TORSION OF CIRCULAR SHAFTS

* Theory of pure torsion.
* Derivation of Torsion equation.

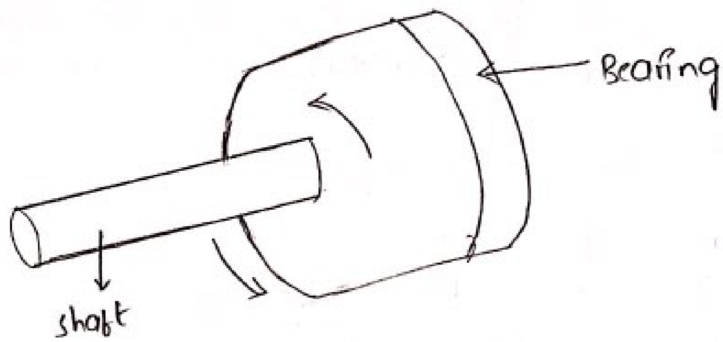
$$\frac{T}{J} = \frac{q}{r} = \frac{C\theta}{L}$$

* Assumptions made in the throry of pure tension.
* Torsion Moment of Resistance.
* polar section modulus.
* power transmitted by shafts.
* combined bending and torsion and end thrust.
* Design of shafts according to theories of failures.

→ Springs :

* Introduction.
* Types of springs.
* Deflection of closed and open coil helical springs under axial pull and axial couple.
* springs in series and parallel.
* carriage or leay springs.

→ Torque :-



A force that tends to cause rotation is nothing but torque. (or) Moment of force.

→ pure torsion:- If the shaft is subjected to two opposite turning moments, it is said to be in pure torsion.

$$\boxed{T = FR}$$

f ⟶ axial force

R ⟶ Radius.

Because of two unequal torques torsion will develops.

→ Shaft :- It is cylindrical in to section, solid or hallow.

They are made of mild steel, alloy steel, copper alloyes.

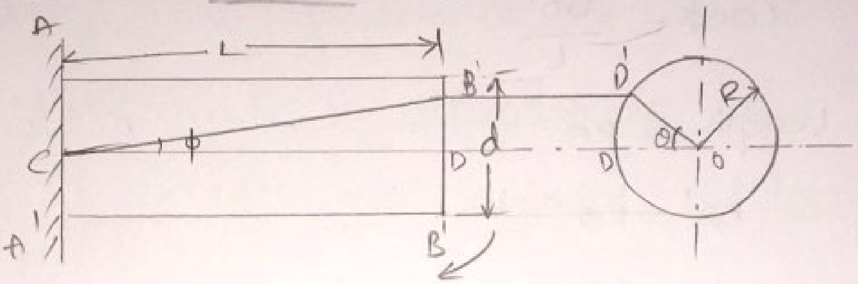
shafts may be subjected to.

* Torsion load
* Bending load
* Axial load

* Combination of above three load

The shafts are designed on the basis of strength and rigidity.

→ Derivation of shear stress produced in a circular shaft subjected to torsion :-



shaft is fixed at one end and torque being applied on the other end.

If a line CD is drawn on the shaft it will distorted to CD' on the application of torque. Thus c/s^n will be twisted through angle 'θ' and surface by angle 'ϕ'.

τ – shear stress induced at the surface of shaft due to torque 'T'.

L – Length of shaft.

R – Radius of shaft.

T – Torque applied at the end BB.

C – Modulus of Rigidity of the material of the shaft.

ϕ – Angle ∠CDD'

θ – ∠DOD' is also called Angle of twist.

→ Distortion at the outer surface due to Torque

$\gamma = DD'$.

→ shear strain at outer surface.

$$\phi = \frac{DD'}{L}$$

$$\tan\phi = \frac{DD'}{L}$$

→ ARC Length $= R\theta$

$$DD' = R\theta$$

$$\phi = \frac{R\theta}{L}$$

$$C = \frac{\tau}{\phi} = \frac{\tau L}{R\theta}$$

$$C = \frac{\tau L}{R\theta} \Rightarrow \boxed{\frac{C\theta}{L} = \frac{\tau}{R}}$$

C, θ, L are constants.

Hence shear stress produced is proportional to R.

$$\therefore \frac{\tau}{R} = \frac{C\theta}{L}$$

If 'q' is the shear stress induced at a radius 'r' from the centre of the shaft.

$$\frac{\tau}{R} = \frac{q}{r}$$

$$\therefore \boxed{\frac{\tau}{R} = \frac{q}{r} = \frac{C\theta}{L}}$$

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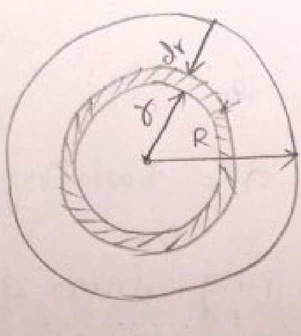
It is clear that shear stress at any point in the shaft is prpportional to the distance of the point from the axis of the shaft.

Hence shear stress is maximum at outer surface and shear stress is zero at the axis of the shaft.

⇒ Assumptions made in derivation of shear stress produced in circular shaft subjected to torsion:-

* The material of the shaft is uniform throughout.
* The twist along the shaft is uniform.
* The shaft is uniform circular section throughout.
* c/s of the shaft, which are plane before twist remains plane after twist.
* All radii which are straight before twist remains straight after twist.

→ Maximum torque transmitted by a circular shaft:-



Maximum torque transmitted by a circular solid shaft is obtained from the maximum shear stress induced at the outer surface of the solid shaft.

consider a shaft subjected to torque 'T'.

τ – Max. shear stress induced at the outer surface.

R – Radius of the circular shaft.

r – Radius of the elementary circular ring.

q – shear stress at the radius 'r'

dr – thickness of the elementary circular ring.

$$\frac{\tau}{R} = \frac{q}{r}$$

$$q = \frac{\tau}{R} \times r$$

Turning force on the elementary ring = shear stress acting on the ring x Area of the ring.

$$\Rightarrow \quad f = q \times A$$

$$= \frac{\tau}{R} \times r \times 2\pi \times r . dr$$

$$= \frac{\tau}{R} \times 2\pi r^2 . dr$$

Then turning moment dT = twisting force x distance of the ring from axis.

$$dT = f \times r$$

$$= \frac{\tau}{R} \times 2\pi r^2 . dr \times r$$

$$dT = \frac{\tau}{R} \times 2\pi r^3 . dr$$

Total torque 'T' is obtained by integrating above equation.

$$\int_0^R dT = \int_0^R \frac{\tau}{R} \times 2\pi r^3 . dr$$

$$= \frac{\tau}{R} \int_0^R 2\pi r^3 . dr$$

$$= \frac{\tau}{R} 2\pi \int_0^R r^3 . dr$$

$$= \frac{\tau}{R} . 2\pi \left[\frac{r^4}{4}\right]_0^R$$

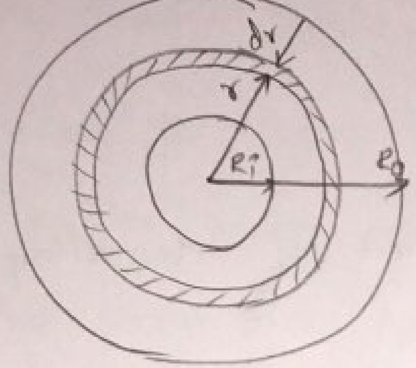
$$= \frac{\tau}{R} 2\pi \left[\frac{R^4}{4}\right]$$

$$= \frac{R^3 \, 2\pi \, \tau}{4} = \frac{\tau \pi R^3}{2}$$

$$\therefore \quad R = \frac{D}{2}$$

$$T = \frac{\tau \pi D^3}{16}.$$

→ Maximum torque transmitted by hollow circular shaft:-



Maximum torque transmitted by a hallow circular.

$$\frac{\tau}{R_0} = \frac{q}{r}$$

$$q = \frac{\tau r}{R_0}$$

Turning force on the elementary ring = shear stress acting on the ring x Area of the ring.

$$\Rightarrow \quad F = q \times A$$

$$= \frac{\tau r}{R_0} \times A$$

$$= \frac{\tau r}{R_0} \times 2\pi r \cdot dr$$

$$= \frac{\tau r}{R_0} \times 2\pi r^2 \cdot dr$$

Twisting moment 'dT' = twisting force x distance of the ring from axis

$$dT = F \times r$$

$$dT = \frac{\tau}{R_0} \times 2\pi r^3 \cdot dr$$

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Total (turning force) torque 'T' is obtained by integrating above eqn.

$$\int_{R_i}^{R_o} dT = \int_{R_i}^{R_o} \frac{\tau}{R_o} \times 2\pi r^3 . dr$$

$$T = \frac{\tau}{R_o} \int_{R_i}^{R_o} 2\pi r^3 . dr$$

$$= \frac{2\pi\tau}{R_o} \int_{R_i}^{R_o} r^3 . dr$$

$$= \frac{2\pi\tau}{R_o} \left[\frac{r^4}{4}\right]_{R_i}^{R_o}$$

$$= \frac{2\pi\tau}{R_o} \left[\frac{R_o^4}{4} - \frac{R_i^4}{4}\right] \qquad \left\{ \because R_o = \frac{D_o}{2},\ R_i = \frac{D_i}{2} \right\}$$

$$= \frac{2\pi\tau}{D_o/2} \left[\frac{(D_o/2)^4}{4} - \frac{(D_i/2)^4}{4}\right]$$

$$= \frac{4\pi\tau}{D_o} \left[\frac{D_o^4}{64} - \frac{D_i^4}{64}\right]$$

$$= \frac{\cancel{4}\pi\tau}{\cancel{64}D_o} \left[D_o^4 - D_i^4\right]$$

$$\therefore \boxed{T = \frac{\pi\tau}{16 D_o}\left[D_o^4 - D_i^4\right]}$$

problems :-

→ A solid shaft of 150mm ϕ is used to transmit torque. Find the maximum torque transmitted by the shaft if the max. shear stress induced to the shaft is 45 N/mm^2.

sol:- Given data.

$d = 150mm$

$\tau = 45 N/mm^2$

$$T = \frac{\tau \pi R^3}{2}$$

$$= \frac{45 \times \pi \times 75^3}{2}$$

$T = 29.82 \times 10^6$ N-mm.

→ The shearing stress of solid shaft is not to exceed 40 N/mm^2 when the torque transmitted 20,000 N-m. Determine the min dia of the shaft ?

sol:- Given data

$\tau = 40 N/mm^2$

$T = 20,000$ N-m

$D = ?$

$$T = \frac{\pi D^3 \tau}{16}$$

$$20{,}000 = \frac{\pi \times 40 \times D^3}{16}$$

$$20 \times 10^3 = 7.85\, D^3$$

$$\boxed{D = 13.65m}$$

→ POWER TRANSMITTED BY SHAFTS:

Let N – rpm of shafts

T = Mean torque transmitted in N–m

ω = Angular speed.

Then power $P = \omega \times T$

$$P = \frac{2\pi N T}{60} \text{ watts} \qquad \left[\because \omega = \frac{2\pi N}{60}\right]$$

problem :-

→ In a hallow circular shaft of outer and inner dia of 20cm and 10cm respectively, the shear stress is not to exceed $40\ N/mm^2$. find the Max. torque which the shaft can safely transmitt.

sol:- Given data

$D_o = 2000mm$

$D_i = 1000\ mm$

$\tau = 40\ N/mm^2$

$$T = \frac{\pi \tau}{16 D_o}\left[D_o^4 - D_i^4\right]$$

$$= \frac{\pi \times 40}{16 \times 200}\left[200^4 - 100^4\right]$$

$$T = 58.90 \times 10^6 \text{ N-mm.}$$

→ Two shafts of same material of same lengths are subjected to same torque, if the first shaft is of a solid secn and the second shaft outside dia and the max. shear stress developed in each shaft is same. Compare the weights of the shaft.

sol:-

$$D_o = D_o$$

$$D_i = \tfrac{2}{3} D_o$$

$$T_H = \frac{\pi \tau}{16 D_o}\left[D_o^4 - D_i^4\right]$$

$$= \frac{\pi \tau}{16 D_o}\left[D_o^4 - \left(\frac{2}{3} D_o\right)^4\right]$$

$$= \frac{\pi \tau}{16 D_o}\left[D_o^4 - \frac{16}{81} D_o^4\right]$$

$$= \frac{\pi \tau}{16 D_o}\left[D_o^4 - \left(1 - \frac{16}{81}\right)\right]$$

$$= \frac{\pi \tau}{16 D_o}\left[D_o^4 \times 0.802\right]$$

$$T_H = \frac{\pi \tau}{16 D_0}\left[0.802 D_0^4\right]$$

$$T_S = T_H$$

$$\frac{\cancel{\tau} \cancel{\pi} D^3}{16} = \frac{\cancel{\tau}\cancel{\pi}}{16 D_0}\left[0.802 D_0^4\right]$$

$$D^3 = \frac{0.802 D_0^{\cancel{4}}}{\cancel{D_0}}$$

$$D^3 = 0.802 D_0^3$$

$$\boxed{D = 0.929 D_0}$$

weight of solid shaft = wt density x vol of shaft

$= w \times A \times L$

$= w \times \frac{\pi}{4} D^2 \times L$

wt of hallow shaft = wt density x vol. of shaft

$= w \times A \times L$

$= w \times \frac{\pi}{4}\left[D_0^2 - D_i^2\right] \times L$

$$W_S = W_H$$

$$\cancel{w} \times \frac{\cancel{\pi}}{\cancel{4}} D^2 \times \cancel{L} = \cancel{w} \times \frac{\cancel{\pi}}{\cancel{4}}\left[D_0^2 - D_i^2\right] \times \cancel{L}$$

$$D^2 = D_0^2 - D_i^2$$

$$\frac{W_S}{W_H} = \frac{D^2}{D_0^2 - D_i^2}$$

$$= \frac{(0.929 D_o)^2}{D_o^2 - D_i^2}$$

$$= \frac{0.86 D_o^2}{D_o^2 - (\frac{2}{3} D_o)^2}$$

$$= \frac{0.86 D_o^2}{(1 - 0.44) D_o^2}$$

$$= \frac{0.86}{0.56}$$

$$= 1.53$$

→ A solid circular shaft and a hollow circular shaft whose inside dia is $3/4^{th}$ of outside dia, are of some materia, of equal lengths and are required to transmit a given torque. compare the wts of there two shafts, if max. shear stress developed in the two shafts are equal.

sol:-

$$D_o = D_o$$

$$D_i = \frac{3}{4} D_o$$

$$T_H = \frac{\pi \tau}{16 D_o} [D_o^4 - D_i^4]$$

$$= \frac{\pi \tau}{16 D_o} \left[D_o^4 - \left(\frac{3}{4} D_o\right)^4\right]$$

$$= \frac{\pi \tau}{16 D_o} \left[D_o^4 - (1 - (3/4)^4)\right]$$

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$$= \frac{\pi \tau}{16 D_0} \left[0.68 D_0^4 \right]$$

$$T_S = T_H$$

$$\frac{\cancel{\tau} \cancel{\pi} D^3}{16} = \frac{\cancel{\tau} \cancel{\pi}}{16 D_0} \left[0.68 D_0^4 \right]$$

$$D^3 = 0.68 D_0^3$$

$$D = 0.87 D_0$$

wt of solid shaft = wt density $\times$ vol. of shaft.

$$= w \times A \times L$$

$$= w \times \frac{\pi D^2}{4} \times L$$

wt of hallow shaft = wt density $\times$ vol of shaft

$$= w \times A \times L$$

$$= w \times \frac{\pi}{4} (D_0^2 - D_i^2) \times L$$

$$\frac{W_S}{W_H} = \frac{w \times \frac{\pi D^2}{4} \times L}{w \times \frac{\pi}{4} (D_0^2 - D_i^2) \times L}$$

$$= \frac{D^2}{D_0^2 - D_i^2}$$

$$= \frac{(0.87 D_0)^2}{D_0^2 - D_i^2}$$

$$= \frac{0.75 D_0^2}{D_0^2 - (3/4 D_0)^2}$$

$$= \frac{0.75 D_o^2}{0.43 D_o^2}$$

$$\boxed{\frac{W_S}{W_H} = 1.74}$$

→ polar Moment of Interia :-

It is defined as the moment of interea about an axis perpendicular to the plane and passing through the C.G of the area.

It is denoted by 'J' and units are mm^4.

→ Derivation:-

The moment 'dT' on the circular ring is given by

$$dT = \frac{\tau}{R} \times 2\pi r^3 . dr$$

$$= \frac{\tau}{R} r^2 . dA$$

$$\text{Total torque} = \int_0^R dT$$

$$T = \frac{\tau}{R} \int_0^R r^2 . dA$$

But $r^2.dA$ = moment of interia of the elementary ring about an axis perpendicular to the plane and passing through centre of circule.

$\int_0^R r^2 . dA$ = MOI of circle about an axis ⊥er to the

plane of circle and passing through the centre of axis.
circle.

polar MOI of solid circle 'J' = $\frac{\pi}{32} d^4$.

proof:-

$$= \int_0^R r^2 . dA$$

$$= \int_0^R r^2 . 2\pi r . dr$$

$$= 2\pi \left[\frac{r^4}{4}\right]_0^R$$

$$= 2\pi \left(\frac{R^4}{4}\right)$$

$$= \frac{\pi R^4}{2}$$

$$\boxed{J = \frac{\pi d^4}{32}}$$

→ For Hollow shaft:

$$J = \frac{\pi}{32}(d_0^4 - d_I^4)$$

proof:-

$$= \int_{R_i}^{R_0} 2\pi r^3 . dr$$

$$= 2\pi \left[\frac{r^4}{4}\right]_{R_i}^{R_0}$$

$$= \frac{\cancel{2}\pi}{\cancel{2}4}\left[R_0^4 - R_i^4\right]$$

$$= \frac{\pi}{2}\left[\left(\frac{d_o}{2}\right)^4 - \left(\frac{d_i}{2}\right)^4\right]$$

$$= \frac{\pi}{2}\left[\frac{d_o^4}{16} - \frac{d_i^4}{16}\right]$$

$$= \frac{\pi}{16 \times 2}\left[d_o^4 - d_i^4\right]$$

$$\boxed{J = \frac{\pi}{32}\left(d_o^4 - d_i^4\right]}$$

→ TORSIONAL RIGIDITY:-

It is defined as the product of modulus of rigidity 'c' and polar moment of ~~interia~~ inertia of the shaft 'J'

$$T.R = CJ$$

Torsional rigidity is also defined as the torque required to produce a twist of one radian per unit length of the shaft.

→ The strength of the shaft means the maximum torque or maximum power the shaft can transmit.

Let a twisting moment 'T' produces a twist of 'θ' radians in a shaft of length 'L', then

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\boxed{CJ = \frac{TL}{\theta}}$$

$$\boxed{C.J = T} \qquad \left\{ \begin{array}{l} \because L = 1m \\ \theta = 1r \end{array} \right\}$$

Problem :-

1) A hallow circular shaft 20mm thick transmits 300kw at 200 rpm. Determine the external dia of shaft is shear stress due to torsion is not to exceed 0.00186. Take 'c' = 8×10^4 N(mm^2) [1 watt = 1Nm/s]

Sol:- Given data

$t = 20mm$

$P = 300$ kw @ 200rpm

$C = 8\times10^4 \; N/mm^2$

$\phi = 0.0086$

$$P = \frac{2\pi n T}{60} = \frac{2\times\pi\times200\times300T}{60}$$

$$300 = \frac{2\times\pi\times200\times T}{60}$$

$$300 = 20.94T$$

$$P = \frac{2\pi n T}{60}$$

$$300 = \frac{2 \times \pi \times 200 \times T}{60}$$

$$300 = 20.94T$$

$$T = 14.32 \text{ kN-m}$$

$$T = 14326.64 \text{ N-m}$$

Max torque produce by hallow shaft

$$T = \frac{\tau \pi}{16 D_o}\left[D_o^4 - D_i^4\right]$$

$$14326.64 = \frac{\tau \times \pi}{16 D_o}\left[D_o^4 - D_i^4\right]$$

$$C = \frac{\tau}{\phi}$$

$$\tau = C\phi$$

$$= 8 \times 10^4 \times 0.00086$$

$$\tau = 68.8 \text{ N/mm}^2$$

$$D_o = D_i + t + t$$

$$= D_i + 2t$$

$$= D_i + 2(20)$$

$$\boxed{D_i = D_o - 40}$$

$$\frac{\pi}{16} \times 68.8\left[\frac{D_o^4 - D_i^4}{D_o}\right] = 14326.64$$

$$D_o^4 - D_i^4 = \frac{14326.64 \times 16 D_o}{\pi \times 68.8}$$

$$(D_o^2)^2 - (D_i^2)^2 = 1060.53 D_o$$

$$(D_0^2 - D_i^2)(D_0^2 + D_i^2) = 1060.53 D_0$$

$$(D_0^2 - (D_0 - 40)^2)(D_0^2 + (D_0 - 40)^2) = 1060.53 D_0$$

$$(D_0^2 - D_0^2 - 1600 + 80D_0)(D_0^2 + D_0^2 + 1600 - 80D_0) = 1060.53 D_0$$

$$80(D_0 - 20)\, 2\,(D_0^2 + 800 - 40D_0) = 1060.53 D_0$$

$$160(D_0^2 + 800 - 40D_0)(D_0 - 20) = 1060.53 D_0$$

$$(D_0^2 + 800 - 40D_0)(D_0 - 20) = \frac{1060.53 D_0}{160}$$

$$(D_0^2 + 800 - 40D_0)(D_0 - 20) = 6628.31 D_0$$

$$D_0^3 - 20D_0^2 + 800D_0 - 16000 - 40D_0^2 + 800D_0 = 6628.31 D_0$$

$$D_0^3 - 60D_0^2 - 16000 + 1600D_0 - 6628.31 D_0 = 0$$

$$D_0^3 - 60D_0^2 - 5027.9D_0 - 16000 = 0$$

$$\boxed{D_0 = 107.94\,mm}$$

2Q) A hallow shaft of external diameter 120mm transmit 300KW power at 200 rpm. determine max internal dia of the max stress in the shaft is not to exceed $60N/mm^2$.

sol:- Given data.

$D_0 = 120 mm$

$P = 300 kW$ @ 200 rpm

$\tau = 60 \ N/mm^2$

$$P = \frac{2\pi n T}{60}$$

$$T = \frac{300 \times 10^3 \times 60}{2 \times \pi \times 200}$$

$$\boxed{T = 14.32 \times 10^3 \ N-m}$$

$$T = \frac{\pi \tau}{16 D_0}(D_0^4 - D_i^4)$$

$$14.32 \times 10^3 = \frac{\pi \times 60}{16 \times 120}(120^4 - D_i^4)$$

$$\frac{14.32 \times 10^3 \times 16 \times 120}{\pi \times 60} = 207.36 \times 10^6 - D_i^4$$

$$145.90 \times 10^6 = 207.36 \times 10^6 - D_i^4$$

$$\boxed{D_i = 88.54 mm}$$

3Q) Determine the diameter of the solid shaft which will transmit 90kw at 160 rpm. Also determine the length of the shaft if the twist must not to exceed 1° over the entire length. The max shear stress is limited to $60 N/mm^2$. Take $C = 8 \times 10^4 N/mm^2$ and also find polar moment of Inertia?

Sol:- Given data

$P = 90 kw$

$n = 160 rpm$

$\theta = t = 1^{\circ} = \frac{\pi}{180} rad$

$C = 8 \times 10^4 N/mm^2$

$\tau = 60 N/mm^2$

$$P = \frac{2\pi n T}{60}$$

$$T = \frac{P \times 10^6 \times 60}{2 \times \pi \times 160} = \frac{90 \times 10^6 \times 60}{2 \times \pi \times 160}$$

$$\boxed{T = 5.37 \times 10^6}$$

$$T = \frac{\tau \pi D^3}{16}$$

$$5.37 \times 10^6 = \frac{60 \times \pi \times D^3}{16}$$

$$D^3 = \frac{5.37 \times 10^6 \times 16}{60 \times \pi}$$

$$\boxed{D = 76.95 \text{ mm}}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi \times 76.95^4}{32}$$

$$\boxed{J = 3.44 \times 10^6 \text{ mm}^4}$$

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$L = \frac{C\theta \times J}{T}$$

$$= \frac{8 \times 10^4 \times \frac{\pi}{180} \times 3.44 \times 10^6}{5.37 \times 10^6}$$

$$\boxed{L = 895.04 \text{ mm}}$$

→ principal plane :-

The plane in which shear stress is zero is called principal plane.

The stresses which are acting on principal plane is known as principal stresses.

The position of principal stresses acting on the plane is obtained by

$$\tan 2\theta = \frac{2\tau}{\sigma}$$

→ Combined bending and torsion:-

When a shaft is transmitting torque 'or' power, it is subjected to shear stresses, At the same time, the shaft is also subjected to bending moment due to self weight, gravity and interial loads. Due to bending moment, the bending stresses are also setup in the shafts. Hence each particle in the shaft is subjected to shear stress and bending stresses. for the design purpose, it is necessary to find principal stresses, max shear stress and strain energy.

consider any point on the c/s of the shaft

Let
$T \to$ Torque @ section.

$D \to$ Dia. of shaft

$M \to$ B.M @ c/s

The torque 'T' will produce shear stress at the point, whereas the BM will produce bending stress.

Let
'q' → shear stresses @ point produced by torque.

$$\boxed{D = 76.95\ mm}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi \times 76.95^4}{32}$$

$$\boxed{J = 3.44 \times 10^6\ mm^4}$$

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$L = \frac{C\theta \times J}{T}$$

$$= \frac{8 \times 10^4 \times \frac{\pi}{180} \times 3.44 \times 10^6}{5.37 \times 10^6}$$

$$\boxed{L = 895.04\ mm}$$

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The torque 'T' will produce shear stress at the point, whereas the BM will produce bending stress.

Let

'q' → shear stresses @ point produced by torque.

$\sigma \rightarrow$ B.S @ point produced by BM.

$\rightarrow$ shear stress @ any point due to torque.

$$\Rightarrow \frac{q}{r} = \frac{T}{J}$$

$$\Rightarrow q = \frac{Tr}{J}$$

$\rightarrow$ The B.S @ any point due to BM

$$\Rightarrow \frac{M}{I} = \frac{\sigma}{y}$$

$$\Rightarrow \sigma = \frac{My}{I}$$

$\rightarrow$ The B.S and S.S is max @ outer surface of the shaft

$$r = R = D/2$$

and $y = D/2$

$$\sigma = \frac{M \times D}{2I}$$

It is a solid shaft

$$= \frac{M\cancel{D}}{2 \times \frac{\pi}{64} D^{\cancel{4}}}$$

$$= \frac{64M}{2\pi D^3} = \boxed{\frac{32M}{\pi D^3} = \sigma_b}$$

$$\frac{q}{r} = \frac{T}{J}$$

$$q = \frac{Tr}{J}$$

$$= \frac{\frac{\tau \pi D^3}{16} \times \frac{D}{2}}{J}$$

$$= \frac{\tau \pi D^4}{32 J}$$

$$q = \frac{Td}{2J}$$

$$q = \frac{Td}{2 \times \frac{\pi d^4}{32}}$$

$$\boxed{q = \frac{16T}{\pi d^3}}$$

$$\tan 2\theta = \frac{2\tau}{\sigma}$$

$$= \frac{\frac{2 \times 16 T}{\pi d^3}}{\frac{32 M}{\pi d^3}} = \frac{T}{M}$$

$$\boxed{\tan 2\theta = \frac{T}{M}}$$

→ Major principal stresse-

$$MPS = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{32M}{2 \times \pi D^3} + \sqrt{\left(\frac{32 M}{2 \times \pi D^3}\right)^2 + \left(\frac{16 T}{\pi D^3}\right)^2}$$

$$= \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + \tau^2}\right)$$

→ Minor principal stress :-

$$= \frac{16}{\pi D^3}\left(M - \sqrt{M^2+T^2}\right)$$

→ Max. shear stress $= \dfrac{\text{Major ps} - \text{Minor ps}}{2}$

$$= \frac{16}{\pi D^3}\left[\sqrt{M^2+T^2}\right]$$

* for hallow shaft

→ Major p.s $= \dfrac{16 D_o}{\pi (D_o^4 - D_i^4)}\left[M + \sqrt{M^2+T^2}\right]$

→ Minor p.s $= \dfrac{16 D_o}{\pi (D_o^4 - D_i^4)}\left[M - \sqrt{M^2+T^2}\right]$

→ Max. shear stress $= \dfrac{16 D_o}{\pi [D_o^4 - D_i^4]}\left[\sqrt{M^2+T^2}\right]$

→ spring:-

The springs are elastic bodies which absorbs energy due to "resilience".

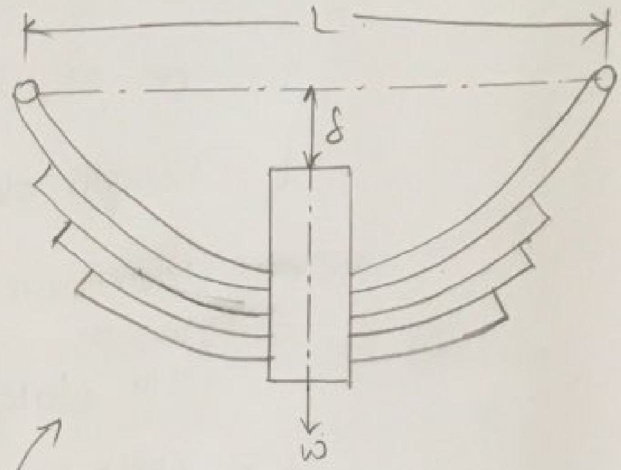
* Resilience:- The strain energy stored in a body within elastic limit.

The absorbed energy may be released as and when required. A spring which is capable of observing the greatest amount of energy for the given stress, without getting permenetly distorted is knowns as Best spring.

→ Types of springs:-

springs are two types

* Laminated or Leas springs.
* Helical springs.



→ Laminated or Leas springs:-

The springs are used to observe the shocks in railway wagons, coaches, and road vehicles [Lorry, tractors].

Laminated spring consists of number of parallel strips of a metal having different lengths and same width, placed one over other. Intially all plates bend into the same radius and are free to slide one over other.

Which is having same central deflection of 'δ'. The spring rest on the axis of vehicle and its top plate pinned at the ends to the chass of the vehicle. When the spring is loaded to the design load 'ω'. All the plates become flat & central deflection 'δ' will disappears.

Let b = width of each plate

η = no. of plates.

l = Length of span

σ = Maximum bending stress developed in the plates.

t = thickness of each plate.

W = point load acting at the centre of the spring.

δ = original deflection of the spring.

→ Expression for Maximum Bending stress developed in the plate 'σ' :-

The load 'w' is acting at the centre of the lower most plate, will be shade equally on the two ends on the top.

B.M @ centre $= \frac{WL}{4}$ [Load @ one end $\times \frac{1}{2}$]

MOI of each plate $= \frac{Bt^3}{12}$

By the bending equation.

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{\sigma I}{y}$$

$$= \frac{\sigma \times Bt^3}{12 \times \frac{t}{2}}$$

$$= \frac{\sigma B t^2}{6}$$

Total resisting moment by 'n' plates.

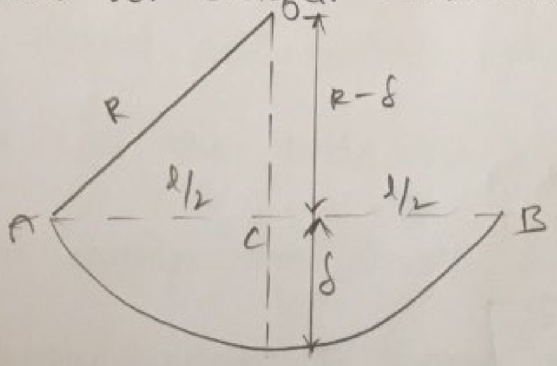
$$= \frac{n \sigma B t^2}{6}$$

As per B.M due to loads is equals to total resisting moment.

$$\frac{WL}{4} = \frac{n\sigma B t^2}{6}$$

$$\Rightarrow \quad \sigma = \frac{6WL}{4Bt^2 n} = \boxed{\frac{3WL}{2Bt^2 n} = \sigma}$$

→ Expression for central deflection 'δ':



R — Radius of the plate to which they are bent

from $\angle ACO = OA^2 =$

$$AO^2 = AC^2 + OC^2$$

$$R^2 = (l/2)^2 + (R-\delta)^2$$

'δ' is small quantity, neglect the 'δ'.

$$R^2 = \frac{l^2}{4} + R^2 + \delta^2 - 2R\delta$$

$$= \frac{l^2}{4} - 2R\delta$$

$$\boxed{\delta = \frac{l^2}{8R}}$$

By the bending equation.

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$R = \frac{Ey}{\sigma}$$

$$R = \frac{Et}{2\sigma}$$

$$\delta = \frac{l^2}{\frac{8Et}{2\sigma}}$$

$$\delta = \frac{2\sigma l^2}{8Et}$$

$$\boxed{\delta = \frac{\sigma l^2}{4Et}}$$

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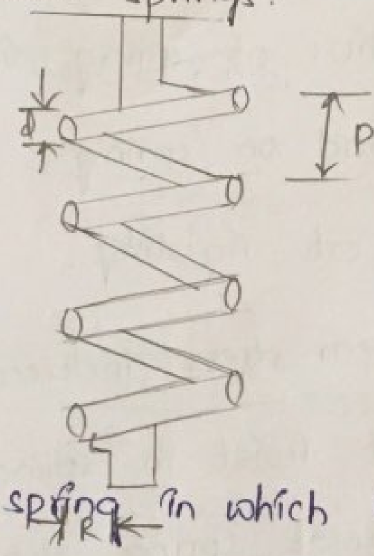
→ HELICAL SPRINGS:-

These are the thick wire coiled into a helix.

Those are two types.

* closed coil helical springs.
* open coil helical springs.

→ closed coil helical springs:-



It is the spring in which helix angle is very small. A closed coil helical spring is carries an axial load, as the helix angle in case of closed coil helical springs are small, hence the bending effect on the spring is ignore and we assume that the coil of closed helical springs are to stand purely torsional stress.

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→ Expression for maximum stress induced in wire:

closed coil helical spring is subjected to an axial load.

let 'd' → dia of wire.

P → pitch of helical spring.

n → no. of turns (or) coils.

R → Mean radius of spring coil.

W → Axial load on spring.

C → Modulus of rigidity.

τ → Max. shear stress induced in wire.

θ → Angle of twist in spring wire.

δ → deflection of spring due to axial load.

L → Length of wire [nxd]

Twisting moment of wire $T = W \times R$

But twisting moment 'T' $= \frac{\pi \tau d^3}{16}$.

$$W \times R = \frac{\pi \tau d^3}{16}$$

$$\boxed{\tau = \frac{16 WR}{\pi d^3}}$$

→ Expression for deflection of spring:

Length of 1 coil $= \pi d \Rightarrow 2\pi r$

Then Length of total wire $= n \times 2\pi r$

as the every section of wire is subjected to torsion hence the strain energy stored by the spring due to torsion is given by.

$$U = \frac{\tau^2}{4C} \times V$$

$$= \frac{\left(\frac{16WR}{\pi d^3}\right)^2}{4C} \times V \qquad \left[\because V = A \times L\right]$$

$$= \frac{\left(\frac{256W^2R^2}{\pi d^6}\right)}{4C} \times 2\pi r n \times \frac{\pi}{4} d^2$$

$$= \frac{256W^2R^2}{4C\pi^2 d^6} \times 2\pi r n \times \frac{\pi}{4} d^2$$

$$= \frac{64W^2R^2}{Cd^4} \times R n \frac{\pi}{2}$$

$$\boxed{U = \frac{32W^2R^3 n\pi}{Cd^4}}$$

Work done on the spring = Avg load × deflection

$$= \frac{W}{2} \times \delta$$

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$$\frac{32n\pi W^2R^3}{Cd^4} = \frac{W}{2}\times\delta$$

$$\boxed{\delta = \frac{64n\pi WR^3}{Cd^4}}$$

→ Expression for stiffness of the spring:

$$S = \frac{\text{load}}{\text{deflection}} = \frac{W}{\delta}$$

$$= \frac{WCd^4}{64n\pi WR^3}$$

$$\boxed{S = \frac{Cd^4}{64nR^3}}$$

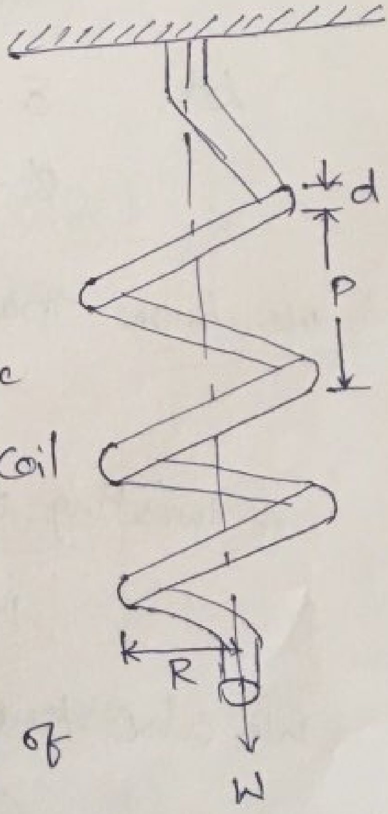
Note:-

* The solid length of the spring means the distance between coils when the coils are touching each other, There is no gap between the coils i.e $(n\times d)$.

Open coiled helical Spring:

In an open helical spring, the Spring wire is coiled in Such way, that there is large gap between the two Consecutive turns. As a result of this, the Spring Can take Compressive load also. An open helical Spring, like a Closed helical Spring, may subjected to

1) axial load
2) axial twist.



Now Consider an open helical Spring Subjected to axial load

Let $d \rightarrow$ Diameter of Spring wire

$R \rightarrow$ Mean radius of spring Coil

$p \rightarrow$ pitch of spring coil

$n \rightarrow$ no. of Coils

$C \rightarrow$ Modulus of Rigidity of Spring Materials

$W \rightarrow$ axial load on the Spring

$\tau \rightarrow$ Max. Shear stress induced in the spring wire due to loading.

$\sigma_b \rightarrow$ Bending Stress induced in the spring wire due to bending.

$\delta \rightarrow$ Deflection of the Spring as a result of axial load &

$\alpha \rightarrow$ Angle of helix.

A little Consideration will show that the load "W" will Cause a moment WR.

This moment will resolve into two Components.

$T = WR$ ~~Sin~~ $\cos\alpha$ (It Causes twisting Coils)

$M = WR \sin\alpha$ (It Causes bending Coils)

Let, $\delta \rightarrow$ Angle of twist, as a result twisting Moment

$\phi \rightarrow$ Angle of bend, as a result bending moment.

We know that the length of the Spring wire,

$$l = 2\pi nR \sec\alpha.$$

& twisting Moment

$$W \cdot R \cdot \cos\alpha = \frac{\pi}{16} \tau d^3$$

We also know bending stress,

$$\sigma_b = \frac{M \cdot y}{I} \Rightarrow \frac{WR \sin\alpha \cdot \frac{d}{2}}{\frac{\pi}{64} \times d^4}$$

$$= \frac{32 WR \sin\alpha}{\pi d^3}$$

$$\text{angle of twist} = \theta = \frac{Tl}{Jc} = \frac{WR \cos\alpha \cdot l}{Jc}$$

We have also seen in previous article, that angle of bend due to bending moment,

$$\phi = \frac{Ml}{EI} = \frac{WR\sin\alpha \cdot l}{EI}$$

We know that work done by the load in deflecting the spring, is Equal to the stress energy of the Spring.

$$\frac{1}{2} W \cdot \delta = \frac{1}{2} T\theta + \frac{1}{2} M\phi$$

$$W \cdot \delta = T \cdot \theta + M\phi$$

$$= \left[WR\cos\alpha \times \frac{WR \cancel{\sin\alpha}\cos\alpha \cdot l}{Jc} \right] + \left[WR\sin\alpha \times \frac{WR\sin\alpha \cdot l}{EI} \right]$$

$$\delta = WR^2 l \left[\frac{\cos^2\alpha}{Jc} + \frac{\sin^2\alpha}{EI} \right]$$

Now Sub. values of $l = 2\pi R n \sec\alpha$,

$$J = \frac{\pi}{32} (d)^4$$

$$I = \frac{\pi}{64} d^4$$ in above

$$\delta = WR^2 \times 2\pi R n \sec\alpha \left(\frac{\cos^2\alpha}{\frac{\pi}{32} d^4 c} + \frac{\sin^2\alpha}{E \cdot \frac{\pi}{64} d^4} \right)$$

$$= \frac{64 WR^3 n \sec\alpha}{d^4} \left[\frac{\cos^2\alpha}{c} + \frac{2\sin^2\alpha}{E} \right]$$

Note:- If we sub. $\alpha = 0$ in previous Eqn.

It Gives deflection of a Closed coil spring.

$$\boxed{\text{i.e.,} \quad \delta = \frac{64 WR^2 n}{Cd^4}}$$

Springs in Parallel & Series:-

Series:

In this Case, the two springs Connected in series. Each spring is subjected to the same load applied at the end of the spring. A little Consideration will show that the total extension of the assembly is Equal to the algebraic sum of the extensions of the two springs.

Parallel:

In this Case, the two springs are Connected in parallel. The extension of each spring is the same. A little Consideration will show that the load applied on the assembly is shared by the two springs.

problems:-

1Q) A closed coil helical spring is to carry a load of 500N. Its mean coil dia is to be 10 times that of wire diameter. calculate these diameters if maximum shear stress in the material of the spring is to be $80 N/mm^2$?

sol:- Given data

$W = 500N$

$D = 10d$

$\tau = 80 N/mm^2$

$$\tau = \frac{16WR}{\pi d^3}$$

$$80 = \frac{16 \times 500 \times 5\cancel{d}}{\pi \times d^{\cancel{3}}}$$

$$80 = \frac{12732.39}{d^2}$$

$$\boxed{d = 12.61 mm}$$

$D = 10 \times 12.61$

$$\boxed{D = 126.15 mm}$$

BQ) If the stiffness of the spring is 20N/mm deflection and modulus of rigidity $8.4 \times 10^4 N/mm^2$. find the no. of coils in the closed coil helical spring.

Sol:- Given data

$\delta = 20$ N/mm

$C = 8.4 \times 10^4$ N/mm^2

$$\delta = \frac{Cd^4}{64\,nR^3}$$

$$20 = \frac{8.4 \times 10^4 \times (12.61)^4}{64 \times n \times (63.075)^3}$$

$$20 = \frac{132.24}{n}$$

$$\boxed{n = 6.612}$$

$$\delta = \frac{64\,WR^3 n}{Gd^4} = \frac{64 \times 500 \times (63.075)^3 \times 6.612}{8 \times 10^4 \times (12.61)^4}$$

$$\boxed{\delta = 24.99\text{mm}}$$

2Q) A closely coiled helical spring of mean diameter 20cm is made of 3cm dia rod and has 16 turns. A weight of 3KN is dropped on the spring. find the height by which the weight should be dropped before stringing the spring so that the spring may compressed by 18cm. Take $C = 8 \times 10^4$ N/mm^2.

sol:-

$D = 20cm = 200mm$

$d = 3cm = 30mm$

$n = 16$

$W = 3kN$

$\delta = 18cm = 180mm$

$C = 8 \times 10^4 \ N/mm^2$

$$\delta = \frac{64 WR^3 n}{cd^4}$$

$$180 = \frac{64 \times W \times 100^3 \times 16}{8 \times 10^4 \times 30^4}$$

$$\boxed{W = 11390.625 N}$$

Work done by the falling weight on the spring.

$\frac{W}{2} \times \delta =$ work done.

$$\frac{11390.625}{2} \times 180 = 3 \times 10^3 (H + 180)$$

$$1.025 \times 10^6 = 540 \times 10^3 + 3 \times 10^3 H$$

$$H = \frac{485000}{3 \times 10^3}$$

$$\boxed{H = 161.67 mm}$$

4Q) stiffness of closely coiled helical spring is 1.5 N/mm of compression under a maximum load of 60N. The maximum shearing stress produced in the wire of the spring 125 N/mm². The solid length of the spring when the coils are touching is given as 5cm. find.

(i) Dia of wire.

(ii) Mean Dia of coil.

(iii) No. of coils required

Take $c = 4.5 \times 10^5$ N/mm².

sol: Given data

$S = 1.5$ N/mm²

$W = 60N$

$\tau = 125$ N/mm²

$L = 5cm = 50mm$

$$\delta = \frac{W}{S} = \frac{60}{1.5}$$

$$\boxed{\delta = 40mm}$$

$$S = \frac{cd^4}{64nR^3}$$

$$1.5 = \frac{4.5 \times 10^5 \times d^4}{64n \times R^3}$$

$$d^4 = \frac{64 \times 1.5 \times nR^3}{4.5 \times 10^5}$$

$$d^4 = 2.133 \times 10^{-4} \, nR^3 \longrightarrow (1)$$

$$d^4 = 2.133 \times 10^{-4} \, n \left[\frac{D}{2}\right]^3$$

$$d^4 = 2.666 \times 10^{-5} \, D^3 n$$

$$\tau = \frac{16WR}{\pi d^3}$$

$$R = \frac{\tau \times \pi d^3}{16W}$$

$$= \frac{125 \times \pi d^3}{16 \times 60}$$

$R = 0.409d^3$ sub in (1)

$$d^4 = 2.133 \times 10^{-4} \times (0.409d^3)^3 n$$

$$d^4 = 1.459 \times 10^{-5} d^9 n$$

$$\frac{d^4}{d^9 n} = 1.459 \times 10^{-5}$$

$$d^5 n = \frac{1}{1.459 \times 10^{-5}}$$

$$d^5 n = 68540.09$$

$$L = n \times d$$

$$50 = n \times d$$

$$n = \frac{50}{d}$$

$$d^5 \times \frac{50}{d} = 68540.09$$

$$d^4 = \frac{68540.09}{50} = 1370.80 \text{ mm}.$$

$$\boxed{d = 6.08 \text{m}}$$

$$n = \frac{50}{d} = \frac{50}{6.08}$$

$$\boxed{n = 8.22}$$

$$R = 0.409 \times (6.08)^3$$

$$\boxed{R = 91.92 \text{mm}}$$

$$D = 2R \Rightarrow 2 \times 91.92$$

$$\boxed{D = 183.85 \text{ mm}}$$

→ closed coil helical spring subjected to axial twist:-

When a twisting couple is applied to the spring parallel to the axis. It produces a bending effect on it. Depending upon the direction of twisting couple or turning moment the spring coil will open-out. In both cases the radius of coil is changes and bending stress will induced

$n_1 \longrightarrow$ no. of coils before the twist.

$n_2 \rightarrow$ no. of coils after the twist.

$\phi \rightarrow$ angle of rotation.

$I \longrightarrow$ MOI

$R_1 \rightarrow$ Mean Radius.

$R_2 \rightarrow$ changed radius

$\sigma_b \rightarrow$ Bending stress

$\epsilon \rightarrow$ young's modulus

$\rightarrow$ Initial curvature $= \frac{1}{R_1}$

$\rightarrow$ final curvature $= \frac{1}{R_2}$

$\rightarrow$ changed in curvature $= \frac{1}{R_2} - \frac{1}{R_1}$

By Bending eqn

$$\frac{M}{I} = \frac{\epsilon}{R}$$

$$\frac{1}{R} = \frac{M}{\epsilon I}$$

$$\frac{1}{R_2} - \frac{1}{R_1} = \frac{M}{\epsilon I}$$

since, the length of wire remains unchanged before and after applying the twisting couple then.

$$\therefore l = 2\pi R_1 n_1 = 2\pi R_2 n_2$$

$\therefore \phi$ = final helix angle - Initial helix angle

$$R_1 = \frac{l}{2\pi n_1} \quad ; \quad R_2 = \frac{l}{2\pi n_2}$$

$$\frac{2\pi n_2}{l} - \frac{2\pi n_1}{l} = \frac{M}{EI}$$

$$\frac{M}{EI} = \frac{2\pi}{l}(n_2 - n_1)$$

$$\phi = 2\pi n_2 - 2\pi n_1$$

$$\frac{M}{EI} = \frac{\phi}{l}$$

$$\phi = \frac{Ml}{EI}$$

$$\phi = \frac{M\, 2\pi R n}{E \times \frac{\pi d^4}{64}}$$

$$\phi = \frac{128 R n M}{E d^4}$$

then $\sigma_b = \frac{M}{EI}$; $Z = \frac{I}{y}$

$$\sigma_b = \frac{My}{I}$$

$$= \frac{Md}{2I} \qquad \left[\because y = \frac{d}{2}\right]$$

$$= \frac{Md}{2 \times \frac{\pi d^4}{64}}$$

$$\sigma_b = \frac{32M}{\pi d^3} \Rightarrow M = \frac{\sigma_b \pi d^3}{32}$$

Strain energy stored $U = \frac{1}{2} \times m \times \phi$

$$= \frac{1}{2} \times m \times \frac{Ml}{EI}$$

$$= \frac{M^2 l}{2EI}$$

$$= \frac{\left(\frac{\sigma_b \pi d^3}{32}\right)^2 \times l}{2E \times \frac{\pi d^4}{64}}$$

$$= \frac{\sigma_b^2 \times \pi^2 \times d^6 \times l \times 32}{32^2 \times E \times \pi d^4}$$

$$= \frac{\sigma_b^2 \times \pi \times d^2 \times l}{32 E}$$

$$U = \frac{\sigma_b^2}{8} \times \frac{\pi}{4} d^2 \times l$$

$$U = \frac{\sigma_b^2}{8} \times V$$

problems:

1Q) A closed coil helical spring made of wire 5mm dia and having inside dia of 40mm joints two shafts the effective no. of coils b/w the shafts is 15 and 0.735 kw is transmitted through the spring at 1000 rpm. calculate the Relative axial twist in degrees b/w the ends of spring and also intensity of bearing stress in the material. Take $E = 200 \, GN/m^2$.

sol:- Given data

$d = 50 \text{ mm}$

$n = 15$

$P = 0.735 \times 10^3 \, W$

$R = 5mm$

$n = 1000\ rpm$

$E = 200 \times 10^9\ N/m^2$

Mean dia $= d + 2t = 40 + 2 \times 5$

$D = 50mm$

$$P = \frac{2\pi n t}{60}$$

$$T = \frac{0.735 \times 10^3 \times 60}{2 \times \pi \times 1000}$$

$T = 7.01 N\text{-}mm$

$$\phi = \frac{128 R n M}{E d^4}$$

$$= \frac{128 \times 25 \times 15 \times 7.01}{200 \times 10^3 \times (50)^4}$$

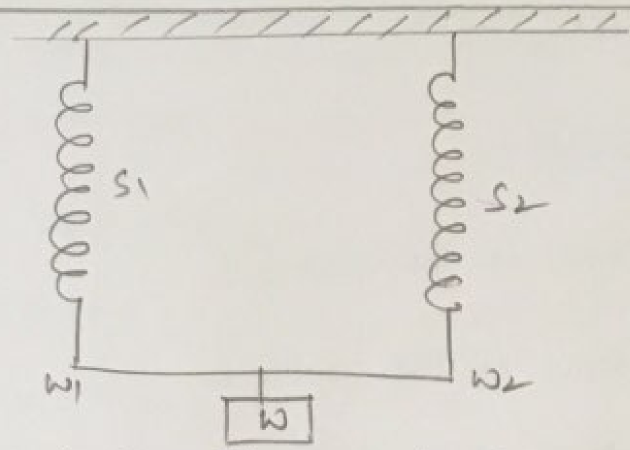
$$= 2.69 \times 10^{-3}\ rad \times \frac{180}{\pi}$$

$$\boxed{\phi = 0^\circ 9'}$$

$$\sigma_b = \frac{32M}{\pi d^3}$$

$$= \frac{32 \times 7.01}{\pi \times 5^3}$$

$$\boxed{\sigma_b = 0.57\ N/mm^2}$$



then the load will be shade

$$W = W_1 + W_2$$

then $\delta \times S = \delta \times S_1 + \delta \times S_2$

$$S = S_1 + S_2$$

problem:

1Q:- A composite spring has two closed coil springs connected in series, one spring has 12 coils of a mean dia of 25mm and wire dia 2.5mm. find the wire dia of the other spring, if it has 15 coils of mean dia 40mm. The stiffness of composite spring is 1.5 KN/m. Determine the greatest load that can be carried by composite spring and the correspadily extension if max. shear stress is 250 M N/m², C = 80 GN/m².

sol:- Given data

$n_1 = 12$

$D_1 = 25mm$

$R_1 = 2.5mm$

$n_2 = 15$

$D_2 = 40mm$

$S = 1.5 KN/m$

$\sigma = 250 MN/m^2$

$C = 80 GN/m^2$

$R_2 = 40mm$

$$S_1 = \frac{cd_1^4}{64 n_1 R_1^3}$$

$$= \frac{80\times10^3 \times 2.5^4}{64 \times 12 \times 12.5^3}$$

$S = 2.08$ N-mm

$$S_2 = \frac{cd_2^4}{64 n_2 R_2^3}$$

$$= \frac{80\times10^3 \times d_2^4}{64 \times 15 \times 20}$$

$S_2 = 0.01 d_2^4$

$$\frac{1}{S} = N\left[\frac{1}{S_1} + \frac{1}{S_2}\right]$$

$$\frac{1}{1.5} = \frac{1}{2.09} + \frac{1}{0.01 d_2^4}$$

$$0.67 = \frac{100.48}{d_2^4}$$

$d_2 = 3.49$ mm.

$$\delta = \frac{64 W R_2^3 n_2}{C d_2^4}$$

$$= \frac{64 W \times 20^3 \times 15}{80 \times 10^3 \times (3.5)^4}$$

$$\delta = 0.63 W$$

$$\tau = \frac{16 W R}{\pi d^4}$$

$$W = \frac{\tau \pi d_2^3}{16 R_2}$$

$$= \frac{250 \times \pi \times (3.5)^3}{16 \times 20}$$

$$W = 105.23 N$$

$$\delta = 0.63 \times 105.23$$

$$\boxed{\delta = 66.29 mm}$$

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UNIT- II

COLUMNS AND STRUCTS

Syllabus:

* Introduction
* Types of columns
* short, medium and columns
* Axially loaded compression members.
* Crushing load.
* Assumptions
* Derivation of euler's critical load formula for various end conditions.
* Equivalent length of a column.
* Equivalent length of a column.
* Slenderness ratio.
* euler's critical stress.
* Limitations of euler's theory.
* Rankine - Gordon formula.
* Long columns subjected to eccentric loading
* secant formula - Empirical formula - straight line formula - proof for perry's formula.

→ BEAM COLUMNS:

* Laterally loaded structs subjected to u.d.l and concentrated loads.

* Maximum B.M and stress due to transverse and lateral loading.

columns and structs:

A member of structure or bar which carries axially compressive load is called "struct". If the struct is vertical i.e., inclined at 90° to the horizontal is known as "column".

Generally a member in any position other than vertical subjected to a compressive load is called struct, and vertical member is subjected to compressive load is called column eg: vertical pillar b/w roof & floor.

* The difference b/w struct and column is struct may have its one or both the ends are fixed rigidly or hinged or pinned while column will have both ends are fixed rigidly eg: piston rods, connecting rods.

failure occur in struct and column

* By pure compression
* By Buckling.

* By combination of Buckling and pure compression

⇒ Definations:

→ column: It is a long vertical slender bar (or) vertical member, subjected to an axial compressive load and fixed rigidly at both ends.

→ Struct: It is a slender bar (or) member in any position other then vertical. subjected to compressive load and fixed rigidly or hinged or pinned at one or both ends.

→ Slenderness ratio (K):- It is the ratio of unsupported length of column to the minimum radius of gyration of the c/s ends of the columns. It has "no units".

→ Buckling factor:- The maximum limiting load at which the column tends to have lateral displacement or tends to buckling or crippling load.

The Buckling takes place having minimum radius rad of gyration or least moment of interia

Radius of gyration $r_{min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{mm^4}{mm^2}} = mm$

→ Safe load :- It is the load to which is actually subjected to and is well below the buckling load. It is obtained by dividing the bucklings loads by a suitable factor of safety.

$$\text{safe load} = \frac{\text{Buckling load}}{\text{F.O.S}}$$

$$\text{F.O.S} = \frac{\text{B.L}}{\text{S.F.}}$$

⇒ Classification of columns :-

Depending upon slenderness ratio or length to diameter ratio, columns can be divided into 3 types. They are

* Short columns
* Medium columns
* Long columns

→ Short columns :- columns which have length less than 8 times their respective diameter or slenderness ratio (K) is less than 30 are called short columns (or) "stocky" structs.

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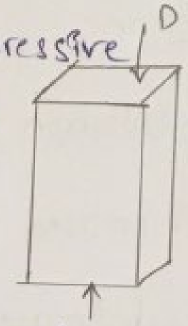
When short columns are subjected to compressive loads, their buckling is generally negligable and as such the buckling stress are very small as compared with direct compressive stress. Therefore, it is assumed that short columns are always subjected to direct compressive stresses only.



$l < 8d$ (or)

$k < 30$

→ MEDIUM COLUMNS: The columns which have their lengths varies from 8 times their diameter to 30 times their respective diameter (or) their slenderness ratio lying between 30 to 120 are called medium columns (or) Intermediate columns.

In these columns, Both are buckling as well as direct stresses are of significant values.

∴ In design of intermediate columns, both these stresses are taken into account.

→ Long columns: The columns having their lengths more than 30 times of their respective diameter or slenderness ratio (k) is greater than 120 are called long columns.

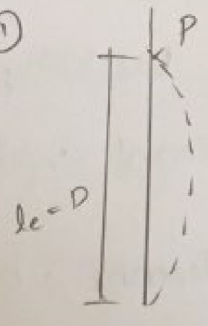
They are usually subjected to buckling stress only. Direct compressive stress is very small as compared with buckling load. Hence it is negligable.

→ strength of column:

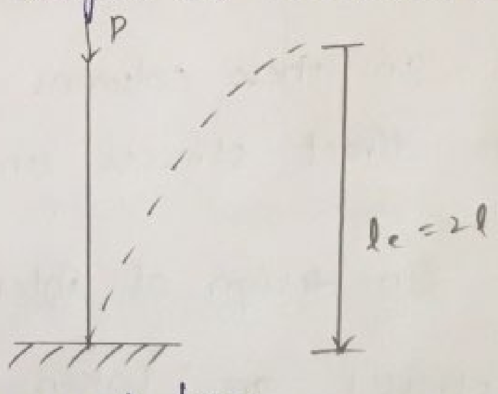
The strength of column depends upon slenderness ratio. If 'K' is increased the compressive strength of a column is decreases as the tendency to buckle is increases. The strength of column depends upon end conditions also.

→ End conditions:-

* Both ends are pinned (or) hinged (or) rounded (or) free. ①



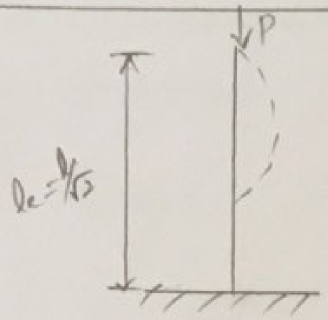
②



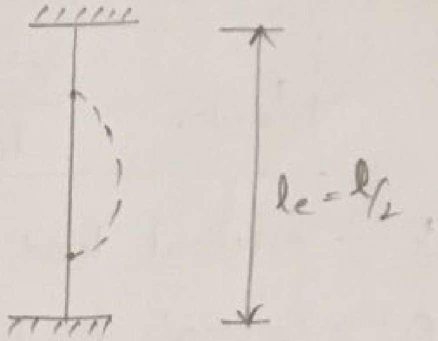
* one end fixed and other end free.

* One end fixed and other end pinjointed.

(3)



(4)



* Both ends are fixed.

→ Euler's theory Assumptions:

for long column:-

* The column is intially straight & of uniform lateral dimension.
* The compressive is exactly axial and it passes through the centroid of the column section.
* The material of the column is perfectly homo-geneous and isotrophic.
* pin joints are friction less and fixed ends are perfectly rigid.
* The wt. of the column itself is neglected.
* The column fails by buckling alone.
* Limit of proportionality is not exceeded.

→ Euler's formula:-

It is used for calculating the critical load for a column or struct.

$$\boxed{P_{euler} = \frac{\pi^2 EI}{le^2}}$$

$P \rightarrow$ critical load

$E \rightarrow$ young's modulus.

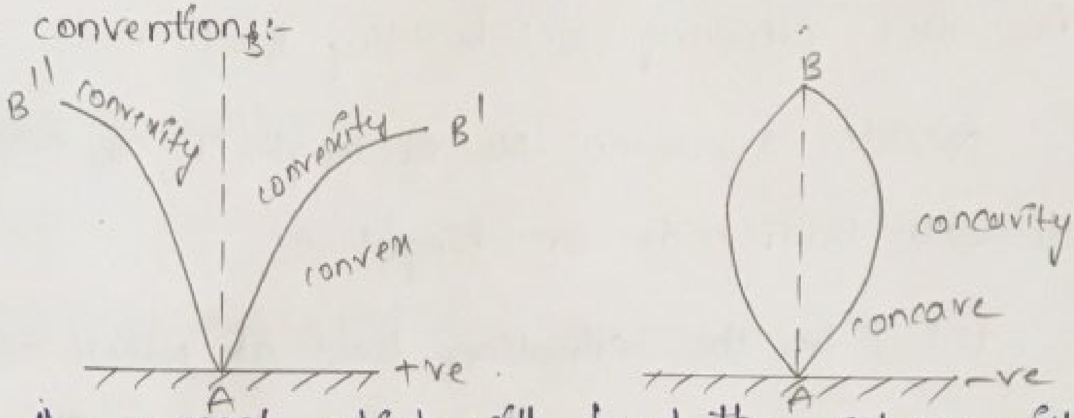
$I \rightarrow$ least moment of Inertia of section of column.

$le \rightarrow$ effective length of the struct (or) equivalent length.

A column of given length, c/s^n and material will have different values of buckling loads for different end condition.

case	End condition	Equivalent length (le)	Buckling load (Euler's) 'P'
1.	Both ends hinged (or) pin jointed (or) rounded (or) free	l	$\frac{\pi^2 EI}{l^2}$
2.	one end is fixed & other end is free.	$2l$	$\frac{\pi^2 EI}{(2l)^2} = \frac{\pi^2 EI}{4l^2}$
3.	one end is hinged & other end is free	$\frac{l}{\sqrt{2}}$	$\frac{\pi^2 EI}{\left(\frac{l}{\sqrt{2}}\right)^2} = \frac{2\pi^2 EI}{l^2}$
4.	Both ends are fixed (or) encastered.	$l/2$	$\frac{\pi^2 EI}{(l/2)^2} = \frac{4\pi^2 EI}{l^2}$

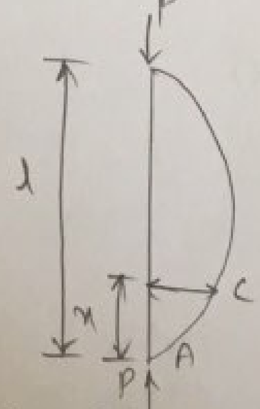
→ sign conventions:-



A moment which will bend the column with its convexity towards its intial central live is taken '+ve', AB represents the intial central line of column . wheather the column bends taking the shape AB' or AB'' the moment producing curvature is '+ve'.

The moment which will bend column with its concavity towards initial central line is taken as '-ve'.

→ Expression for crippling load when the both ends are hinged :-



The load at which the column just buckle is called crippling (or) buckling load.

consider a column AB of length 'L' of uniform c/s with both ends are hinged.

Let p be the crippling load at which column just buckles. Due to the crippling load the column will defect into a curved from "AEB". consider any section at a distance of 'x' from end 'A'.

Let 'y' deflection at section (or) lateral displace-ment then the moment due to crippling load

$$M = -P \times y \longrightarrow ①$$

But we know $M = \frac{d^2y}{dx^2} EI \longrightarrow ②$

Equating ① and ②

$$-Py = \frac{d^2y}{dx^2} EI$$

$$\frac{d^2y}{dx^2} EI + Py = 0$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0$$

The solution of above eqn is

$$y = c_1 \cos\left[x\sqrt{\frac{P}{EI}}\right] + c_2 \sin\left[x\sqrt{\frac{P}{EI}}\right] \longrightarrow ③$$

c_1, c_2 = integration constants are obtained by boundary conditions.

At end 'A' $x=0, y=0$

$$0 = c_1 \cos(0) + c_2 \sin(0)$$

$$0 = c_1 \cdot 1 + c_2(0)$$

$$c_1 = 0$$

At $x = l$, $y=0$; $c_1 = 0$ sub in (3)

$$0 = 0 + c_2 \sin\left[l\sqrt{\frac{P}{EI}}\right]$$

$$c_2 = 0 \quad (\text{or}) \quad \sin\left[l\sqrt{\frac{P}{EI}}\right] = 0$$

As $c_1 = 0$, if $c_2 = 0$ then from (3)

$$y = 0$$

This means that the bending of the column will be zero or the column will not bend at all which is not true.

$$\sin\left[l\sqrt{\frac{P}{EI}}\right] = 0$$

$$\cancel{\sin}\left[l\sqrt{\frac{P}{EI}}\right] = \cancel{\sin}\,0 \quad (\text{or}) \quad \sin\pi \quad (\text{or}) \quad \sin 2\pi$$

then

$$l\sqrt{\frac{P}{EI}} = 0 \quad (\text{or}) \quad \pi \quad (\text{or}) \quad 2\pi$$

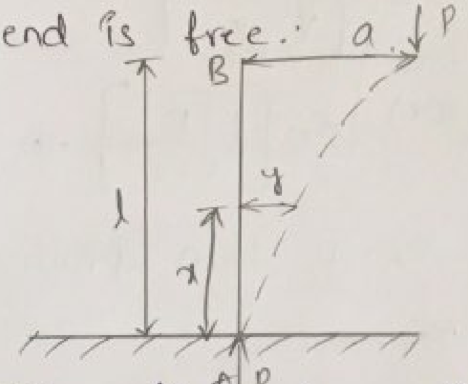
$$l\sqrt{\frac{P}{EI}} = \pi$$

$$\sqrt{\frac{P}{EI}} = \frac{\pi}{l}$$

$$\frac{P}{EI} = \frac{\pi^2}{l^2}$$

$$\boxed{P = \frac{\pi^2 EI}{l^2}} \qquad \left[\because l_e = l\right]$$

→ Expression for crippling load which one end is fixed and other end is free.:



consider any section at a distance of 'x'. from end 'A'. Let 'y' be the deflection at the section 'a' - deflection at the free end 'B'. Then the moment at the section

$$M = P(a-y) \quad — ①$$

But, we know that

$$M = \frac{d^2y}{dx^2} EI \quad —— ②$$

Equating ① and ②

$$P(a-y) = \frac{d^2y}{dx^2} EI$$

$$\frac{d^2y}{dx^2} EI + Py = Pa$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{P}{EI} a \qquad [\div EI]$$

final solution for above equation is

$$y = C_1 \cos\left[x\sqrt{\frac{P}{EI}}\right] + C_2 \sin\left[x\sqrt{\frac{P}{EI}}\right] + a \longrightarrow (3)$$

$$\frac{dy}{dx} = -C_1 \sin\left[x\sqrt{\frac{P}{EI}}\right]\sqrt{\frac{P}{EI}} + C_2 \cos\left[x\sqrt{\frac{P}{EI}}\right]\sqrt{\frac{P}{EI}} + 0 \longrightarrow (4)$$

Apply Boundary conditions,

at 'A' $x=0$, $y=0$ sub in (3)

$$0 = C_1 \cos(0) + C_2 \sin(0) + a$$

$$0 = C_1 + a$$

$$C_1 = -a$$

At 'A' $x=0$, $\frac{dy}{dx} = 0$, sub in (4)

$$0 = -C_1(0) + C_2 \cos(0)\sqrt{\frac{P}{EI}}$$

$$C_2 = 0 \text{ (or) } \sqrt{\frac{P}{EI}} = 0$$

But for crippling load 'P', $\sqrt{\frac{P}{EI}}$ can't be zero

then $C_2 = 0$

sub values c_1 and c_2 in eqn ③

$$y = -a\cos\left(x\sqrt{\frac{P}{EI}}\right) + 0 + a$$

$$y = -a\cos\left(x\sqrt{\frac{P}{EI}}\right) + a \quad \text{— ⑤}$$

At B, $x = l$, $y = a$ sub in ⑤

$$a = -a\cos\left(l\sqrt{\frac{P}{EI}}\right) + a$$

$$= -a\cos\left[l\sqrt{\frac{P}{EI}}\right]$$

$$0 = \cos\left[l\sqrt{\frac{P}{EI}}\right] \quad \text{or } a = 0$$

But can't be equal to '0' then

$$\cos\left(l\sqrt{\frac{P}{EI}}\right) = 0$$

$$\cancel{\cos}\left(l\sqrt{\frac{P}{EI}}\right) = \cancel{\cos}\,0 \text{ (or) } \cos\frac{\pi}{2} \text{ (or) } \cos\frac{3\pi}{2}$$

$$l\sqrt{\frac{P}{EI}} = \frac{\pi}{2}$$

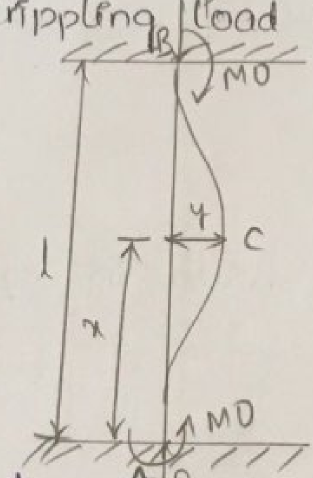
$$\sqrt{\frac{P}{EI}} = \frac{\pi}{2l}$$

$$\frac{P}{EI} = \frac{\pi^2}{(2l)^2}$$

$$\boxed{P = \frac{\pi^2 EI}{4l^2}}$$

$\therefore$ $l_e = 2l$.

→ Expression for crippling load when both ends are fixed:



Let 'M_0' fixed end moment. At 'A' and 'B' then the moment of section.

$$M = -Py + M_0 \longrightarrow (1)$$

But we know that

$$M = \frac{d^2y}{dx^2} EI \longrightarrow (2)$$

Equating (1) and (2)

$$EI \frac{d^2y}{dx^2} = M_0 - Py$$

$$EI \frac{d^2y}{dx^2} + Py = M_0$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = M_0$$

The final solution of above equation is

$$y = C_1 \cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x\sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} \longrightarrow (3)$$

$$\frac{dy}{dx} = -C_1 \sin\left(\frac{x\sqrt{P}}{\sqrt{EI}}\right) \sqrt{\frac{P}{EI}} + C_2 \cos\left(x\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} + 0 \longrightarrow (4)$$

At A, $x = 0$, $y = 0$ sub in (3)

$$0 = C_1 + \frac{M_0}{P}$$

$$C_1 = -\frac{M_0}{P}$$

At $x = 0$, $\frac{dy}{dx} = 0$, sub in eqn ④

$$0 = C_2 \cdot 1 \cdot \sqrt{\frac{P}{EI}}$$

$$C_2 = \sqrt{\frac{P}{EI}} \text{ (or) } 0.$$

sub C_1 and C_2 in ③

$$y = -\frac{M_0}{P}\cos\left(x\sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P} \longrightarrow ⑤$$

At B, $x = l$, $y = 0$ sub in ⑤

$$0 = \frac{-M_0}{P} \cdot \cos\left(l\sqrt{\frac{P}{EI}}\right) + \frac{M_0}{P}$$

$$-\frac{M_0}{P}\left(\cos\left(l\sqrt{\frac{P}{EI}}\right) - 1\right) = 0$$

$$\cos\left(l\sqrt{\frac{P}{EI}} - 1\right) = 0$$

$$\cancel{\cos}\left(l\sqrt{\frac{P}{EI}}\right) = \cancel{\cos}(0) = \cos 2\pi = \cos 4\pi$$

$$l\sqrt{\frac{P}{EI}} = 2\pi$$

$$\sqrt{P/EI} = 2\pi/l$$

$$P/EI = (2\pi)^2/l^2$$

$$\boxed{P = 4\pi^2 EI/l^2} \qquad \left[\because l_e = l/2\right]$$

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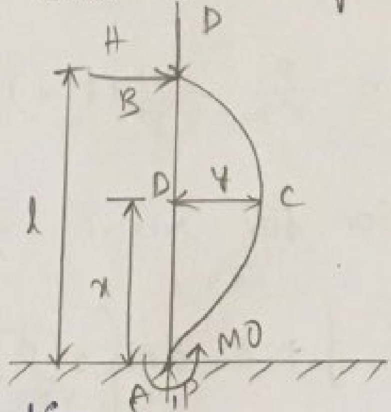
→ Expression for crippling load when one end is fixed and other end is hinged :-



consider a section 'x' at A and M_0 fixed end moment at 'A' and 'H' horizontal reaction at 'B'.

There will be a fixed end moment 'M_0' at end 'A'. this will try to bring back the slope of deflected column is '0' at 'A'. Hence will be acting anti-clockwise at 'A'. Fixed end moment 'M_0' is to be balanced. This will be balanced by a horizontal reaction at the top end 'B'.

The moment at the section 'D'.

But we know that $M = -Py + H(l-x)$ — ①

$$M = \frac{d^2y}{dx^2} EI \quad \text{—} ②$$

Equate ① + ②

$$\frac{d^2y}{dx^2} EI = -Py + H(l-x)$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{HP}{EIP}(l-x)$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{P}{EI} \cdot \frac{H}{P}(l-x)$$

The final pu solution for above eqn is

$$y = c_1 \cos\left[x\sqrt{\frac{P}{EI}}\right] + c_2 \sin\left[x\sqrt{\frac{P}{EI}}\right] + \frac{H}{P}(l-x) \longrightarrow ③$$

$$\frac{dy}{dx} = -c_1 \sin\left(x\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} + c_2 \cos\left(x\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} - \frac{H}{P} \longrightarrow ④$$

Apply. Boundary condition.

At 'A' $x=0$, $y=0$ sub in eqn ③

$$0 = c_1 \cos(0) + c_2 \sin(0) + \frac{H}{P}(l-0)$$

$$= c_1 + \frac{H}{P}(l)$$

$$c_1 = -\frac{H}{P}(l)$$

At 'A' $x=0$, $\frac{dy}{dx}=0$, sub in ④

$$0 = -c_1(0) + c_2\cos(1) \cdot \frac{\sqrt{P}}{\sqrt{EI}} - \frac{H}{P}$$

$$= c_2\sqrt{\frac{P}{EI}} - \frac{H}{P}$$

$$\boxed{c_2 = \frac{H}{P}\sqrt{\frac{EI}{P}}}$$

Sub c_1 and c_2 values in ③

$$y = -\frac{H}{P} l \cos\left[x\sqrt{\frac{P}{EI}}\right] + \frac{H}{P}\sqrt{\frac{EI}{P}} \sin\left(x\sqrt{\frac{P}{EI}}\right] + \frac{H}{P}(l-x) \rightarrow ⑤$$

At B $x = l$, $y = 0$, sub in (5)

$$0 = -\frac{H}{P} l \cos\left[l\sqrt{\frac{P}{EI}}\right] + \frac{H}{P}\sqrt{\frac{EI}{P}} \sin\left[l\sqrt{\frac{P}{EI}}\right] + \frac{H}{P}(l-x)$$

$$\frac{H}{P} \cdot l \cos\left[l\sqrt{\frac{P}{EI}}\right] = \frac{H}{P}\sqrt{\frac{EI}{P}} \sin\left[l\sqrt{\frac{P}{EI}}\right]$$

$$l \cos\left(l\sqrt{\frac{P}{EI}}\right) = \sqrt{\frac{EI}{P}} \sin\left(l\sqrt{\frac{P}{EI}}\right)$$

$$l\sqrt{\frac{P}{EI}} = \frac{\sin\left(l\sqrt{\frac{P}{EI}}\right)}{\cos\left(l\sqrt{\frac{P}{EI}}\right)}$$

$$l\sqrt{\frac{P}{EI}} = \tan\left(l\sqrt{\frac{P}{EI}}\right)$$

The solution for above eqn is

$$l\sqrt{\frac{P}{EI}} = 4.5 \text{ radians.}$$

equaring o.b.s

$$\frac{P}{EI} = \left(\frac{4.5}{l}\right)^2$$

$$P = \frac{20.25\, EI}{l^2}$$

$$P = \frac{2\pi^2 EI}{l^2} \qquad \left[\because l_e = \frac{l}{\sqrt{2}}\right]$$

→ Critical stress :- (OR) CRIPPLING STRESS :-

The stress which is produced by crippling load (or) critical load is known as crippling stress (or) critical stress.

$$\text{critical stress} = \frac{\text{crippling load}}{\text{area}}$$

→ crippling stress in terms of effective length and Radius of gyration 'k'.

$$k = \sqrt{\frac{I}{A}}$$

$$I = Ak^2$$

The MOI is expressed in terms of Radius of gyration 'k' as $I = Ak^2$

Now, crippling load 'p' in terms of effective length is given by

$$P = \frac{\pi^2 EI}{le^2}$$

$$P = \frac{\pi^2 EAk^2}{le^2}$$

$$P = \frac{\pi^2 EA}{\left(\frac{le}{k}\right)^2}$$

critical stress $= \frac{P}{A}$

$$= \frac{\pi^2 E A}{n\left(\frac{l_e}{k}\right)^2}$$

$$C.S = \frac{\pi^2 E}{\left(\frac{l_e}{k}\right)^2}$$

→ Limitations of Euler's formula :-

* crippling stress $= \frac{\pi^2 E I}{\left(\frac{l_e}{k}\right)^2}$, if column with both ends hinged, then effective length $l_e = l$, then C.S becomes $\frac{\pi^2 E I}{\left(\frac{l}{k}\right)^2}$, then $\left(\frac{l}{k}\right)$ is the slenderness ratio.
* If slenderness ratio is small, then crippling stress is more. But for the column material the crippling stress cannot be greater than the crushing stress. Hence the slenderness ratio is less than a certain limit Euler's formula gives a value of crippling stress greater than the crushing stress. In the limiting case, we can find the value of (l/k) for which crippling stress is

equal to crushing stress.

→ limitations of euler's formula :-

* crippling stress = $\frac{\pi^2 EI}{\left(\frac{le}{k}\right)^2}$, if column with both ends hinged, then effective length $le = l$, then cos becomes $\frac{\pi^2 EI}{(l/k)^2}$, then (l/k) is the slenderness ratio.

* If slenderness ratio is small, then crippling stress is more. But for the column material the crippling stress cannot be greater than the crushing stress. Hence the slenderness ratio is less than a certain limit, euler's formula gives a value of crippling stress greater than the crushing stress. In the limiting case, we can find the value of (l/k) for which crippling stress is equal to crushing stress.

Ex:-

* for mild steel column, Both ends are hinged crushing stress = $330\ N/mm^2$, $E = 2.1 \times 10^5\ N/mm^2$

$$C.S = \frac{\pi^2 E}{(l/k)^2}$$

$$330 = \frac{\pi^2 \times 2.1 \times 10^5}{(l/k)^2}$$

$$(l/k)^2 = 6280.65$$

$$l/k = 79.25 \cong 80$$

Hence, if slenderness ratio is less than 80, for mild steel column, Both ends are hinged, then euler's formula will not be varied.

problems:-

1Q) A solid round bar 3m long and 5cm in dia. is used as a struct with both ends hinged. Determine the crippling load. Take $E = 2 \times 10^5$ N/mm².

sol: Given data.

$L = 3m$ [hinged]

$d = 5cm = 0.5m$

$E = 2 \times 10^5$ N/mm²

$$P = \frac{\pi^2 E I}{l_e^2}$$

$$I = \frac{\pi}{64}(50)^4$$

$= 306.79 \times 10^3$ mm

$$P = \frac{\pi^2 \times 2 \times 10^5 \times 306.79 \times 10^3}{(3 \times 10^3)^2}$$

$$\boxed{P = 67.28 \text{ kN}}$$

→ One end fixed, other end is free:-

$L_e = 2l$

$$P = \frac{\pi^2 \times 2 \times 10^5 \times \frac{\pi}{64}(50)^4}{(2 \times 3 \times 10^3)^2}$$

$$\boxed{P = 16.82 \text{ kN}}$$

→ One end is hinged, other end is free:

$$L_e = \frac{L}{\sqrt{2}}$$

$$P = \frac{\pi^2 \times 2 \times 10^5 \times \frac{\pi}{64}(50)^4}{\left(\frac{3 \times 10^3}{\sqrt{2}}\right)^2}$$

$$\boxed{P = 134.57 \text{ kN}}$$

→ two ends are fixed:

$$L_e = \frac{L}{2}$$

$$P = \frac{\pi^2 \times 2 \times 10^5 \times \pi/64\,(50)^4}{(3 \times 10^3/2)^2}$$

$$\boxed{P = 269.15\,KN}$$

2(Q) A column of timber section 15x20 cm is 6m long both ends being fixed. If E of timber 17.5 KN/mm². Determine.

* crippling load.
* safe load for the column if F.O.S = 3.

sol:- Given data.

$B \times D = 15 \times 20\ cm = 150 \times 200\ mm$

$L = 6m$ [fixed $L_e = L/2$] = 6000

$E = 17.5 \times 10^3\ N/mm^2$

$$P = \frac{\pi^2 E I}{l_e^2}$$

$$I_{xx} = \frac{BD^3}{12} = \frac{150 \times 200^3}{12} = 100 \times 10^6\ mm^2$$

$$I_{yy} = \cancel{\frac{BD^3}{1}}\ \frac{DB^3}{12} = \frac{200 \times 150^3}{12} = 56.25 \times 10^6\ mm^2$$

$$P = \frac{\pi^2 \times 17.5 \times 10^3 \times 56.25 \times 10^6}{3000^2}$$

$$\boxed{P = 1079.48\,KN}$$

$$\text{safe load} = P/f.o.s = \frac{1079.48}{3}$$

$$\boxed{s.f = 359.82\,kN}$$

→ RANKINE'S FORMULA:-

We have learnt that euler's formula gives correct results for only very long columns. But, if column is very short is not to very long.

On the basis of results of experiment, perfor-med by Rankine, he established empirical formula which is applicable to all columns whether they are long (or) short. The emperical formula which is given by Rankine's is called Rankine's formula

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E} \quad \text{——} (1)$$

$P \rightarrow$ crippling load by Rankine's formula.

$P_c \rightarrow$ Crushing load $= \sigma_c \times A$

$\sigma_c \rightarrow$ Ultimate crushing stress

$A \rightarrow$ Area of C/s

$P_E \rightarrow$ crippling load by euler's formula.

$\rightarrow \dfrac{\pi^2 EI}{le^2}$

For a given material, the crushing stress 'σ_c' is constant.

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Hence, the crushing load 'P_c' will also be constant for a given c/s^n area of column, P_c is constant and hence value of 'p' depends upon value of P_E, but for a given column material and given c/s^n area, the value of P_E is depends upon the effective length of column.

If the column is short, which means the value of ~~for~~ ~~is~~ Le is small, then the value of P_E will be large, then $\frac{1}{P_E}$ is small enough and is negligable as compared to the value of $\frac{1}{P_C}$.

$$\frac{1}{P} \rightarrow \frac{1}{P_c}$$

then $P = P_c$

Hence, crippling load by Ranking formula is approximately equal to crushing load. Because the short column will be failed by crushing.

If the column is long, which means the value of Le is large, then the value of P_E will be small and the value of $\frac{1}{P_E}$ will be large enough compared with $\frac{1}{P_C}$, hence the value of $\frac{1}{P_C}$ will be neglected, then the

$$\frac{1}{P} = \frac{1}{P_E}$$

Hence, crippling load by Rankine's formula for long column is approximately equal to crippling load by euler's formula.

Hence, Ranking formula is gives the satisfactory results for all lengths of columns, ranging from short to long columns.

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_E}$$

$$\frac{1}{P} = \frac{P_E + P_c}{P_c \cdot P_E}$$

$$P = \frac{P_c \cdot P_E}{P_E + P_c}$$

$$= \frac{P_c}{P_E + \frac{P_c}{P_E}}$$

$$\boxed{P = \frac{P_c}{1 + \frac{P_c}{P_E}}}$$

$$P = \frac{\sigma_c \cdot A}{1 + \left[\frac{\sigma_c \cdot A}{\left(\frac{\pi^2 E I}{l_e^2}\right)}\right]}$$

But $I = Ak^2$

k = least R.O.G

$$P = \left[\frac{\sigma_c \cdot A}{1 + \frac{\sigma_c \cdot A l_e^2}{\pi^2 \cdot E A k^2}}\right]$$

$$P = \left[\frac{\sigma_c \cdot A}{1 + \frac{\sigma_c}{\pi^2 E}\left(\frac{l_e^2}{k}\right)}\right]$$

$$\boxed{P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l_e}{k}\right)^2}}$$

$\because a = \frac{\sigma_c}{\pi^2 E}$

$\therefore a$ = Rankine's constant.

S.NO	Material	σ_c in N/mm^2	a	f.o.s
1.	wrought Iron	250	$\frac{1}{9000}$	3
2.	cast iron	550	$\frac{1}{1600}$	5
3.	Mild steel	320	$\frac{1}{7500}$	3
4.	Timber	150	$\frac{1}{750}$	6.

problems:

1Q) The external diameter and Internal diameter of hallow cast iron column 5cm and 4cm respectively. If the length of the column is 3m and both ends are fixed. Determine the crippling load using Rankine's formula. Take $\sigma_c = 550\ N/mm^2$ & $a = \frac{1}{1600}$.

Sol:

Given data

$D = 5cm = 50\ mm$

$d = 4cm = 40\ mm$

$\sigma_c = 550\ N/mm^2$

$L = 3m = 3000\ mm$

$$a = \frac{1}{1600}$$

$$P = \frac{\sigma_c \cdot A}{1+a\left[\frac{l}{k}\right]^2}$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}(50^4-40^4)}{\frac{\pi}{4}(50^2-40^2)}}$$

$k = 16$

$$A = \frac{\pi}{4}(50^2-40^2) = 706.85\ mm^4$$

$$P = \frac{550 \times 706.85}{1+\frac{1}{1600}\left[\frac{3000/2}{16}\right]^2}$$

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$$\boxed{P = 59.37\ KN}$$

20Q) A hallow cylindrical cast iron column is 4m long with both ends are fixed. Determine the minimum dia of the column if it has to carry a safe load of 250KN with a f.o.s = 5. Take Internal dia as 0.8 times of External dia. Take $\sigma_c = 550\ N/mm^2$ and $a = \frac{1}{1600}$ in Rankine's formula.

Sol:-

Given data

$L = 4m$ [fixed]

$S.f = 250\ KN$

$F.OS = 5$

$d_i = 0.8 d_0$

$\sigma_c = 550\ N/mm^2$, $a = \frac{1}{1600}$

$P = S.f \times F.O.S$

$= 250 \times 5$

$P = 1250 KN$

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}(d_0^4 - 0.8 d_0^4)}{\frac{\pi}{64}(d_0^2 - 0.8 d_0^2)}}$$

$K = 0.32 d_0$.

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$$A = \frac{\pi}{4}(d_0^2 - 0.8 d_0^2)$$

$$A = 0.28 d_0^2$$

$$P = \frac{\sigma_c \cdot A}{1 + a\left(\frac{l}{k}\right)^2}$$

$$1250 \times 10^3 = \frac{550 \times 0.28\, d_0^2}{1 + \frac{1}{1600}\left[\frac{4000/2}{0.32\, d_0}\right]^2}$$

$$1250 \times 10^3 = \frac{154 d_0^2}{1 + \frac{24414.06}{d_0^2}}$$

$$1250 \times 10^3 = \frac{154 d_0^2}{\frac{d_0^2 + 24414.06}{d_0^2}}$$

$$1250 \times 10^3 = \frac{154 d_0^4}{d_0^2 + 24414.06}$$

$$1250 \times 10^3 (d_0^2 + 24414.06) = 154 d_0^4$$

$$154 d_0^4 - 1250 \times 10^3 d_0^2 - 3.05 \times 10^{10} = 0$$

$$d_0^2 = 18705 \text{ mm}$$

$$\boxed{d_0 = 136 \text{ mm}}$$

$$d_i = 0.8 \times 136$$

$$\boxed{d_i = 108.8 \text{ mm}}$$

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(Q) Find the Euler's crushing load for a hollow cylindrical cast iron column having external dia 20cm & thickness 25mm if it is 6m long and is hinged at its both ends. Take $E = 1.2 \times 10^5$ N/mm². compare the load with crushing load given by the Rankine's formula.

$\sigma_c = 550$ N/mm², $a = \frac{1}{1600}$ for what length of the column would these two formulas gives the same crushing load.

Sol:- Given data.

$d_o = 20cm \Rightarrow 200mm$

$t = 25mm$

$L = 6m \Rightarrow 6000mm$

$E = 1.2 \times 10^5$ N/mm²

$\sigma_c = 550$ N/mm²

$a = \frac{1}{1600}$

$d_i = d_o - 2t$; $I = \frac{\pi}{64}(d_o^4 - d_i^4)$

$= 200 - 2(25)$; $= \frac{\pi}{64}(200^4 - 150^4)$

$d_i = 150$ mm ; $I = 53.68 \times 10^6$ mm⁴

$$P = \frac{\pi^2 EI}{le^2} = \frac{\pi^2 \times 1.2 \times 10^5 \times 53.68 \times 10^6}{6000^2}$$

Euler's $\longrightarrow$ $\boxed{P = 1766 \times 10^3 \, N}$

By the Rankine's formula

$$P = \frac{\sigma_c A}{1+a\left(\frac{l}{k}\right)^2}$$

$$A = \frac{\pi}{4}(200^2 - 150^2) \; ; \quad k = \sqrt{\frac{I}{A}} = \sqrt{\frac{53.67 \times 10^6}{13.74 \times 10^3}}$$

$$= 13.74 \times 10^3 mm^2 \quad ; \quad k = 62.504 \, mm.$$

$$P = \frac{550 \times 13.74 \times 10^3}{1 + \frac{1}{1600}\left[\frac{6000}{62.504}\right]^2}$$

$\boxed{P = 1118.02 \, KN}$

$\rightarrow$ E.L = R.L $\quad [Le = L]$

$$\frac{\pi^2 EI}{le^2} = \frac{\sigma_c A}{1+a\left(l/k\right)^2}$$

$$\frac{\pi^2 \times 1.2 \times 10^5 \times 53.67 \times 10^6}{l^2} = \frac{550 \times 13.74 \times 10^3}{1+\frac{1}{1600}\left[\frac{l^2}{62.504}\right]^2}$$

$$\frac{6.35 \times 10^{13}}{l^2} = \frac{7.55 \times 10^6}{1 + 1.59 \times 10^{-7} l^2}$$

$$8.459 \times 10^6 = \frac{l^2}{1+1.59 \times 10^{-7} l^2}$$

$$8.459 \times 10^6 + 1.34 l^2 = l^2$$

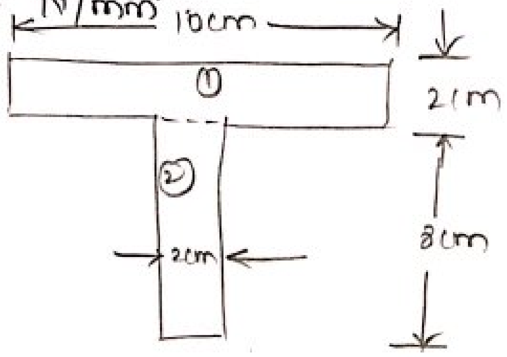
$$8.459 \times 10^6 + 1.34 l^2 - l^2 = 0$$

$$8.459 \times 10^6 + 0.34 l^2 = 0$$

$$\boxed{l = 4.9m}$$

Q) Determine the crippling load for a T-section of dimensions 10cm x 10cm x 2cm and of length 5m. when it is used as a struct with both of its hinged Take $E = 2 \times 10^5$ N/mm²

Sol:-



Given data

L = 5000mm → both ends hinged

$E = 2 \times 10^5$ N/mm²

$$P = \frac{\pi^2 E I}{l_e^2}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$a_1 = 10 \times 2 = 200cm \rightarrow 2000$ mm²

$y_1 = 8 + \frac{2}{2} = 9\,cm \Rightarrow 90\,mm$

$a_2 = 80 \times 20 = 1600\,cm \Rightarrow 1600\,mm^2$

$y_2 = \frac{8}{2} = 4\,cm \Rightarrow 40\,mm$.

$$\bar{y} = \frac{2000 \times 90 + 1600 \times 40}{2000 + 1600}$$

$$\bar{y} = 67.7\,mm$$

$$I_{xx} = \frac{b_1 d_1^3}{12} + A_1 h_1^2 + \frac{b_2 d_2^3}{12} + A_2 h_2^3$$

$$= \left[\frac{100 \times 20^3}{12} + 2000 \times 22.3^2 + \frac{20 \times 80^3}{12} + 1600 \times 27.7^2\right]$$

$$\boxed{I_{xx} = 3.14 \times 10^6\,mm^4}$$

$$I_{yy} = \frac{bd^3}{12} + \frac{bd^3}{12}$$

$$= \frac{20 \times 100^3}{12} + \frac{80 \times 10^3}{12}$$

$$I_{yy} = 1.72 \times 10^6\,mm^4$$

$$P = \frac{\pi^2 E I_{yy}}{l_e^2}$$

$$= \frac{\pi^2 \times 2 \times 10^5 \times 1.72 \times 10^6}{5000^2}$$

$$\boxed{P = 135.80 \times 10^3\,N}$$

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Q) A hallow alloy tube 5m long with enternal and internal diameters 40mm and 25mm respectively was found to extend 6.4mm under a tensile load of 60KN. find the buckling load for the tube when used as a column with both ends hinged. Also find the safe load for the tube, taking a factor of safety = 4.

sol:- Given data

$L = 5m$ [Hinged]

$D = 40mm$

$d = 25mm$

$\delta l = 6.4mm$

$W = 60KN$

$f.o.s = 4$

$$P = \frac{\pi^2 E I}{le^2}$$

$$A = \frac{\pi}{4}(40^2 - 25^2)$$

$$\boxed{A = 765.76\ mm^2}$$

$$I = \frac{\pi}{64}(40^4 - 25^4)$$

$$\boxed{I = 106.48 \times 10^3 mm^4}$$

Modulus of Elasticity E

$$E = \frac{\text{stress } \sigma}{\text{strain } e} = \frac{W/A}{\delta l / l} = \frac{60\times10^3 / 765.76}{64/5000}$$

$$E = \frac{78.35}{0.0128}$$

$$E = 61.92\times10^3 \text{ N/mm}^2$$

$$P = \frac{\pi^2 \times 61.92\times10^3 \times 106.48\times10^3}{5000^2}$$

$$\boxed{P = 2602.26 \text{ N}}$$

$$\text{safe load} = \frac{P}{\text{f.o.s}}$$

$$= \frac{2602.06}{4}$$

$$\boxed{S.F = 650.31 \text{N}}$$

Q) A hallow cast iron whose outside dia is 200mm has a thickness of 20mm. It is 4.5m long and fixed at its both ends. calculate the safe load by Rankine's formula using f.o.s = 4. calculate the slenderness ratio and ratio of Euler's and Rankine's critical load. Take $\sigma_c = 550$ N/mm², $a = \frac{1}{1600}$, $E = 9.4\times10^4$ N/mm².

Sol:

Given data

$l = 4500 = \frac{4500}{2} = 2250\ mm$

$D = 200\ mm$

$t = 20\ mm$

$d = D - 2t = 200 - 2(20)$

$= 160\ mm$

f.o.s = 4

$\sigma_c = 550\ N/mm^2$

$a = \frac{1}{1600}$

$E = 9.4 \times 10^4\ N/mm^2$

$A = \frac{\pi}{4}(200^2 - 160^2)$

$= 11.30 \times 10^3\ mm^2$

$I = \frac{\pi}{64}(200^4 - 160^4)$

$I = \cancel{4}46.36 \times 10^4\ mm^4$

$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{46.36 \times 10^6}{11.30 \times 10^3}}$

$\boxed{K = 64.05}$

$P_R = \frac{\sigma_c A}{1 + a(l/k)^2}$

$$= \frac{550 \times 11.30 \times 10^3}{1 + \frac{1}{1600}\left[\frac{2250}{64.05}\right]^2}$$

$$\boxed{P_R = 3.5 \times 10^6 \text{ N}}$$

$$\text{safe load} = \frac{P_R}{\text{f.o.s}} = \frac{3.5 \times 10^6}{4}$$

$$\boxed{\text{s.f} = 877.19 \text{ kN}}$$

$$P_E = \frac{\pi^2 E I}{le^2} = \frac{\pi^2 \times 9.4 \times 10^4 \times 46.36 \times 10^6}{(2250)^2}$$

$$\boxed{P_E = 8.49 \times 10^6 \text{N}}$$

$$\frac{P_E}{P_R} = \frac{8.49 \times \cancel{10^6}}{3.5 \times \cancel{10^6}} = 2.42$$

$$\boxed{\frac{P_E}{P_R} = 2.42}$$

$$k = l/k$$

$$= \frac{4500}{64.05}$$

$$\boxed{k = 72.5}$$

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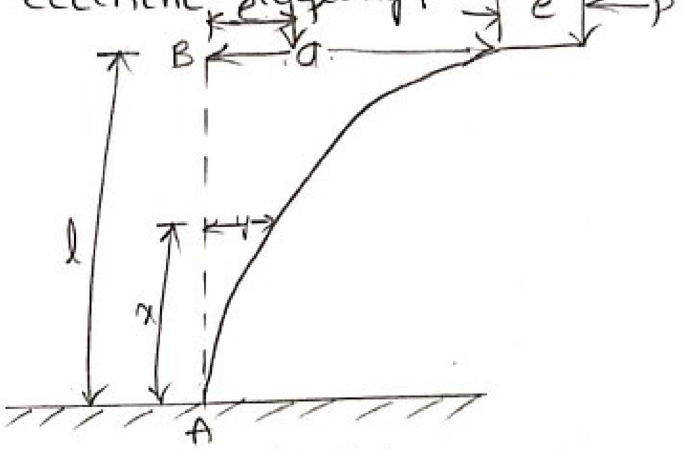
→ columns with eccentric loading :-



A column AB of length 'l' fixed at one end 'B' and free at 'A'. A column is subjected to load 'P' which is an eccentric by amount of 'e'. The free end sway side ways by amount of 'a' and the column will deflect.

where,

a ⟶ deflection @ end 'B'.

e ⟶ eccentricity.

A ⟶ Area of the column.

consider any section 'x' from end 'A'.

Let y ⟶ deflection @ section

Moment $M = P(a+e-y) \longrightarrow (1)$

w.k.T

$$M = EI\frac{d^2y}{dx^2} \longrightarrow (2)$$

$$P(a+e-y) = EI\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{P}{EI}(a+e)$$

The final soln for above eqn is

$$y = C_1 \cos\left[x\sqrt{\frac{P}{EI}}\right] + C_2 \sin\left(x\sqrt{\frac{P}{EI}}\right] + (a+e) \longrightarrow (3)$$

$$\frac{dy}{dx} = -C_1 \sin\left(x\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} + C_2 \cos\left(x\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} + 0 \longrightarrow (4)$$

At A, $x=0$ & $y=0$, $\frac{dy}{dx}=0$ in (3)

$$0 = C_1(0) + (a+e)$$

$$\Rightarrow C_1 = -(a+e)$$

$$\Rightarrow C_2 = 0.$$

sub C_1 and C_2 values in (3)

$$y = -(a+e)\cos\left(x\sqrt{\frac{P}{EI}}\right) + (a+e) \longrightarrow (5)$$

At B, $x=l$, $y=a$ sub in (5)

$$a = -(a+e)\cos\left(l\sqrt{\frac{P}{EI}}\right) + (a+e)$$

$$\cancel{a} - \cancel{a} + e = -(a+e)\cos\left(l\sqrt{\frac{P}{EI}}\right)$$

$$a+e = \frac{e}{\cos\left(l\sqrt{\frac{P}{EI}}\right)} \quad \text{(or)}$$

$$\boxed{a+e = e\sec\left(l\sqrt{\frac{P}{EI}}\right)}$$

→ Max stress:-

$$\sigma_{max} = \sigma_c + \sigma_b$$

where

σ_c = crushing stress (or) direct stress $\Rightarrow \frac{P}{A}$

σ_b = bending stress

so, the σ_{max} will be at the section where B.M will be max.

B.M max at fixed support i.e, at end A then the moment $M = P(a+e)$

$$M = P.e\sec\left(l\sqrt{\frac{P}{EI}}\right)$$

But WKT BM eqn in terms stress

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$M = \frac{\sigma_b I}{y}$$

$$\sigma_b = \frac{P.e\sec\left(l\sqrt{\frac{P}{EI}}\right) \cdot y}{I}$$

But

$$Z = \frac{I}{y} \Rightarrow \frac{y}{I} = \frac{1}{Z}$$

$$\sigma_b = \frac{P.e.\sec\left(\sqrt{\frac{P}{EI}} \cdot l\right)}{Z}$$

$$\sigma_{max} = \sigma_c + \sigma_b$$

$$= \frac{P}{A} + \frac{P.e \sec\left(l\sqrt{\frac{P}{EI}}\right)}{Z}$$

sub $l_e = 2l$ in above eqn

$$\sigma_{max} = \frac{P}{A} + \frac{P.e \sec\left(\frac{l_e}{2} \cdot \sqrt{\frac{P}{EI}}\right)}{Z}$$

problems:-

Q) A column of circular section is subjected to a load of 120kN. The Load is $\parallel^{rl}$ to the axis but eccentric by an amount of 2.5mm. The external and Internal dia of column are 60mm and 50mm respectively. If both ends of the column are fixed and column is 2.1m long. Then determine Max stress in the column $E = 200$ Gpa.

sol:- Given data

$P = 120$ kN

$e = 2.5$ mm

$D = 60$ mm

$d = 50$ mm

$L = 2.1$ m [fixed end so, $l = \frac{l_e}{2}$]

$$e = 200\times10^{5}\ N/mm^2$$

$$\sigma_{max} = \frac{P}{A} + \frac{P.e\sec\left(\frac{l_e}{2}\sqrt{\frac{P}{EI}}\right)}{Z}$$

$$A = \frac{\pi}{4}(60^2-50^2) = 863.93\ mm^2$$

$$= \frac{120\times10^3}{863.93} + \frac{120\times10^3\times2.5\times\sec\left(\frac{2100}{2}\times\sqrt{\frac{P}{EI}}\right)}{Z}$$

$$I = \frac{\pi}{4}(60^4-50^4) = 329.37\times10^6\ mm^4$$

$$\sigma_{max} = \frac{120\times10^3}{863.93} + \frac{120\times10^3\times2.5\times\sec\left(\frac{2100}{2}\right)\times\sqrt{\frac{12\times10^3}{200\times10^5\times5.27\times10^6}}}{\frac{5.27\times10^6}{30}}$$

$$= 138.900 + \frac{25.78\times10^6}{10.979\times10^3}.$$

=

Straight line Method:-

The Euler's formular and Rankine's formula gives only approximate value's of crippling load due to following reasons.

* The pin joints are ~~an~~ not practically friction less.

* The end fixation is never perfectly rigid.

* In case of Euler's formula, the effect of direct compression is neglected.

* The load is not exactly applied as designed.

* The members are never perfectly straight and uniform is section.

* The material of the member is not homogeneous and isotrophic.

On the account of this, the emperical straight line formula are commonly used in pratical designing.

$$\boxed{P = (\sigma_c . A) - \eta \left(\frac{l_e}{k}\right) A}$$

where,

$P \rightarrow$ crippling load on the column.

$\sigma_c \rightarrow$ compressive yield stress.

$A \rightarrow$ c/sn Area of the column.

$\frac{l_e}{k} \rightarrow$ slenderness ratio

$\eta \rightarrow$ a constant whose value depends upon the material of the column.

In the above eqn, If 'p' is plotted against '$\frac{l_e}{k}$', we will get a straight line and hence above eqn represents the straight line eqn (or) formula.

$$\boxed{\frac{P}{A} = \sigma_c \cdot A - \eta\left(\frac{l_e}{k}\right)}$$

→ **prof. PERRY's FORMULA:-**

In case where we have to determine safe load that can be applied on a column that a given eccen--tricity.

$\sigma_0 \rightarrow$ stress due to direct load $= \frac{P}{A}$

$\sigma_{max} \rightarrow$ permissiable stress

$l_e \longrightarrow$ effective length

$\sigma_b \rightarrow$ Max. compressive stress due to B.M.

from the bending eqn.

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma_b = \frac{M y_c}{A k^2}$$

where,

$y_c \longrightarrow$ distance from N.A to outermost layer in compression.

$M \longrightarrow P_e \sec\left(\frac{l_e}{2}\right)\sqrt{\frac{P}{EI}}$

$$\sigma_b = \frac{P_e \sec\left(\frac{l_e}{2}\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right) \times y_c}{Ak^2}$$

$$\sigma_b = \frac{P_e \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{euler}}} \times \frac{180}{\pi}\right) \times y_c}{Ak^2}$$

from euler's formula.

$$P = \frac{\pi^2 EI}{le^2}$$

$$= \frac{P_e \sec\left[\frac{\pi}{2}\sqrt{\frac{P}{\frac{\pi^2 EI}{le^2}}} \times \frac{180}{\pi}\right] \times y_c}{Ak^2}$$

$$= \frac{P_e \sec\left(\frac{\pi}{2}\frac{\sqrt{P}\, l_e}{\pi\sqrt{EI}} \times \frac{180}{\pi}\right) \times y_c}{Ak^2}$$

$$\sigma_b = \frac{P \times e \times \sec\left(\frac{l_e}{2}\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right) \times y_c}{Ak^2}$$

$$\sigma_0 = \frac{P}{A}$$

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$= \frac{P}{A} + \frac{P_e \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{euler}}} \times \frac{180}{\pi}\right) \times y_c}{Ak^2}$$

$$\sigma_{max} = \frac{P}{A}\left[\frac{1+e\sec\frac{\pi}{2}\sqrt{\frac{P}{P_{euler}}}\times y_c}{k^2}\right]$$

According to perry's formula.

$$\sec\frac{\pi}{2}\sqrt{\frac{P}{P_E}} = \frac{1.2P_E}{P_E - P} \quad \text{[Approximately]}$$

$$\sigma_E = \frac{P_E}{A} \; ; \; \sigma_0 = \frac{P}{A}$$

$$P_E = \sigma_E \cdot A \; ; \; P = \sigma_0 \cdot A$$

$$\frac{1.2\sigma_E A}{\sigma_E \cdot A - \sigma_0 A} = \frac{1.2\sigma_E}{\sigma_E - \sigma_0}$$

$$\sigma_{max} = \sigma_0\left[1+\frac{ey_c}{k^2}\cdot\frac{1.2\sigma_E}{\sigma_E-\sigma_0}\right]$$

$$\frac{\sigma_{max}}{\sigma_0} - 1 = \frac{ey_c}{k^2}\,\frac{1.2\sigma_E}{\sigma_E-\sigma_0}$$

$$\boxed{\left(\frac{\sigma_{max}}{\sigma_0}-1\right)\left(\frac{\sigma_E-\sigma_0}{\sigma_E}\right) = \frac{1.2ey_c}{k^2}}$$

→ Laterally ($\perp^r$) loaded structures

Columns carrying axially compressive loads. If the columns are also subjected to transverse loads, then they are called beam column.

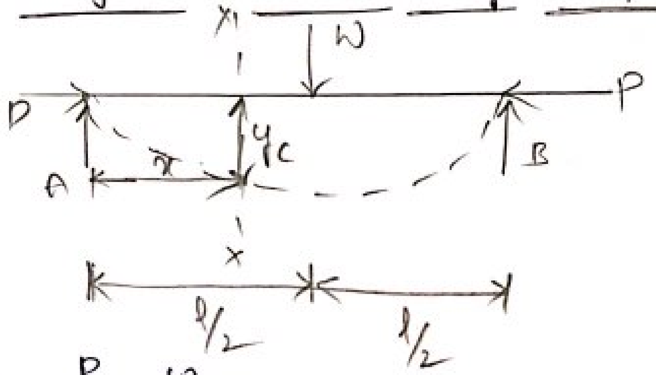
The traverse load is generally uniformly distributed

But.

* Transverse load is generally uniformly a point load and acts at the centre.

* Transverse load is uniformly distributed.

Struct subjected to axially compressive load:-



$R_A = R_B = \frac{W}{2}$

$$M = -Py = -\frac{W}{2}x \longrightarrow ①$$

Then WKT

$$M = \frac{d^2y}{dx^2} \cdot EI \longrightarrow ②$$

Equating ① & ②

$$-Py - \frac{W}{2}x = \frac{d^2y}{dx^2} EI$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{-W}{2EI} \cdot x$$

The final eqn for above eqn

$$y = C_1 \cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \sin\left(x\sqrt{\frac{P}{EI}}\right) - \frac{Wx}{2P} \longrightarrow ③$$

$$\frac{dy}{dx} = -C_1 \sin\left(x\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} + C_2 \cos\left(x\sqrt{\frac{P}{EI}}\right)\sqrt{\frac{P}{EI}} - \frac{W}{2P} \longrightarrow (4)$$

At A, $x=0$, $y=0$ in eqn (3)

$C_1 = 0$

At C, $x = \frac{l}{2}$, $\frac{dy}{dx} = 0$

$$C_2 = \frac{W}{2P}\sqrt{\frac{EI}{P}} \times \frac{1}{\cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)}$$

Sub C_1 and C_2 in (3)

$$y = \frac{W}{2P}\sqrt{\frac{EI}{P}} \times \frac{1}{\cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)} \times \sin\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right) - \frac{WL}{4P}$$

$x = \frac{l}{2}$, $y = y_{max}$

$$\boxed{y_{max} = \frac{W}{2P}\sqrt{\frac{EI}{P}} \times \tan\left[\frac{l}{2}\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right] - \frac{WL}{4P}}$$

Max B.M :-

The max B.M occurs at middle of the section so,

$y = y_{max}$ and $x = l/2$.

$$M = -\left(P y_{max} + \frac{W}{2}x\right)$$

$$M = -\left[P \frac{W}{2P}\sqrt{\frac{EI}{P}} \times \tan\left[\frac{l}{2}\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right] - \frac{WL}{4P} + \frac{Wl}{4}\right]$$

$$= -\frac{W}{2}\sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right) \cancel{- \frac{Wl}{4P}} + \cancel{\frac{Wl}{4}}\Big]$$

$$\boxed{M = -\frac{W}{2}\sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right)}$$

-ve sign due to sign convention.

Hence the magnitude of the max. B.M is

$$M = -\frac{W}{2}\sqrt{\frac{EI}{P}} \times \tan\left(\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right)$$

$$\boxed{M = -\frac{W}{2}\sqrt{\frac{EI}{P}} \times \tan\left[\frac{l}{2}\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right]}$$

Max. stress:-

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$\sigma_0 = \frac{P}{A} \; ; \; \sigma_b = \frac{My}{Ak^2}$$

$$\sigma_b = \frac{\frac{W}{2}\sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right)}{Ak^2}$$

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{\frac{W}{2}\sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right)}{Ak^2}}$$

problems:-

Q) Determine the max. stress induced in a cylindrical steel strut of length 1.2m. And dia 30mm. The strut is hinged at both ends and subjected to an axial thrust of 20kN at its ends.

And transverse point load of 1.8KN at centre.

$E = 208$ GPa ?.

Sol:-

Given data

$L = 1.2m \Rightarrow 1200mm$

$d = 30mm$

$P = 20KN \Rightarrow 20 \times 10^3 N$

$\omega = 1.8KN \Rightarrow 1.8 \times 10^3 N$

$E = 208\ Gpa \Rightarrow 208 \times 10^3\ N/mm^2$.

$$A = \frac{\pi}{4} \times 30^2$$
$$= 706.85\ mm^2$$

$$I = \frac{\pi}{64} \times 30^4$$
$$= 39.76 \times 10^3\ mm^4$$

$$\sigma_{max} = \frac{P}{A} + \frac{My}{I}$$

$$M = \frac{\omega}{2}\sqrt{\frac{EI}{P}} \times \tan\left(\frac{l}{2}\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right)$$

$$= \frac{1.8 \times 10^3}{2}\sqrt{\frac{208 \times 10^3 \times 39.76 \times 10^3}{20 \times 10^3}} \times \tan\left(\frac{1200}{2}\sqrt{\frac{20 \times 10^3}{208 \times 10^3 \times 39.76 \times 10^3}}\right)$$

$$= 5.78 \times 10^5 \times 1.628 \times 10^{-3}$$

$$\boxed{M = 9.411 \times 10^3\ N\text{-}mm}$$

$$\sigma_{max} = \frac{20 \times 10^3}{706.85} + \frac{9.411 \times 10^3 \times 15}{39.76 \times 10^3} \qquad [\because y = 15m$$

$$\boxed{\sigma_{max} = 322.56\ N/mm^2}$$

Q) A steel tube having 38mm outer dia, 66mm inner dia and 2.8m long is used as a struct with both ends hinged. The load is parallel to axis of the struct but it is eccentric. Find the max value of eccentricity so that crippling load on the struct is 60% of euler's crippling load. Take $E = 210\, GN/m^2$ and yield strength $320\, MN/m^2$?

sol:- Given data

$D = 88\, mm$

$d = 6.6\, mm$

$L = 2.8m \Rightarrow 2800mm$

$P = 60\%\, P_E$

$E = 210\, GN/m^2 \Rightarrow 210 \times 10^3\, N/mm^2$

$\sigma_{max} = 320\, MN/m^2 \Rightarrow 320\, N/mm^2$

$A = \frac{\pi}{4}(88^2 - 66^2)$; $I = \frac{\pi}{64}(88^4 - 66^4)$

$= 2660.92\, mm^2$; $= 2.01 \times 10^6\, mm^4$

$$P_E = \frac{\pi^2 E I}{L_e^2} \qquad [L_e = L]$$

$$= \frac{\pi^2 \times 210 \times 10^3 \times 2.01 \times 10^6}{2800^2}$$

$P_E = 531.37 \times 10^3\, N$

$P = 60\%$ of P_E

$$P = 531.37 \times 10^3 \times 60\%$$

$$\boxed{P = 318.82 \times 10^3\ N}$$

$$\sigma_0 = \frac{P}{A} = \frac{318.82 \times 10^3}{2660.92}$$

$$\boxed{\sigma_0 = 119.81\ N/mm^2}$$

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$\sigma_b = \frac{Pe\sec\left(\frac{le}{2}\sqrt{\frac{P}{EI}} \times \frac{180}{\pi}\right) \times y_c}{AK^2} \qquad K^2 = \frac{I}{A} = 755.37$$

$$= \frac{318.82 \times 10^3 \times e \times \sec\left(\frac{2800}{2}\sqrt{\frac{318.82 \times 10^3}{210 \times 10^3 \times 2.01 \times 10^6}} \times \frac{180}{\pi}\right) \times 44}{2660.92 \times 755.37}$$

$$= \frac{318.82 \times 10^3 \times e \times \sec(1400 \times 0.049) \times 44}{2.00 \times 10^6}$$

$$= \frac{318.82 \times 10^3 \times e \times 2.76 \times 44}{2.00 \times 10^6}$$

$$\sigma_b = 19.35e$$

$$\sigma_{max} = 119.81 + 19.35e$$

$$320 = 139.16e$$

$$e = \frac{320}{139.16} \Rightarrow \boxed{e = 2.29\ mm}$$

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UNIT-III

DIRECT AND BENDING STRESSES

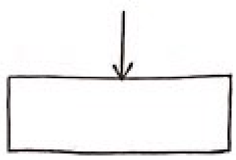
* Direct stress :-

Direct stress alone is produced in a body when it is subjected to an axial tensile <or> compressive load.

* Bending stress:-

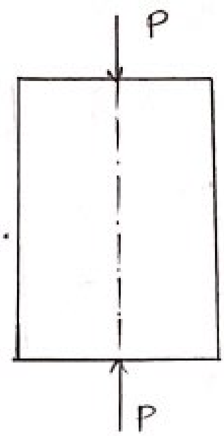
It is produced in a the body, when it is subjected to a bending moment.

* But, if a body is subjected to axial loads and also BM, Then both the stresses (i.e. bending & direct stresses) will be produced in the body.
* Both these stresses act normal to a c/s, hence the two stresses may be horizontally added into a single resultant stress.



* Combined bending & direct stresses :-

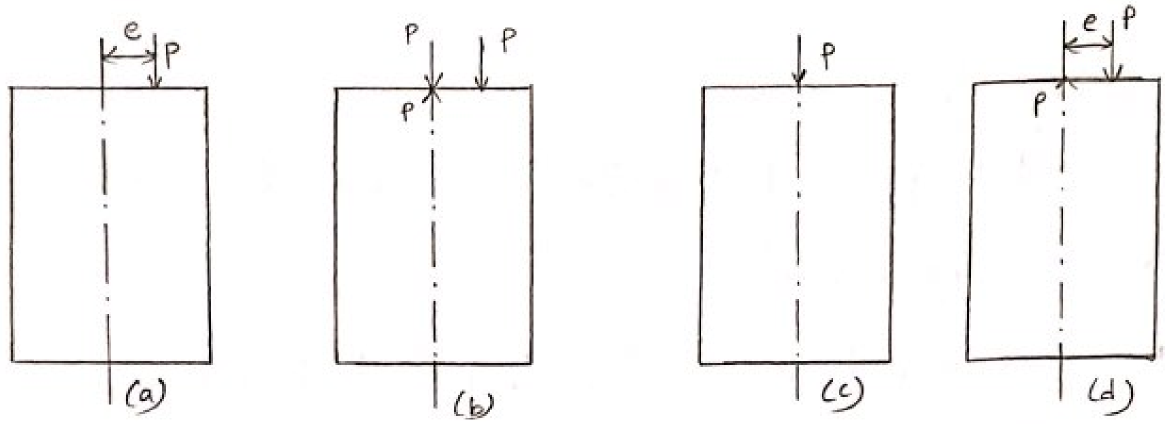
* Consider a column subjected by a compressive load (P) acting along the axis of the column. This load will cause a direct compressive stress. whose intensity will be uniform across the c/s of the column.



$$\sigma_0 = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

where, σ_0 = Intensity of stress

A = Area of c/s.



* Now, consider the case of a column subjected by compressive load (P) whose line of action is at a distance of e from the axis of column.

Here, e is known as eccentricity of the load. The eccentric load will cause direct stress & bending stress.

1) (b) we have applied, along the axis of column, two equal and opposite forces p. Thus 3 forces are acting now on the column.

2) (c) The force is acting along the axis of column and hence this force will produce a direct stress.

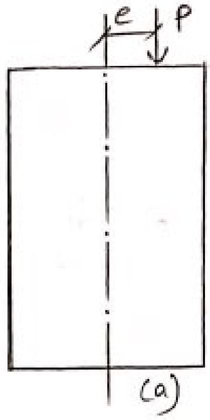
3) (d) The forces will form a couple, whose moment will be $P \times e$. This couple will produce a bending stress.

⇒ Hence, an eccentric load will produce a direct stress as well as bending stress.

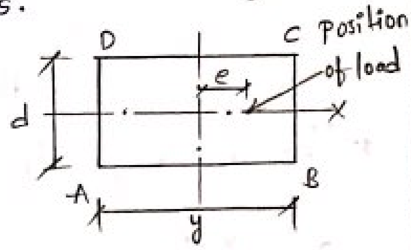
* Resultant stress when a column of rectangular section is subjected to an eccentric load :-

* A column of rectangular section is subjected to an eccentric load.

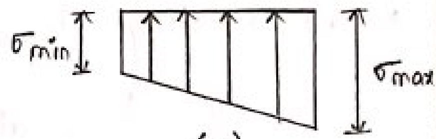
* Let, the load is eccentric with respect to an axis y-y. That an eccentric load causes direct stress as well as bending stress.



(a)



Plan (b)



(c)

* Let, P = Eccentric load on column

e = Eccentricity of the load.

σ_0 = direct stress

σ_b = bending stress

b = width of the column

d = depth of the column.

Area, $A = bd$

* Now, Moment due to eccentric load, 'P' given by

$$M = P \times e$$

* The direct stress is given by $\sigma_0 = \frac{P}{A}$

* This stress is uniform along the c/s of the column.

* The bending stress σ_b due to moment at any point of the column section at a distance y from the neutral axis Y-Y.

$$\frac{M}{I} = \frac{\sigma_b}{\pm y}$$

$$\sigma_b = \frac{\pm My}{I}$$

I = M.O.I of column section about N.A Y-Y

$$= \frac{db^3}{12}$$

Sub. in above

$$\sigma_b = \frac{\pm 12My}{db^3}$$

* The bending stress depends upon the value of y from axis Y-Y.

* The bending stress at extreme is obtained by

$y = \frac{b}{2}$ in above eqn

$$\sigma_b = \pm\frac{6M}{db^2}$$

$\therefore$ Sub. $M = P \times e$

$$\sigma_b = \frac{6Pe}{db^2}$$

Hence, $A = b \times d$ sub. in above

$$\boxed{\sigma = \frac{6Pe}{Ab}}$$

* The resultant stress at any point will be alzebraic sum of σ_0, σ_b.

* Iyb y is taken as '+ve' on the same side of Y-Y as the load, then bending stress will be of same type of the direct stress. Here, direct stress is compressive and thence bending stress will also be compressive towards the right of the axis Y-Y.

* Similarly bending stress will be tensile towards the left of the axis Y-Y. Taking compressive load as +ve and tensile load as -ve. we can find max. & min. stress at extremities of the section.

* The stress will be max. along BC min. along AD.

Then, σ_{max} = direct stress + bending stress

$$= \frac{P}{A} + \frac{6pe}{Ab}$$

$$= \frac{P}{A}\left[1 + \frac{6e}{b}\right]$$

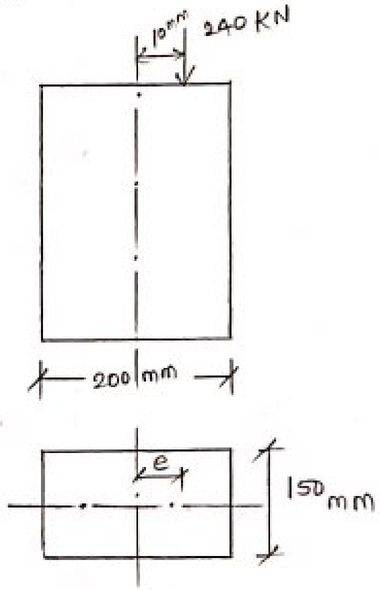
σ_{min} = direct stress − bending stress

$$= \frac{P}{A}\left[1 - \frac{6e}{b}\right]$$

* The resultant stress along the width of the column will varied by a strain line law.

* σ_{min} is -ve, then stress along the layer AD will be tensile. If σ_{min} is 0, then there will be no tensile stress along the width of the column. If σ_{min} is +ve then there will be only compressive stress along the width of the column.

1) A Rectangular column of width 200mm and of thickness 150mm carries a point load of 240 KN at an eccentricity of 10mm. Determine the max. & min. stresses on the section.



Sol:-

$$A = bd = 200 \times 150$$

$$= 30 \times 10^3 \ mm^2$$

$$e = 10 \ mm$$

$$P = 240 \times 10^3 \ N$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{6e}{b}\right]$$

$$= \frac{240 \times 10^3}{30 \times 10^3}\left[1 + \frac{6 \times 10}{200}\right]$$

$$= 10.4 \ N/mm^2$$

$$\sigma_{min} = \frac{P}{A}\left[1 - \frac{6e}{b}\right]$$

$$= \frac{240 \times 10^3}{30 \times 10^3}\left[1 - \frac{6 \times 10}{200}\right]$$

$$= 5.6 \ N/mm^2.$$

For the above problem min. stress on the section is given 0. Then find eccentricity of the point load 240 KN acting on the rectangular column also calculate the corresponding max. stress on the section.

Sol:-

$$\sigma_{min} = \frac{P}{A}\left[1 - \frac{6e}{b}\right]$$

$$0 = \frac{240 \times 10^3}{30 \times 10^3}\left[1 - \frac{6e}{200}\right]$$

$$\frac{6e}{200} = 1 \Rightarrow e = \frac{200}{6} = 33.3 \text{ mm}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{6e}{b}\right] = \frac{240 \times 10^3}{30 \times 10^3}\left[1 + \frac{6 \times 33.33}{200}\right]$$

$$= 15.9 \text{ N/mm}^2.$$

For the previous problem the e is given 50 mm instead of 10mm. Then find max. & min. stresses on the section. Also plot these stresses along the width of the section.

Sol:-

$$\sigma_{max} = \frac{240 \times 10^3}{30 \times 10^3}\left[1 + \frac{6 \times 50}{200}\right]$$

$$= 20 \text{ N/mm}^2$$

$$\sigma_{min.} = \frac{240 \times 10^3}{30 \times 10^3}\left[1 - \frac{6 \times 50}{200}\right]$$

$$= -4 \text{ N/mm}^2$$

'-ve' sign means tensile stress.

Note:- The min. stress is '0' when $e = \frac{b}{6}$ mm.

* The min. stress is '+'ve (Compressive) when $e > \frac{b}{6}$.

* The min. stress is '−'ve (tensile) when $e < \frac{b}{6}$.

2) The line of thrust, in a compression test on specimen 15mm diameter, is parallel to the axis of specimen but is displaced from it. Calculate the distance of the line of thrust from the axis when the max. stress is 20% greater than the mean stress on a normal section.

Sol:-

$$d = 15 \text{ mm}$$

$$A = \frac{\pi}{4}(15)^2 = 176.7 \text{ mm}^2$$

$$M = P \times e$$

Now, the bending stress

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{My}{I}$$

Max. bending stress will be when $y = \pm \frac{d}{2}$

Hence, the max. bending stress is given by

$$\sigma_b = \frac{M}{I}\left(\pm \frac{d}{2}\right) = \pm \frac{M}{I}\left(\frac{d}{2}\right)$$

$$= \pm \frac{M}{\frac{\pi}{64} d^4} \times \frac{d}{2} = \pm \frac{32M}{\pi d^3}$$

$$\sigma_b = \pm \frac{32Pe}{\pi d^3}$$

Direct stress due to load

$$\sigma_0 = \frac{P}{A} = \frac{P}{176.714}$$

Max. stress = Direct stress + Bending stress

$$= \sigma_0 + \sigma_b$$

$$\sigma_{max} = \frac{P}{176.714} + \frac{32Pe}{\pi d^3}$$

we know,

$$\sigma_{max} = 1.2 \times \text{mean stress}$$

$$= 1.2 \times \frac{P}{176.714} \quad \text{sub in } ①$$

$$1.2 \times \frac{P}{176.714} = \frac{P}{176.714} + \frac{32Pe}{\pi d^3}$$

$$\frac{1.2}{176.714} = \frac{1}{176.714} + \frac{32e}{\pi (15)^3}$$

$$\frac{32e}{\pi (15)^3} = \frac{100}{88357}$$

$$e = 0.37 \text{ mm}$$

3) A hollow rectangular column of external depth 1m and external width 0.8m, e is 10cm thick. Calculate the max. & min. stress in the section of column if a vertical load of 200 KN is acting with an eccentricity of 15cm.

Sol:- Given data:-

$D = 1m = 1000\ mm$

$B = 0.8\ m = 800mm$

$d = D - 2t = 1000 - 2(100) = 800mm$

$b = B - 2t = 800 - 2(100) = 600\ mm$

$A = BD - bd = 1000 \times 800 - 800 \times 600 = 32 \times 10^4\ mm^2$

$e = 15\ cm = 150\ mm$

$P = 200 \times 10^3\ N$

σ_{max} $\quad I = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{800 \times 1000^3}{12} - \frac{600 \times 800^3}{12}$

$= 4.106 \times 10^{10}\ mm^4$

$\sigma_b = \frac{My}{I} = \frac{Pe \times y}{I} = \frac{200 \times 10^3 \times 150 \times \frac{1000}{2}}{4.106 \times 10^{10}}$

$= 0.365\ N/mm^2$

$\sigma_0 = \frac{P}{A} = \frac{200 \times 10^3}{32 \times 10^4} = 0.625\ N/mm^2$

$\sigma_{max} = \sigma_0 + \sigma_b$

$= 0.625 + 0.365$

$= 0.99\ N/mm^2$

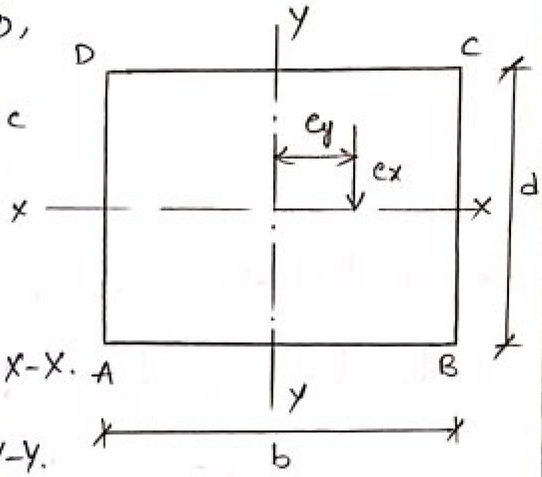
$\sigma_{min} = \sigma_0 - \sigma_0$

$= 0.625 - 0.365$

$= 0.26\ N/mm^2$

* Resultant stress when a column of rectangular section is subjected to a load which is eccentric to both axis:-

* A column of rectangular section ABCD, subjected to a load which is eccentric both axis.



* Let, P = Eccentric load on column

e_x = Eccentricity of load about X-X.

e_y = Eccentricity of load about Y-Y.

b = width of column

d = depth of column

σ_0 = direct stress (due to ec)

σ_{bx} = bending stress due to eccentricity e_x.

σ_{by} = bending stress due to eccentricity e_y.

M_x = Moment of load about x-x axis

$= P \times e_x$

M_y = Moment of load about y-y axis

$= P \times e_y$

I_{xx} = Moment of Inertia about x-x axis $= \frac{bd^3}{12}$

I_{yy} = M.O.I about y-y axis $= \frac{db^3}{12}$

* The direct stress, $\sigma_0 = \frac{P}{A}$

* The bending stress due to eccentricity ey is given by

$$\sigma_{by} = \frac{M_y \times x}{I_{yy}}$$

$$= \frac{P \times e_y \times x}{I_{yy}}$$

* In the above eqn x - varies from $-\frac{b}{2}$ to $+\frac{b}{2}$.
* The bending stress due to eccentricity e_x is given by,

$$\sigma_{bx} = \frac{M_x \times y}{I_{xx}} = \frac{P \times e_x \times y}{I_{xx}}$$

* In the above eqn y varies from $+\frac{d}{2}$ to $-\frac{d}{2}$.
* The resultant stress at any point on the section

$$= \sigma_0 \pm \sigma_{by} \pm \sigma_{bx}$$

$$= \frac{P}{A} \pm \frac{P \times e_y \times x}{I_{yy}} \pm \frac{P \times e_x \times y}{I_{xx}}$$

$$= \frac{P}{A} \pm \frac{M_y \times x}{I_{yy}} \pm \frac{M_x \times y}{I_{xx}}$$

* At pointc c the coordinates x & y are positive. Hence, the resultant stress will be max.
* At point A, the coordinates x & y are negative then the resultant stress will be min.

* At the point B, x is +ve & y is −ve. Hence resultant stress will be

$$\sigma_0 + \sigma_{by} - \sigma_{bx}$$

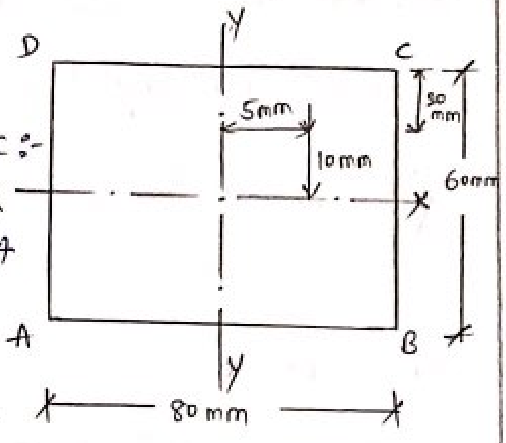
$$\frac{P}{A} + \frac{M_y \times x}{I_{yy}} - \frac{M_x \times y}{I_{xx}}$$

* At the point D, x is −ve, & y is +ve. Hence the resultant stress will be

$$\sigma_0 - \sigma_{by} + \sigma_{bx}$$

$$\frac{P}{A} - \frac{M_y \times x}{I_{yy}} + \frac{M_x \times y}{I_{xx}}$$

1) A short column of rectangular cross section 80 mm × 60mm carries a load of 40 kN at a point 20 mm from the longer side & 35mm from the shorter side. Determine max. compressive & min. tensile stresses in the section.



Sol: Max. Compressive ~~load~~ stress at point C:-

$$I_{xx} = \frac{bd^3}{12} = \frac{80 \times 60^3}{12} = 144 \times 10^4 \text{ mm}^4$$

$$I_{yy} = \frac{db^3}{12} = \frac{60 \times 800^3}{12} = 256 \times 10^4 \text{ mm}^4$$

$$\sigma_{max.c} = \sigma_0 + \sigma_{by} + \sigma_{bx}$$

$$= \frac{P_1}{A} + \frac{P \times e_y \times x}{I_{yy}} + \frac{P \times e_x \times y}{I_{xx}}$$

$$= \frac{40\times10^3}{60\times80} + \frac{40\times10^3\times5\times40}{256\times10^4} + \frac{40\times10^3\times10\times30}{144\times10^4}$$

$$= 19.7\ N/mm^2$$

Max. Tensile stress at point A :

$$\sigma_{max.T} = \sigma_0 - \sigma_{by} - \sigma_{bx}$$

$$= \frac{40\times10^3}{60\times80} - \frac{40\times10^3\times5\times40}{256\times10^4} - \frac{40\times10^3\times10\times30}{144\times10^4}$$

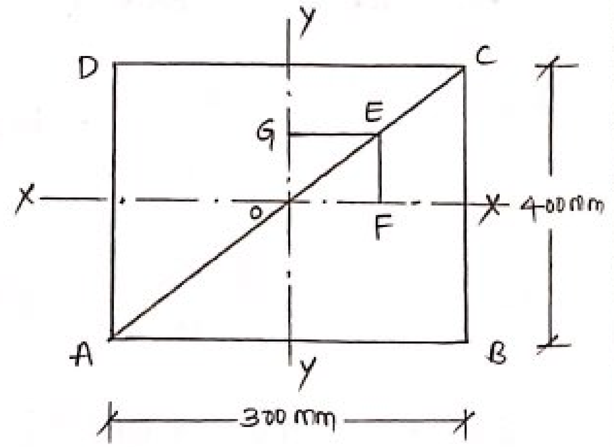
$$= -3.125\ N/mm^2.$$

'-ve' indicates tensile.

2) A column is rectangular in c/s of 300 mm x 400 mm in dimensions. The column carries an eccentric point load of 360 KN on one diagonal at a distance of quarter diagonal length from a corner. Calculate the stresses at all corners. Draw stress distribution diagrams for any two adjacent sides.

Sol:- $AC = \sqrt{AB^2+BC^2} = \sqrt{300^2+400^2}$

$= 500$ mm



In ΔCAB,

$\tan\theta = \frac{400}{300} = \frac{4}{3}$, $\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{300}{500} = \frac{3}{5}$

$OE = EC = \frac{1}{4}AC = \frac{1}{4}\times500 = 125mm$

$$e_x = EF = OE\sin\theta = 125 \times \frac{4}{5} = 100\ mm$$

$$e_y = OE\cos\theta = 125 \times \frac{3}{5} = 75\ mm$$

$$I_{xx} = \frac{BD^3}{12} = \frac{300 \times 400^3}{12} = 16 \times 10^8\ mm^4$$

$$I_{yy} = \frac{DB^3}{12} = \frac{400 \times 300^3}{12} = 9 \times 10^8\ mm^4.$$

At point C: $x \to +ve,\ y \to +ve$

$$\sigma_{max} = \frac{P}{A} + \frac{P \times e_y \times x}{I_{yy}} + \frac{P \times e_x \times y}{I_{xx}}$$

$$= \frac{360 \times 10^3}{300 \times 400} + \frac{360 \times 10^3 \times 75 \times 150}{9 \times 10^8} + \frac{360 \times 10^3 \times 100 \times 200}{16 \times 10^8}$$

$$= 12\ N/mm^2$$

At point D:- $x \to -ve,\ y \to +ve$

$$\sigma_{max} = \frac{360 \times 10^3}{300 \times 400} - \frac{360 \times 10^3 \times 75 \times 150}{9 \times 10^8} + \frac{360 \times 10^3 \times 100 \times 200}{16 \times 10^8}$$

$$= 3\ N/mm^2.$$

At point A:- $x \to -ve,\ y \to -ve$

$$\sigma_{max} = \frac{360 \times 10^3}{300 \times 400} - \frac{360 \times 10^3 \times 75 \times 150}{9 \times 10^8} - \frac{360 \times 10^3 \times 100 \times 200}{16 \times 10^8}$$

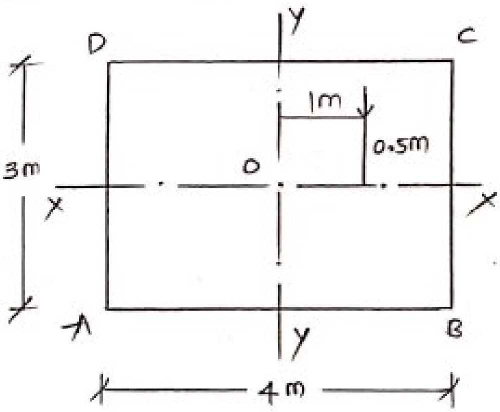
$$= -6\ N/mm^2.$$

At point B :- $x \rightarrow +ve$, $y \rightarrow -ve$

$$\sigma_{max} = \frac{360\times10^3}{12\times10^4} + \frac{360\times10^3\times75\times150}{9\times10^8} - \frac{360\times10^3\times100\times200}{16\times10^8}$$

$$= 3\ N/mm^2.$$

3) A masonry pire of 4m x3m supports a vertical load of 80KN
(i) Find the stresses developed at each corner of pire.
(ii) what additional load should be placed at the centre of the pire. so, there is no tension any where in the pire section.
(iii) What are the stresses at the corners with the additional load in the centre.



Sol:-

$P = 80\times10^3\ N$

$$I_{xx} = \frac{4\times3^2}{12} = 9\ m^4$$

$$I_{yy} = \frac{3\times4^3}{12} = 16\ m^4$$

At point A :- $x \rightarrow -ve$, $y \rightarrow -ve$

$$\sigma_{max} = \frac{80}{3\times4} - \frac{80\times1\times2}{16} - \frac{80\times0.5\times3/2}{9} = -10\ KN/m^2$$

At point B :- $x \rightarrow +ve$, $y \rightarrow -ve$

$$\sigma_{max} = \frac{80}{3\times4} + \frac{80\times1\times2}{16} - \frac{80\times0.5\times1.5}{9} = 10\ KN/m^2.$$

At point C :- $x \rightarrow +ve, \; y \rightarrow +ve.$

$$\sigma_{max} = \frac{80}{12} + \frac{80 \times 1 \times 2}{16} + \frac{80 \times 0.5 \times 1.5}{9} = 23.33 \text{ KN/m}^2$$

At point D :- $x \rightarrow -ve, \; y \rightarrow +ve$

$$\sigma_{max} = \frac{80}{12} - \frac{80 \times 1 \times 2}{16} + \frac{80 \times 0.5 \times 1.5}{9} = 3.33 \text{ KN/m}^2$$

ii) W = Compressive load additionally added at the centre for no tension any where in the pire.

* Load is compressive & will cause a compressive stress.

$$\therefore \frac{w}{A} = \frac{w}{12}$$

* As a load is placed at the centre it will produce a uniform compressive stress across the section of pire.
* But we know there is no tensile stress at a point A, having magnitude $= 10 \text{ KN/m}^2$.
* Hence, the com. stress due to load w should be equal to tensile stress. at A, $\frac{w}{12} = 10$

$$w = 120 \text{ KN}$$

$$\text{Compressive stress} = \frac{w}{12} = \frac{120}{12} = 10 \text{ KN/m}^2$$

stress due to additional load is 10 KN/m^2.

iii) At point A :

$$\sigma_{max} = -10 + 10 = 0 \ KN/m^2$$

At point B :

$$\sigma_{max} = 10 + 10 = 20 \ KN/m^2$$

At point C :

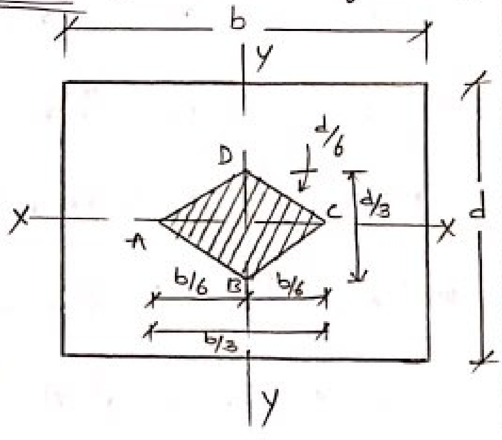
$$\sigma_{max} = 23.33 + 10 = 33.33 \ KN/m^2$$

At point D :

$$\sigma_{max} = 3.33 + 10 = 13.33 \ KN/m^2.$$

* Core <or> kernel :- The load may be applied any where so, as not to produce tensile stress in any part of the entire rectangular section is called core <or> kernel of the section.

* Middle Third Rule for rectangular sections (i.e. kernel of section) :-



* Cement concrete columns are weak in tension. Hence, the load must be applied on these columns in such a way that there is no tensile stress anywhere in the section but when an eccentric load acting on a column it will produces direct stress as well as bending stress.

* Consider a rectangular section of width 'b' & depth 'd'.

* Let the section is subjected to a load which is eccentric to the axis y-y.

* Let P = Eccentric load

 e = Eccentricity

 A = Area of section

* But we know the min. stress as $\sigma_{min} = \frac{P}{A}\left[1-\frac{6e}{b}\right]$

* σ_{min} is negative then the stress will be tensile. But if σ_{min} is zero <or> +ve then there will be no tensile stress along the width of the column.

Then, $\sigma_{min} \geq 0$

$$\frac{P}{A}\left[1-\frac{6e}{b}\right] \geq 0$$

$$1 - \frac{6e}{b} \geq 0$$

$$1 \geq \frac{6e}{b}$$

$$\boxed{e \leq \frac{b}{6}}$$

* The above result shows that eccentricity is must be $< \frac{b}{6}$. Hence the greatest eccentricity of the load is $\frac{b}{6}$ from the axis y-y. Hence the load is applied at any distance $< \frac{b}{6}$ from the axis any side of the axis y-y, the stresses are wholly compressive.

* Hence, the range with in which the load can be applied so, as not to produce any tensile stress, is with in the middle third of the base.

* Similarly, if the load had be eccentric w.r.to the axis X-X, the condition that tensile stress will not occur is when the eccentricity of the load w.r.to the axis X-X does not exceed $\frac{d}{6}$.

* Hence, the range with in which load may be applied is with in the middle third of the depth.

* If it is possible that load not likely to be eccentric about both axis X-X and Y-Y. The condition that tensile stress will not occur is when the load is applied anywhere with in the rhombus ABCD whose diagonals are $AC = b/3$ & $BD = \frac{d}{3}$ with in which the load may be applied anywhere so, as not to produce tensile stress in any part of the entire rectangular section is called core <or> kernel of the section.

<u>Note</u>:- If σ_0 is equal to σ_b then the tensile stress will be '0'.

* If $\sigma_0 > \sigma_b$ then the stress throughout the section will be compressive.

* $\sigma_0 < \sigma_b$ then there will be tensile stress.

* Hence, for no tensile stress, $\sigma_0 \geq \sigma_b$.

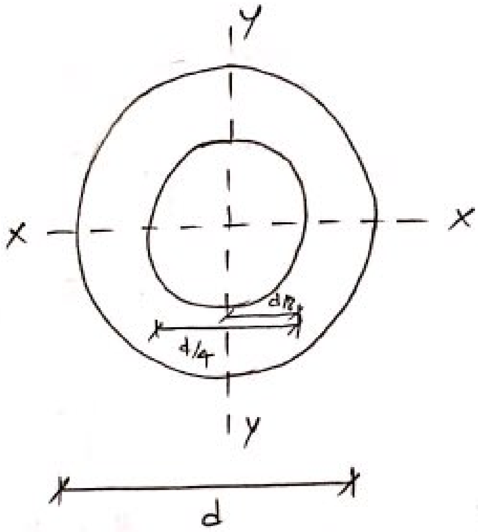
* Middle Quarter Rule for circular section :-

i.e, kernal section :-

d = diameter

p = Eccentric load

e = Eccentricity ~~load~~



$A = Area = \frac{\pi}{4} d^2$

$$\sigma_0 = \frac{4P}{\pi d^2}$$

But, $M = p \times e$

$$\sigma_b = \frac{My}{I}$$

Max. Bending stress will be when, $y = \pm \frac{d}{2}$

Max. Bending stress is given by

$$\sigma_b = \frac{M}{I}\left(\pm \frac{d}{2}\right)$$

$$= \pm \frac{p \times e \times \frac{d}{2}}{\frac{\pi}{64} \times d^4} = \pm \frac{32pe}{\pi d^3}$$

Now, Min. stress is given by,

$$\sigma_{min} = \sigma_0 - \sigma_b$$

$$= \frac{4P}{\pi d^2} - \frac{32pe}{\pi d^3}$$

For no tensile stress, $\sigma_{min} \geq 0$

$$\frac{4P}{\pi d^2} - \frac{32Pe}{\pi d^3} \geq 0$$

$$\frac{4P}{\pi d^2}\left[1-\frac{8e}{d}\right] = 0$$

$$1-\frac{8e}{d} \geq 0$$

$$\boxed{e \geq \frac{d}{8}}$$

* Above eqn. means that the load can be eccentric on any side of the centre of the circle by an amount $= \frac{d}{8}$.
* Thus, If the line of action of the load is with in a circle of diameter $= \frac{1}{4}$th of the main circle. Then the stress will be compressive throughout the circular section.

* Kernel of Hollow Circular section: <or> Value of Eccentricity for hollow circular section.

Let, D_o = External dia.

D_i = Internal dia

P = Eccentric load

A = Area of section

$= \frac{\pi}{4}(D_o^2 - D_i^2)$

$M = P \times e$

$$z = I/y_{max} = \frac{\frac{\pi}{64}\left[D_o^4 - D_i^4\right]}{\left(\frac{D_o}{2}\right)} = \frac{\pi}{32D_o}\left[D_o^4 - D_i^4\right]$$

$$\sigma_o = \frac{P}{A}$$

* The σ_o is compressive & uniform throughout the section.

$$\sigma_b = \frac{M}{Z}$$

* The σ_o may be compressive / Tensile.
* The resultant stress at any point is the alzebraic sum of direct & bending stress.
* There will be no tensile stress at any point if the bending is less than/Equal to σ_o at that point.

Hence, for no tensile stress

$$\sigma_b \le \sigma_o$$

$$\frac{M}{z} \le \frac{P}{A}$$

$$\frac{\cancel{P} \times e}{z} \le \frac{\cancel{P}}{A} \Rightarrow \boxed{e \le \frac{Z}{A}}$$

$$\le \frac{\frac{\pi}{32D_o}\left[D_o^4 - D_i^4\right]}{\frac{\pi}{4}\left[D_o^2 - D_i^2\right]}$$

$$\le \frac{4\pi}{32\pi D_o}\left[\frac{(D_o^2 + D_i^2)(D_o^2 - D_i^2)}{(D_o^2 - D_i^2)}\right]$$

$$\leq \frac{1}{8D_o}\left[D_o^2 + D_i^2\right]$$

* It means that the load can be eccentric, on any site of the centre of circle, by an amount equal to $\frac{(D_o^2+D_i^2)}{8D_o}$.

* Thus if the line of action of the load whith is a circle of dia. equal $\frac{(D_o^2+D_i^2)}{4D_o}$ then the stress will be compressive throughout.

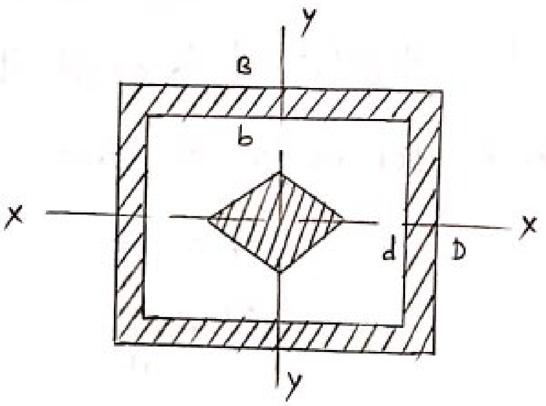
* Kernel of Rectangular section <or> Value of Eccentricity for hollow Rectangular section :-

B = Outlet width

b = Inlet width

d = Inner depth

D = Outer depth



$$A = BD - db$$

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$y_{max} = D/2$$

$$Z_{xx} = \frac{I_{xx}}{y_{max}} = \frac{\left(\frac{BD^3 - bd^3}{12}\right)}{D/2} = \frac{BD^3 - bd^3}{6D}$$

Similarly,

$$Z_{yy} = \frac{DB^3 - db^3}{6B}$$

For no tensile @ any s/n, the value of 'e' is given be eqn

$$e \leq \frac{Z}{A} \quad \langle or \rangle \quad e_{xx} \leq \frac{Z_{xx}}{A}$$

$$e \leq \frac{\left(\frac{BD^3 - bd^3}{6D}\right)}{BD - bd} \leq \frac{BD^3 - bd^3}{6D(BD - bd)}$$

Similarly, $e_y = \frac{DB^3 - db^3}{6B(BD - bd)}$

It means that load can be eccentric on either side of geometric axis by an amount equal to $\frac{BD^3 - bd^3}{6D(BD - bd)}$ & $\frac{DB^3 - db^3}{6B(DB - db)}$ along x axis & y-axis respectively.

* chimney's :- Chimney's are tall structures subjected to horizontal wind pressure. The base of the chimney's are subjected to bending moment due to horizontal wind force. The B-M at the base produces bending stresses. The base of the chimney is also subjected to direct stress due to self weight of the chimney. Hence, at the base of the chimney, bending stress, direct stresses are acting.

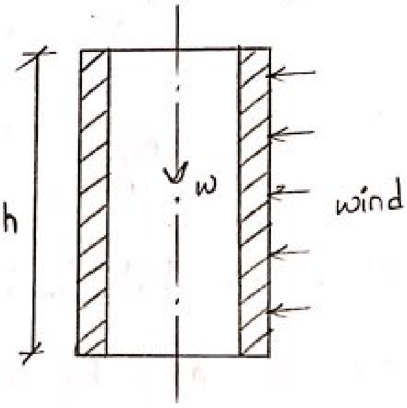
* The direct stress is given by $= \frac{\text{weight of the chimney}}{\text{Area of s/n at the base}}$

$$= \frac{W}{A}$$

$$\sigma_b = \frac{M}{Z} = \frac{My}{I}$$

* The wind force, F acting in the horizontal direction of the surface of the chimney is given by $F = K \times P \times A$

  K = Coefficient of wind resistance which depends upon the shape of area exposed to wind

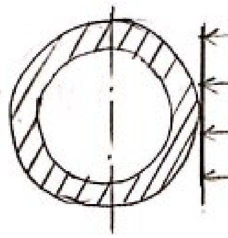


  $K = 1 \rightarrow$ for rectangular

  $K = \frac{2}{3} \rightarrow$ Circular

  $A = D \times h$ <or> $B \times h$

* The wind force F will acting at $\frac{h}{2}$.
* The moment of F at the base of the chimney is $F \times \frac{h}{2}$.



* Hence, BM, $F = F \times \frac{h}{2}$.

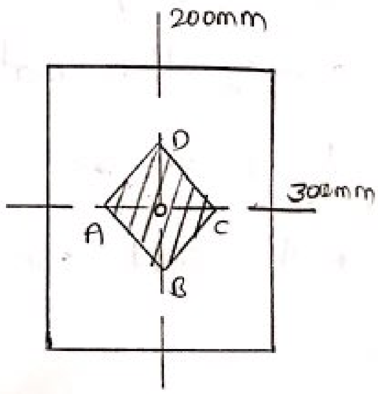
1) Draw neat sketch of kernal of the following cross-sections.

i) Rectangular section 200 mm x 300 mm.

ii) Hollow Circular cylinder with external dia. 300 mm thickness 50 mm.

iii) Kernel for square sec. 400 cm^2

iv) Hollow Rectangular section internal 100 x 150 mm.



Sol: i) The value of e for no tensile stress along width is given by $e \leq \frac{b}{6}$

$$e \leq \frac{200}{6} = 33.33 \text{ mm}$$

Take, $OA = OC = 33.33$ mm

The value of e = no tensile stress along the depth is

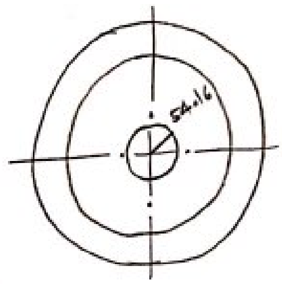
$$e \leq \frac{d}{6} = \frac{300}{6} = 50$$

$OD = OB = 50$ mm

ii) $D = 300$ mm

$d = 300 - 2(50) = 200$ mm

$$e \leq \frac{1}{8D_o}\left[D_o^2 + D_i^2\right] = \frac{1}{8 \times 300}\left[300^2 + 200^2\right]$$



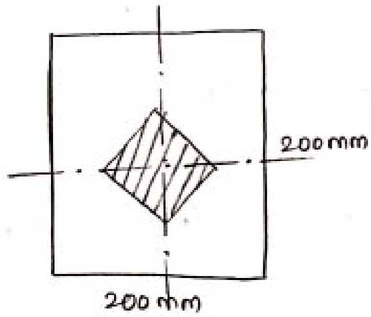
$e \leq 54.16$ mm

iii) $A = 400\text{cm}^2$

$a = \sqrt{400} = 20\text{cm} = 200$ mm

$$e \leq \frac{a}{6}$$

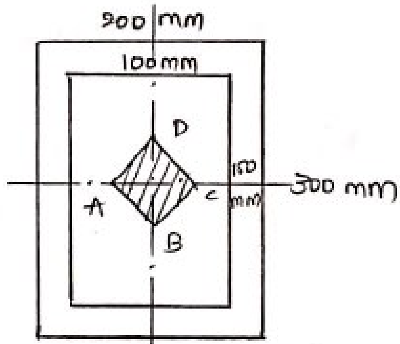
$$e \leq \frac{200}{6} = 33.33 \text{ mm}$$



iv) $$e_x \leq \frac{BD^3 - bd^3}{6D(BD - bd)}$$

$$e_x \leq \frac{200 \times 300^3 - 100 \times 15^3}{6 \times 300\,(200 \times 300 - 100 \times 150)}$$

$e_x \leq 62.5$ mm



$$e_y = \frac{DB^3 - db^3}{6B(BD - bd)} = \frac{300 \times 200^3 - 150 \times 100^3}{6 \times 200\,(200 \times 300 - 100 \times 150)}$$

$= 41.66$ mm

Hence, take, $OD = OB = 62.5$ mm

2) Determine the max. & min. stresses at the base of hollow circular chimney of height 20 m with a external dia. 4m & Internal dia. 2m The chimney is subjected to a horizontal wind pressure of intensity 1 KN/m. The sp. wt of the material of chimney is 22 KN/m³.

Sol:- weight density = sp. wt × volume

$$= 22 \times \frac{\pi}{4}(4^2 - 2^2) \times 20$$

$$= 4146.9 \text{ KN.}$$

$$\sigma_0 = \frac{W}{A} = \frac{4146.9}{\frac{\pi}{4}(4^2 - 2^2)} = 440 \text{ KN/m}^2.$$

$$\sigma_b = \frac{My}{I}.$$

$$M = F \times \frac{h}{2} = KPA \times \frac{h}{2}$$

$$= \frac{2}{3} \times 1 \times 4 \times 20 \times \frac{20}{2}$$

$$= 533 \text{ KN-m}$$

$$\sigma_b = \frac{533 \times \frac{4}{2}}{\frac{\pi}{64}[4^4 - 2^4]} = 90.54 \text{ KN/m}^2.$$

$$\sigma_{min} = \sigma_0 - \sigma_b = 440 - 90.54 = 349.46 \text{ KN/m}^2$$

$$\sigma_{max} = \sigma_0 + \sigma_b = 440 + 90.54 = 530.54 \text{ KN/m}^2.$$

* Dams and Retaining walls:-

⇒ Dam is constructed to store the water.

⇒ A large quantity of water is required for irrigation & power generation through out the year.

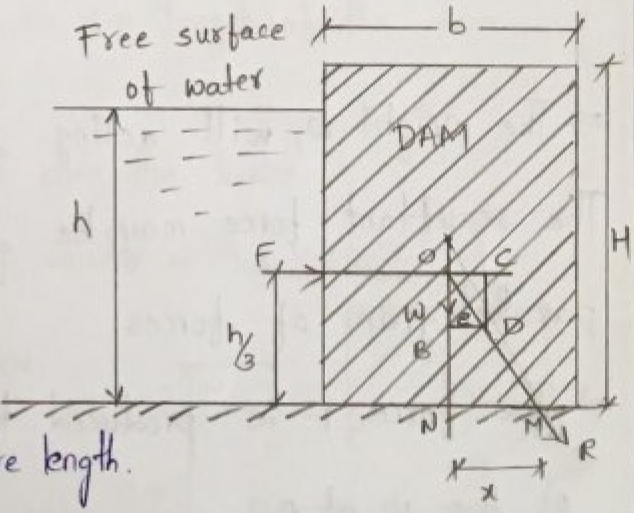
⇒ A Retaining wall is constructed to retain in hilly areas.

⇒ The water stored in a dam, exerts pressure force on the face the dam incontact with water similarly the earth, retained by a retaining wall, exerts peressure on the retaining wall.

* Types of dams:-

1) Rectangular dam
2) Trapezoidal dam
   i) water face vertical
   ii) water face inclined.

1) Rectangular dam:-



h = Height of water

F = Force exerted by water on the side of the dam.

W = weight of the dam per metre length.

H = Height of dam

b = width of dam

W_0 = weight density of dam.

⇒ Consider 1 m length of dam.

The forces acting on dam are.

1) The force F due to water incontact with side of the dam.

$F = wA\bar{h}$

σ = wt. acting on element / area of c/s.

$= \frac{wAx}{A} = wx$

$F = \sigma \times A$

$F = wA\bar{h} = w(h \times 1) = \frac{h}{2}$

$$\boxed{F = \frac{wh^2}{2}}$$

2) The weight w of the dam, w = wt. density x volume

$= w_0 \times V$

$= w_0 \times A \times V$

$= w_0 \times b \times H \times 1$

⇒ The weight w, will acting downwards through C.G of the dam, The resultant force may be determined by method of parallelogram of forces.

⇒ The force F is produced to intersect the line of action of the w at O.

⇒ Take OC = F & OB = W to some scale. Then diagonal OD represents resultant R.

$R = \sqrt{F^2 + w^2}$

⇒ Angle made by the resultant with vertical.

$$\tan\theta = \frac{F}{W}$$

Let, x = distance of MN. It is obtained by similar triangles i.e. $\frac{MN}{ON} = \frac{BD}{OB} \Rightarrow \frac{x}{h/3} = \frac{F}{w}$

$$x = \frac{Fh}{3W}$$

The distance x can also calculated by taking moments of all forces about the point M.

$$F \times \frac{h}{3} = W \times x$$

$$\boxed{x = \frac{Fh}{3W}}$$

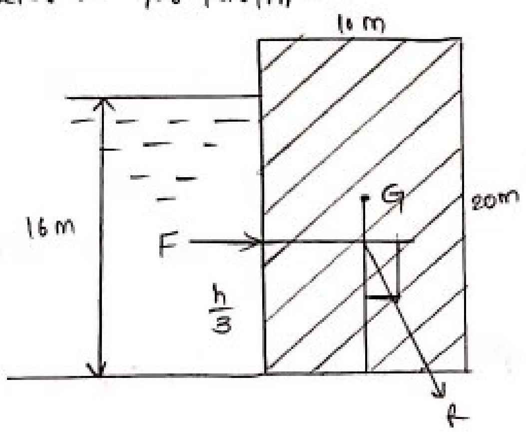
1) A masonry dam of rec. section is 20m high & 10m wide, as water upto a height of 16m on its one side. Find.

i) Pressure force due to water on 1m length of the dam.

ii) Position of centre of pressure.

iii) The point at which resultant cuts the base.

Take, $w_0 = 19.62\ KN/m^3$ and of water $= 9.8\ KN/m^3$.



sol:- i) $F = \frac{wh^2}{2} = \frac{9.8 \times 16^2}{2} = 1254.4 KN$

$W = w_0 \times b \times H \times L$

$= 19.62 \times 10 \times 20 \times 1$

$= 3924\ KN$

ii) Position of centre of pressure:

The point at which force F is acting is known as centre of pressure. F is acting horizontally at the height of $h/3$ above the base.

$$h/3 = 16/3 = 5.33$$

iii) The point at which resultant cuts the base = x.

$$W = w_0 \times b \times H \times 1 = 19.62 \times 10 \times 20 \times 1$$

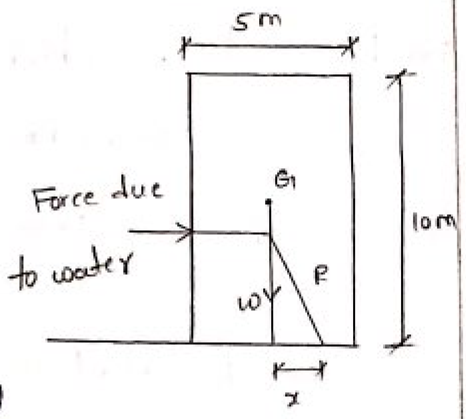
$$= 3924 \text{ KN}$$

$$x = \frac{Fh}{3W} = \frac{1254.4 \times 16}{3 \times 3924} = 1.7\text{m}$$

2) A masonry dam of rec. section 10 m high & 5 m wide has water upto the top on its one side if the wt. density of masonry 21.582 KN/m3. Find

i) Pressure force due to water per metre length of dam.

ii) Resultant force and the point at which it cuts the base of the dam.



Sol: i) $F = wA\bar{h} = 9.81 \times 1000 \times (10 \times 1) \times \frac{10}{2}$

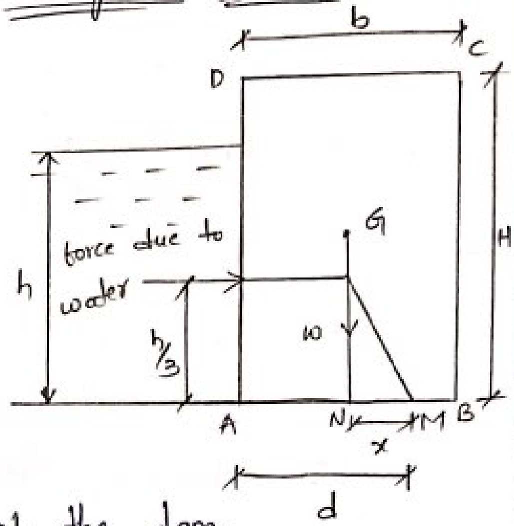
$$= 490500 \text{ N}.$$

ii) $R = \sqrt{F^2 + W^2}$

$$W = w_0 \times V = 21.582 \times 10 \times 5 \times 1 = 1079.1 \text{ KN}$$

$$R = \sqrt{(490500)^2 + (1079.1 \times 10^3)^2} = 1185346.81 \text{ N}$$

* Stresses across the section of a Rectangular section:-



⇒ A Rec. dam of height H, width 'b'.

h = water depth in dam

⇒ The forces acting on the dam.

1) Force due to water at a height of $h/3$ above the base of the dam.

2) The wt. W of the dam at the C.G of the dam.

⇒ The Resultant force R is cutting the base of the dam at M.

Let, $x = \frac{Fh}{3W}$

d = The distance b/w A & point M.

$= AN + NM$

$= \frac{b}{2} + x = \frac{b}{2} + \frac{Fh}{3W}$

⇒ The resultant force 'R' acting at M may be dissolved in vertical & Horizontal components.

W = vertical component

F = horizontal component

⇒ The horizontal component W acting at point M on the base of the dam is eccentric load as it is not acting at the middle of the base.

⇒ But an eccentric load produces & bending & direct stress.

Eccentricity of W = distance NM = x

Then, $e = d - \frac{b}{2}$

$\Rightarrow$ Now, moment on the base $= W \times e$

But, we know $\frac{M}{I} = \frac{\sigma_b}{y} \rightarrow$ (1)

Take, $d = 1$

$$I = \frac{db^3}{12} - \frac{b^3}{12}$$

σ_b = bending stress

$$y = \pm \frac{b}{2}$$

sub. the values in eqn (1)

$$\sigma_b = \pm \frac{My}{I} = \pm \frac{M \times b}{2 \times \frac{b^3}{12}}$$

$$\sigma_b = \pm \frac{6M}{b^2} = \pm \frac{6We}{b^2}$$

$$\boxed{\sigma_b = \pm \frac{6we}{b^2}}$$

$$\sigma_b \text{ at } B = \pm \frac{6we}{b^2}$$

$$\sigma_b \text{ at } A = -\frac{6we}{b^2}$$

$\Rightarrow$ But direct stress, $\sigma_0 = \frac{w}{A} = \frac{w}{1 \times b} = \frac{w}{b}$

$\Rightarrow$ Total stress across the dam,

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$= \frac{w}{b} + \frac{6we}{b^2}$$

$$\sigma_{max} = \frac{W}{b}\left[1+\frac{6e}{b}\right]$$

$$\sigma_{min} = \frac{W}{b}\left[1-\frac{6e}{b}\right]$$

1) A masonry dam of rec. section 20 m high & 10 m wide as water upto a height of 16 m on its one side. Find max. & Min. stress intensity at the base of the dam. Take $w_0 = 19620\ N/m^3$.

Sol:- $F = wA\bar{h}$

$= 9.81 \times 1000 \times 16 \times 1 \times \frac{16}{2}$

$= 1255680\ N$

$W = w_0 \times V = 19620 \times 20 \times 10 \times 1 = 3924000\ N$

$x = \frac{Fh}{3W} = \frac{1255680 \times 16}{3 \times 3924000} = 1.706\ m = e$

$\sigma_{max} = \frac{W}{b}\left[1+\frac{6e}{b}\right] = \frac{3924000}{10}\left[1+\frac{6 \times 1.706}{10}\right]$

$= 794.06\ KN/m^2$ (Compressive)

$\sigma_{min} = \frac{W}{b}\left[1-\frac{6e}{b}\right] = \frac{3924000}{10}\left[1-\frac{6 \times 1.706}{10}\right]$

$= 9.26\ KN/m^2$ (Tensile)

* ## A Trapezoidal dam having water face vertical :-

H = Height of dam

h = height of water

a = Top width of dam

b = bottom width of dam

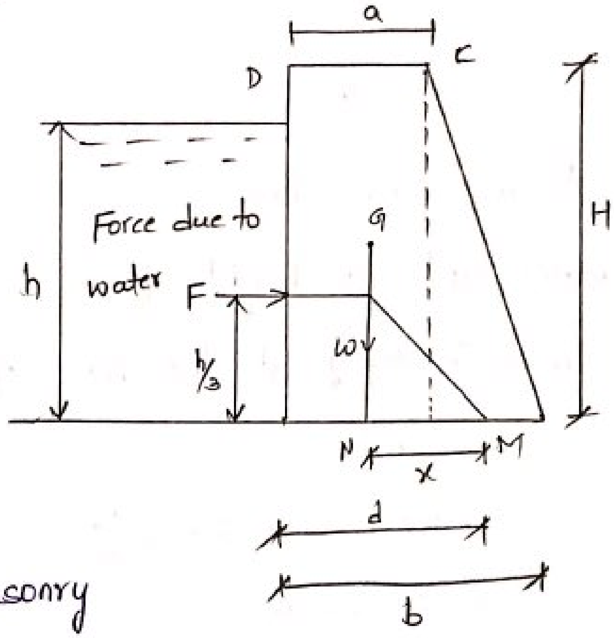
w = wt. density of water

$= \rho \times g = 9.81 \times 1000 \ N/m^3$.

w_0 = wt. density of dam masonry

F = Force exerted by water

W = wt. of dam per metere length.



1) Force exerted by water, $F = wA\bar{h} = w \times h \times 1 \times \frac{h}{2}$

$$= \frac{wh^2}{2}$$

F will be acting horizontally at a height of $h/3$ above the base.

2) W = wt. density $\times$ Volume

$$= w_0 \times \left(\frac{a+b}{2}\right) H \times 1$$

The wt W will acting downwards through the C.G of the dam.

⇒ The distance of CG of trapezoidal section from the vertical face AD is obtained by splitting into a rectangle and a triangle, taking the moments of their areas about the line AD & equating the same width the moment of the total area

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of the trapezoidal section about the line AD.

$$(a\times H)\times\frac{a}{2}+(b-a)\frac{H}{2}\times\left[\left(\frac{b-a}{3}\right)+a\right]=\left(\frac{a+b}{2}\right)\times H\times AN.$$

⇒ From the above eqn distance AN can be calculated.

⇒ The distance AN can also calculated by using the relation

$$AN=\frac{a^2+ab+b^2}{3(a+b)}$$

⇒ Then x = The distance MN & is given by $\frac{Fh}{3W}$.

⇒ Now, Eccentricity, $e=d-\frac{b}{2}$.

⇒ Then the total stress across the base of the dam at point B,

$$\sigma_{max}=\frac{W}{b}\left[1+\frac{6e}{b}\right]$$

$$\sigma_{min}=\frac{W}{b}\left[1-\frac{6e}{b}\right]$$

1) A Trapezoidal dam is of 18m height, the dam is having water upto a depth of 15m on its vertical side. The top & bottom width of the dam are 4m & 8m respectively. The $w_0 = 19.62\ KN/m^3$. Determine

i) The Resultant force per metre length.

ii) The point where resultant cuts the base.

iii) The max. & min. stress intensities at the base.

Sol:- i) $F=\frac{wh^2}{2}=\frac{9.81\times1000\times15^2}{2}$

$= 1103625\ N.$

$$W = w_0 \times V = 19.62 \times \left(\frac{4+8}{2}\right) \times 18 \times 1 = 2118.96 \text{ KN}.$$

$$R = \sqrt{F^2 + W^2} = \sqrt{(1103625)^2 + (2118.96 \times 10^3)^2}$$

$$= 2389137.84 \text{ N}.$$

ii) $$AN = \frac{a^2+ab+b^2}{3(a+b)} = \frac{4^2+4\times 8+8^2}{3(4+8)} = 3.11 \text{ m}$$

$$x = \frac{Fh}{3W} = \frac{1103625 \times 15}{3 \times 2118.96 \times 10^3} = 2.6084 \text{ m}.$$

$$d = AN + x = 3.11 + 2.604 = 5.714$$

$$e = d - \frac{b}{2} = 5.714 - \frac{8}{2} = 1.714.$$

iii) $$\sigma_{max} = \frac{W}{b}\left[1 + \frac{6e}{b}\right]$$

$$= \frac{2118.96 \times 10^3}{8}\left[1 + \frac{6 \times 1.714}{8}\right]$$

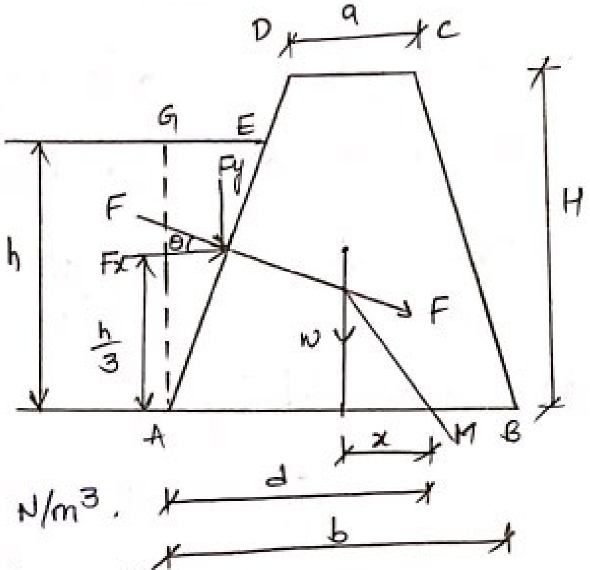
$$= 605.360 \text{ KN/m}^2 \text{ (Compressive)}$$

$$\sigma_{min.} = \frac{W}{b}\left[1 - \frac{6e}{b}\right]$$

$$= \frac{2118.96 \times 10^3}{8}\left[1 - \frac{6 \times 1.714}{8}\right]$$

$$= 75.620 \text{ KN/m}^2 \text{ (Tensile)}.$$

* Trapezoidal dam having water face inclined :-



H = Ht. of the dam

a = Top width of the dam

b = Bottom width of the dam

w_0 = wt. density of dam masonry

h = ht. of the water

w = wt. of the water $= 9.81 \times 1000 \ N/m^3$.

F = Force exerted by water on face AD.

Effects component of F in y-dir.^{n}, $F_y = F\sin\theta$

x-dir.^{n}, $F_x = F\cos\theta$

Inclination of face AD with vertical

W = wt. of the dam per metre length

$$= w_0 \times \frac{(a+b)}{2} \times H \times 1$$

From Δ^{le} AGE,

$$\cos\theta = \frac{AG}{AE} \qquad \because AG = h$$

Then, $AE = \dfrac{h}{\cos\theta}$

$\Rightarrow$ The force exerted by the water on face AE =

$$F = wg \times A \times \bar{h}$$

Area of face, AE = AE × 1

$$= \frac{h}{\cos\theta} \times 1$$

$$F = w \times \frac{h}{\cos\theta} \times \frac{h}{2} = \frac{wh^2}{2\cos\theta}$$

⇒ The force, F is acts perpendicular to face AE at a ht. of $h/3$ above the base.

Then, $F_x = F\cos\theta = \frac{wh^2}{2\cos\theta} \times \cos\theta = \frac{wh^2}{2}$

⇒ Force exerted by water on face AE in vertical.

$$F_y' = F\sin\theta = \frac{wh^2}{2\cos\theta} \times \sin\theta = \frac{wh^2}{2}\tan\theta$$

$$\therefore F_y = \frac{wh^2}{2} \times \frac{GE}{AG} = \frac{wh^2}{2} \times \frac{GE}{h}$$

$$= \frac{wh}{2} \times GE = \left[\frac{w \times h \times GE}{2}\right]$$

$$= w' \times \text{Area of } \Delta^{le}\ AEG \times 1$$

⇒ Hence, The force F acting on inclined face AE is equivalent to force F_x acting on the vertical face AG & the face F_y which is equal to the wt. of water in the wedge AEG.

⇒ F_y acts through C.G of the Δ AEG wt. of the dam per meter length is given by

$$W = \left(\frac{a+b}{2}\right) H \times w_0$$

⇒ Now, the force R, which is the resultant of force F & W, cuts the base of the dam at point M. The distance AM can be calculated by taking all moments of all forces i.e.

F_x, F_y & W about the point M but the distance AM = d.

⇒ Now, the eccentricity, $e = x = d - \frac{b}{2}$.

⇒ Then the total stress across the base of the dam at point B,

$$\sigma_{max} = \frac{V}{b}\left(1 + \frac{6e}{b}\right)$$

@ point A, $\sigma_{min} = \frac{V}{b}\left(1 - \frac{6e}{b}\right)$

where, V = Sum of vertical forces acting on the dam.

$$V = F_y + W$$

1) A masonry dam of trapezoidal section is 10m high it has top width of 1m & bottom width 7m. The force exposed to water has a slope of 1 horizontal to 10 vertical. Calculate the max. & min. stresses of the base. when the water level coinsides with the top of the dam. $w_0 = 19.62$ KN/m³.

Sol:

$$F_x = \frac{wh^2}{2} = \frac{9810 \times 10^2}{2} = 490500 \text{ N}.$$

$$F_y = w \times \text{Area of } \Delta^{le} \text{ AGE} \times 1$$

$$= 9810 \times \frac{1}{2} \times 10 \times 1 \times 10 = 49050 \text{ N}.$$

$$W = \left(\frac{a+b}{2}\right) H \times w_0 = \left(\frac{1+7}{2}\right) \times 10 \times 19.62 = 784.8 \text{ KN}$$

$$V = F_y + W = 49050 + 784.8 \times 10^3 = 833850 \text{ N}.$$

AN = Moment of individual section about A = Total moment of Trapezoidal about point A.

$$\left(\frac{10\times1}{2}\times\frac{2}{3}\right)+(10\times1)(1.5)+\frac{10\times5}{2}\times\left(\frac{5}{3}+2\right)=\frac{1+7}{2}\times10\times AN$$

$NM = AM - AN$

$AN = 2.75m$

$$\left[F_x\times\frac{10}{3}\right]-F_y\left[Am-\frac{1}{3}\right]-w(NM)=0$$

$$490500\times\frac{10}{3}-49050\left[d-\frac{1}{3}\right]-784.8\times10^3\left[d-AN\right]=0$$

$$163500-49050d+16350-784.8\times10^3d+2158200=0$$

$$833850d = 3809550$$

$$d = 4.56\ m$$

$$e=d-\frac{b}{2}=4.56-\frac{7}{2}=1.06$$

$$\sigma_{max}=\frac{V}{b}\left[1+\frac{6e}{b}\right]$$

$$=\frac{833850}{7}\left[1+\frac{6\times1.06}{7}\right]$$

$$=227.35\ KN/m^2\ (\text{compressive})$$

$$\sigma_{min}=\frac{V}{b}\left[1-\frac{6e}{b}\right]$$

$$=\frac{833850}{7}\left[1-\frac{6\times1.06}{7}\right]$$

$$=10.86\ KN/m^2\ (\text{Compressive}).$$

4. THIN END THICK CYLINDER

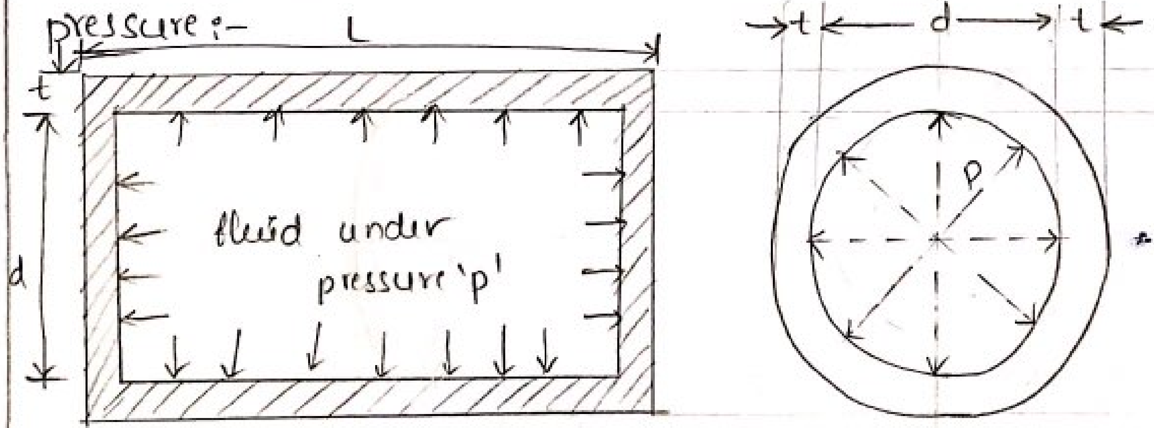
→ THIN CYLINDER :-

The vessels ~~vol~~ such as boilers , compressing air receivers etc, are of cylindrical or spherical forms. These vessels are generally used for storing fluids [liquid (or) gas] under pressure.

The values of such vessels are thin as compared to their diameters. If the thickness of the wall of the cylinderical vessel is less than $\frac{1}{15}$ to $\frac{1}{20}$ of its internal diameter, the cylindrical vessel is known as 'thin cylinder'.

In case of thin cylinder, the stress distribution is assumed uniform over the thickness of the wall.

The cylindrical vessel subjected to an internal fluid pressure :-



Here ;

p → Internal pressure

L → Length of the cylinder

d → Internal dia of the cylinder

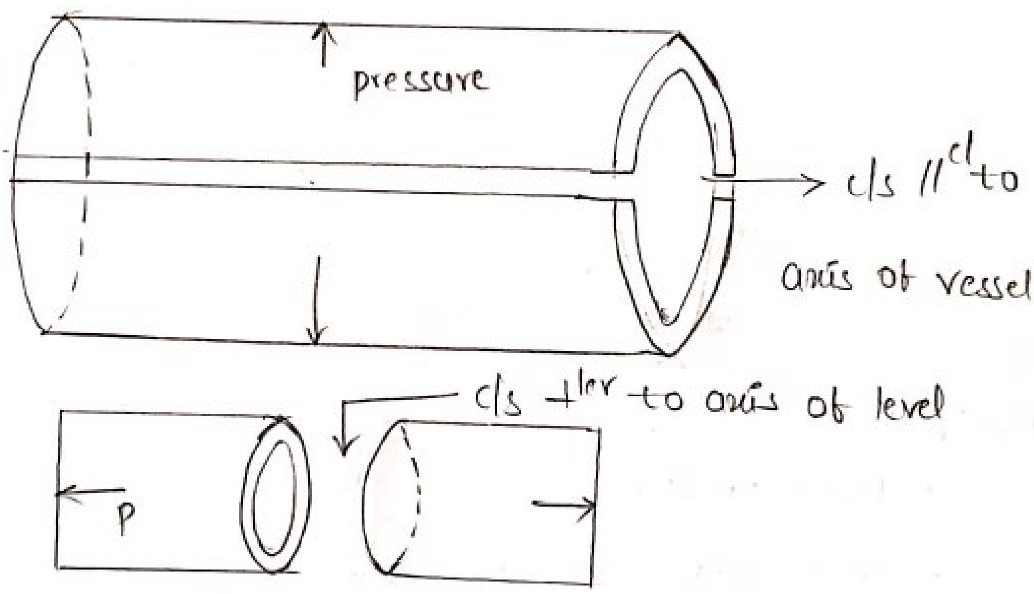
t → thickness " " "

On the account of internal fluid pressure "p", the cylindrical vessel may fail by spliting up in any one of the two ways.

The forces, due to the pressure of the fluid acting vertically upwards and downwards. on the cylinder, tend to burst the cylinder.

The forces, due to the pressure of the fluid acting vertically upwards and at the ends of the thin cylinder, tend to burst the cylinder.

Stresses in a thin cylin



Stresses in a thin cylindrical vessel subjected to internal pressure:-

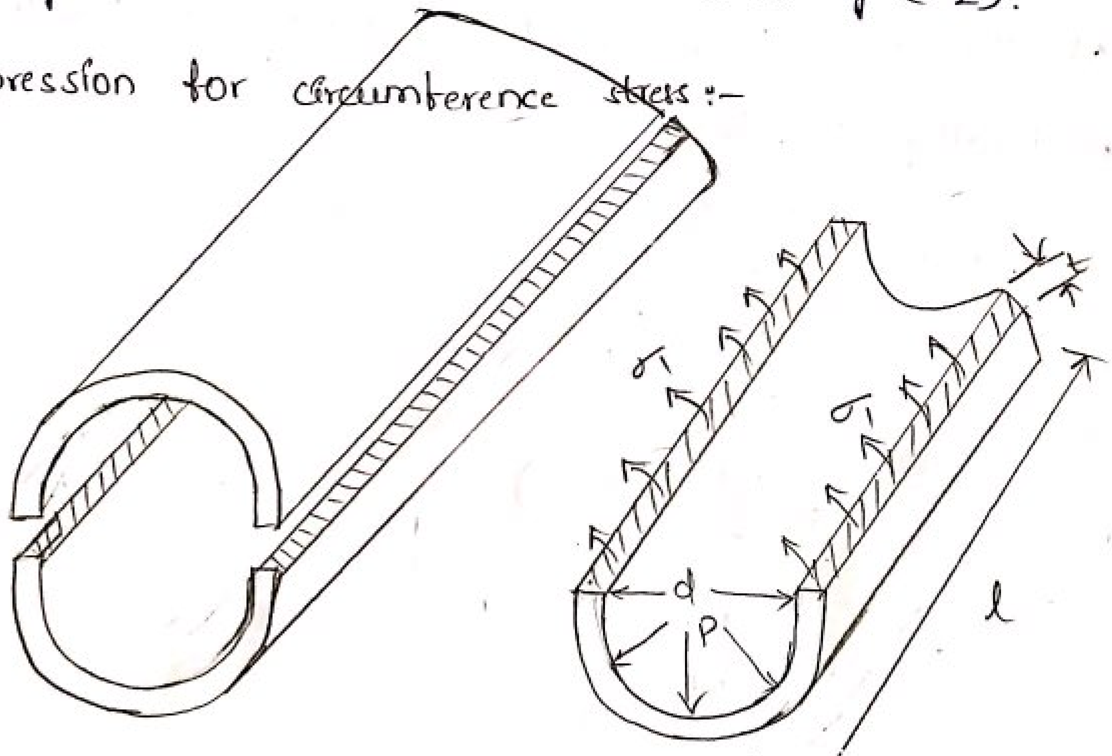
There are two types:-

* Circumferential stress (or) hoop's stress.
* Longitudinal stress.

Circumferential stress:- The stresses which are acting in the direction of circumference of the cylinder is called "circumferential stress" (or) "hoop stress". It is indicated by (σ_1).

Longitudinal stress:- The stresses which are acting longitudinal the direction of longitudinal axis is called "Longitudinal stress". It is indicated by (σ_2).

Expression for circumference stress:-



consider a thin cylindrical vessel subjected to internal fluid pressure the circumferential stress will be setup in the material of cylinder, If the brusting cylinder takes place, →continuation,

Max shear stress:-

At any point in the material of the cylindrical shell, there are two stresses, namely circumferential stress of magnitude.

$\sigma_1 = \frac{Pd}{2t}$ acting circumferential and longitudinal stress of magnitude.

$\sigma_2 = \frac{Pd}{4t}$ acting parallel to the (longitudinal)axis of the shell. There are two stresses are tensile & mutually $\perp^r$, then.

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{\frac{Pd}{2t} - \frac{Pd}{4t}}{2}$$

$$= \frac{\frac{Pd}{t}\left(\frac{1}{2} - \frac{1}{4}\right)}{2}$$

$$\boxed{\tau_{max} = \frac{Pd}{8t}}$$

→ continue.

then the expression for hoop stress is obtained.

$p \rightarrow$ pressure

$d \rightarrow$ diameter

$\sigma_1 \rightarrow$ Circumferential stress.

The brusting will takes place if force due to fluid pressure is more than the resisting force. Due to the circumferential stress setup in the material. If the limiting case the two forces should be equal.

Force due to fluid pressure = $p \times$ area on which 'p' is acting.

$= 2p(Lt + Lt)$

$= 2pLt$ (or) $p \times L \times t \rightarrow (1)$

Force due to circumferential stress = $\sigma_1 \times$ area on which σ_1 is acting.

$= \sigma_1 (L \times t + L \times t)$

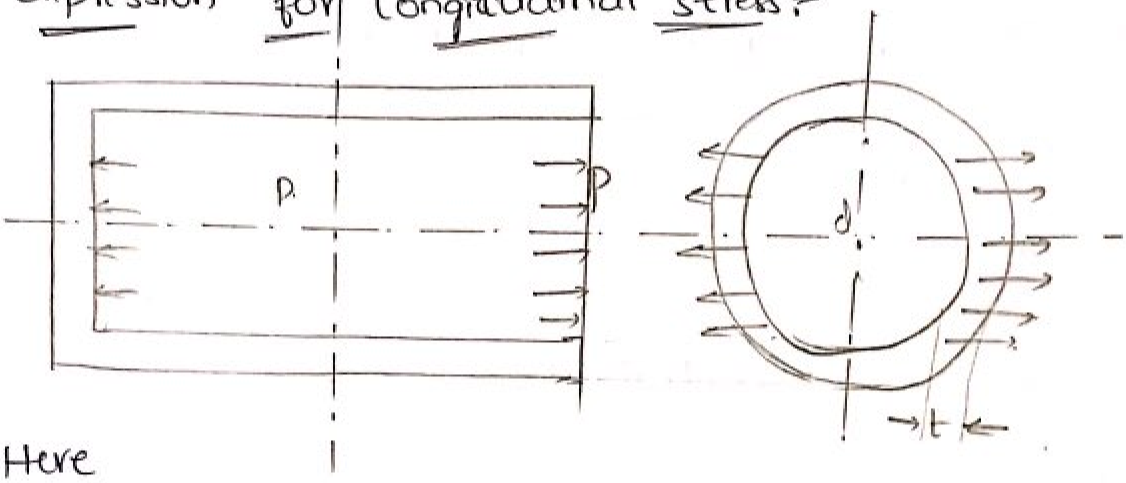
$= 2\sigma_1 Lt \rightarrow (2)$

Equating (1) & (2)

$p \times d = 2\sigma_1 Lt$

$\boxed{\sigma_1 = \frac{pd}{2t}} \rightarrow$ Tensile.

→ Expression for longitudinal stress:-



Here

$\sigma_2 \rightarrow$ longitudinal stress

Force due to fluid pressure = p × area on which p is acting.

$$= p \times \frac{\pi}{4} d^2 \longrightarrow (1)$$

Resisting force = σ_2 × area on which σ_2 is acting.

$$= \sigma_2 \times \pi d t \longrightarrow (2)$$

Equating (1) & (2)

$$p \times \frac{\pi}{4} d^2 = \sigma_2 \times \pi d t$$

$$\sigma_2 = \frac{pd}{4t}$$

Longitudinal stress = $\frac{1}{2}$ of the circumferential stress

Hence in the material of the cylinder, the permissible stress ~~of the~~ should be less than the circumferential stress.

∴ If max. permissible stress in the material is given, this stress should be taken as circumferential stress.

By using above equation 'σ_1' and 'σ_2' are the same units in N/mm^2. 'd' and 't' in 'mm'.

If the thickness of thin cylinder is to be deter-mined by the equation 'σ_1'.

Problems:-

Q) A cylindrical of pipe dia 1.5m and thickness 1.5cm is subjected to an internal fluid pressure of $1.2\ N/mm^2$. Determine. (i) Longitudinal stress developed in the pipe & (ii) circumferential stress developed in the pipe.

sol:- Given data

$d = 1.5m \Rightarrow 1500mm$

$t = 1.5cm = 15mm$

$P = 1.2\ N/mm^2$

(i) Longitudinal stress

$$\sigma_2 = \frac{pd}{4t} = \frac{1.2 \times 1500}{4 \times 15}$$

$$\boxed{\sigma_2 = 30\ N/mm^2}$$

(ii) circumferential stress

$$\sigma_1 = \frac{pd}{2t} = \frac{1.2 \times 1500}{2 \times 15} \Rightarrow \boxed{\sigma_1 = 60\ N/mm^2}$$

Q) A cylindrical of internal dia 2.5m and of thickness 5cm contains a gas. If the tensile stress in the material is not to exceed 80 N/mm², Determine internal pressure of fluid?

sol:- Given data

$d = 2.5m \Rightarrow 2500mm$

$t = 5cm \Rightarrow 50mm$

$\sigma_1 = 80\ N/mm^2$

$P = ?$

$$\sigma_1 = \frac{Pd}{2t}$$

$$P = \frac{80 \times 2 \times 50}{2500}$$

$$\boxed{P = 3.2\ N/mm^2}$$

Q) A water main 80cm dia contains water at a pressure head of 100m. If the wt density of water 9810 N/m³, find the thickness of the metal required for the water main, given the permissible stress as 20N/mm²?

pressure of water inside the water main = $\rho g h$.

sol:- wt density of water $w = \rho \times g$

$= 1000 \times 9.81 = 9810\ N/m^3$

$= 9.81\ N/mm^3$.

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pressure of water inside the water main $= \rho g h$

$$= 9810 \times 100 \times 10^3$$

$$= 981 \times 10^6.$$

$$\sigma_1 = \frac{pd}{2t}$$

$$20 = \frac{0.981 \times 800}{2 \times t}$$

$$20 = 392.4\left(\frac{1}{t}\right)$$

$$\boxed{t = 19.62\,mm \simeq 20\,mm}$$

Efficiency of a Joint:-

The cylindrical shell such as Boilers are having two types of joints namely.

* longitudinal joint.
* circumferential joint.

In case of joint, holes are made in the material of the shell for the rivets due to the holes the area offering resistance is decreased due to decrease in area, the stress developed in the material of the shall will be more. Hence, incase of riveted the circumferential and longitudinal joint are given. Then the circumferential and longitudinal stress are obtained by

$$\sigma_1 = \frac{pd}{2t\eta_l}$$

$$\sigma_2 = \frac{Pd}{4t\eta_c}$$

where

$\eta_l \rightarrow$ efficiency of longitudinal joint.

$\eta_c \rightarrow$ " " circumferential "

Note :-

→ In longitudinal joint, the circumferenti stress is developed

→ In circumferential joint, the longitudinal stress is developed.

→ The efficiency of joint means, the efficiency of longitudinal joint.

→ The efficiencies of joint.

problems:-

A boiler is subjected to an internal steam of $2 N/mm^2$. The thickness of the boiler plate is 2cm and permissible tensile stress is $120 N/mm^2$. findout the max. dia, when efficiency of longitudinal joint is 90% and that circum-ferential joint is 40%?

sol:-

Given data

$p = 2 N/mm^2$

$t = 2 cm \Rightarrow 20mm$

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$\sigma = 120\ N/mm^2$

$$\sigma_1 = \frac{Pd}{2t\eta_c}$$

$$d = \frac{\sigma_1 2t\eta_c}{P}$$

$$d = \frac{120\times2\times20\times0.9}{2}$$

$$\boxed{d = 2160\ mm}$$

$$\sigma_2 = \frac{Pd}{4t\eta_l}$$

$$d = \frac{120\times4\times20\times0.4}{2}$$

$$\boxed{d = 1920mm}$$

The main ∅ min. dia is max. dia of the cylinder because the stress ∝ dia.

Q) A cylinder of thickness 1.5 cm, has to withstand max. internal pressure of 1.5 N/mm^2. If the ultimate max tensile stress in the material of the cylinder is 300 N/mm^2, Fos is 3 and joint efficiency 80%. Determine the dia of cylinder.

Sol:- [The range of fos 3-5] and [possion's ratio 0.2 to 0.5]

Given data.

$t = 1.5 cm$

$P = 1.5 N/mm^2$

$\sigma = 300 N/mm^2$.

$f.o.s = 3$

$\eta = 80\%$.

$$f.o.s = \frac{\text{Ultimate stress}}{\text{working stress}} \Rightarrow \sigma_1 = \frac{300}{3} = 100 N/mm^2$$

$$\sigma_1 = \frac{Pd}{2t\eta_l}$$

$\sigma_1 = 100 N/mm^2$

$$100 = \frac{1.5 \times d}{2 \times 1.5 \times 0.8}$$

$$\boxed{d = 1600mm}$$

Effect of Internal pressure on the dimensions of thin cylindrical vessel:-

When a fluid having internal pressure 'p' is stored in a thin cylindrical shell due to internal pressure of fluid, the stresses set-up at any point of the material of the shell are

* hook or circumferential stress acting on longitudinal section.

* Longitudinal stress acting on circumferential section.

→ These stresses are principal stresses, as they are acting on principal plains. The stress in the third principal plain is zero. As the thickness 't' of the cylinder is very small.

→ Actually the stress in the third principal plane is radial stress which is very small for thin cylinder and can be neglected.

Let p → Internal fluid pressure.

L → Length of cylinder

d → dia of " "

t → thickness of cylinder

E → young's modulus of the material.

σ_1 → hoop's or circumferential stress

σ_2 → Longitudinal stress

μ → poisson's ratio

δt → change in dia due to stresses set up in the material.

δl, δr → change in length & change in volume.

$e_1 \rightarrow$ circumferential strain

$e_2 \rightarrow$ longitudinal strain

$$e_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$= \frac{pd}{2tE} - \mu \frac{pd}{4tE}$$

$$\boxed{e_1 = \frac{pd}{2tE}\left[1 - \frac{\mu}{2}\right]}$$

$$e_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

$$\boxed{e_2 = \frac{pd}{2tE}\left[\frac{1}{2} - \mu\right]}$$

But circumferential strain is also given as

$$e_1 = \frac{\text{change in circumberence due to pressure}}{\text{original circumference.}}$$

$$e_1 = \frac{\text{final} - \text{Initial circumference}}{\text{original circumference}}$$

then
$$e_1 = \frac{\pi(d + \delta d) - \pi d}{\pi d}$$

$$= \frac{\cancel{\pi d} + \cancel{\pi}\,\delta d - \cancel{\pi d}}{\cancel{\pi} d}$$

$$e_1 = \frac{\cancel{\pi}\,\delta d}{\cancel{\pi} d} \Rightarrow \boxed{e_1 = \frac{\delta d}{d}}$$

Similarly $\boxed{e_2 = \frac{\delta l}{l}}$

$$\frac{\delta d}{d} = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$\delta d = \frac{Pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]$$

llly $\delta l = \frac{Pdl}{2tE}\left[\frac{1}{2} - \mu\right]$

Volumetric strain:-

Intial volume $= \frac{\pi}{4} d^2 \times l$

Final volume $= \frac{\pi}{4}(d + \delta d)^2 \times (l \times \delta l)$

$$= \frac{\pi}{4}(d^2 + \delta d^2 + 2d\,\delta d) \times (l \times \delta l)$$

$$= \frac{\pi}{4} d^2 l + \delta l + \cancel{\delta d^2 \cdot l} + \delta l + \cancel{2d\,\delta d} \cdot l + \delta l$$

neglecting the small quantities

$$= \frac{\pi}{4}(d^2 \cdot \delta l + l d^2 + 2 l d\, \delta d)$$

$$e_v = F.V - I.V$$

$$= \frac{\pi}{4} d^2 l - \frac{\pi}{4} d^2 \cdot \delta l + l d^2 + 2 l d\, \delta d$$

$$\boxed{e_v = \frac{\pi}{4}(d^2 \delta l + 2 l d\, \delta d)}$$

→ volumetric strain $= \frac{\delta v}{V}$

$$= \frac{\cancel{\frac{\pi}{4}}(d^2\delta l + 2\,l d\,\delta d)}{\cancel{\frac{\pi}{4}} d^2 \times l}$$

$$= \frac{d^2\delta l + 2 l d\,\delta d}{d^2 l}$$

$$= \frac{\cancel{d^2}\delta l}{\cancel{d^2} l} + \frac{2\cancel{l}\cancel{d}\,\delta d}{d^{\cancel{2}}\cancel{l}}$$

$$= \frac{\delta l}{l} + 2\frac{\delta d}{d}$$

$$\boxed{e_v = e_2 + 2e_2}$$

$$= \cancel{2}\left[\frac{pd}{\cancel{2}tE}\left[1 - \frac{\mu}{2}\right]\right] + \frac{2d}{2tE}\left[\frac{1}{2} - \mu\right]$$

$$= \frac{pd}{tE}\left[1 - \frac{\mu}{2} + \frac{1}{4} - \frac{\mu}{2}\right]$$

$$= \frac{pd}{2tE}\left[\frac{2-\mu}{2} + \frac{1-2\mu}{4}\right]$$

$$= \frac{pd}{2tE}\left[\frac{4-2\mu+1-2\mu}{2}\right]$$

$$e_v = \frac{pd}{2tE}\left[\frac{5}{2} - 2\mu\right]$$

problem:-

1) Calculate (i) change in dia 'δd'.

(ii) change in length 'δl'

(iii) change in volume 'δv' of thin cylinder

shell 100cm dia, 1cm thick, 5m long when subjected to internal pressure of $3 N/mm^2$. Take $E = 2\times10^5 N/mm^2$ & $\mu = 0.3$

sol:- Given data

$d = 100\,cm$

$t = 1\,cm$

$l = 5\,m$

$E = 2\times10^5\,N/mm^2$

$\mu = 0.3$

$$\frac{\delta d}{d} = \frac{pd^2}{2tE}\left[1-\frac{\mu}{2}\right]$$

$$= \frac{3\times1000^2}{2\times10\times2\times10^5}\left[1-\frac{0.3}{2}\right]$$

$$\boxed{\delta d = 0.6\,mm}$$

$$\delta l = \frac{pdl}{2tE}\left[\frac{1}{2}-\mu\right]$$

$$= \frac{3\times1000\times5000}{2\times10\times2\times10^5}\left[\frac{1}{2}-0.3\right]$$

$$\boxed{\delta l = 0.75\,mm}$$

$$\delta v = \frac{\pi}{4}\left(d^2\delta l + 2ld\,\delta d\right)$$

$$= \frac{\pi}{4}\left(1000^2\times0.75 + 2\times5000\times1000\times0.6\right)$$

$$\boxed{\delta v = 5.3\times10^6\,mm^3}$$

2) A cylindrical shell 90cm long and 20cm dia having thickness of metal as 8mm is filled with the fluid atm. pressure, If an additional 20cm³ of fluid is pumped into the

the cylinder, find

(i) pressure exerted by the fluid on cylinder

(ii) hoop's stress induced. Take $E = 2\times10^5 N/mm^2$ & $\mu = 0.3$

sol:- Given data, $L = 90cm \Rightarrow 900mm$

$d = 20cm \Rightarrow 200mm$

$t = 8mm$

$\delta v = 20cm^3 \Rightarrow 200mm^3$

$E = 2\times10^5 N/mm^2$

$\mu = 0.3$

$v = \frac{\pi}{4} d^2 \times l = \frac{\pi}{4} \times 200^2 \times 900$

$v = 28.27\times10^6 mm^3$

$$\frac{\delta v}{v} = \frac{pd}{2tE}\left[\frac{5}{2} - 2\mu\right]$$

$$\frac{20\times10^3}{28.27\times10^6} = \frac{P\times200}{2\times8\times2\times10^5}\left[\frac{5}{2} - 2\times0.3\right]$$

$$7.07\times10^{-4} = 118.75\times10^{-6} P$$

$$\boxed{P = 5.95\ N/mm^2}$$

$$\sigma_1 = \frac{Pd}{2te}$$

$$= \frac{5.95\times200}{2\times8\times}$$

$$\boxed{\sigma_1 = 74.375\ N/mm^2}$$

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Thin cylinder is subjected to internal pressure 'p' & Torque 'T':

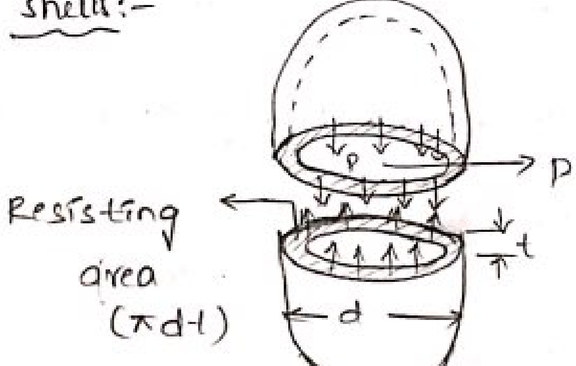
Because of acting torque, we find shear stress also, hence at any point in the material of cylindrical vessel. there will be two tensile stresses mutually ⊥r to each other accompined by shear stress, the major principal stress, minor principal stress and max shear stress will obtained by

$$\text{major p.s} = \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$$

$$\text{minor p.s} = \frac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$$

$$\text{Max shear stress} = \frac{1}{2}(\text{major p.s} - \text{minor p.s})$$

spherical shell:-



A thin spherical shell of internal dia 'd' and thickness 't' and subjected to an internal fluid pressure 'p' the fluid inside the shell has a tendency of split into hemi spheres along x-x axis.

The force 'p' which has a tendency to split the shell is equal to p×A.

$$f = p \times A \quad \left[A = \frac{\pi}{4} d^2\right]$$

→ The area resisting this force = $\pi d t$

∴ hoop or circumferential stress (σ_1) induced in the material of the shell is given by.

$$\sigma_1 = \frac{\text{Force}}{\text{Area of resist.}}$$

$$= \frac{p \times \frac{\pi}{4} d^2}{\pi d t}.$$

$$\boxed{\sigma_1 = \frac{pd}{4t}}$$

The stress 'σ_1' is tensile in nature

→ The fluid inside the shell is also having tendency to split, The shell into two heavy spheres along y-y axis, then it can be shown as the tensile hoop stress will also be $\frac{pd}{4t}$.

$$\boxed{\sigma_1 = \sigma_2 = \frac{pd}{4t}}$$

→ The stress 'σ_2' will be right angles to σ_1.

problems:

1) A vessel in the shape of spherical shell of 1.20m internal dia and 12mm shell thickness is subjected to pressure of 1.6 N/mm^2. Determine the stress induced in the material of the shell ?

sol:- Given data $d = 1.2m = 1200mm$

$t = 12mm$

$P = 1.6\ N/mm^2$

$$a = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 1200^2 = 1.13 \times 10^6\ mm^2$$

$$\sigma_1 = \frac{Pd}{4t}$$

$$= \frac{1.6 \times 1200}{4 \times 12}$$

$$\boxed{\sigma_1 = 40\ N/mm^2}$$

2) A spherical vessel 1.5m dia is subjected to an internal pressure of $2N/mm^2$. find the thickness of the plate required if max stress is not to exceed 150 N/mm^2 and joint efficiency is 75% ?

sol:- Given data $d = 1500\ mm$

$P = 2\ N/mm^2$

$\sigma = 150\ N/mm^2$

$\eta_c = 75\ \%$

$$\sigma = \frac{Pd}{4t\eta_c}$$

$$t = \frac{pd}{4\sigma \eta_c}$$

$$t = \frac{2 \times 1500}{4 \times 150 \times 0.75}$$

$$\boxed{t = 6.67 \text{ mm}}$$

change in dimensions of a thin spherical shell due to internal pressure :-

There is no shear stress at any point in the shell.

Then max shear stress $= \frac{\sigma_1 - \sigma_2}{2}$

$$= \frac{\frac{pd}{4t} - \frac{pd}{4t}}{2}$$

$$\tau_{max} = 0$$

Then the stress σ_1 and σ_2 are acting right angles to each other (or) mutually perpendicular.

$\therefore$ strain in any are direction is given by

$$e_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_1}{E}$$

$$= \frac{pd}{4tE} - \mu \frac{pd}{4tE} \qquad [\because \sigma_1 = \sigma_2]$$

$$e_1 = \frac{pd}{4tE}[1 - \mu]$$

$$\boxed{\frac{\delta d}{d} = \frac{pd}{4tE}[1 - \mu]}$$

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volumetric strain:-

If $v \rightarrow$ original volume

$$v = \frac{4}{3}\pi r^3 \text{ (or) } \frac{\pi}{6} d^3$$

$$\delta v = \frac{\pi}{6} 3d^2 \cdot \delta d$$

$$\frac{\delta v}{v} = \frac{\pi/6 \; 3d^2 \cdot \delta d}{\pi/6 \; d^3}$$

$$= \frac{3d \; \delta d}{d^2}$$

$$\boxed{e_v = \frac{3 \cdot \delta d}{d}} \rightarrow = 3e_1$$

$$\boxed{\delta v = \frac{3 \cdot \delta d \, v}{d}}$$

problems:-

1) A spherical shell of internal dia 0.9m and of thick-ness 10mm is subjected to an internal pressure of 1.4 N/mm². Determine the δd and δv. Take $E = 2 \times 10^5$ N/mm² and $\mu = 0.3$.

Sol:- Given data

$d = 0.9m = 900$ mm

$t = 10mm$

$p = 1.4$ N/mm²

$E = 2 \times 10^5$ N/mm²

$\mu = 0.3$

$$\delta d = \frac{pd^2}{4tE}[1-\mu]$$

$$= \frac{1.4 \times 900^2}{4 \times 10 \times 2 \times 10^5}[1-0.3]$$

$$\boxed{\delta d = 0.0994 \, mm}$$

$$\delta v = \frac{3 \, \delta d \, v}{d}$$

$$v = \pi/6 \, d^3$$

$$= \pi/6 \times 900^3$$

$$= 381.703 \times 10^6.$$

$$= \frac{3 \times 0.0994 \times 381.703 \times 10^6}{900}$$

$$\boxed{\delta v = 126.49 \times 10^3 \, mm^3}$$

THICK CYLINDERS:

The thick cylinders are the cylindrical vessels containing fluid under pressure and whose wall thickness is not small, i.e, $\frac{t}{d} > \frac{1}{20}$.

Unlike thin shells, the radial stress in the wall thickness is not negligable, rather it varies from the inner surface where it is equals to the magnitude of the fluid pressure to the outer surface where usually it is equals to zero. if exposed to atmosp-here.

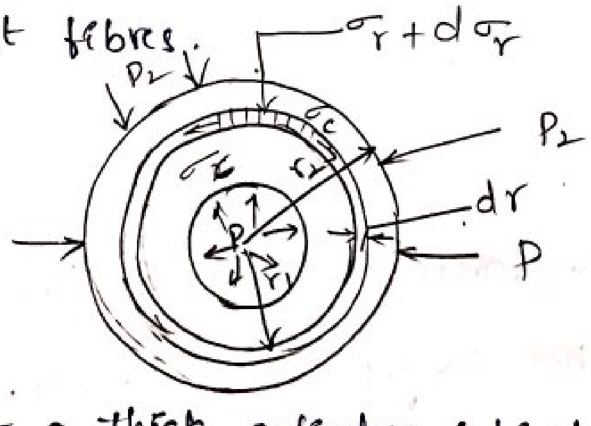
→ Circumferential stress is also various along the thick-ness.

→ The variation in the radial, as well as circumferential stress across the thickness are obtained with the help of Lame's theory.

Lame's theory :-

→ Assumptions:-

* The material is homogenous and isotropic.
* plain sections ⊥r to the longitudinal axis of the cylinder. Remain plain after application of internal pressure.
* The material is stressed within elastic limit only.
* All the fibres of the material are free to expand or contract independently without being constrained by the adjacent fibres.



consider a thick cylinder subjected to internal and external stress [pressure].

consider an elemental ring of internal radius 'r' & thickness 'dr'.

$r_1 \rightarrow$ Internal radius of the thick cylinder.

$r_2 \rightarrow$ external radius of the thick cylinder.

$L \rightarrow$ length of the cylinder.

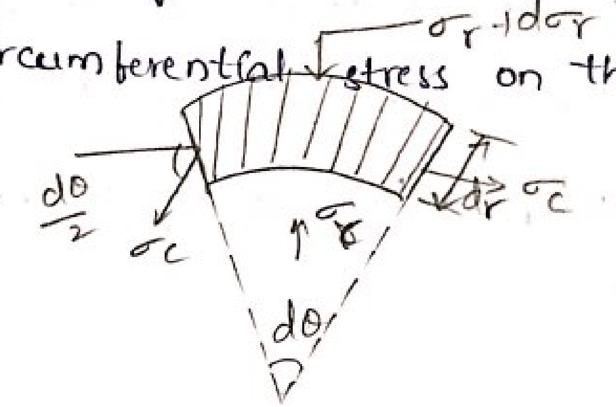
$P_1 \rightarrow$ pressure on the inner surface of the cylinder.

$P_2 \rightarrow$ pressure on the outer surface of the cylinder.

$\sigma_r \rightarrow$ Internal radial stress (pressure) on the elemental ring.

$\sigma_r + d\sigma_r \rightarrow$ external radial stress (pressure) on the elemental ring.

$\sigma_c \rightarrow$ circumferential stress on the elemental by



→ Resolving the forces in x-direction

so, neglecting the longitudinal stresses.

→ Resolving the forces in y-direction.

$$(\sigma_r + d\sigma_r)(r+dr)\,d\theta \times l + 2\sigma_c \sin\frac{d\theta}{2} \times dr \times l = \sigma_r \times r \times d\theta \times l$$

$$(\sigma_r r + \sigma_r . dr + d\sigma_r . r + d\sigma_r . dr) d\theta \times 1 + 2\sigma_c \sin\frac{d\theta}{2} \times dr \times 1$$

$$= \sigma_r \times r \times d\theta \times 1$$

$$d\theta(\sigma_r . r + \sigma_r dr + d\sigma_r . r) + 2\sigma_c \frac{d\theta}{2} \times dr = \sigma_r \times r \times d\theta$$

$$\sigma_r . r + \sigma_r dr + d\sigma_r . r + \sigma_c dr = \sigma_r r$$

$$\sigma_r dr + d\sigma_r . r + \sigma_c dr = 0$$

$$\boxed{dr[\sigma_c + \sigma_r] = d\sigma_r . r} \rightarrow (1)$$

Hence at any point in the section of elemental ring the following 3 principal stresses exists.

→ The radial stress (σ_1)

→ The circumferential stress (σ_c)

→ The longitudinal tensile stress (σ_l)

Then longitudinal strain 'e_l' is constant, then

$$e_l = \frac{\sigma_l}{E} - \mu \frac{\sigma_c}{E} - \mu \frac{\sigma_r}{E} = \text{constant}$$

σ_l, E, μ are constants, then

$$(\sigma_c - \sigma_r) = \text{constant}$$

Let the constant value is '$2a$'.

$$\sigma_c - \sigma_r = 2a$$

$$\sigma_c = 2a + \sigma_r$$

continuation →

sub σ_c in (1)

$$dr[\sigma_c+\sigma_r] = -r \cdot d\sigma_r$$

$$\sigma_c = -\sigma_r - r\frac{d\sigma_r}{dr}$$

$$(\sigma_r+2a) = -\sigma_r - r\frac{d\sigma_r}{dr}$$

$$\frac{d\sigma_r}{dr} = \frac{2(\sigma_r+a)}{r}$$

$$\frac{d\sigma_r}{\sigma_r+a} = \frac{2dr}{r}$$

Apply integration o.b.s

$$\log_e(\sigma_r+a) = -2\log_e r + \log_e b$$

$$\log_e(\sigma_r+a) = \log_e\left[\frac{b}{r^2}\right]$$

$$\sigma_r+a = \frac{b}{r^2}$$

$$\boxed{\sigma_r = \frac{b}{r^2}-a} \Rightarrow p_x = \frac{b}{x^2}-a \longrightarrow ①$$

$$\boxed{\sigma_c = \frac{b}{r^2}+a} \Rightarrow \sigma_x = \frac{b}{x^2}+a \longrightarrow ③$$

The above Eqn are known as Lame's eqn.

→ The constants 'a' and 'b' can be evaluated from the known internal and external p. radial pressure and radius.

→ It may be noted that σ_r is compressions & σ_c is tensile.

→ Eqn ② → gives radial pressure p_x.

→ Eqn ③ → gives hoop's stress at any radius x.

The constants 'a' and 'b' are obtained from the boundary conditions are.

* at $x = r_1$, $p_x = p_0$ [Inside fluide pressure].

* at $x = r_2$, $p_x = 0$ [Atmospheric pressure].

After knowing the values of 'a' and 'b', the hoop's stress can be calculated at any radius.

problems:-

1) Determine the max and min hoop's stress across the section of a pipe of 400mm of internal dia and 100mm thick, when the pipe contains a fluid at a pressure of $8 N/mm^2$.

sol:- Given data, $t = 100mm \Rightarrow 0.1m$

$d_i = 400mm \Rightarrow 0.4\,m$

$d_o = d_i + 2t$

$= 400 + 2 \times 100$

$d_o = 600\,mm \Rightarrow 0.6m$

$r_1 = \frac{d_i}{2} = \frac{200}{2} = 200mm$

$r_2 = \frac{d_o}{2} = \frac{600}{2} = 300\,mm$

Apply Boundary conditions.

$$P_x = \frac{b}{x^2} - a$$

→ At $x = r_1$, $P_x = P_0$

$$P_0 = \frac{b}{200^2} - a$$

$$8 = \frac{b}{200^2} - a \rightarrow (1)$$

→ At $x = r_2$, $P_x = 0$

$$\frac{b}{300^2} - a = 0 \rightarrow (2)$$

$$8 = \frac{b}{200^2} - \frac{b}{300^2}$$

$$b = 576 \times 10^3.$$

sub b in (2)

$$\frac{576 \times 10^3}{300^2} = a$$

$$6.4 = a$$

→ radial stress

$$\sigma_r = \frac{b}{r_1^2} - a \; ; \; \frac{b}{r_2^2} - a$$

$$= \frac{576 \times 10^3}{200^2} - 6.4$$

$$\sigma_r = 8\ N/mm^2$$

and $$\frac{576 \times 10^3}{300^2} - 6.4$$

$$\sigma_r = 0\ N/mm^2$$

→ circumferential stresses

$$\sigma_c = \frac{b}{r_1^2} + a \quad ; \quad \frac{b}{r_2^2} + a$$

$$= \frac{576 \times 10^3}{200^2} + 6.4 \; ; \; \frac{576 \times 10^3}{300^2} + 6.4$$

$$= 20.8 \; N/mm^2 \quad ; \quad 12.8 \; N/mm^2.$$

Q> Find the thickness of metal necessary for a cylindrical shell of internal diameter 160mm to withstand an internal pressure of $8 N/mm^2$. The max. hoop's stress in the section is not to exceed $35 N/mm^2$?

Sol:-

$$d_i = 160 mm$$

$$p = 8 \; N/mm^2$$

$$\sigma_c = 35 N/mm^2$$

$$P_x = \frac{b}{x^2} - a$$

Apply boundary conditions

at $x = r_2$; $P_x = P_0$.

$$r_2 = \frac{d_i}{2} = \frac{160}{2} = 80 mm$$

$$8 = \frac{b}{80^2} - a$$

$$8 = \frac{b}{6400} - a \quad \text{—— (1)}$$

$\sigma_c = 35$, $x = 80 = r_1$

$$\sigma_c = \frac{b}{x^2} + a$$

$$35 = \frac{b}{80^2} + a \rightarrow (2)$$

(2) - (1) $\quad 2a = 27$

$$a = 13.5$$

Sub a in (2)

$$35 = \frac{b}{80^2} + 13.5$$

$$b = 137.6 \times 10^3$$

$$\sigma_c = \frac{b}{r_2^2} + a$$

$$35 = \frac{137.6 \times 10^3}{r_2^2} + 13.5$$

$$r_2 =$$

At $x = r_2$, $P_x = 0$

$$P_x = \frac{b}{x^2} - a$$

$$\cancel{0} = \frac{137.6 \times 10^3}{r_2^2} = 13.5$$

$$r_2 = 100.95 \text{ mm}$$

$$d_o = 2r_2 = 201.91 \text{ mm}$$

$$d_o = d_i + 2t$$

$$d_o - d_i = 2t \Rightarrow 160 - 201.91 = 2t$$

$$41.91 = 2t$$

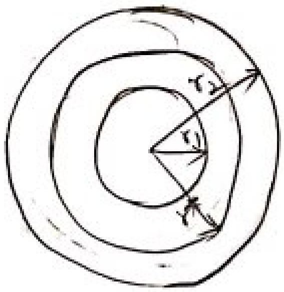
$$\boxed{t = 20.955 \text{ mm}}$$

Stresses in compound thick cylinders:-

We find that the hoop's stress is maximum at the inner radius and it decreases towards the outer radius. The hoop's stress is tensile in nature & it is caused by the internal fluid pressure inside the cylinder. The maximum hoop stress at the inner radius is always greater than the internal fluid pressure. Hence the maximum fluid pressure. Hence the maximum fluid pressure inside the cylender is limited corresponding to the condition. that the hoop's stress at the inner radius reaches the permissible value. Incase of cylinders which have to carrying high internal fluid pressures, some methods of reducing the hoop's stress have to devised.

One method is to wind strong steel wire under tension on the cylinder. The effect of the wire is to put the cylinder wall under initial compressive stress.

Another method is to shrink one cylinder over the other. Due to initial compression where as the outer cylinder will be put into initial tension. If now the compound cylinder is subjected to internal fluid pressure, both the inner & outer cylinders will be subjected to hoop tensile stress. The net effect of initial stresses due to shrinking and those due to internal fluid pressure is to make the resultant stresses more (or) less uniform.



$r_2 \rightarrow$ outer radius of the compound cylinder.

$r_1 \rightarrow$ Inner radius " " "

$r^* \rightarrow$ Radius @ junction of two cylinder [outer radius of ra, inner cylinder (or) Inner radius of outer cylinder].

$p^* \rightarrow$ Radial pressure at the junction of two cylinders.

Let us now apply Lame's eqn [after shrinking the outer cylinder over the inner cylinder and fluid under pressure is not admitted into inner cylinder].

* for outer cylinder the lame's eqn at a radius 'x':

$$P_x = \frac{b_1}{x^2} - a_1 \longrightarrow (1)$$

* $$\sigma_x = \frac{b_1}{x^2} + a_1 \longrightarrow (2)$$

where a_1 and b_1 are constants for outer cylinder

at $x = r_2$, $P_x = 0$ and $x = r^*$, $P_x = p^*$

$$0 = \frac{b}{r_2^2} - a_1 \longrightarrow (3)$$

$$p^* = \frac{b}{r^{*2}} - a_1 \longrightarrow (4)$$

from eqn (3) and (4), the constants a_1 & b_1 can be determined. Then hoop's stresses in the outer cylinder due to shrinking can be obtained.

→ for inner cylinder, the lame's eqn at radius x

$$P_x = \frac{b_2}{x^2} - a_2 \longrightarrow (5)$$

$$\sigma_x = \frac{b_2}{x^2} + a_2 \longrightarrow (6)$$

→ At $x = r_1$, $P_x = 0$. As fluid under pressure is not admitted into inner cylinder and At $x = r^*$, $P_x = p^*$

$$0 = \frac{b_2}{r_1^2} - a_2 \longrightarrow (7)$$

$$p^* = \frac{b_2}{r^{*2}} - a_2 \longrightarrow (8)$$

From (7) & (8), the constant a_2 & b_2 can be determined. Then hoop's stress in the inner cylinder due to shrinking can be obtained.

→ Hoop's stresses in compound cylinder due to internal fluid pressure alone :-

When the fluid under pressure is admitted into the compound cylinder, the hoop stresses are set in the compound cylinder.

To find these stresses, the inner cylinder & outer cylinder will together be considered as one thick shell,

let $p \rightarrow$ Internal fluid pressure

Now, the lame's eqn

$$P_x = \frac{B}{x^2} - A \text{ and}$$

$$\sigma_x = \frac{B}{x^2} + A$$

where A & B are the constants for single thick shell due to internal fluid pressure.

$\rightarrow$ at $x = r_2$, $P_x = 0$

$$0 = \frac{B}{r_2^2} - A$$

$\rightarrow$ At $x = r_1$, $P_x = p$

$$p = \frac{B}{r_1^2} - A$$

From these eqn, we can find A & B from A + B, we can find σ_x, P_x.

The resultant hoop's stresses will be the algebraic sum of hoop's stresses caused due to shrinking & those due to internal fluid pressure.

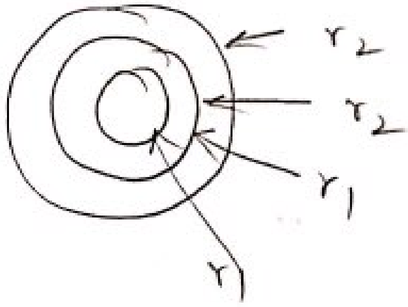
problems:-

Q) The compound cylinder is made by shrinking a cylinder of external dia 300mm and internal dia of 250mm over another cylinder of external dia 250 mm and internal dia 200mm. The radial pressure at the junction after shrinking is 8 N/mm². Find the final stresses set up across the section when the compound cylinder is subjected to an internal fluid pressure of 84.5 N/mm².

sol:- Given data.

$$2\begin{cases} D_i = 300mm \\ D_o = 250mm \end{cases}$$

$$1\begin{cases} d_i = 250mm \\ d_o = 200mm \end{cases}$$



$p^* = 8 N/mm^2$

$P = 84.5 N/mm^2$

$$P_x = \frac{b}{x^2} - a$$

$\rightarrow$ At $x = r_2$, $P_x = 0$ $[r_2 = 150]$

$$0 = \frac{b_1}{r_2^2} - a_1$$

$r_2 = 150mm$

$r_1 = 125mm$

$$= \frac{b_1}{150^2} - a_1$$

$$\frac{b_1}{150^2} = a_1 \longrightarrow (1)$$

$\rightarrow$ At $x = r^*$, $P_x = p^*$ $[r^* = 125]$

$$p^* = \frac{b_1}{x^2} - a_1$$

$$8 = \frac{b_1}{125^2} - a_1$$

$$8 + a_1 = \frac{b_1}{125^2}$$

$$8 = \frac{b_1}{150^2} - \frac{b_1}{125^2}$$

$$\boxed{b_1 = 409090.9091}$$

sub b_1 in (1)

$$\frac{b_1}{150^2} = a_1$$

$$\boxed{a_1 = 18.18}$$

$$\sigma_x = \frac{b_1}{x^2} + a_1$$

$$\sigma_x = \frac{409090.9091}{125^2} + 18.18$$

$$\boxed{\sigma_{x@125} = 44.36 \ N/mm^2}$$

$$\sigma_{x@150} = \frac{409090.9091}{150^2} + 18.18$$

$$\boxed{\sigma_{x@150} = 36.36 \ N/mm^2}$$

$$\rightarrow \quad P_x = \frac{b_2}{x^2} - a_2$$

At $x = r_1$, $P_x = 0$ $\quad [r_1 = 100]$

$$0 = \frac{b_2}{r_1^2} - a_2$$

$$\frac{b_2}{100^2} = a_2 \longrightarrow (2)$$

At $x = r^*$, $P_x = P^*$ $\quad [r^* = 125]$

$$P^* = \frac{b_2}{(r^*)^2} - a_2$$

$$8 = \frac{b_2}{125^2} - a_2$$

$$8 = \frac{b_2}{125^2} - \frac{b_2}{125^2}$$

$$\boxed{b_2 = 222222.2}$$

sub b_2 in ②

$$\frac{222222.2}{100^2} = a_2$$

$$\boxed{a_2 = 22.22}$$

$$\sigma_{x@100} = \frac{b_2}{x_2^2} + a_1$$

$$= \frac{222222.2}{100^2} + 22.22$$

$$\boxed{\sigma_{x@100} = 44.44 \ N/mm^2}$$

$$\sigma_{x@125} = \frac{222222.22}{125^2} + 22.22$$

$$\boxed{\sigma_{x@125} = -36.44 \ N/mm^2}$$

→ Hoop stresses

$$P_x = \frac{B}{x^2} - A$$

→ at $x = r_2$, $P_x = 0$ $[r_2 = 150]$

$$0 = \frac{B}{150^2} - A$$

$$\frac{B}{150^2} = A \longrightarrow ③$$

$\rightarrow$ At $x = r_1$, $P_x = P$ $[r_1 = 100]$

$$94.5 = \frac{B}{100^2} - A$$

$$94.5 = \frac{B}{150^2} - \frac{B}{100^2}$$

$$\boxed{B = 1521000}$$

sub B in (3)

$$\frac{1521000}{150^2} = A$$

$$\boxed{A = 67.6}$$

$$\sigma_{x@150} = \frac{B}{x^2} + A$$

$$= \frac{1521000}{150^2} + 67.6$$

$$\boxed{\sigma_{x@150} = 135.2\ N/mm^2}$$

$$\sigma_{x@100} = \frac{B}{x^2} + A$$

$$= \frac{1521000}{100^2} + 67.6$$

$$\boxed{\sigma_{x@100} = 219.7\ N/mm^2}$$

$$\sigma_{x@125} = \frac{B}{x^2} + A$$

$$= \frac{1521000}{125^2} + 67.6$$

$$\boxed{\sigma_{x@125} = 164.944\ N/mm^2}$$

The resultant hoop stresses will be the algebraic sum of hoop's stress caused due to shrinkage and those due to internal fluid pressure.

→ for Inner cylinder

$f_{100} = \sigma_{@100}$ due to shrinkage + σ_{100} due to internal pressure.

$= 44.44 + 219.7$

$= 264.14.$

$f_{125} = \sigma_{125}$ due to shrinkage + σ_{125} due to internal pressure.

$= -36.44 + 164.944$

$\boxed{f_{125} = 128.504 \text{ N/mm}^2}$

→ for outer cylinder

$f_{150} = \sigma_{150}$ due to shrinkage + σ_{150} due to internal pressure.

$= 36.36 + 135.2$

$\boxed{f_{150} = 171.5 \text{ N/mm}^2}$

$f_{125} = \sigma_{125}$ due to shrinkage + σ_{125} due to internal pressure

$= 44.06 + 164.944$

$$\boxed{f_{b5} = 209.30 \ N/mm^2}$$

Initial difference in radii at the junction of a compound cylinder for shrinkage:-

By shrinking the outer cylinder over the inner cylinder, some compressive stresses are produced in the inner cylinder. Inorder to shrink the outer cylin-der over the inner cylinder. The inner diameter of the outer cylinder should be slightly less than the outer diameter of the inner cylinder. Now the outer cylinder is heated and inner cylinder is inserted into that. After cooling, the outer cylinder shrinks over the inner cylinder. Thus inner cylinder is put into compre-ssion and outer cylinder is put into tension. After shrinking the outer radius of inner cylinder decreases where as the inner radius of outer cylinder is increases from the initial values.

Let, $r_2 \rightarrow$ outer radius of the outer cylinder

$r_1 \rightarrow$ Inner radius of Inner cylinder

$r^* \rightarrow$ Radius of junction after shrinking (or) it is common radius after shrinking.

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Before shr

p^* → Radial pressure at the junction aftershrinking.

Before shrinking the outer radius of inner cylinder is slightly more than "r^*" and inner radius of outer cylinder is slightly less than the "r^*".

→ for the outer cylinder, the Lame's eq. is

$$P_x = \frac{b}{x^2} - a$$

$$\sigma_x = \frac{b}{x^2} + a$$

The values of a, b constants are different for each cylinder.

→ Let the constants for inner cylinder be a_2, b_2 and for outer cylinder a_1, b_1.

→ The radial pressure at the junction (p^*) is same for outer cylinder and inner cylinder.

→ All the junction $x = r^*$, $P_x = p^*$. Hence the radial the pressure at the junction.

$$p^* = \frac{b_1}{(r^*)^2} - a_1 \Rightarrow \frac{b_2}{(r^*)^2} - a_2$$

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$$P_r^* = \frac{b_1}{(r^*)^2} - a_1 \longrightarrow (1)$$

$$P_x = \frac{b_2}{(r*)^2} - a_2 \longrightarrow (2)$$

(1) - (2)

$$\frac{b_1}{(r^*)^2} - a_1 - \frac{b_2}{(r*)^2} + a_2 = 0$$

$$\boxed{b_1 - b_2 = (a_1 - a_2)\, r^{*2}}$$

Now hoop strain [circumferential strain] in the cy cylinder at any point.

$$e_c = \frac{\sigma_x}{E} + \frac{P_x}{mE}$$

But circumferential strain = $\frac{\text{Increase circumference}}{\text{original circumferential}}$

$$= \frac{2\pi(r+dr) \, 2\pi r}{2\pi r} = \frac{dr}{r}$$

$$\boxed{\frac{dr}{r} = \text{Radial strain}}$$

$$\frac{dr}{r} = \frac{\sigma_x}{E} + \frac{P_x}{mE}$$

Hence equating the values of circumfrentral strain on shrinking at the junction there is an extension in the inner radius of outer cylinder & compression.

In the outer radius of inner cylinder

→ At the function, $x = r^*$, Increase in the inner of outer cylinder.

$$dr = \left[\frac{\sigma_x}{E} + \frac{P_x}{mE}\right] r^*$$

$(r = r^*)$

→ But for the outer cylinder,

$$\sigma_x = \frac{b_1}{x^2} + a_1$$

$$\sigma_x = \frac{b_1}{(r^*)^2} + a_1 \qquad (x = r^*)$$

$$P_x = \frac{b_1}{(r^*)^2} - a_1$$

$$dr = r^* \left[\frac{b_1}{(r^*)^2 E} + a_1 + \frac{b_1}{(r^*)^2 mE} - a_1\right] \rightarrow ③$$

$$= r^* \left[\frac{b_1}{(r^*)^2 E} - \left(\frac{1}{m}\right)\right]$$

$$= \frac{b_1}{r^* E}\left[1 + \frac{1}{m}\right].$$

$||^{le}$ decrease in the outer Radius of inner cylinder is obtained.

$$dr = -\left[r^* \left[\frac{\sigma_x}{E} + \frac{P_x}{mE}\right]\right]$$

But for inner cylinder.

$$P_x = \frac{b_2}{(r^*)^2} - a_2$$

$$\sigma_x = \frac{b_2}{(r^*)^2} + a_2$$

$$dr = -\left[r^* \left[\frac{\frac{b_2}{(r^*)^2} + a_2}{E} + \frac{\frac{b_2}{(r^*)^2} - a_2}{mE}\right]\right]$$

$$= -\frac{r^* b_2}{r^{*2} E}\left[1 + \frac{1}{m}\right]$$

$$= -\left[\frac{b_2}{r^* E}\left[1 + \frac{1}{m}\right]\right]$$

But for the original difference ③ + ④

$$= r^* \left[\frac{1}{E}\left[\frac{b_1}{r^{*2}} + a_1\right] + \frac{1}{mE}\left[\frac{b_1}{r^{*2}} - a_1\right] - r^*\left[\frac{1}{E}\left[\frac{b_2}{r^{*2}} + a_2\right.\right.\right.$$

$$\left.\left. - \frac{1}{mE}\left[\frac{b}{r^{*2}} - a_2\right]\right]\right]$$

$$\Rightarrow \frac{r^*}{E}\left[\left[\frac{b_1}{r^{*2}} + a_1\right] - \left[\frac{b_2}{r^{*2}} + a_2\right]\right] + \frac{r^*}{mE}\left[\left[\frac{b_1}{r^{*2}} - a_1\right] - \left(\frac{b_2}{r^{*2}} - a_2\right)\right]$$

Hence, But eqn $\frac{b_1}{r^{*2}} - a_1 = \frac{b_2}{r^{*2}} - a_2$

Hence, second part of above eqn is zero. Hence above eqn becomes original difference of radii

@ Junction.

$$= \frac{r^*}{E}\left[\left[\frac{b_1}{r^{*2}} + a_1\right] - \left[\frac{b_2}{r^{*2}} + a_2\right]\right]$$

$$= \frac{r^*}{e}\left[\left(\frac{b_1-b_2}{r^{*2}}\right)\left[a_1-a_2\right]\right]$$

$$= \frac{r^*}{e}\left[(a_1-a_2)(a_1-a_2)\right]$$

$$\boxed{dr = \frac{2r^*}{e}\left[(a_1-a_2)\right]}$$

The values of a_1 and a_2 are obtained from the given condition the value of a_1 is for outer cylinder where as a_2 is for inner cylinder.

problems:-

Q) A steel cylinder of 300mm external dia is to shrunk to another steel cylinder of 150mm internal dia. After shrinking the diameter at the junction is 250mm and Radial pressure at the common junction is 28 N/mm². Find the original difference in the radii at the junction.

Take $E = 2\times10^5$ N/mm².

sol:-

$D_0 = 300$ mm $\Rightarrow R_0 = 150$ mm

$d_i = 150$mm $\Rightarrow R_i = 75$ mm

$D^* = 250$ mm $\Rightarrow R^* = 125$mm

$p^* = 28$ N/mm²

$E = 2\times10^5$ N/mm²

from Lame's eqn

$$P_x = \frac{b_1}{x^2} - a_1$$

→ For outer cylinder

→ At $x = r_2^*$, $P_x = p^*$

$$28 = \frac{b_1}{125^2} - a_1 \longrightarrow ①$$

$$28 + a_1 = \frac{b_1}{125^2}$$

→ At $x = r_2$, $P_x = P$

$$0 = \frac{b_1}{150^2} - a_1$$

$$\frac{b_1}{150^2} - a_1 \rightarrow ②$$

$$28 = \frac{b_1}{125^2} - \frac{b_1}{150^2}$$

$$\boxed{b_1 = 1431818.182}$$

sub b_1 in ②

$$\frac{1431818.182}{150^2} = a_1$$

$$\boxed{a_1 = 63.63}$$

→ for inner cylinder

$x = r_1$, $P_x = 0$

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$$0 = \frac{b_2}{75^2} - a_2 \rightarrow (3)$$

$$x = r_1^*, \quad P_x = p^*$$

$$28 = \frac{b_2}{125^2} - a_2 \rightarrow (4)$$

$$28 = \frac{b_2}{125^2} - \frac{b_2}{75^2}$$

$$\boxed{b_2 = -246093.75}$$

$$\boxed{a_2 = -43.75}$$

$$dr \Rightarrow \frac{2r^*}{E}\left[(a_1 - a_2)\right]$$

$$= \frac{2 \times 125}{2 \times 10^5}(63.63 + 43.75)$$

$$\boxed{dr = 0.13\ mm}$$

Q) A steel tube of 200mm external dia is to be strong on to another steel tube of 60mm internal dia. The dia at the junction after shrinking is 120mm. Before shrinking on, the difference of dia at the junction 0.08mm. calculate the radial pressure at the junction and hoop stresses developed in the two tubes after shrinking on. Take $E = 2 \times 10^5\ N/mm^2$.

sol:-

Given data

$d_2 = 200mm \Rightarrow r_2 = 100mm$

$d_1 = 60mm \Rightarrow r_1 = 30mm$

$d^* = 120mm \Rightarrow r^* = 60mm$

$dr = 0.67mm$

$E = 2 \times 10^5 \, N/mm^2$

$\rightarrow$ for outer cylinder

At $x = r_2$, $P_x = 0$ $\quad P_x = \frac{b}{x^2} - a_1$

$$0 = \frac{b_1}{100^2} - a_1$$

$$\frac{b_1}{100^2} = a_1 \rightarrow (1)$$

$\rightarrow$ At $x = r^*$, $P_x = p^*$

$$p^* = \frac{b_1}{60^2} - a_1 \rightarrow (2)$$

$$p^* = \frac{b_1}{100^2} - \frac{b_1}{60^2}$$

$$p^* = 1.77 \times 10^{-4} b_1$$

$\rightarrow$ for inner cylinder

At $x = r^*$, $P_x = p^*$

$$p^* = \frac{b_2}{60^2} - a_2 \rightarrow (3)$$

At $x = r_1$, $P_x = 0$

$$0 = \frac{b_2}{30^2} - a_2 \rightarrow (4)$$

$$\frac{b_1}{100^2} - a_1 = \frac{b_2}{30^2} - a_2$$

$$\frac{b_1}{100^2} - \frac{b_2}{30^2} = a_2 + a_1$$

$b_2 - b_1 =$

sub (3) in (2)

$$\frac{b_2}{60^2} - a_2 = \frac{b_1}{60^2} - a_1$$

$$(b_1 - b_2) = (a_1 - a_2)\, 60^2$$

$$\frac{2\gamma^*}{\epsilon}(a_1 - a_2) = 0.08\,m$$

$$(a_1 - a_2) = \frac{\epsilon}{2\gamma^*}(0.08)$$

$$= \frac{2 \times 10^5}{2 \times 60}(0.08)$$

$$(a_1 - a_2) = 133.33$$

$$\hookrightarrow a_1 = 133.33 + a_2$$

$$(b_1 - b_2) = 133.33 \times 60^2$$

$$(b_1 - b_2) = 480000$$

$$480000 = ((133.33 + a_2) - a_2)\, 60^2$$

$480000 =$

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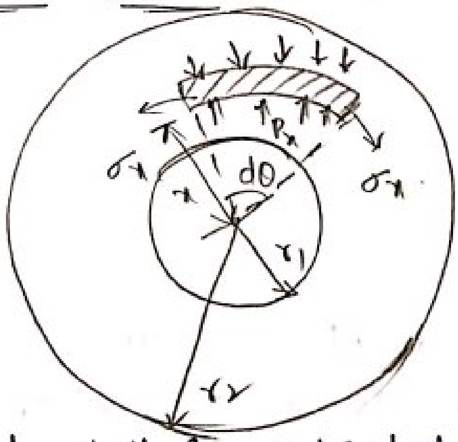
THICK SPHERICAL SHELLS:-



A spherical shell is subjected to internal pressure 'p'

Let $r_2 \rightarrow$ external radius

$r_1 \rightarrow$ Internal radius

consider an elemental strip of spherical shell of thickness 'dx' at a radius 'x'.

Let this elemental strip subjected an angle 'dθ' at Centre. Due to the internal fluid pressure,

Let the radius 'x' increased to 'x+u' and increases its thickness 'dx' to 'du'.

Let $e_y \rightarrow$ circumferential strain along y-axis.

$e_x \rightarrow$ Radial strain.

Now, increase in radius $= u$

final radius $= x+u$

$$\text{circumferential strain} = \frac{\text{final} - \text{Initial circumference}}{\text{original circumference}}$$

$$= \frac{2\pi(x+u) - 2\pi x}{2\pi x}$$

$$e_y = \frac{u}{x} \longrightarrow (1)$$

The original thickness of element $= dx$

Final thickness of element $= dx + du$

Then Radial strain $= \dfrac{\text{final} - \text{initial thickness}}{\text{original thickness}}$

$$= \frac{\cancel{dx} + du - \cancel{dx}}{dx}$$

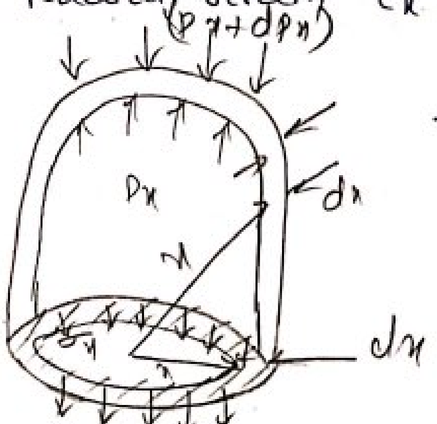
$$e_x = \frac{du}{dx} \longrightarrow (2)$$

But from eqn (1)

$u = x \cdot e_y$ [sub in eqn (2)]

then

Radial strain $e_x = \dfrac{d(x \cdot e_y)}{dx}$



$$e_x = e_y + x\frac{de_y}{dx} \longrightarrow (3)$$

Now consider a elemental spherical shell of radius 'x' and thickness 'x'.

Let P_x and $P_x + dp_x \longrightarrow$ radial pressure at radii 'x' and $x+dx$ respectively.

$\sigma_x \rightarrow$ circumferential tensile stress which is equal in all directions in a spherical shell.

considering, the equilibrium of half of the elemtary spherical shell on which the following external forces are acting.

→ An upward force $\pi x^2 \times p_x$ due to internal radial pressure 'p'

→ A downward force of $\pi(x+dx)^2(p_x+dp_x)$ due to radial pressure (p_x+dp_x)

→ A downward resisting force $[\sigma_x \cdot 2\pi x \cdot dx]$.

Equating upward and downwards force.

$$\pi x^2 p_x = \pi(x+dx)^2(p_x+dp_x)+(\sigma_x \cdot 2\pi x\, dx)$$

$$\pi x^2 \cdot p_x = \pi(x^2+dx^2+2x\,dx)(p_x+dp_x)+(\sigma_x\, 2\pi x\, dx)$$

$$\pi x^2 p_x = \pi x^2+\pi dx^2+\pi 2x\, dx \cdot p_x+dp_x+\sigma_x\, 2\pi x\, dx$$

$$2x \cdot \sigma_x\, dx = -2x \cdot dx\, p_x - x^2 dp_x$$

$\div x$

$$2\sigma_x dx = -2\,dx\, p_x - x\,dp_x$$

$$2\sigma_x = -2p_x - x\frac{dp_x}{dx}$$

$$\sigma_x = -p_x - \frac{x}{2}\frac{dp_x}{dx} \rightarrow (4)$$

differentiate above eqn w.r.t.'x'.

$$\frac{d}{dx}(\sigma_x) = \frac{d}{dx}\left(-p_x - \frac{x}{2}\frac{dp_x}{dx}\right)$$

$$\frac{-dp_x}{dx} = \frac{1}{2}\left[x\frac{d^2p}{dx^2} + \frac{dp_x}{dx}\right]$$

At any point in the elementary spherical shell there are 3 principal stresses.

→ Radial pressure 'p_x' which is compressive.

→ circumferential (or) hoop's stress 'σ_x' which is tensile.

→ circumferential stress 'σ_x' which is tensile of same magnitude of radial strain and on a plane at right angles to the plane of 'σ_x' of radial strain.

→ Radial strain $e_x = \frac{p_x}{E} + \frac{\sigma_x}{mE} + \frac{\sigma_x}{mE}$

$$= \frac{p_x}{E} + \frac{2\sigma_x}{mE} \quad [\text{compressive}]$$

$$e_x = -\left[\frac{p_x}{E} + \frac{2\sigma_x}{mE}\right] \quad [\text{tensile}]$$

→ circumferential strain $e_y = \frac{\sigma_x}{E} - \frac{\sigma_x}{mE} + \frac{p_x}{mE}$

$$= \frac{1}{E}\left[\sigma_x - \frac{\sigma_x}{m} + \frac{p_x}{m}\right]$$

$$e_y = \frac{1}{E}\left[\sigma_x\left(\frac{m-1}{m}\right) + \frac{p_x}{m}\right] \quad (\text{tensile})$$

sub e_y and e_x in ③

$$-\left[\frac{P_x}{E} + \frac{2\sigma_x}{E}\right] = \frac{1}{E}\left[\sigma_x\left(\frac{m-1}{m}\right) + \frac{P_x}{m}\right] + x\frac{d}{dx}\left[\frac{1}{E}\left(\sigma_x\left(\frac{m-1}{m}\right) + \frac{P_x}{m}\right)\right]$$

$$(m+1)(P_x + \sigma_x) + x(m-1)\frac{d\sigma_x}{dx} + x\frac{dP_x}{dx} = 0$$

$$(m+1)\left(P_x - P_x - \frac{x}{2}\cdot\frac{dP_x}{dx}\right) + x(m-1)x\left[-\frac{dP_x}{dx} - \frac{1}{2}\left[x\frac{d^2P}{dx^2} + \frac{dP_x}{dx}\right]\right] + x\frac{dP_x}{dx} = 0$$

$$4\frac{dP_x}{dx} + x\frac{d^2P_x}{dx^2} = 0$$

$$4z + x\frac{dz}{dx} = 0 \qquad \left[\frac{dP_x}{dx} = z\right]$$

$$4z = -x\frac{dz}{dx}$$

$$\frac{dz}{z} = -4\frac{dx}{x}$$

Integrate

$$\log_e z = -4\log_e x + \log_e c_1$$

$$\log_e z = \log_e x^{-4} + \log_e c_1$$

$$\log_e z = \log_e\left(\frac{c_1}{x^4}\right)$$

$$z = \frac{c_1}{x^4}$$

$$\frac{dP_x}{dx} = \frac{c_1}{x^4}$$

$$dP_x = \frac{c_1}{x^4}\cdot dx$$

Integrate,

$$P_x = \frac{c_1}{x^3} + c_2 \quad \text{sub in } \sigma_x$$

$$\sigma_x = \left(\frac{-c_1}{3x^3} + c_2\right) - \frac{x}{2}\frac{dP_x}{dx}$$

$$\Rightarrow \frac{c_1}{3x^3} - c_2 - \frac{x}{2}\cdot\frac{c_1}{x^4}$$

$$= \frac{-c_1}{6x^3} - c_2$$

If we substitute

$c_1 = -6b$, $c_2 = -a$

$$P_x = \frac{-6b}{3x^3} + (-a)$$

$$\boxed{P_x = \frac{2b}{x^3} - a}$$

$$\sigma_x = -\frac{(-6b)}{6x^3} - (-a)$$

$$\boxed{\sigma_x = \frac{b}{x^3} + a}$$

Problems:-

Q) A thick spherical shell of 200mm internal dia is subjected to an internal fluid pressure of 7N/mm². If permissible tensile stress in shell material is 8N/mm². Find the thickness of the shell and also find the max hoop's stress?

Sol:- Given data.

$d_1 = 200mm \Rightarrow r_1 = 100mm$

$p_x = 7 N/mm^2$

$\sigma_x = 8 N/mm^2$

$p_x = \frac{2b}{x^3} - a$; $\sigma_x = \frac{b}{x^3} + a$

At $x = r$, $p_x = p$; At $x = r$, $\sigma_x = \sigma_0$

$7 = \frac{2b}{100^3} - a$; $8 = \frac{b}{100^3} + a$

① + ②

$$7 = \frac{2b}{100^3} - \cancel{a}$$

$$8 = \frac{b}{100^3} + \cancel{a}$$

$$15 = \frac{3b}{100^3}$$

$$\boxed{b = 5 \times 10^6}$$

sub in σ_x

$$8 = \frac{b}{100^3} + a$$

$$8 = \frac{5 \times 10^6}{100^3} - a$$

$$8 = 5 - a$$

$$\boxed{a = 3}$$

→ At $p_x = 0$, $x = r_2$

$$0 = \frac{2b}{(r_2)^3} - a$$

$$\frac{2b}{r_2^3} = a$$

$$\frac{2 \times 5 \times 10^6}{r_2^3} = a$$

$$\boxed{r_2 = 149.38\,mm}$$

$d_2 = 298.76\,mm$

$= 300\,mm$

$d_o = d_i + 2t$

$300 - 200 = 2t$

$100 = 2t$

$$\boxed{t = 50\,mm}$$

σ_x at r_2

$$\sigma_x = \frac{b}{x^3} + a$$

At $x = r_2$,

$$\sigma_x = \frac{5 \times 10^6}{150^3} + 3$$

$$\boxed{\sigma_x = 4.48\,N/mm^2}$$

→ UNSYMMETRICAL BENDING AND SHEAR CENTRE :-

It is assumed that natural axis of c/s^n of the beam is $\perp^r$ to the plane of loading. This means that the plane of loading is $\parallel^{el}$ to the plane containg the principal centroidal axis of the interia of c/s^n of the beam. This type of bending is known as sym symmetrical bending.

This type of bending is known as symmetrical bending If the plane of loading or plane of bending doesnot live [or parallel] a plane that contains the principal centroidal axis of the c/sn, that bending is known as unsymmetrical bending.

In case of unsymmetrical bending, the N.A is not $\perp^r$ to the plane of bending.

The unsymmetrical bending will be when.

* section is symmetrical [such as □^{lr}, O^{lr}, I-section] but load line inclined to both the principal axis.
* The section is unsymmetrical [such as 'L' section (or) [section], and load line is along centroidal axis.

→ **properties of beam c/sn :-**

* The integral $\int xy \cdot dA$ is known as product of interia and pair of axis, for which it is zero, are known as principal axis of c/sn.

* The moments of interia of an area about its principal axis are known as principal moments of interia.

* The B.M about any other axis is known as unsymmetrical bending.

→ **principal moments of interia :-**

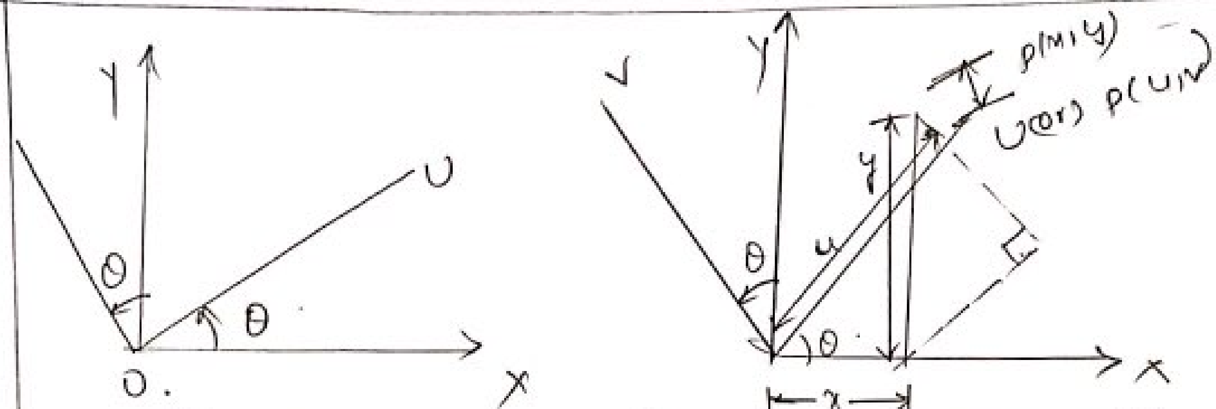
* The principal axis of any area are those axis about which the product of interia $[I_{xy}]$ is zero.

* Axis of symmetry through centroid are automatically principal axes, as the product of moments for opposite co-ordinates cancelling each other out.

→ **condition for principal axis :-**

* $$\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{xx}}$$

* principal moments of inertia about an axis OU and OV are.



$$I_{vu} = \frac{1}{2}(I_{xx}+I_{yy}) + \frac{1}{2}(I_{xx}-I_{yy}) - I_{xy}\sin 2\theta \quad \text{—①}$$

$$I_{Uv} = \frac{1}{2}(I_{xx}+I_{yy}) + \frac{1}{2}(I_{xx}-I_{yy})\sec 2\theta$$

$$I_{vv} = \frac{1}{2}(I_{xx}+I_{yy}) - \frac{1}{2}(I_{xx}-I_{yy})\sec 2\theta$$

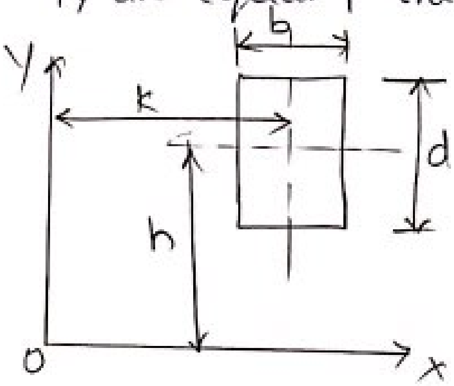
$$I_{uu} + I_{vv} = I_{xx} + I_{yy}$$

→ The co-ordinates u, v relative to ou, ov and x, y relative to ox, oy of any point 'p' are given by

$u = x\cos\theta + y\sin\theta$

$v = y\cos\theta - x\sin\theta$.

→ ~~If~~ I_{xx} and ~~I_{yy} are equal~~, ~~the~~



→ If I_{xx} and I_{yy} are equal, then $2\theta = 90^\circ$

and $\cos 2\theta = 0^\circ$ & $\sec 2\theta = \infty$.

Hence the above eqns will not be give correct result for I_{uu}. Hence, to find I_{uu} in such cases, the eqns ① is used.

→ A rectangle of width 'b' and depth 'd'. The sides of rectangle is parallel to principal axis. The product of inertia 'I_{xy}' will be

$$I_{xy} = \int xy\, dA$$

$$= \iint xy\, dx.dy$$

$$= \int_{k-b/2}^{k+b/2} x\, dx . \int_{h-d/2}^{h+d/2} y\, dy$$

$$= \left[\frac{x^2}{2}\right]_{k-b/2}^{k+b/2} \left[\frac{y^2}{2}\right]_{h-d/2}^{h+d/2}$$

$$= \left[\frac{(k+b/2)^2}{2} - \frac{(k-b/2)^2}{2}\right]\left[\frac{(h+d/2)^2}{2} - \frac{(h-d/2)^2}{2}\right]$$

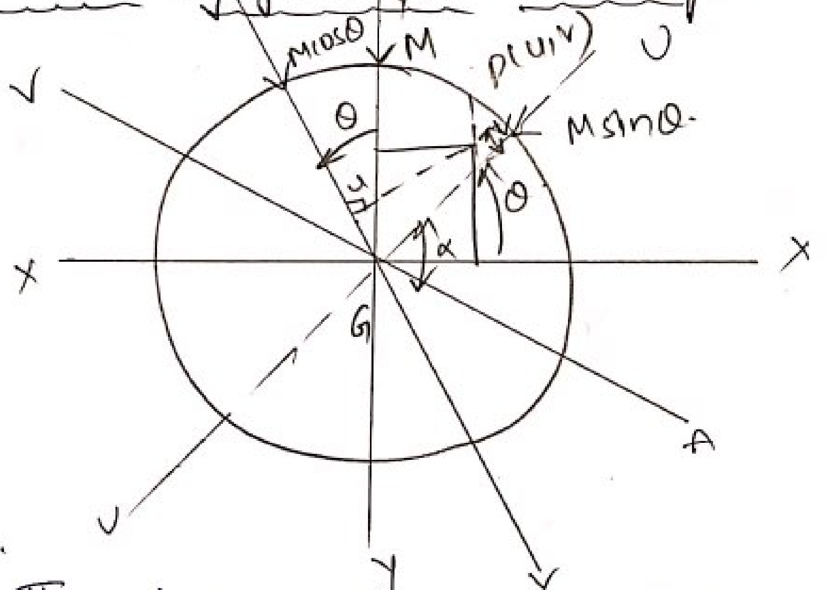
$$= kb \times hd$$

$$= b \times d \times h \times k$$

$$\boxed{I_{xy} = Ahk}$$

Hence, the product of inertia of a rectangle whose sides are ||'el to the axis is equal to area of rectangle × distance of its C.G from x-axis × distance of C.G from y-axis.

Stresses in unsymmetrical bending:-



The c/sn of a beam subjected to B.M of 'M' in the plane of y-y. The co-ordinate axis' x-x & y-y pass through the centroid 'G' of the section. Let UU, VV are the principal axis passes through 'G' and inclined at angle 'θ' to xx and yy axis respectively.

It is required to find resultant stress at any point 'p' having co-ordinates x, y w.r.t axis xx, yy and u, v.

To find stress distribution over the section the moment 'M' in plane yy is resolving into components in the plane UU and VV.

The moment in a plane UU, the moment is $M\sin\theta$, the moment in a plane VV, the moment is $M\cos\theta$.

The moment in a plane UU, will bend the beam about an axis VV, The bending stress σ_b, due to this moment will be equal to

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma_b = \frac{My}{I}$$

$$\boxed{\sigma_b = \frac{M\sin\theta \times u}{I}}$$

M^{ly}, The moment in the plane VV, will bend the beam about axis UU. The B.S due to this moment will be.

$$\boxed{\sigma_b = \frac{M\cos\theta}{I_{UU}} \times V}$$

Then the resultant B.S at any point, p(u,v) will be given by.

$$\sigma_b = \frac{M\sin\theta \times u}{I_{vv}} + \frac{M\cos\theta \times v}{I_{uu}}$$

$$= M\left[\frac{u\sin\theta}{I_{vv}} + \frac{v\cos\theta}{I_{uu}}\right]$$

In the above eqn, the signs of u and v will determine the nature of B.S. If the coordinate of a point w.r.t xx, yy axis are known then the co-ordinates of the same point w.r. of uu, vv axis will be given by the

$$u = x\cos\theta + y\sin\theta$$

$$v = y\cos\theta - x\sin\theta$$

where, θ = inclination of principal axis uu, vv with axis xx, yy.

Neutral axis:-

At the NA, the resultant B.S is zero, Hence the eqn of N.A is obtained by substituting the value of $\sigma_b = 0$

$$M\left[\frac{u\sin\theta}{I_{vv}} + \frac{v\cos\theta}{I_{uu}}\right] = 0$$

$$\frac{u\sin\theta}{I_{vv}} + \frac{v\cos\theta}{I_{uu}} = 0$$

$$\frac{v\cos\theta}{I_{uu}} = -\frac{u\sin\theta}{I_{vv}}$$

$$v = \frac{-u\sin\theta}{I_{vv}} \times \frac{I_{uu}}{\cos\theta}$$

$$v = -u\left[\frac{I_{uu}}{I_{vv}} \times \frac{\sin\theta}{\cos\theta}\right]$$

$$v = -u\left[\frac{I_{uu}}{I_{vv}} \tan\theta\right] \longrightarrow (1)$$

Above eqn is the equation of straight line $[y = mx]$ passing through the centroid 'G' of the section. Here

$M = -\left[\frac{I_{uu}}{I_{vv}} \tan\theta\right]$ is the slope of NA.

→ Slope of NA:-

Let α = angle made by the NA with axis 'UU', then

$\tan\alpha$ = slope of NA

$$\tan\alpha = M$$

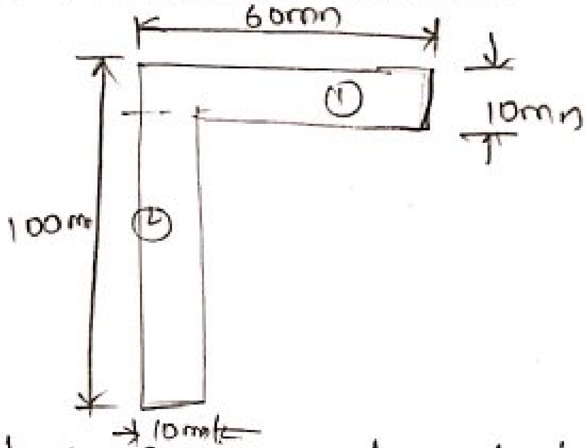
$$\tan\alpha = -\left[\frac{I_{uu}}{I_{vv}} \cdot \tan\theta\right]$$

$$\alpha = \tan^{-1}\left\{-\left[\frac{I_{uu}}{I_{vv}} \tan\theta\right]\right\}$$

Note:- The nature of stress on one side of NA will be same where as on the other side on NA, the stress will be of opposite nature.

→ The stress will be maximum at a point which is having maximum distance from NA.

problems:-



. Diagram shows an unequal angle of dimensions 100mm x 60mm and 10mm thick. Determine.

* position of principal axis and.
* Magnitude of principal MOI for the given angle.

sol:-

$x_1 = \frac{60}{2} = 30mm$

$A_1 = 10 \times 60 = 600 \, mm^2$

$x_2 = \frac{10}{2} = 5mm$

$A_2 = 90 \times 10 = 900 \, mm^2$

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{600 \times 5 + 900 \times 45}{600 + 900}$$

$\boxed{\bar{x} = 15mm}$

$y_1 = \frac{10}{2} = 5mm$, $y_2 = 10 + \frac{90}{2} = 55mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{600 \times 5 + 900 \times 55}{600 + 900}$$

$$\boxed{\bar{y} = 35 mm}$$

$$I_{xx①} = \frac{BD^3}{12} + Ah^2$$

$$= \frac{60 \times 10^3}{12} + 60 \times 10 [35-5]^2$$

$$I_{xx} = 545 \times 10^3 mm^4.$$

$$I_{xx②} = \frac{BD^3}{12} + Ah^2$$

$$= \frac{10 \times 90^3}{12} + 10 \times 90 [35-55]^2$$

$$= 967.5 \times 10^3 mm^4$$

$$I_{xx} = 1.5125 \times 10^6 mm^4$$

$$I_{yy} = \frac{10 \times 60^3}{12} + 10 \times 60 [15-30]^2 + \frac{90 \times 10^3}{12} + 90 \times 10 [15-5]^2$$

$$I_{yy} = 412.5 \times 10^3 mm^4.$$

$$I_{xy} = A_1 h_1 k_1 + A_2 h_2 k_2$$

k_1 = Horizontal distance of C.G of $\square^k$ from yy axis.

$$k_1 = \frac{60}{2} - 15 = 15$$

k_1 = vertical distance of C.G of $\square^k$ from xx axis

$$h_1 = 35 - \frac{10}{2} = 30$$

$$k_2 = \frac{10}{2} - 15 = 10$$

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$h_2 = 35-55 = -20mm$

$A_1 = 600mm^2$

$A_2 = 900mm^2$

$\bar{I}_{xy} = 600 \times 15 \times 30 + 900 \times (-10) \times (-20) = 450 \times 10^3 mm^4$.

→ position of principal axis

$$\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{xx}}$$

$$= \frac{2 \times 450 \times 10^3}{412.5 \times 10^3 - 1.5125 \times 10^6}$$

$\tan 2\theta = -0.818$.

$\tan 2\theta$ is '-ve' in II co-ordinate

$2\theta = \tan^{-1}(-0.818)$

$2\theta = -39.28$

$2\theta = 180 - 39.28$

$2\theta = 140.72$

$$\theta = \frac{140.72}{2}$$

$\boxed{\theta = 70.36^\circ}$ uu and vv.

→ The axis uu will be obtained by drawing uu @ an angle $70^\circ.36'$ with xx-axis through 'g' in anticlockwise direction. This axis vv is at right angles to UU through G. The axis UU and VV are the principal axis. ⟶ continuation

Let 'δu' ⟶ deflection due to load $W\sin\theta$ along line UU'. Deflection of beams in unsymmetrical bending :-

δv ⟶ deflection due to load $W\cos\theta$ along line VV'.

$$\delta u = \frac{K(W\sin\theta)L^3}{EI_{VV}}$$

$$\delta v = \frac{K(W\cos\theta)L^3}{EI_{UU}}$$

Here K ⟶ a constant depending upon the end conditions of the beam and position of the load along beam.

L ⟶ length of beam.

The resultant deflection $\delta = \sqrt{\delta u^2 + \delta v^2}$

$$= \sqrt{\left[\frac{K(W\sin\theta)L^3}{EI_{VV}}\right]^2 + \left[\frac{K(W\cos\theta)L^3}{EI_{UU}}\right]^2}$$

$$= \frac{KWL^3}{E}\sqrt{\left[\frac{\sin^2\theta}{I_{VV}^2} + \frac{\cos^2\theta}{I_{UU}^2}\right]}$$

Here, $K = \frac{1}{48}$ for S.S.B carrying a point load @ Centre.

The angle β made by resultant deflection 'δ' with the line GV' is given by —

continue.

$$I_{UU} = \frac{1}{2}(I_{xx} + I_{yy}) + \frac{1}{2}(I_{xx} - I_{yy}) \sec 2\theta$$

$$= \frac{1}{2}(1.5125\times10^6 + 4125\times10^3) + \frac{1}{2}(1.5125\times10^5 - 412.5\times10^3)\times \sec 2(70^\circ 36')$$

$$= 962500 + 550000 \times -1.29$$

$$= (-205.49\times10^3)\times -1.291$$

$$= 253000$$

$$I_{VV} = \frac{1}{2}(I_{xx} + I_{yy}) - \frac{1}{2}(I_{xx} - I_{yy}) \sec 2\theta$$

$$= \frac{1}{2}(1.5125\times10^6 + 412.5\times10^3) - \frac{1}{2}(1.5125\times10^6 - 412.5\times10^3)\times \sec 2(70^\circ 36')$$

$$= 962500 - 550000 * (-1.29)$$

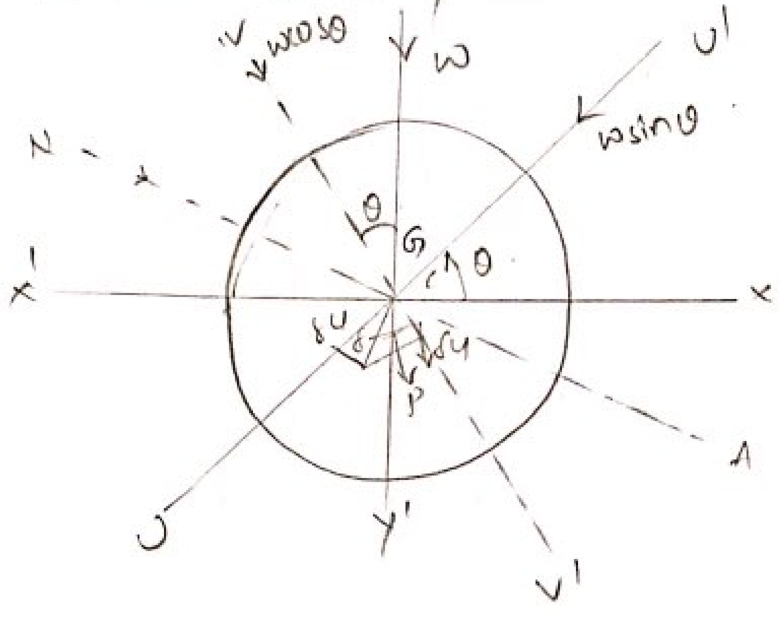
$$= 1672000$$

$$I_{UU} + I_{VV} = I_{xx} + I_{yy}$$

$$253000 + 1672000 = 1.5125\times10^6 + 412.5\times10^3$$

$$1925000 = 1925000$$

Deflection of beams in unsymmetrical bending :-



A transverse section of beam with centroid 'G' along with rectangular co-ordinate axis x-x and y-y'. The principal axis UU' and VV' inclined at an angle 'θ' to xy set of co-ordinate axis. 'w' is the load acting along with the line y-y'. This load can be resolved into two components.

$w \sin\theta$ along UG

$w \cos\theta$ along VG

The component $w \sin\theta$ will be bend the beam along about VV' axis. where as $w \cos\theta$ will be bent about the axis UU'.) → continuation

$$\tan\beta = \frac{\delta u}{\delta v}$$

$$\tan\beta = \frac{\frac{\cancel{k}(w\sin\theta)\cancel{L^3}}{\cancel{E}I_{vv}}}{\frac{\cancel{k}(w\cos\theta)\cancel{L^3}}{\cancel{E}I_{UU}}}$$

$$= \frac{\sin\theta}{I_{vv}} \times \frac{I_{UU}}{\cos\theta}$$

$$\tan\beta = \tan\theta \frac{I_{UU}}{I_{vv}}$$

$$\tan\alpha = -\left[\frac{I_{UU}}{I_{vv}} \tan\theta\right]$$

From above eqn, It is clear that magnitude of angles i.e, β and α are the same. They are measured from ⊥r line GU and GV in the same deflection. 'α' gives the direction of NA, and 'β' gives the direction of resultant deflection.

Hence the resultant deflection will be in a direction of ⊥r to N.A.

→ Method for finding bending stress in unsymmetrical bending :-

* Find C.G of the given section. Draw the horizontal and vertical lines XGX' and YGY' through 'G'.

Then xx' and yy' represent the rectangular co-ordinate axis.

* Determine I_{xx} and I_{yy} and I_{xy} of the given section.

* calculate the value of 'θ' from

$$\boxed{\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{xy}}}$$

→ If the value of 'θ' is "+ve", the principal axis uu' will be in counter clock wise axis direction with x-axis, Now find the location of vv' axis which is right angle to uu' - axis

* find the values of I_{uu} and I_{vv} by using

$$I_{uu} = \frac{1}{2}(I_{xx} + I_{yy}) + \frac{1}{2}(I_{xx} - I_{yy}) \sec 2\theta$$

$$I_{vv} = \frac{1}{2}(I_{xx} + I_{yy}) - \frac{1}{2}(I_{xx} - I_{yy}) \sec 2\theta$$

If I_{xx} is equal to I_{yy}, then the values of I_{uu} will be obtained from.

$$I_{uu} = \frac{1}{2}(I_{xx} + I_{yy}) + \frac{1}{2}(I_{xx} - I_{yy}) \cos 2\theta - I_{xy} \sin 2\theta.$$

* find 'M' and its components along principal axis 'Gu' and 'Gv'.

* find the resultant B.S i.e, $\sigma_b = M.\sigma_s$

$$\boxed{\sigma_b = M\left[\frac{U\sin\theta}{I_{VV}} + \frac{V\cos\theta}{I_{UU}}\right]}$$

Q) A cantilever of length 1cm carries a point load of 2000N at the free end. The c/sn of the cantilever is an unequal angle of direction 100mm x 60mm and 10mm thick. The small length of angle is horizontal. The load passes through the centroid of the c/sn. Determine

(i) The position of N.A

(ii) The magnitude of max stress setup, at the fixed section of the cantilever.

(iii) Draw culber's L-section ?

Sol:-

Given data

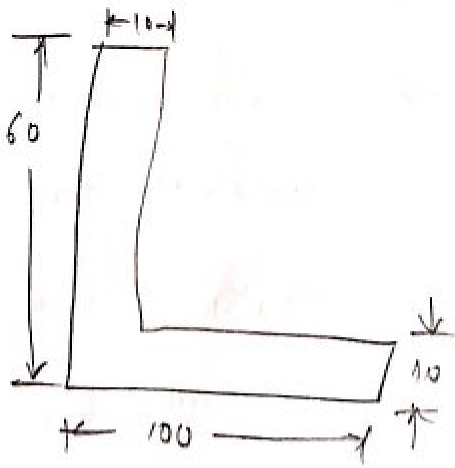
$L = 1cm = 1000mm$

$W = 2000N = 2KN$

$t = 10mm$

$B = 100mm$

$D = 60mm$



$A_1 = 60 \times 10 = 600mm^2$

$A_2 = 90 \times 10 = 900mm^2$

$$x_1 = \frac{60}{2} = 30mm$$

$$x_2 = \frac{10}{2} = 5mm$$

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{600 \times 30 + 900 \times 5}{600 \times 900} = 15mm$$

$$y_1 = \frac{10}{2} = 5mm, \quad y_2 = 10 + \frac{90}{2} = 55mm$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{600 \times 5 + 900 \times 55}{600 + 900} = 35mm$$

$$I_{xx①} = \frac{BD^3}{12} + Ah^2 = \frac{60 \times 10^3}{12} + (60 \times 10)(35-5)^2$$

$$= 545 \times 10^3 \; mm^4$$

$$I_{xx②} = \frac{10 \times 90^3}{12} + (10 \times 90)(35-55)^2 = 967.5 \times 10^3 \; mm^4$$

$$I_{yy} = \frac{10 \times 60^3}{12} + (60 \times 10)(15-30)^2 + \frac{90 \times 10^3}{12} + (90 \times 10)(15-5)^2$$

$$I_{xy} = 412.5 \times 10^3 \; mm^4$$

$$I_{xx} = I_{xx①} + I_{xx②}$$

$$= 1.5125 \times 10^6 \; mm^4$$

$$I_{xy} = A_1h_1k_1 + A_2h_2k_2$$

$$h_1 = 35 - 5 = 30mm \quad [\bar{y} - y_1]$$

$$h_2 = 35 - 55 = -20mm \quad [\bar{y} - y_2]$$

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$K_1 = \frac{60}{2} - 15 = 15mm$ $[x_1 - \bar{x}]$

$K_2 = \frac{10}{2} - 15 = -10mm$ $[x_2 - \bar{x}]$

$I_{xy} = 600 \times 30 \times 15 + 900 \times (-20) \times (-10)$

$= 450 \times 10^3 mm^4.$

→ position of principal axis

$\tan 2\theta = \frac{2 I_{xy}}{I_{yy} - I_{xx}}$

$= \frac{2 \times 450 \times 10^3}{412.5 \times 10^3 - 1.5125 \times 10^6}$

$\tan 2\theta = -0.81$

$2\theta = \tan^{-1}(-0.81)$

$= -39.00$

$2\theta = 180 - 39.00$

$2\theta = 141$

$\theta = \frac{141}{2}$

$\boxed{\theta = 70.5}$

→ I_{UU} and I_{VV}

$I_{UU} = \frac{1}{2}(I_{xx} + I_{yy}) + \frac{1}{2}(I_{xx} - I_{yy}) \sec 2\theta$

$= \frac{1}{2}(1.5125 \times 10^6 + 412.5 \times 10^3) + \frac{1}{2}(1.5125 \times 10^6 - 412.5 \times 10^3) \times \sec(70.5)$

$= 962500 + 550000(-1.29)$

$$\boxed{I_{UU} = 253 \times 10^3 \text{ mm}^4}$$

$$I_{VV} = \frac{1}{2}(I_{xx}+I_{yy}) - \frac{1}{2}(I_{xx}-I_{yy})\sec 2\theta$$

$$= \frac{1}{2}(1.5125\times10^6 + 412.5\times10^3) - \frac{1}{2}(1.5125\times10^6 - 412.5\times10^3)$$

$$\times \sec 2(70^\circ 51')$$

$$= 962500 - 550000(-1.29)$$

$$\boxed{I_{VV} = 1672000 \text{ mm}^4}$$

$\rightarrow$ $U = x\cos\theta + y\sin\theta$

$\rightarrow$ $V = y\cos\theta - x\sin\theta$

$$M = W \times L$$

$$= 2\times10^3 \times 1000$$

$$M = 21\times10^6 \text{ N-mm}$$

$$\sigma_b = \frac{M u \sin\theta}{I_{VV}} + \frac{V\cos\theta}{I_{UU}}$$

$$= \frac{21\times10^6 \times \sin(70.5)\times 4}{1672000} + \frac{\cos(70.5)\times V}{253\times10^3}$$

$$\sigma_b = 21\times10^6 \times 5.63\times10^{-7} 4 + 1.99\times10^{-7} V$$

$$\sigma_b = 1.1274 + 2.66\,V$$

position of N.A = where $\sigma_b = 0$

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$\sigma_b = 0$

$1.127u + 2.66v = 0$

$1.127u = -2.66v$

$u = \frac{2.66v}{1.127}$

$u = 2.36v$

$\boxed{v = -0.425u}$

The eqn of straight line passing through 'G' with m = -0.425 $\tan\alpha = -0.425$

$\Rightarrow \alpha = -23.05$

Hence the N.A will be inclined to -23.05 to u-axis at this case of cantilever, the stress will be tensile above the N.A and compressive below the N.A. The point 'L' is having M is having more distance below the N.A. Hence at a point 'L', there will be max tensile stress. where as point M, there will be max. compressive stress.

Let us find the values of u, v.

→ At point L:-

→ $u = x\cos\theta + y\sin\theta$

$x = -15mm$, $y = 35mm$

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$u = -15 \times \cos(70.5) + 35 \times \sin(70.5)$

$u = 27.98$

$v = y\cos\theta - x\sin\theta$

$= 35 \times \cos(70.5) - (-15)\sin(70.5)$

$= 25.822$

→ At point M :-

$x = -(\bar{x} - 10)$

$= -(15 - 10) = -5$

$y = -\bar{y} = -35\,mm$

$u = -5\cos(70.5) + (-35)\sin(70.5)$

$= -34.66$

$v = -35\cos(70.5) + 5\sin(70.5)$

$= -6.97$

The stress will be max at the top of NA

$$\sigma_b = M\left[\frac{u\sin\theta}{I_{vv}} + \frac{v\cos\theta}{I_{uu}}\right]$$

$$= 2\times10^6\left[\frac{27.98 \times \sin(70.5)}{1672000} + \frac{25.822 \times \cos(70.5)}{253\times10^3}\right]$$

$\sigma_b = 99.68\ N/mm^2$ [In Tension]

$$\sigma_b = 2\times10^6\left[\frac{-34.66 \times \sin(70.5)}{1672000} + \frac{-6.97\cos(70.5)}{253\times10^3}\right]$$

$\sigma_b = -57.47 \ N/mm^2$ [In compression]

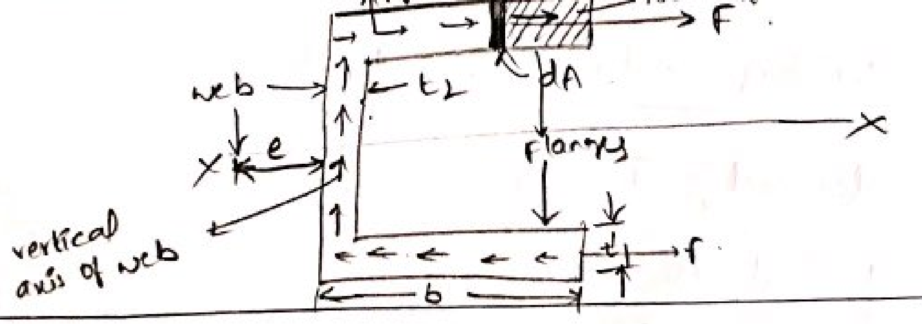
→ **SHEAR CENTER:-**

shear center is a point [in or outside a section] through which the applied shear force produces no torsion or twist of the member. If the load is not applied through the shear center, there will be a twisting of the beam due to unbalanced moment caused by the shear force acting on the section.

* The shear center is lies on the axis of symmetry, If the section is symmetrical about one axis. The shear centre doesnot coinside with the centroid in this case.
* For sections having two axis of symmetry, the shear centre lies on the intersection of these axis and thus coinsides with the centroid.

* shear center is also known as centre of twist.

→ Determination of shear centre for a channel section:-



The given section is symmetrical about the axis x-x.

Hence shear centre will ~~be~~ lie on this axis (x-x).

let $F \rightarrow$ applied s.f on the section.

$t_1 \rightarrow$ thickness of flanges.

$t_2 \rightarrow$ thickness of web.

$F^* \rightarrow$ s.f produced in flanges.

$F_v \rightarrow$ vertical s.f produced in the web

$b \rightarrow$ length of flange

$h \rightarrow$ distance b/w lines of action of F^*

WKT.

shear stress follows the direction of boundary and hence shear stress distribution is horizontal in flanges and vertical in web. The shear force due to these shear stresses will be horizontal in flanges and vertical in web.

The total shear force in the web must be equal to the applied vertical shear force. Hence the vertical s.f is balanced. But the s.f in flanges are unbalanced. They are equal and opposite. Hence produce a clockwise couple of magnitude $[F^* \times h]$.

If the applied force 'F' acts through the vertical axis of the web and passes through 'o' [i.e., the centre of web]. Then there will be no moments due to vertical forces [i.e, due to applied force 'F' and due to S.F produced in the web]. But there is a clockwise moment $F^* \times h$" which is unbalanced and can twist the section of the channel. Now, line of application of applied vertical force 'F' is displaced to the left by a distance of 'e' from the vertical axis of the web, then the unbalanced clockwise moment $F^* \times h$ can be balanced by the counter clockwise moment due to applied force 'F' and vertical force produced in the web.

Taking moments of all forces about point 'o' [The moment of force fbe produced in the web] is zero as it passes through 'o'.

$$F \times e = F^* \times h \longrightarrow (1)$$

$$F^* = \frac{F \times e}{h}$$

$$e = \frac{f^* h}{F}$$

The above eqn is gives the location of shear center, Here.

f^* = shear force produced in flanges.

$$= \int \tau \, dA$$

$$\tau = \frac{FA\bar{y}}{Ib}$$

where,

τ = shear stress in the flange.

f = applied force

A = Area of shaded portion of flange $[= x t_1]$

b = Actual width of flange $[t_1]$

I = MOI about axis of symmetry $[I_{xx}]$

h = distance b/w horizontal s.f in flanges.

→ for finding s.f f^*

consider an element at a dist 'x' from right hand edge of top flange.

where $A\bar{y}$ = moment of shaded area about x-x axis.

$$\tau = \frac{f \times x \times t_1 \times h/2}{I_{xx} b}$$

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$$= \frac{f x t_1 h}{2 I_{xx} b}$$

The shear force the elementary area dA is given by $= \tau\, dA$.

$$= \tau \times dx \times t_1$$

The total s.f $= \int_0^b \tau\, dA \Rightarrow \int_0^b \tau \times dx \times t_1 = F^*$

$$= \int_0^b \frac{f x h t_1}{2 I_{xx} b} \times dx \times t_1$$

$$= \frac{f h t_1}{2 I_{xx}} \int_0^b x\, dx$$

$$= \frac{f h t_1}{2 I_{xx}} \left[\frac{x^2}{2}\right]_0^b$$

$$= \frac{f \times h \times t_1 . b^2}{2 I_{xx} . 2}$$

$$\boxed{F^* = \frac{f h t_1 b^2}{4 I_{xx}}}$$

The s.f in the bottom flange will be also equal to 'F^*' but in opposite direction.

→ <u>Location of shear center</u>:-

Let 'e' → distance of shear centre along the axis of symmetry $(x-x)$

from eqn (1)

$$f \times e = f^* h$$

$$e = \frac{f^* h}{F}$$

$$e = \frac{\cancel{f} h t_1 b^2}{4 I_{xx}} \times \frac{h}{\cancel{F}}$$

$$e = \frac{h^2 t_1 b^2}{4 I_{xx}}$$

where

$$I_{xx} = 2\left[\frac{b \times t^3}{12} + b t_1 \left(\frac{h}{2}\right)^2\right] \text{ for two flanges} + \frac{t_2 h^3}{12} \text{ for web}$$

$$I_{xx} = \left[\frac{b t_1^3}{6} + \frac{b t_1 h^3}{2} + \frac{t_2 h^3}{12}\right]$$

problems:-

→ Determine the position of shear centre for a channel section of 120mm × 120mm outside and 10mm thick?

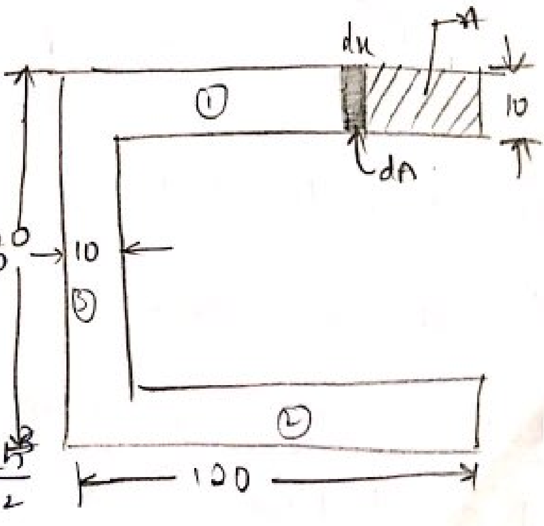
Sol:- $b = 120mm$

$A =$

$b = 120 - \frac{10}{2} = 115mm$

$h = 120 - \frac{10}{2} - \frac{10}{2} = 110\ mm$

$\bar{y} = \frac{h}{2} = \frac{110}{2} = 55mm$



$$I_{xx} = \frac{b t^3}{6} + \frac{b t_1 h^2}{2} + \frac{t_2 h^3}{12}$$

$$= \frac{120\times10^3}{6} + \frac{120\times10\times110^2}{2} + \frac{10\times110^3}{12}$$

$=$

$$I_{xx} = \frac{b\cdot t^3}{12} + Ah^2 = 2\left[\frac{120\times10^3}{12} + 120\times10\times55^2\right] + \frac{10\times100^3}{12} + 1000$$

$= 8114333.333\ mm^4.$

$$e = \frac{h^2 t_1 b^2}{4\times I_{xx}} = \frac{110^2\times10\times120^2}{4\times 8.11\times10^6} = 53.71\,mm.$$

$$\tau = \frac{FA\bar{y}}{I_{xx}\,b}$$

$$= \frac{F\times x\times10\times55}{8.11\times10^6\times120}$$

$$= 6.78\times10^{-6} F\times x$$

$$F^* = \int_0^b \tau\, dA$$

$$= \int_0^b \tau\, dx\cdot 10$$

$$= \int_0^{115} 6.78\times10^{-6} F\times x\cdot dx\times10$$

$$= \int_0^{115} 6.78\times10^{-5}\ F\times x\cdot dx$$

$= 6.678 \times 10^{-5} \left[\frac{x^2}{2}\right]_0^{115} f$.

$= 6.78 \times 10^{-5} \cdot \frac{115^2}{2} \cdot f$

$f^* = 0.44f$

$f \times e = f^* \times h$

$f^* = \frac{f \times e}{h}$

$0.44f = \frac{f \times e}{110}$

$0.44 = \frac{e}{110}$; $e = 48.4\,mm$.