = reflection coefficient.

$$\rho = \frac{P_{incident}}{P_{reflected}}$$



Low VSWR measurement (S≤10):-

Fig 2. shows the setup which is used to measure low VSWR i.e., less than 10 and the readings are directly taken from the VSWR meter.

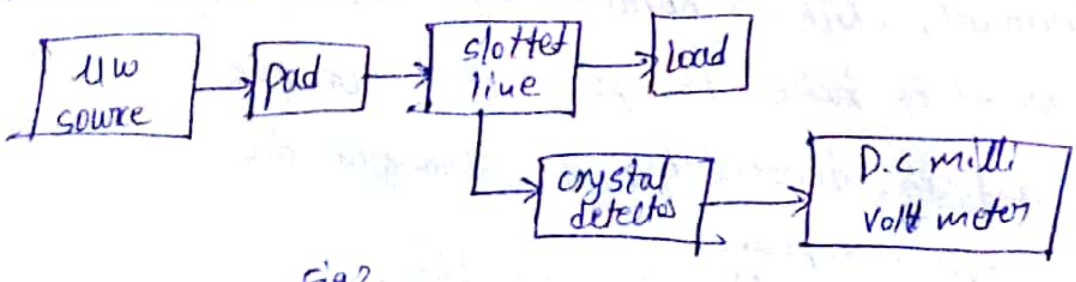


Fig 2.

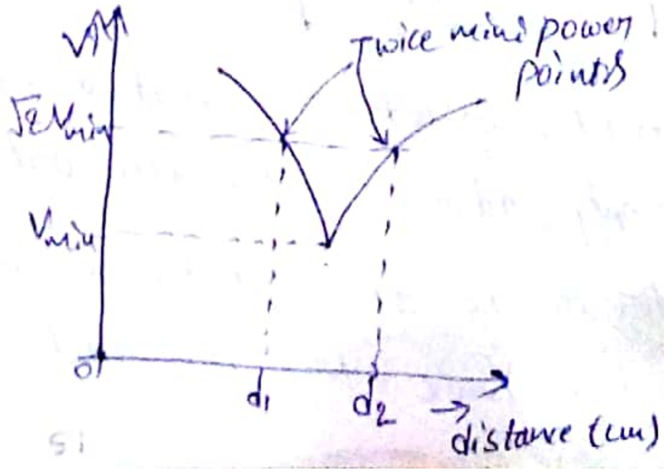
In this method of measurement, an adequate reading on D.c milli voltmeter is taken by simply adjusting the attenuator. The max reading on the meter can be obtained by moving the probe on slotted waveguide i.e., V_{max}. Now the full scale readings is noted down by adjusting the value of attenuator. Next, the mini reading on the meter i.e., V_{min} is obtained by adjusting the probe on the slotted line. Then the ratio of first reading i.e., V_{max} to the second reading i.e., V_{min} gives VSWR.

When the meter is calibrated in terms of VSWR, the max deflection on the VSWR can be obtained by adjusting the pad.

When VSWR >10, the meter will be congested and accurate measurement is not possible. So, the above method is not suitable for VSWR >10.

High VSWR measurement (S >10):

The method which is used to measure high VSWRs i.e., greater than 10 is called as double minimum method. In this method, the minimum deflection on the VSWR meter can be read by inserting the probe to the required depth. Let d_1 be the position where the power is twice the minimum, which is obtained by moving the probe. Now, the probe is moved to twice the power point on the other side of the minimum and is denoted by d_2 shown in fig 3.



$$2P_{min} \propto V_x^2$$

$$\frac{1}{2} = \frac{V_{min}^2}{V_x^2}$$

$$V_x^2 = 2V_{min}^2$$

$$V_x = \sqrt{2}V_{min}$$

Scanned by CamScanner

$$\lambda_c = 2a$$

$$\lambda_0 = \frac{c}{f}$$

where $$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$VSWR = \frac{\lambda_g}{\pi(d_2-d_1)}$$

Ex: A slotted line is used to determine the SWR value of a waveguide. Adjacent null positions are located at 13.31 cm 15.45 cm. If the separation b/w the two minimum power points is 2mm, What is the value of the SWR?

Given that, for a slotted line,

adjacent null positions are located at,

$z_1 = 13.31\text{cm}$.

$z_2 = 15.45\text{ cm}$.

separation b/w the two mini power points is $(d_2-d_1) = 2\text{mm} = 2\times10^{-1}\text{cm}$.

Then, $$VSWR = \frac{\lambda_g}{\pi(d_2-d_1)}$$

$$\lambda_g = 2[z_2-z_1] = 2[15.45-13.31]$$

$$\lambda_g = 4.28\text{ cm}.$$

$$SWR = \frac{4.28}{\pi(2\times10^{-1})} = 6.8118$$

Scanned by CamScanner

Frequency Measurement:-

The freq can be calculated from measured guide wavelength in a voltage standing wave pattern along a short circuited line by using a slotted line.

There are two techniques to measure the μw freq.

1) Mechanical 2) Electronic.

Mechanical Technique:-

1) slotted line Technique.

2) Resonant cavity "

3) spectrum anlyzer method.

Slotted line Technique:-

→ Bench set up diagram.

→ when a waveguide is created mismatched by a load a standing wave is created in the waveguide. The distance b/w the two adjacent maxima or minima is one half of the wavelength. The freq can be determined from the measured wavelength.

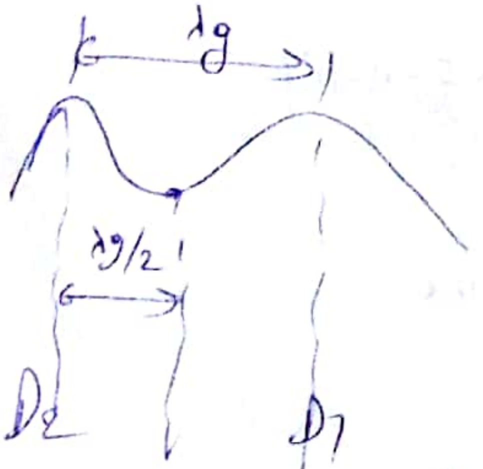


Fig1: Maxima & minima of a wave.

$$\lambda_g/2 = D_2 - D_1 = \Delta D$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

TE_{10} $\lambda_c = 2a$

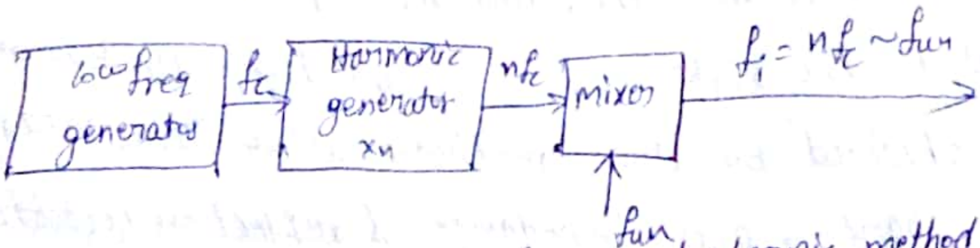
$$\lambda_0 = \frac{c}{f}$$

$$f = \frac{c}{\lambda_0}.$$

Scanned by CamScanner

Electronic Techniques:-

These techniques generally are more accurate but expensive. Frequency counters or high freq heterodyne systems can be used. Here the unknown freq is compared with harmonics of a known lower freq is compared with harmonics of a known lower freq by use of a low freq generator, a harmonic generator and a mixer as shown in fig below.



measurement of freq - electronic method

Measurement of Impedance:-

Impedance at microwave frequencies can be measured using any of the following 3 methods.

a) using Magic T
b) using slotted line
c) Using reflectometer.

b) Using slotted line:-

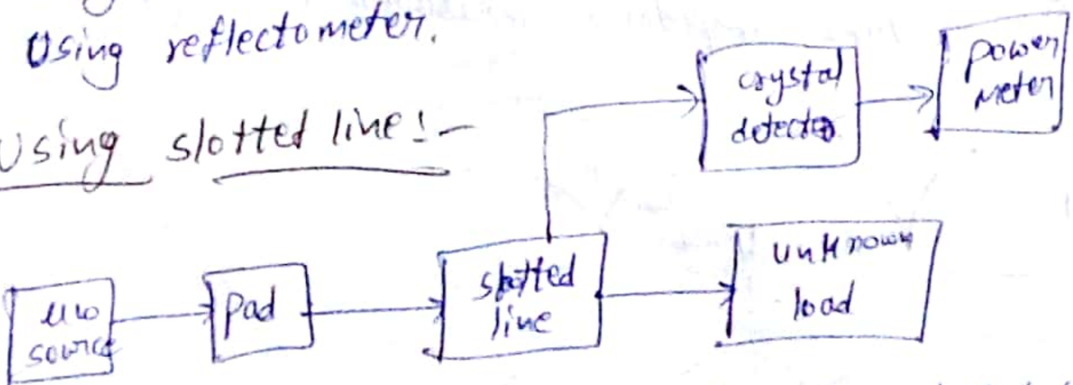


Fig: setup 1, Impedance measurement using slotted line.

Incident and reflected waves will be present propotional to the mismatch of the load under test resulting in standing waves. Using slotted waveguide and with the load Z_L in the circuit given by fig above, the position of V_{max} & V_{min} can be accurately determined. Now the load Z_L is replaced by a short circuit as shown by fig below and the shift in minimum is measured. If the minimum is shifted to the left, then the impedance is inductive and if it shifts to the right, it is capacitive. Unknown impedance can be obtained by usual methods using the data recorded and a smitch chart. Both impedance & reflection coefficient can be obtained in magnitude and phase.

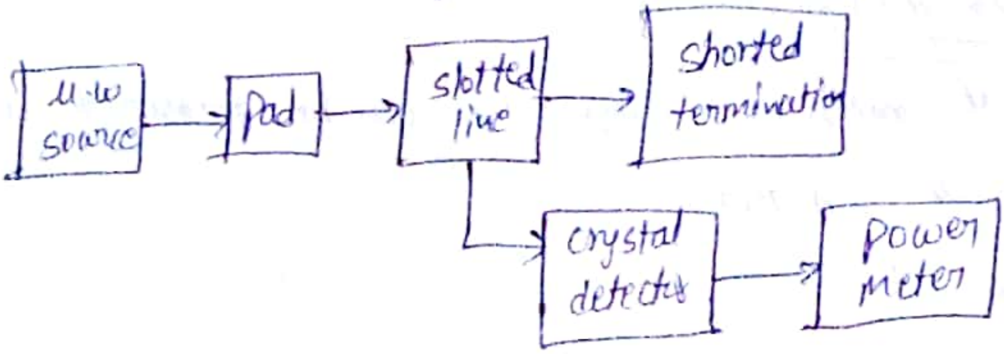


Fig2: setup2, impedance measurement using slotted line.

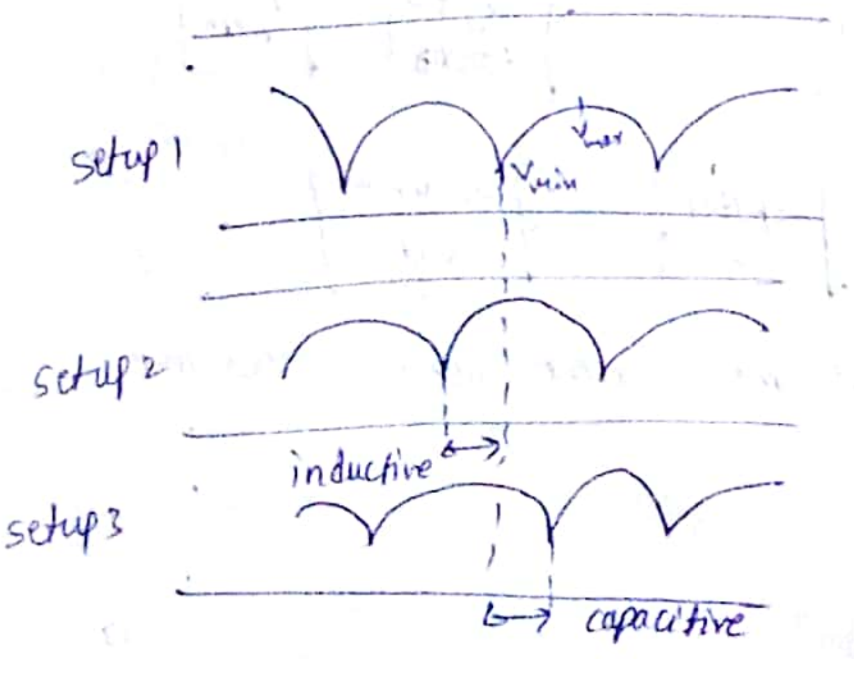
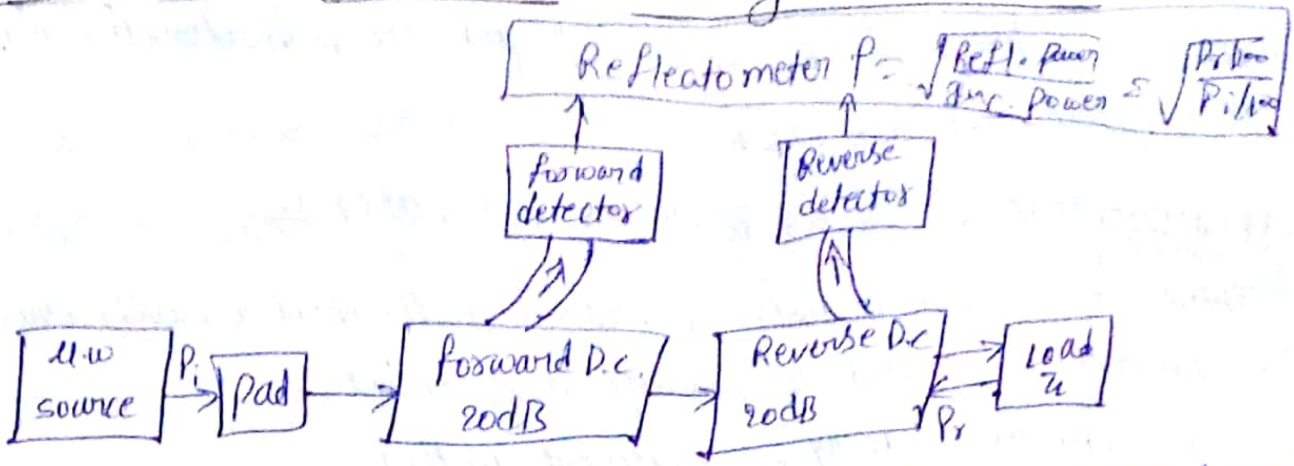


Fig3: o/p standing waves of setup1 & 2.

Measurement of Impedance using Reflectometer:-



The reflectometer indicates magnitude of impedance but not the phase angle, whereas a slotted line waveguide measurement gives both. A typical set up for reflectometer technique is shown in fig above. where two directional couplers are used to sample the incident power P_i & the reflected power P_r from load. Both the directional couplers are identical. The magnitude of the reflection coefficient ρ can be directly obtained on the reflectometer from which impedance can be calculated.

From reflectometer reading we have,

$$\rho = \sqrt{\frac{P_r}{P_i}}$$

Knowing ρ we can calculate VSWR and impedance by using the relations $S = \frac{1+\rho}{1-\rho}$ and $\frac{Z - Z_g}{Z + Z_g} = \rho$

where Z_g is the known wave impedance & Z is unknown impedance. Due to directional property of the couplers, there will be no inter ference b/w forward and reverse waves. The i/p power is kept to a low level by means of pad. The reflectometer accuracy is greatest at low VSWR.

Measurement of Q of a cavity Resonator :-

These are several methods for measuring the Q of a cavity resonator.
1. Transmission Method 2. Impedence measurement &
3. Transient decay or decrement method.

The setup for transmission method of measuring Q is shown in fig below.

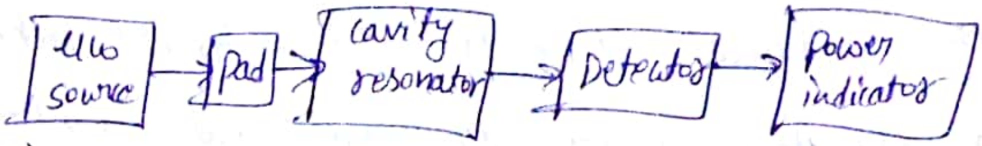
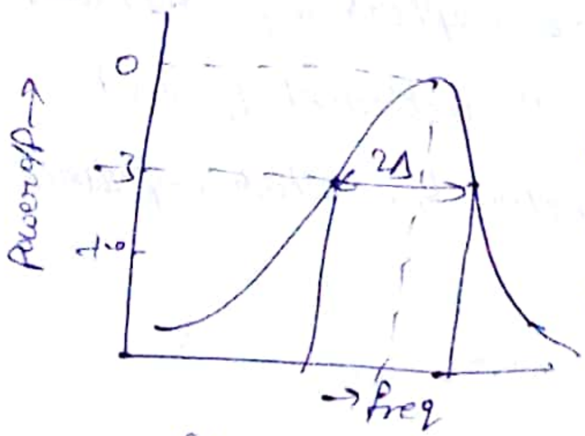


Fig: measurement of Q of a cavity resonator using transmission method.



Resonance curve

By varying the freq of μw source and keeping signal level constant, the o/p power is measured. Alternately cavity can be tuned by keeping both signal level & freq constant & o/p power measured from the resonance curve half power b.w (2Δ) can be obtained.

$$2\Delta = \pm\frac{1}{Q_L} \qquad \text{where } Q_L = \text{loaded value}$$

$$Q_L = \pm\frac{1}{2\Delta} = \pm\frac{\omega}{2(\omega-\omega_0)}$$

If the coupling b/w μw source & cavity & that b/w detector and cavity are neglected, $Q_L = Q_0$.

Scattering matrix:-

The S-matrix is a matrix which is used to represent all inputs which are applied to the ports of a given network in a matrix form. This is a square matrix which it gives all the relations of power i/p & o/p ports of a μw junction. The elements in this matrix are known as 'scattering coefficients' or scattering parameters.

It is denoted as,

$$[b] = [S][a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & & \ddots & \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

Column matrix [b] corresponding to reflected waves or o/p's.

scattering column matrix [S] of order n×n

matrix [a] corresponding to incident waves (or) i/p's

The scattering parameters are fixed properties of the linear circuits, which describe how the energy couples b/w each pair of ports or transmission lines connected to the circuit.

Formally S-parameters can be defined for linear electronic components. They are algebraically related to the impedance parameters also to the admittance parameters.

Properties:

1. scattering matrix is always a square matrix. i.e., the order of S-matrix is n×n,

2. It is a unitary matrix.

i.e., $[S][S^*] = [I]$

3. It holds symmetrical property.

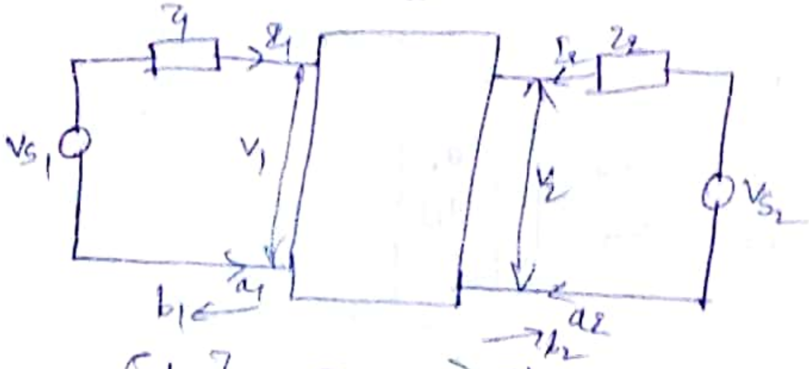
i.e., $S_{ij} = S_{ji}$

4. For, $K = 1, 2, \ldots n$ and

$j = 1, 2, \ldots n$

$\sum_{i=1}^{n} S_{iK} \cdot S_{ij}^* = 0$ for $K \neq j$.

S-matrix for 2-port device:-



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$b_1 = S_{11} a_1 + S_{12} a_2$ — ①

$b_2 = S_{21} a_1 + S_{22} a_2$ — ②

If port ② is '0' then

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0} \quad \& \quad S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0}$$

If port ① is '0' then

$$S_{12} = \frac{b_1}{a_2}\Big|_{a_1=0} \quad \& \quad S_{22} = \frac{b_2}{a_2}\Big|_{a_1=0}$$

→ For a 2-port method n/w the equations of S- parameters are ① & ②

→ Reflection coefficient (a) at port ① when port ② is terminated

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0}.$$

→ Attenuation of wave travelling from port ② to port ① $S_{12} = \frac{b_1}{a_2}\Big|_{a_1=0}$

→ Attenuation of wave " " port ① to port ② $S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0}$

→ Reflection coefficient at port ② when port ① is terminated the

$$S_{22} = \frac{b_2}{a_2}\Big|_{a_1=0}$$

comparison b/w [S], [Z], [Y] parameters:

The common properties of [S], [Z], [Y] matrices are

1) no of elements in each matrices are equal

2) For a resiprocal device the matrixes exhibits resiprocity properties

ex: $[Z_{ij}] = [Z_{ji}]$ $Z_{ij} = Z_{ji}$

3) If a Z matrix is symmetrical its equivalent S matrix [S] is also symmetrical.

4) Few properties of [S] are proved to be advantages over the Z and Y matrixes such as

i) unitary property

ii) By knowing the matrix coefficients all the measurement parameters can be calculated such a direct correspondance is not possible with [Z] and [Y] matrixes

Relation b/w ABCD & Z parameters:-

Z- parameters:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \text{--- (2)}$$

ABCD- parameters:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = A V_2 - B I_2 \quad \text{--- (3)}$$

$$I_1 = C V_2 - D I_2 \quad \text{--- (4)}$$

case I : $V_2 = 0$.

from eqn (3), $V_1 = -B I_2$

$$B = -\frac{V_1}{I_2} = -\frac{(z_{11} I_1 + z_{12} I_2)}{I_2} \quad [\because \text{from eq}^n \text{ (1)}]$$

from eqn (2)

$$0 = z_{21} I_1 + z_{22} I_2$$

$$z_{22} I_2 = -z_{21} I_1$$

$$\frac{z_{22}}{z_{21}} = \frac{-I_1}{I_2} \quad \text{--- (5)}$$

$$B = -\frac{z_{11} I_1}{I_2} - \frac{z_{12} \cancel{I_2}}{\cancel{I_2}} \quad [\cancel{\text{from eq}^n \text{ (5)}}]$$

$$= z_{11} \cdot \frac{z_{22}}{z_{21}} - z_{12} \quad [\because \text{from eq}^n \text{ (5)}]$$

$$B = \frac{z_{11} \cdot z_{22} - z_{12} \cdot z_{21}}{z_{21}}$$

From eqn (4)

$$I_1 = 0 - D I_2$$

$$D = -\frac{I_1}{I_2} = \frac{z_{22}}{z_{21}} \quad [\because \text{from eq}^n \text{ (5)}]$$

case II: $I_2 = 0$.

$V_1 = A V_2$ [from eqn ③]

$A = \frac{V_1}{V_2}$ —⑥

$= \frac{z_{11} \cancel{I_1}}{z_{21} \cancel{I_1}} = \frac{z_{11}}{z_{21}}$.

$V_1 = z_{11} I_1$. [from eqn ①]

$V_2 = z_{21} I_1$ [from eqn ②]

$I_1 = \frac{V_1}{z_{11}} \Rightarrow I_1 = \frac{V_2}{z_{21}}$ —⑦

$I_1 = C V_2$

$C = \frac{I_1}{V_2}$

$C = \frac{\frac{\cancel{V_2}}{z_{21}}}{\cancel{V_2}} = \frac{1}{z_{21}}$ [from eqn ⑦]

Relation b/w ABCD & Y parameters:-

$V_1 = A V_2 - B I_2$ —①

$I_1 = C V_2 - D I_2$ —②

$I_1 = Y_{11} V_1 + Y_{12} V_2$ —③

$I_2 = Y_{21} V_1 + Y_{22} V_2$ —④

case I: $V_2 = 0$

$V_1 = -B I_2$

$B = -\frac{V_1}{I_2}$ —⑤

$I_1 = -D I_2$

$$D = -\frac{I_1}{I_2} \quad \text{—⑥}$$

$$I_1 = Y_{11} V_1$$

$$Y_{11} = \frac{I_1}{V_1} \quad \text{—⑦}$$

$$I_2 = Y_{21} V_1$$

$$Y_{21} = \frac{I_2}{V_1} \quad \text{—⑧}$$

$$B = \frac{-V_1}{Y_{21} \cdot V_1} = \frac{-1}{Y_{21}} \quad \text{—⑨}$$

case II: $V_1 = 0.$

$$0 = AV_2 - BI_2$$

$$AV_2 = BI_2$$

$$A = \frac{BI_2}{V_2} = \frac{-1}{Y_{21}} \cdot \frac{I_2}{V_2} \quad \text{—⑩}$$

from eqn ③

$$I_1 = Y_{12} V_2$$

$$Y_{12} = \frac{I_1}{V_2} \quad \text{—⑪}$$

From eqn ④

$$I_2 = Y_{22} V_2$$

$$Y_{22} = \frac{I_2}{V_2} \quad \text{—⑫}$$

$$A = \frac{-1}{Y_{21}} \cdot Y_{22} = \frac{-Y_{22}}{Y_{21}}$$

$$Y_{12} = \frac{CV_2 - DI_2}{V_2} \quad \text{[from eqn ⑪]}$$

$$Y_{12} = C - D \cdot \frac{I_2}{V_2}$$

$$C = Y_{12} + D \cdot \frac{I_2}{V_2}$$

$$= Y_{12} - \frac{Y_{11}}{Y_{21}} \cdot \frac{I_2}{V_2}$$

$$= Y_{12} - \frac{Y_{11}}{Y_{21}} \cdot Y_{22} \quad \left[\text{from eq}^n \text{ (18)} \right]$$

$$C = \frac{Y_{12} \cdot Y_{21} - Y_{11} Y_{22}}{Y_{21}}$$