UNIT – I

INTRODUCTION TO PRESTRESSED CONCRETE

Definition of Prestress:– Prestress is defined as a method of applying Pre-Compression to Control the stresses resulting due to external loads below the neutral axis of the beam tension developed due to external load which is more than the Permissible limits of the Plain Concrete. The Pre-Compression applied (may be axial (or) eccentric) will induce the Compressive stress below the neutral axis (or) as a whole of the beam c/s. Resulting either no tension (or) Compression.

Basic Concept:– Prestressed Concrete is basically concrete in which internal stresses of a suitable magnitude and distribution are introduced so that the stresses resulting from the external loads are counteracted to a desired degree.

Terminology:–

1) **Tendon:–** A stretched element used in a Concrete member of structure to impart Prestress to the Concrete.

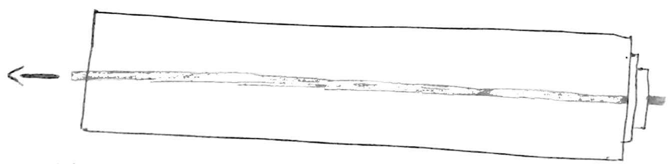
2) **Anchorage:–** A device generally used to enable the tendon to impart and maintain Prestress in Concrete.

3) **Pretensioning** :- A method of Prestressing Concrete in which the tendons are tensioned before the Concrete is Placed. In this method, the Concrete is introduced by bond between steel & Concrete

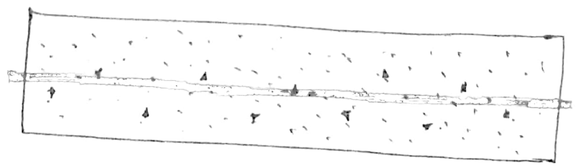
4) **Post-tensioning** :- A method of Prestressing Concrete by tensioning the tendons against hardened Concrete. In this method, the Prestress is imparted to Concrete by bearing.

Historic development of Prestressed concrete :-

Prestressing of Structures was introduced in late 19th Century



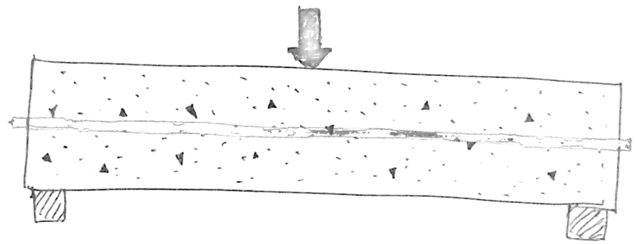
Place and stretch mild steel rods, Prior to Concreting



Release the tension and cut the rods after Concreting

Prestressing of Concrete beams by mild steel rods.

Mild steel rods are stretched and concrete is poured around them. After hardening of concrete, the tension in the rods is released. The rods will try to regain their original length, but this is prevented by the surrounding concrete to which the steel is bonded. Thus, the concrete is now effectively in a state of pre-compression. It is capable of counteracting tensile stress, such as arising from the load.



A Prestressed beam under an external load

But, the early attempts of prestressing were not completely successful. It was observed that the effect of prestress reduced with time. The load resisting capacities of the members were limited. Under sustained loads, the members were found to fail. This was due to the reason that concrete shrinks with time. Moreover under sustained load, the strain in concrete increases with increase in time. This is known as creep strain. The reduction in length due to creep and shrinkage is also applicable to the embedded steel, resulting in significant loss in the tensile strain.

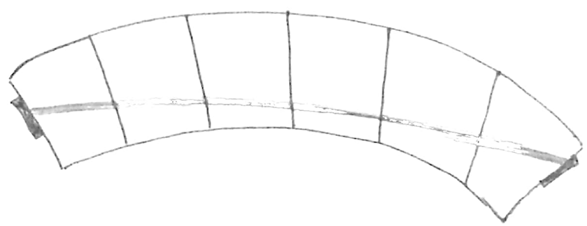
⟹ Before the development of Prestressed Concrete, two significant developments of R/f Concrete are the invention of Portland Cement and introduction of steel in Concrete.

1824 Aspdin. J (England) obtained a Patent for the manufacture of Portland Cement.

1857 Monier. J (France) Introduced steel wires in Concrete to make flower pots, pipes, arches & slabs.

The following events were significant in the development of Prestressed Concrete.

1886 Jackson. P. H (USA) Introduced the Concept of tightening steel tie rods in artificial stone and Concrete arches.



Steel tie rods in arches

1888 Stainer. C.R (USA) Recognised losses due to shrinkage and creep, and suggested retightening the rods to recover lost prestress.

1923 Emperger. F (Austria) Developed a method of winding and pre-tensioning high tensile steel wires around concrete pipes.

1924 Hewett W.H (USA) Introduced hoop stressed horizontal reinforcement around walls of concrete tanks through the use of turnbuckles.

Thousands of liquid storage tanks and concrete pipes were built in the two decades to follow.

1925 Dill R.H (USA) Used high strength unbonded steel rods. The rods were tensioned and anchored after hardening of concrete.

1926 Eugene Freyssinet (France) Used high tensile steel wires, with ultimate strength as high as 1725 MPa and yield stress over 1240 MPa. In 1939, he developed conical wedges for end anchorages for post-tensioning and developed double-acting jacks. He is often referred as the Father of Prestressed Concrete.

1938 Hoyer. E (Germany) Developed 'long line' pre-tensioning method.

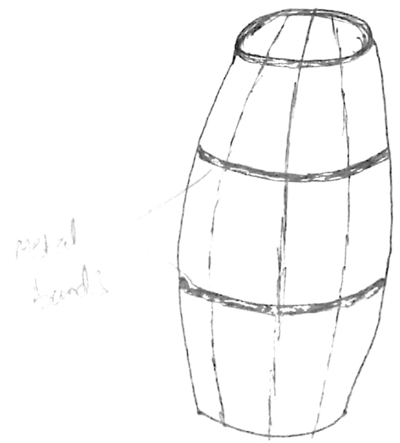
1940 Magnel. G (Belgium) anchoring system for post-tensioning

General Principle of Prestressing

The Prestressing of a structure is not the only instance of Prestressing. The Concept of Prestressing existed before the applications in the Concrete. Two examples of Prestressing before the development of Prestressed Concrete are Provided.

Force - fitting of metal bands on wooden barrels :-

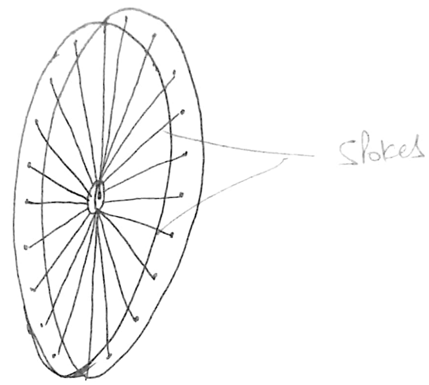
The metal bands induce a state of initial hoop compression, to counteract the hoop tension caused by filling of liquid in the barrels. The Prestressing of a structure is not the only instance of Prestressing. The concept of Prestressing existed before the application in Concrete.



Force fitting metal bands on wooden barrels

Pre-tensioning the Spokes in a bicycle wheel :-

The Pre-tension of a Spoke in a bicycle wheel is applied to such an extent that there will always be a residual tension in the spoke. The tension in Spoke will nullify the applied Compression.



Pre-tensioning the Spokes in a bicycle wheel

For Concrete, internal stresses are induced (by means of tensioned steel) for the following reasons.

* The tensile Strength of Concrete is only about 8% - 14% of its Compressive strength.
* Cracks tend to develop at early stages of loading in flexural members such as beams & slabs.
* To Prevent such Cracks, Compressive force Can suitably be applied in Perpendicular direction.
* Prestressing enhances bending, shear & torsional capacities of flexural members.
* In Piles & liquid Storage tanks, hoop tensile stresses can be effectively Counteracted by circular Prestressing.

Pre-tensioning & Post-tensioning :-

In Pre-tensioning, the tendons are tensioned before the Concrete is Placed. The tendons are temporarily anchored to abutments or stressing beds. Then the Concrete member is Cast between and over the wires. After the Concrete has attained the required strength, the wires are Cut from the bulkhead and Pre-stress is transferred to the Concrete member.

In Post-tensioning the Concrete member is Cast with ducts for the wires. After Concrete has attained Sufficient strength, wires are threaded into the ducts, tensioned from both or one end by means of jacks and at the Precise level of Pre-stress the wires are anchored by means of wedges to the anchorage Plates at the ends.

Advantages of Prestressed Concrete :-

Prestressed Concrete offers great technical advantages in Comparison with other forms of Construction.

1. The Size (or) dimensions of structural members are reduced, which may increase the Clearances (or) reduce Storey heights.
2. It Permits the use of large spans ($>30m$) with Shallow members, Even when heavy load are encountered.

3. In addition to general advantages, such as excellent fire resistance, low maintenance costs, elegance, high corrosion - resistance, adaptability etc, the Prestressed Concrete is found to sustain the effects of impact (or) shock and vibrations.

4. Because of smaller loads due to smaller dimensions being used, there is considerable saving cost of supporting members and foundations.

5. The Prestressing technique has eliminated the weakness of Concrete in tension and hence crack free members of structure are obtained.

6. Because of better material (i.e, Controlled Concrete and high tension steel) being used and nullifying the effect of dead loads, smaller deflections are caused.

Limitations of Prestressing:-

Although Prestressing has advantages, some aspects need to be carefully addressed.

* Prestressing needs skilled technology. Hence, it is not as common as reinforced Concrete.
* The use of high strength materials is costly.
* There is additional cost in auxillary equipments.
* There is need for quality control & inspection
* Prestressed Concrete sections are less fire resistant

⇒ Types of Prestressing

Prestressing of Concrete can be classified in several ways.

I. **Source of Prestressing force :-** This classification is based on the method by which the Prestressing force is generated. There are four sources of Prestressing force : Mechanical, hydraulic, electrical and Chemical.

1) **Hydraulic Prestressing :-** This is the simplest type of Prestressing producing large Prestressing forces. The hydraulic jack used for the tensioning of tendons, Comprises of Calibrated Pressure gauges which directly indicate the magnitude of force developed during the tensioning.

2) **Mechanical Prestressing :-** In this type of Prestressing, the devices includes weights with (or) without lever transmission, geared transmission in Conjunction with Pulley blocks, screw jacks with (or) without gear drives and wire-winding machines. This type of Prestressing is adopted for mass scale Production.

3) **Electrical Prestressing :-** In this type of Prestressing, the steel wires are electrically heated and anchored before placing Concrete in the moulds. This type of Prestressing is also known as thermo-electric Prestressing.

4) **Chemical Prestressing:**– Chemical Prestressing is Produced by introducing an expansive component within a concrete mix constituents. The expansive potential of the concrete mix is utilized to stretch the reinforcement and consequently create the prestressing. Chemical prestressing is also called as self stressing. This can be done by expanding cements which consist of 75% Portland Cement, 15% alumina cement and 10% gypsum which results in formation of calcium sulpho aluminate.

II) **External (or) Internal Prestressing:**– This classification is based on the location of prestressing tendon w.r.t the concrete section.

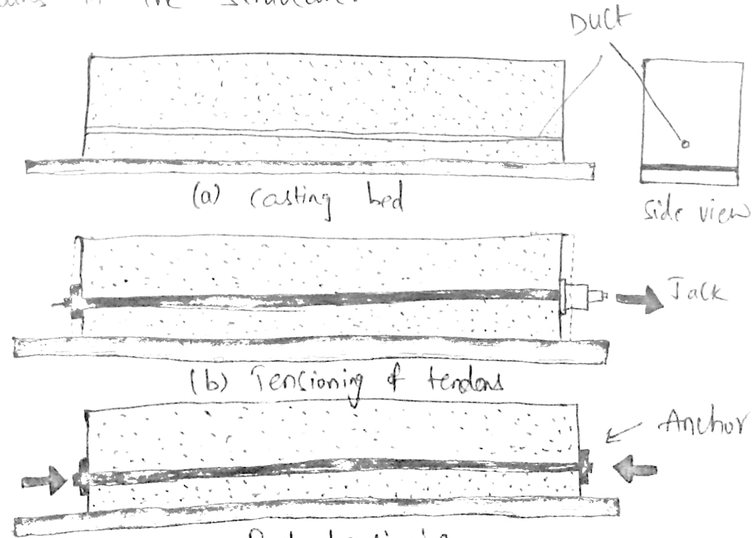
External Prestressing:– When the prestressing is achieved by elements located outside the concrete, it is called external prestressing. The tendons can lie outside the member (or) inside the hollow space of a box girder. This technique is adopted in bridges and strengthening of buildings. ~~In the~~

Internal Prestressing:– When the prestressing is achieved by elements located inside the concrete member (commonly, by embedded tendons), it is called internal prestressing. Most of the applications of prestressing are internal prestressing.

III) **Pre-tensioning (or) Post-tensioning:**– This is the most important classification and is based on the sequence of casting the concrete and applying tension to the tendons.

Pre - tensioning :— Pre-tensioning is accomplished by stressing wires (or) strands, called tendons, to predetermined amount by stretching them between two anchorages prior to placing concrete. The concrete is then placed and tendons become bounded to concrete throughout their length. After concrete has hardened, the tendons are releated by cutting them at the ~~edges~~ anchorages. The tendons tend to regain their original length by shortening and in this process there is a transfer of compressive stress to the concrete through the bond. The tendons are usually stressed by the use of hydraulic jacks. The stress in tendons is maintained during the placing and curing of concrete by anchoring the ends of the tendons to abutments that may be as much as 200m apart. The abutments and other formwork used in this procedure are called prestressing bench (or) bed.

Post-tensioning:- The tension is applied to the tendons (located in a duct) after hardening of the concrete. The pre compression is transmitted from steel to concrete by the anchorage device (at the end blocks). The alternative to pre-tensioning is post-tensioning. In a post-tensioned beam, the tendons are stressed and each end is anchored to the concrete section after the concrete has been cast and has attained sufficient strength to safely withstand the prestressing force. In post-tensioning method, tendons are coated with grease (or) a bituminous material to prevent them from becoming bonded to concrete. Another method used in preventing the tendons from bonding to the concrete during placing and curing of concrete is to encase the tendon in a flexible metal hose before placing it in the forms. The metal hose is referred to as sheath (or) duct and remains in the structure.



Post tensioning

v) **Linear (or) Circular Prestressing:**— This classification is based on the shape of the member Prestressed.

Linear Prestressing:— When the Prestressed members are Straight (or) flat, in the direction of Prestressing, the Prestressing is called linear Prestressing. For example, Prestressing of beams, Piles, Poles and Slabs. The Profile of the Prestressing tendon may be Curved. Railway sleepers are made by linear Prestressing Process.

Circular Prestressing:— When the Prestressed members are Curved, in the direction of Prestressing, the Prestressing is called Circular Prestressing. For example, Circumferential Prestressing of tanks, Silos, Pipes and similar structures. The Circular tanks are the examples for Structures made of Circular Prestressing.

v) **Full, limited (or) Partial Prestressing:**— Based on the amount of Prestressing force, three types of Prestressing are defined.

Full Prestressing:— When the level of Prestressing is such that no tensile stress is allowed in Concrete under service loads; it is called full Prestressing (Type 1 as per IS:1343-2012)

Limited Prestressing:- When the level of Prestressing is such that the tensile stress under service loads is within the cracking stress of concrete, it is called as limited Prestressing. (Type 2)

Partial Prestressing:- When the level of Prestressing is such that under tensile stresses due to service loads, the crack width is within the allowable limit, it is called Partial Prestressing (Type 3).

VI Uniaxial, biaxial (or) Multi-axial Prestressing:- As the names suggest, the classification is based on the directions of Prestressing a member.

Uniaxial Prestressing:- When the Prestressing tendons are parallel to one axis, it is called Uniaxial Prestressing. For example, longitudinal Prestressing of beams.

Biaxial Prestressing:- When there are Prestressing tendons parallel to two axes, it is called Biaxial Prestressing.

Multiaxial Prestressing:- When the Prestressing tendons are parallel to more than two axes, it is called Multiaxial Prestressing. For example, Prestressing of domes

Topic Beyond the Syllabus

Differences of Prestressed Concrete over Reinforced Concrete

1) In Prestress Concrete member steel plays active role. The stress in steel prevails whether external load is there (or) not. But in R.C.C., steel plays a passive role. The stress in steel in R.C.C members depends upon the external loads i.e., no external load, no stress in steel.

2) In Prestressed Concrete, the stresses in steel is almost constant where as in R.C.C the stress in steel is variable with the lever arm.

3) Prestress Concrete has more shear resistance, where as in R.C.C it is less.

4) In Prestressed Concrete members, deflections are less because the eccentric Prestressing force will induce couple which will cause upward deflections, whereas in R.C.C, deflections are more.

5) In Prestress Concrete fatigue resistance is more compared to R.C.C because in R.C.C stress in steel is external load dependant where as in PSC member it is load independant.

6) Prestressed Concrete is more durable as high [illegible]

Concrete is used which are more dense in nature RCC is less durable.

7) In Prestressed Concrete, dimensions are less because external ~~fr~~ stresses are counterbalanced by the internal stress induced by Prestress. Therefore reactions on column & footing are less as a whole the quantity of concrete is reduced by 30% and steel reduced by about 60%-70%. RCC is uneconomical for long span because in R.C.C dimensions of sections are large requiring more concrete & steel. Moreover as self-weight increases, more reactions acted on columns & footings, which requires higher sizes.

Materials for Prestressed Concrete :-

The materials used for Prestressed Concrete are tendons, Anchorage, High strength concrete and High tensile steel. Of these, High Strength Concrete and high tensile steel plays major role ~~in~~ in acquiring full strength in Prestressed Concrete.

Tendon :- A stretched element used in a concrete member of structure to impart Prestress to the concrete. Generally, high-tensile steel wires, bars cables (or) strands are used as tendons.

Anchorage:- A device generally used to enable the tendon to impart and maintain Prestress in the Concrete. The Commonly used anchorages are the Freyssinet, Magnel blaton, Gifford-Udall, Lee Mc Call Systems.

High strength Concrete:- Prestressed Concrete requires Concrete which has a high Compressive strength at a reasonably early age, with Comparatively higher tensile strength than ordinary Concrete. With the development of vibration techniques, it is Possible to Produce high strength Concrete having 28-day Cube Compressive strength in the range of 30-70 N/mm^2 But recent developments Proved that it is even Possible to Produce ultra high strength Concrete with a 28-day Cube Compressive strength ranging from 70-100 N/mm^2.

High strength Concrete mixes can be designed by using

→ Emperical methods

→ British D.O.E Method

→ American Concrete Institute's Mix design

→ IS Code method.

The minimum 28-day cube compressive strength prescribed in the IS code IS : 1343 - 2012 is 40 N/mm² for pre-tensioned members and 30 N/mm² for post-tensioned members. A minimum cement content of 300 - 360 kg/m³ is prescribed mainly to cater to the durability requirements. In high strength concrete mixes, the water content should be as low as possible. The code prescribes that the cement content in the mix should preferably not exceed 530 kg/m³

High - Tensile steel :- For prestressed concrete members, the high tensile steel used generally consists of wires, bars (or) strands. The higher tensile strength is generally achieved by marginally increasing the carbon content in steel in comparison with mild steel. High-tensile steel usually contains 0.6 - 0.85 % carbon, 0.7 - 1.0 % manganese, 0.05 % of sulphur and phosphorous with traces of silicon. The high carbon steel ingots are hot-rolled into rods and cold-drawn through a series of dies to reduce the diameter and increase the tensile strength. The high tensile steel bars commonly employed in prestressing are manufactures in nominal sizes of 10, 12, 16, 20, 22, 25, 28 and 32 mm diameter. The tensile strength decreases with increase in the diameter of the wires.

The IS code prescribes a minimum percentage elongation varying from 2.5% for wires to 10% for bars. For strands, the % elongation measured on a gauge length of not less than 600mm should not be less than 3.5% immediately prior to the fracture of any of the component wire. The IS code specifies the values of the modulus of elasticity of high tensile wires, bars and strands as 210, 200 and 195 kN/mm² respectively.

* ⇒ Why high grade of concrete is used in prestressed concrete?

In pre-stressing, the transfer of pre-stress in pre-tensioning is through bond. To achieve sufficient bond stresses the minimum grade of concrete used is M_{40}.

In post-tensioning, the transfer of prestress is through bearing. So sufficient bearing stresses are available with M_{30} grade concrete.

The problems with shrinkage and creep can be encountered by using concrete with higher value of Young's modulus. Hence, higher grade of concrete should be used which has the higher value of E.

$$E = 5700\sqrt{f_{ck}}$$

To achieve the properties of durability and abrasive resistance, higher grade of concrete is used.

High tensile steel = 2100 MPa — 4000 MP..

* => Why high tension steel is used in Pre-stressed Concrete?

Ordinary steel is not used in Prestressed Concrete (i.e, Fe 250, Fe 415, Fe 500) because they can withstand higher stresses for a longer Period. But high tension steel has an ultimate strength of 2100 MPa and if the loss is even 1000 MPa, there will be still largest stress exists in the tendons.

Need for high strength Concrete & High tensile steel :-

→ High strength Concrete has high resistance in tension, shear, bond and bearing.

→ Large Prestressing forces are applied to the members by tendons and high bearing stresses are developed at the end by anchoring devices so that anchoring devices are designed for high strength Concrete.

→ When high strength Concrete is used shrinkage losses will be less.

→ It has high Modulus of Elasticity and small ultimate creep strain so that loss of Prestress is small.

→ By using high strength Concrete c/s area required will be reduced resulting reduces the less dead weight moment.

→ The mild steel has a stress of 200-300 N/mm².
If initial Prestress is 200 N/mm², the estimated loss of Prestress is 100 N/mm², total tensile stress is 300 N/mm². So that some stress is left over.
Because high tensile steel has an ultimate strength of 2100 N/mm². If initial Prestress is 1000 N/mm², the estimated loss of Prestress due to creep and shrinkage is 200 N/mm². Total stress is 1200 N/mm². There will be a large stress in reinforcement after deducting all losses.

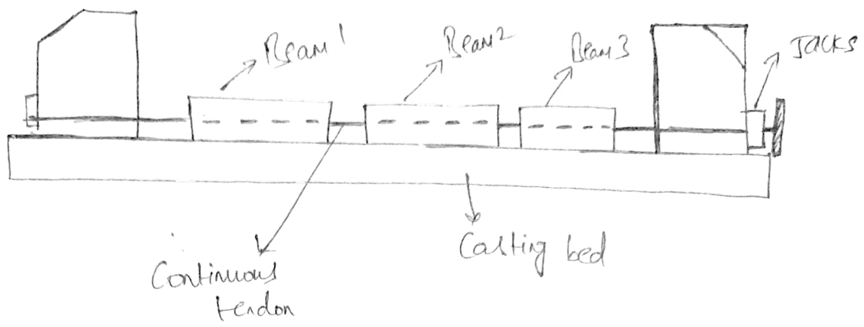
Methods and Systems of Prestressing:-

The two methods of Prestressing are Pre-tensioning and Post-tensioning.

Systems of ~~Prestressing~~ Pretensioning :- Pre-tensioning consists only one system i.e., Hoyer's long line system.

(a) **Hoyer's Long line System :-** Hoyer's System is generally used for mass production like railway sleepers, poles etc. The end abutments are kept sufficient distance apart and several members are cast in a single line. The shutterings are provided at the sides and b/w the members.

This system is also called as long-line method.



Hoyer's long-line system of Pre-tensioning

* Two bulk heads (or) abutments independently anchored to the ground are provided several meters apart, say 100m. Wires are stretched between the bulkheads. Moulds are placed enclosing the wires.
* The concrete is now poured so that a number of beams can be produced in one line.
* After the concrete has hardened, the wires are released from bulkheads and are cut off.
* The Prestress is transferred through the bond between tendons and concrete.
* This system of Prestressing is uneconomical for larger spans.

Systems of Post-tensioning :- Post-tensioning Systems are

a) Freyssinet System
b) Gifford-udall System
c) Lee-Mc call System
d) Magnel blaton System
e) P.S.C Mono wires
f) Electrical Prestressing
g) Chemical Prestressing.

a) **Freyssinet System** :- In 1939, This System was introduced by the French engineer Freyssinet. High strength steel wires of 5mm (or) 7mm dia about 12 in number are grouped into a Cable with a helical Spring inside.

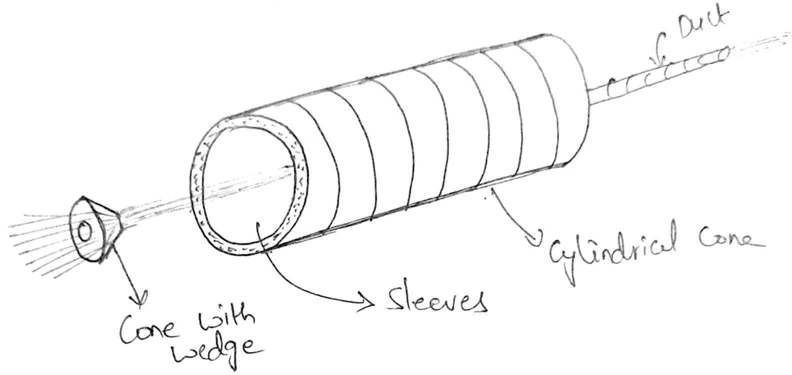
* Spring keeps proper spacing for the wire, and thus provides a channel which can be cement grouted. It further assists to transfer the reaction to the Concrete Cable is inserted in the duct.

* The anchorage device consists of Concrete Cylinder with a Concentric Conical hole and Corrugations on its Surface, and a Conical plug carrying grooves on its Surface.

* These Cylinders are kept in proper ~~Proportion~~ Position and the Conical plugs are pushed into holes after Cables are tightened.

* The central hole passing axially permits cement grout to be injected through it.

* In this way, the space between the wires is filled with the grout and this provides additional restraint against the slipping of the tendons



Freyssinet System

Advantages:-

→ Securing the wires is not expensive

→ Desired stretching force is obtained quickly

→ The plugs may be left in concrete and they don't project beyond the ends of the member.

Disadvantages:-

→ Stresses in wires may not be exactly same (all the wires are stretched together).

→ Jacks used are heavy and expensive.

→ The greatest stretching force available is 250 kN to 500 kN, which is not sufficient.

b) **Gifford- Udall System:-** This system was originated in Great Britain, is widely used in India. This is a single wire system.

* Each wire is stressed independently using a double acting jack.

* Any no.of wires can be grouped together to form a cable in this system. There are 2 types of anchorages devices in this system.

  a) Tube anchorages

  b) Plate anchorages

→ Tube anchorage consists of a bearing plate, anchor wedges and anchor grips.

→ Plate anchorage is may be of square (or) circular and have 8 (or) 12 tapered holes to accommodate the individual prestressing wires.

* These wires are locked into the tapered holes by means of anchor wedges.

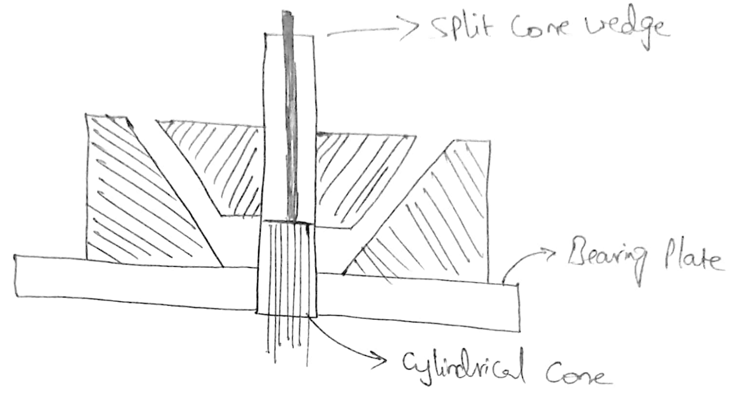
* In addition, grout entry hole is also provided in the bearing plate for grouting.

* Anchor wedges are split cone wedges carrying serrations on its flat surface.

* There is a tube unit which is a fabricated steel component incorporating a thrust plate

tube with a surrounding helix.

* This unit is attached to the end shutters and form an



c) Magnel-Blaton System:- This method was introduced by Prof. Magnel of Belgium.

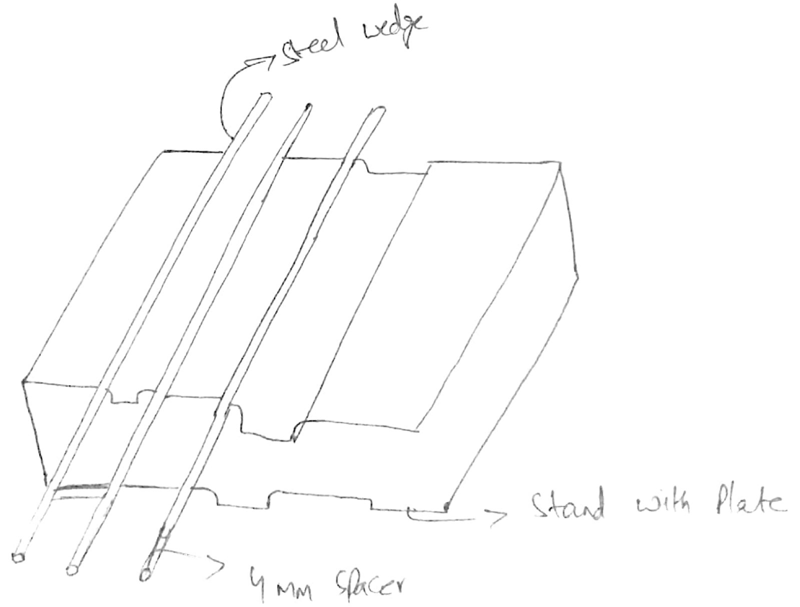
* In Freyssinet system, several wires are stretched at a time. In Magnel-Blaton system, two wires are stretched at a time.

* Cable of rectangular section is provided, which contains layers of wires 5 to 8 mm diameter.

* Cables consists of wires in multiples of 8 wires. Cables with as much as 64 wires are also used under special conditions.

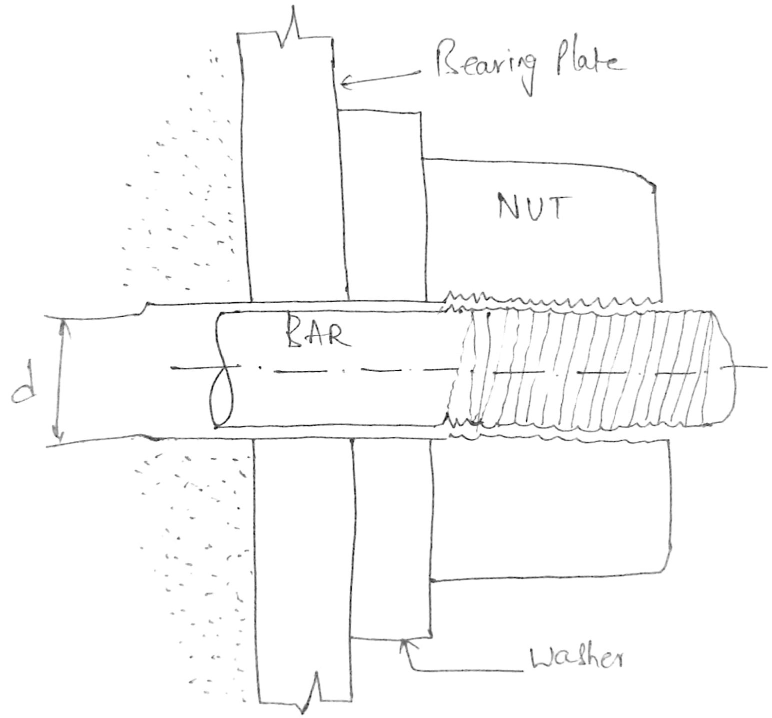
* wires in two adjacent layers are separated with a clearance of 4 mm.

* Wires are maintained in form by Providing Spacers at regular intervals throughout the length of Cable.
* Wires are anchored by wedging, 2 at a time into Sandwitch Plates. These plates are 25 mm thick and are Provided with two wedge shaped grooves On its two faces.
* The Wires are taken two in each groove and tightened. A jack is used to tighten the wires.
* A Steel wedge is driven between the tightened wires to anchor them against the Plate



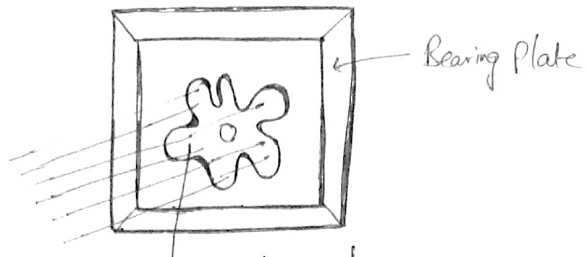
Anchorage of Magnel Blaton System

d) **Lee- Mc Call System**:- This method is used to Prestress Steel bars. The diameter of the bar is between 12 and 40 mm. Bars Provided with threads at the ends are inserted in the Performed ducts. After stretching the bars to the required length, they are tightened using nuts against bearing plates provided at the end sections of the member.



End anchorage for Lee Mc Call System

e) PSC Mono Wires:- Stretching of tendons at a time.



In this system wires are tensioned individually the anchorage system consists of a single piece collect sleeve wedging in a conical cone. The steel guide leads each wire from the cable to the anchor plate along gentle curvature. In addition to the guide the central block is also provided.

UNIT - II

LOSSES OF PRESTRESS

Losses in Prestress:-

The initial Prestressing concrete undergoes a gradual reduction with time from the stages of transfer due to various causes. The force which is used to stretch the wire to the required length must be available all the time as prestressing force if the steel is to be prevented from contracting. contraction of steel wire occurs due to several causes, effecting reduction in the prestress. This reduction in the prestressing force is called loss in prestress. In a prestressed concrete beam, the loss is due to the following:

Types of loss in prestress:-

Pretensioning:- During the process of anchoring, the stressed tendon tends to slip before the full grip is established, thus losing some of its imposed strain (or) in other words, induced stress. This is known as loss due to anchorage draw-in from the time the tendons are anchored until transfer of prestressing force to the concrete, the tendons are held between the two abutments at a constant length. The stretched tendons during this time interval will loose some of its induced stress due to the phenomenon known as relaxation of steel. As soon as the tendons are cut, the stretched tendons tend to go back to their original state, but are prevented from doing so by the interfacial bond developed between the concrete and the tendons.

1. Elastic deformation of concrete
2. Relaxation of stress in steel
3. Shrinkage of concrete
4. Creep of concrete.

<u>Post-tensioning</u>:- The tendons are located inside ducts, and the hydraulic jacks held directly against the member. During stressing operation, the tendons tend to get straightened and slide against the duct, this resulting in the development of a frictional resistance. As a result, the stress in the tendon at a distance away from the jacking end will be smaller than that indicated by the pressure gauge mounted on the jack. This is known as loss due to friction.

With regard to elastic shortening, there will be no loss of prestress if all the tendons are stressed simultaneously because the prestress gauge records the applied stress after the shortening has taken place.

1. No loss due to elastic deformation if all wires are simultaneously tensioned. If the wires are successively tensioned, there will be loss of prestress due to elastic deformation of concrete.

2. Relaxation of stress in steel.

3. Shrinkage of concrete.

4. Creep of concrete

5. Friction

6. Anchorage slip.

1) Loss due to elastic deformation of concrete:-

The loss of prestress due to deformation of concrete depends on the modular ratio & the average stress in concrete at the level of steel.

$$\text{Elastic deformation} = f_c \times \alpha_c$$

$$\alpha_c = \text{Modular ratio} = \frac{E_s}{E_c}$$

f_c = loss of stress in steel.

E_s = Modulus of elasticity of steel

E_c = Modulus of elasticity of concrete

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Strain in Concrete at the level of steel = $\frac{f_c}{E_c}$

Stress in steel corresponding to this strain = $\frac{f_c}{E_c} E_s$

If the initial stress in steel is known, the percentage of loss of stress in steel due to elastic deformation of Concrete can be computed.

2) Loss due to shrinkage of Concrete :-

Loss of Prestress due to Shrinkage = $E_s \times \varepsilon_{cs}$

E_s = Modulus of Elasticity of Steel

ε_{cs} = Shrinkage strain

$E_s = 2.1 \times 10^5$ N/mm²

Shrinkage strain in pre tensioning = 3×10^{-4}

Shrinkage strain in Post-tensioning = $\frac{2 \times 10^{-4}}{\log_{10}(T+2)}$

T = Age of Concrete at transfer in "days".

3) Loss due to Creep of Concrete :- Loss of Prestress due to creep of Concrete = $\phi m f_c$

ϕ = Creep coefficient

m = Modular ratio

f_c = Prestress in steel

Creep coefficient varies from 1.5 to 2.0

$$\text{Creep Coefficient} = \frac{\text{Creep strain}}{\text{Elastic strain}} = \frac{e_c}{e_e}$$

4) Loss due to Relaxation of steel:- Generally, relaxation of steel varies with initial stress. It varies from 0-90 N/mm^2.

5) Loss due to friction :- In case of Post-tensioned members the tendons are placed in ducts. The ducts are either straight (or) curved profile. The loss of prestress due to friction occurs in post tensioning member between the tendon and surrounding concrete. The loss due to friction is classified into 2 types.

a) length effect b) Curvature effect

a) Loss due to length effect :- This effect is due to straight imperfection duct. This loss is also called "wobble" effect (or) wobbing (or) wear effect.

$$P_x = P_0 - P_0 (e^{-\mu\alpha + kx})$$

P_x = Prestressing force at the distance 'x' from jack.

P_0 = Initial Prestress

K = Coefficient (wobbing (or) length) = 0.0015 - 0.0050 depending on type of tendon

e = 2.7183 ; x = Distance from the jack.

b) Loss due to curvature effect

In case of curved duct, the loss of prestress depends upon the radius of curvature of duct

$$P_x = P_0 \cdot e^{(-\mu x + kx)}$$

μ = Coefficient of friction in curve.

6) Loss due to anchorage slip:-

When the cable is tensioned and the jack is released from the tendon to transfer the prestress to concrete the frictional wedges are giving grip to the tendon a little amount of slip may be occurs at the end of the tendon. This loss is called loss due to anchorage slip.

The amount of slip may be taken average slip = 0.75 mm

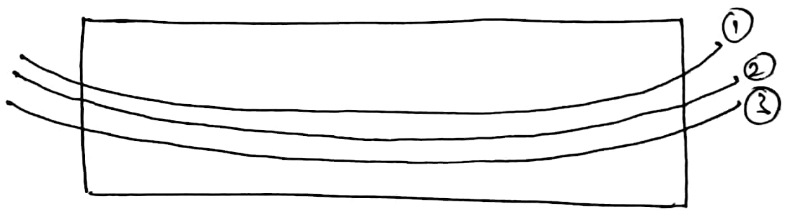
Loss due to anchorage slip $(\delta) = \frac{Pl}{AE_s}$

In Post-tensioning :-

a) loss due to elastic shortening :-

When wires are tensioned By Simultaneously there is no loss due to elastic deformation (Shortening) while Pulling the member gets Compressed and at transfer the elastic shortening has completely occured. The loss due to elastic shortening approaches to one-half of corresponding loss at level of stress at steel.

If the wires are tensioned successively, there is a loss due to elastic shortening.



Cable ① → No loss

Cable ② → loss due to Cable ① = 10%

Cable ③ → loss due to Cable ① & Cable ②

= 10% + 10%.

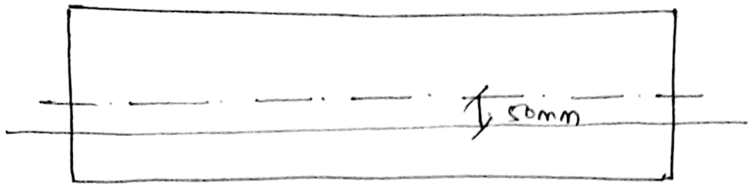
In Post-tensioning of the wires are tensioned successively there is a loss due to elastic shortening when cable ① is tensioned there is no loss due to elastic shortening. When Cable ② is tensioned it doesn't have any loss due to elastic shortening but already Cable ① is stressed and anchored it will undergo loss due to elastic shortening when Cable ③ is tensioned. It doesn't have any loss due to elastic shortening but Cable ① & ② already stressed and anchored it will undergo loss due to elastic deformation. Hence when Cable ① is tightened it will undergo maximum loss of stress due to elastic shortening. When the lost tightened Cable will not undergo loss due elastic shortening.

When the Cables are Curved (In Post-tensioning)

In Post-tensioning max. structures required curved tendons. The loss due to elastic shortening is estimated in Curved tendon by considering the average stress.

Problems on loss due to elastic shortening:-

→ A Pretensioned Concrete beam of 100mm X 300mm is Prestressed by Straight wires carrying an initial force of 150 KN at an eccentricity of 50mm the Modulus of elasticity of steel and concrete are 210 kN/m² and 35 kN/m². Estimate the % loss of stress in steel due to elastic deformation of Concrete. If the area of steel wires is 188 mm².



(e = y)

$b = 100\,mm$

$d = 200\,mm$

$e = 50\,mm$; Force (P) = 150 KN

Area of Steel wires } $a_s = 188\,mm^2$

$E_s = 210\,KN/m^2$

$E_c = 35\,KN/m^2$

$\alpha_c = m = \frac{f_s}{E_c} = \frac{210}{35} = 6$

f_c = stress at a level of steel

$A = 3 \times 10^4\,mm^2$

Stress at a level of steel (f_c):-

$$f_c = \frac{P_o}{A} + \frac{P_o e y}{I} \quad (e = y)$$

$$= \frac{150 \times 10^3}{3 \times 10^4} + \frac{150 \times 10^3 \times (50)^2}{225 \times 10^6}$$

$f_c = 6.67\,N/m^2$

$I = \frac{100 \times 300^3}{12}$

$= 225 \times 10^6\,mm^2$

loss due to elastic shortening $= \alpha_c \times f_c$

$= ~~6.67~~ \times 6 \times 6.67$

$= 40 \text{ N/m}^2$

% loss of stress $= \dfrac{\text{loss in stress}}{\text{Initial stress}}$

$= \dfrac{40}{800} \times 100$

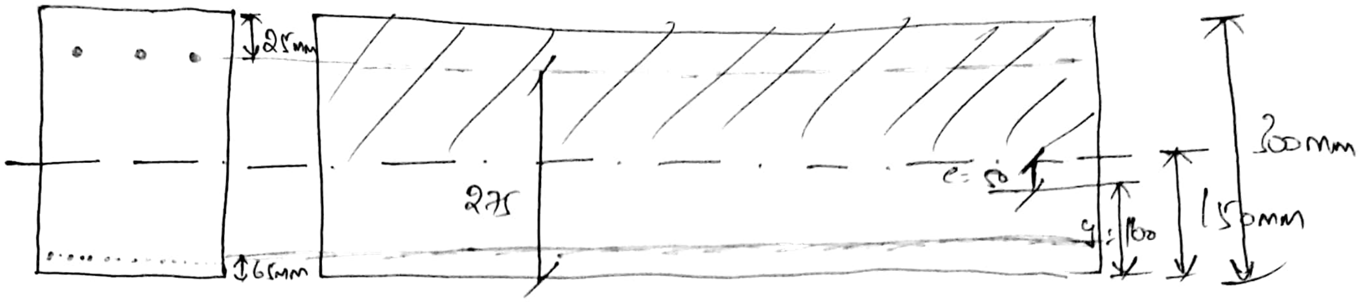
Initial stress $= \dfrac{150 \times 10^3}{188} = 800 \text{ N/mm}^2$

area of steel (188)

So % loss of stress $= \dfrac{40}{800} \times 100$

$= 5\%$

→ A rectangular concrete beam 200 × 300 mm is Prestressed by means of 15 wires of 5 mm dia located 65 mm from the bottom of the beam and 3 wires of 5 mm ϕ located 25 mm from top of beam. If the wires are initially tensioned to a stress of 840 N/mm². Calculate the % loss of stress in steel immediately after transfer allowing for the loss of stress due to elastic deformation of concrete.



Position of bars from centroidal axis to the bottom of beam.

$$y = \frac{3 \times 275 + 15 \times 65}{15+3} \quad (300-25) \Rightarrow y = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{3 \times \frac{\pi}{4}(5)^2 \times 275 + 15 \times \frac{\pi}{4}(5)^2 \times 65}{15 \times \frac{\pi}{4}(5)^2 + 3 \times \frac{\pi}{4}(5)^2}$$

$y = 100$

$$e = \frac{300}{2} - 100 = 50\ mm$$

$E_s = 210\ N/mm^2$

$E_c = 35\ N/mm^2$

$$\alpha_c = \frac{210}{35} = 6$$

Stress in Concrete at level of Steel at top (f_c):-

$$f_c = \frac{P_o}{A} + \frac{P_o e y}{I}$$

e on bottom side so, top tension

$$= \frac{300 \times 10^3}{6 \times 10^4} - \frac{300 \times 10^3 \times 50 \times 125}{45 \times 10^7}$$

$y = 150 - 25 = 125\ mm$

$= 5 - 4.16$

$= 0.83\ N/mm^2$

Loss due to elastic shortening at top = $m \times f_c$

$= 6 \times 0.83$

$= 5.00\ N/mm^2$

Stress in Concrete at level of Steel at bottom (f_c):-

$$= \frac{300 \times 10^3}{6 \times 10^4} + \frac{300 \times 10^3 \times 50 \times 85}{45 \times 10^7} \rightarrow y = 150 - 65 = 85\ mm$$

$= 5 + 2.84$

$= 7.84\ N/mm^2$

% loss of stress $= \frac{5.0}{840} \times 100 = 0.6\%$

Loss due to elastic shortening at top $= 6 \times 7.84$

$= 47.04\ N/mm^2$

% loss of stress $= \frac{47.04}{840} \times 100$

$\Rightarrow 5.6\%$

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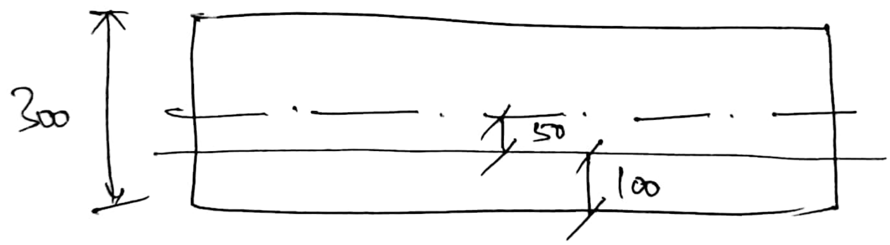
→ A Post-tensioned concrete beam 100 x 300 mm is prestressed by 3 cables each with a c/s area of $50\ mm^2$ and with an initial stress of $1200\ N/mm^2$. All the 3 cables are straight and located 100mm from the soffit of beam. If the modular ratio is '6'. Calculate the loss of stress in the 3 cables due to elastic shortening for the following cases.

<u>Case 1:-</u> Simultaneously tensioning and anchoring all the 3 cables.

<u>Case 2:-</u> successive tensioning of 3 cables



$$y = e = 50mm$$

$$I = 225 \times 10^6\ mm^4$$

<u>Case 1:-</u> $m = 6$

Stress in concrete at level of steel $f_c = \dfrac{P_o}{A} + \dfrac{P_o e y}{I}$

$$= \frac{1200 \times 50}{3 \times 10^4} + \frac{1200 \times 50 \times 50^2}{225 \times 10^6}$$

$$= 6 + 2 = 8\ N/mm^2$$

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Note:- one half of Corresponding loss

If all wires are tensioned simultaneously, there is no loss due to elastic shortening but loss due to elastic shortening Considering half of the Corresponding loss

Case 1:- The loss due to elastic shortening approaches One half of the Corresponding loss with Pre-tensioning

i.e., loss of stress $= \frac{1}{2} \alpha_c f_c$

$= \frac{1}{2} \times 6 \times (2 \times 2.7)$

Case 2:- If all wires are tensioned successively.

For Cable ①: No loss due to elastic shortening

For Cable ②: Loss in cable ①

Loss in cable ① due to elastic shortening

$= m \times f_c$

$= 6 \times 2.67$

$= 16.02 \ N/mm^2$

For Cable ③:- loss in Cable ① & ②

loss in Cable ① due to elastic Shortening $= m \times f_c$

$= 16.02 \ N/mm^2$

Loss in Cable ② due to elastic shortening = $m \times f_c$
= 16.02 N/mm²

<u>Total losses</u>

Cable ① = 16.02 + 16.02
= 32.04 N/mm²

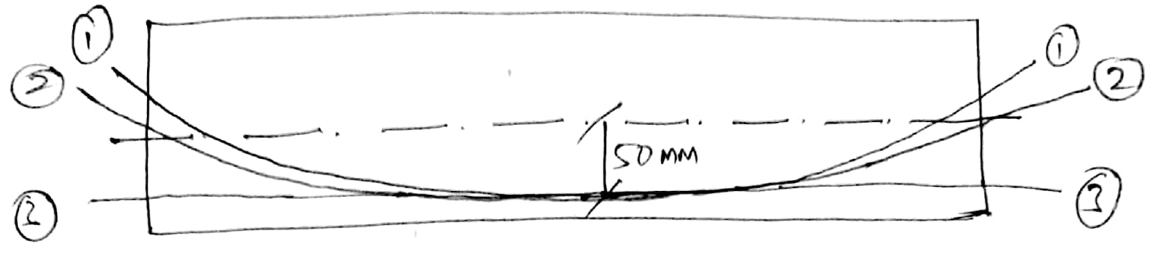
Cable ② = 16.02 N/mm²

Cable ③ = 0 N/mm²

Total Avg loss = $\frac{32.04 + 16.03}{3}$

= 16.02 N/mm²

→ A post-tensioned concrete beam 100 x 300 mm spanning over 10m is stressed by successive tensioning and anchoring of 3 cables 1, 2, 3 respectively. The c/s area of each cable is 200 mm² and the initial stress in the cable is 1200 N/mm². $\alpha_c = 6$. The first cable is parabolic with an eccentricity of 50mm below the centroidal axis at the centre of span and 50mm above the centroidal axis at support section. The second cable is parabolic with zero eccentricity at support and an eccentricity of 50mm at the centre of span. The third cable is straight with a uniform eccentricity of 50mm below centroidal axis (Estimate the % loss of stress in each cable if they are successively tensioned & anchored).



$A = 3 \times 10^4 \ mm^2$

$I = 225 \times 10^6 \ mm^4$

$P_0 = 1200 \times 200 = 240 \times 10^3 \ N$

Cable ①: No loss due to elastic shortening

Cable ②:– Loss in Cable ①

Loss in Cable ① due to elastic shortening at centre

Stress in concrete at level of steel at centre:–

↳ $= m \times f_c$

$$f_c = \frac{P_0}{A} + \frac{P_0 e y}{I}$$

$$= \frac{240 \times 10^3}{3 \times 10^4} + \frac{240 \times 10^3 \times 50^2}{225 \times 10^6}$$

$= 10.67 \ N/mm^2$

Loss in Cable ① due to elastic shortening at supports

$$f_c = \frac{P_0}{A} \quad [\because \text{No eccentricity}]$$

$$= \frac{240 \times 10^3}{3 \times 10^4}$$

$$= 8 \text{ N/mm}^2$$

While Cable '2' is tensioned loss in cable ①:

$$\text{Average stress} = 8 + \frac{2}{3}(10.7 - 8)$$

$$= 9.8 \text{ N/mm}^2$$

$$\text{Loss due to elastic shortening} = m \times f_c$$

$$= 6 \times 9.8$$

$$= 58.8 \text{ N/mm}^2$$

While cable '3' is tensioned loss in Cable ① & ②

For Cable ①

loss in Cable ① at Centre

$$f_c = \frac{P_0}{A} + \frac{P_0 e y}{I}$$

$$= 10.66 \text{ N/mm}^2$$

Loss in Cable ② at support

$$f_c = \frac{P_0}{A} - \frac{P_0 e y}{I} = 5.3 \text{ N/mm}^2$$

Average stress $= 5.3 + \frac{2}{3}(10.66 - 5.3)$

$= 8.87 \ N/mm^2$

Loss due to elastic shortening $= 6 \times 8.88$

$= 53.28 \ N/mm^2$

Loss in Cable ② due to Cable ③ $= 58.8 \ N/mm^2$

Total Losses

In Cable ① $= 58.8 + 53.4$

$= 112.2 \ N/mm^2$

Cable ② $= 58.8 \ N/mm^2$

Cable ③ $= 0$

⇒ A S.S Concrete beam of uniform section is Post-tensioned by means of 2 Cables both of which have an eccentricity of 100 mm below the Centroid of section at Mid span. The Cable ① is Parabolic and is anchored at an eccentricity of 100 mm above the Centroid at each end. The Cable ② is straight and Parallel to the line joining supports. If the c/s area of each Cable is 100 mm². The Concrete beam has a sectional area of $2 \times 10^4 \ mm^4$ and a radius of gyration of 120 mm. Calculate the loss of stress in

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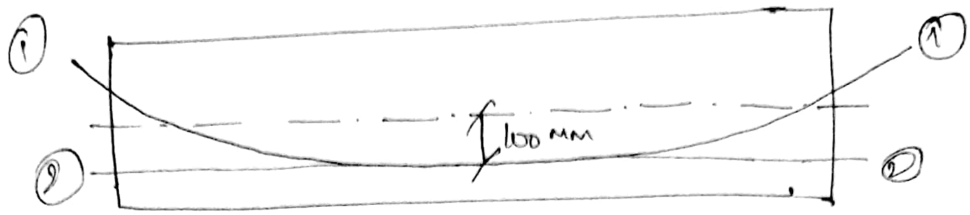
the first Cable. When the second cable is tensioned to a stress of $1200\ N/mm^2$. Take Modular ratio is '6' and neglect friction.



$A = 2\times10^4\ mm^2$

$r\ (or)\ k = 120\ mm$

Initial stress $= 1200\ N/mm^2$; $e = 100\ mm$

$I = Ar^2$

$= 2\times10^4 \times (120)^2$

$= 288\times10^6\ mm^4$

<u>Cable ①</u> :- No loss due to elastic shortening

<u>Cable ②</u> :- Loss in Cable ①

<u>Loss in Cable ① due to elastic shortening of Cable ②</u> :-

Stress in Concrete at level of steel at Centre $= m\times f_c$

$$f_c = \frac{P_o}{A} + \frac{P_o e y}{I}$$

$$= \frac{120\times10^3}{2\times10^4} + \frac{120\times10^3\times100\times100}{288\times10^6}$$

$$= 6 + 4.16 = 10.16\ N/mm^2$$

Stress at Supports

$$= \frac{120\times10^3}{2\times10^4} - \frac{120\times10^3\times100\times100}{288\times10^6}$$

$= 1.84 \ N/mm^2$

Average stress = 7.38 N/mm² → $1.84 + \frac{2}{3}(8.32)$

$= 7.386 \ N/mm^2$

Total loss = 6×7.386 ⇒ $\alpha_e \times f_c$

$= 44.32 \ N/mm^2$

⇒ A Concrete beam is Prestressed by a Cable Carrying an initial Prestressing force of 300 kN. The C/s area of wires in the Cable is 300 mm². Calculate the % loss of stress in the Cable only due to shrinkage of Concrete. Using IS code recommendations. $E_s = 210 \ kN/mm^2$

a) Pre-tensioning

b) Post-tensioning

Assume age of Concrete at transfer = 8 days

Shrinkage strain in Pretensioning $(\varepsilon_{cs}) = 3\times10^{-4}$

" " " Post-tensioning $(\varepsilon_{cs}) = \frac{2\times10^{-4}}{\log_{10}(t+2)}$

Loss due to shrinkage Strain in Prestressing $= \varepsilon_{cs} \times E_s$

$= 3 \times 10^{-4} \times 210 \times 10^3$

$= 63 \ N/mm^2$

Loss due to shrinkage strain in Post tensioning $= \varepsilon_{cs} \times E_s$

$= \frac{2 \times 10^{-4}}{\log_{10}(8+2)} \times 210 \times 10^3$

$= 42 \ N/mm^2$

In Pretensioning

% loss of stress $= \frac{\text{Loss of Stress}}{\text{Initial stress}} \times 100$

Initial stress $= \frac{300 \times 10^3}{300}$

$= 1000 \ N/mm^2$

% loss of stress $= \frac{63}{1000} \times 100$

$= 6.3\%$

In Post tensioning

% loss of stress $= \frac{42}{1000} \times 100$

$= 4.2\%$

⇒ A Component beam of Rectangular section 100 X 300 mm is Prestressed by 5 wires of 7 mm ϕ located at an eccentricity of 50mm. The initial stress in the wires is 1200 N/mm². Estimate the loss of stress in steel due to Creep of concrete using ultimate creep strain method and creep coefficient method. Use following data.

Creep Coefficient is 1.6 ; $E_s = 210$ kN/mm²

$E_c = 35$ kN/mm² ; $\varepsilon_{cc} = 41 \times 10^{-6}$ mm/mm/N/mm²

$e = 50$ mm

Initial stress = 1200 N/mm²

1) Creep Coefficient method:-

loss of stress due to creep $= \phi m f_c$

$$m = \frac{210}{35} = 6$$

f_c = stress in concrete at level of steel

$$f_c = \frac{P_o}{A} + \frac{P_o e y}{I}$$

$$P_o = 1200 \times \frac{a_s}{192.4}$$

$= 230$ kN

$e = y = 50$ mm

$I = 225 \times 10^6$ mm⁴

$A = 3 \times 10^4$ mm²

$f_c = 10.2\ N/mm^2$

$\phi \times m \times f_c = 1.6 \times 6 \times 10.2 = 98.1\ N/mm^2$

2) Ultimate Creep strain Method :- (If given ε_{cc})

*** Loss of stress due to Creep $= \varepsilon_{cc} \times f_c \times E_s$

$= 41 \times 10^{-6} \times 10.2 \times 210 \times 10^3$

$= 87.5\ N/mm^2$

⇒ A Post tensioned Concrete beam of □lar section 100×300mm is stressed by a parabolic cable with zero eccentricity at supports and eccentricity of 50 mm at Centre. The area of Cable is 200 mm². Initial stress in Cable is 1200 N/mm². If the ultimate creep strain is 30×10^{-6} mm/mm/N/mm² of stress of Modulus of elasticity of steel 210 kN/mm². Compute the loss of stress in steel due to creep of Concrete.

$a_s = 200\ mm^2$

$A = 3 \times 10^4\ mm^2$

$P_0 = 1200 \times 200 = 24 \times 10^4\ N = 240\ kN$

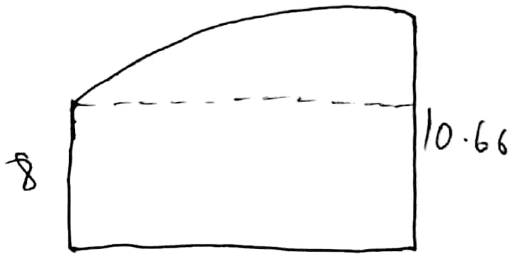
Ultimate creep strain (ε_{cc}) = 30×10^{-6} mm/mm/N/mm²

Modulus of elasticity of steel (E_s) = 210×10^{3} N/mm²

fc at centre [e = 50]

$$f_c = \frac{240 \times 10^3}{3 \times 10^4} + \frac{240 \times 10^3 \times 50^2}{225 \times 10^6}$$

$$= 10.66 \text{ N/mm}^2$$



fc at support [e = 0]

$$f_c = \frac{240 \times 10^3}{3 \times 10^4} = 8 \text{ N/mm}^2$$

Average stress = $8 + \frac{2}{3}(10.66 - 8)$

$$= 9.77 \text{ N/mm}^2$$

Loss of stress due to creep = $\varepsilon_{cc} \times f_c \times E_s$

$$= 30 \times 10^{-6} \times 9.77 \times 210 \times 10^3$$

$$= 9.8 \text{ N/mm}^2$$

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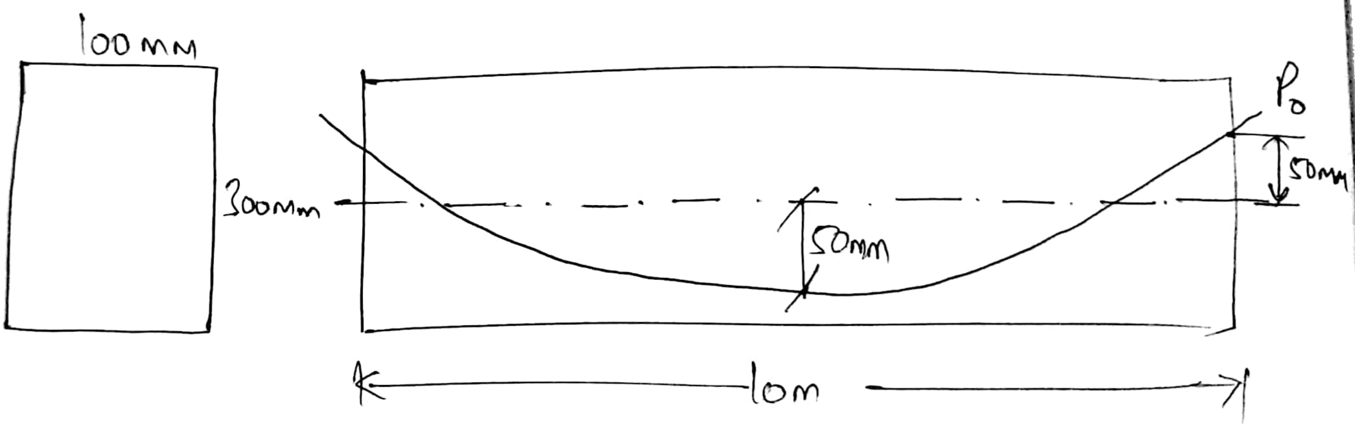
⇒ A Concrete beam of 10m span & 100 x 300 mm is Prestressed by a cable with the c/s area of 200 mm². The Cable profile is Parabolic with an eccentricity of 50 mm above the centroid of the section at the Supports and 50 mm below at Mid span. If the Cable is tensioned from one end only, estimate the loss of stress in the Cable due to the effect of friction. Assume $\mu = 0.35$, $k = 0.0015/m$. The Equation of parabolic Cable $\left[y = \frac{4h}{l^2} x(l - x) \right]$



$a_s = 200\ mm^2$

$l = 10m$

$e = 50 + 50 = 100mm$

To find out 'θ'

$$\text{Slope} = \frac{dy}{dx}$$

Slope at ends $\frac{dy}{dx} = \frac{4h}{l^2}(l - 2x)$

Slope at ends ($x=0$)

$\theta = \frac{4h}{l}$ ($h = e$)

$\theta = \frac{4 \times e}{l} = \frac{4 \times 0.1}{10} = 0.04$ radius

loss of stress due to friction

$\alpha = 2\theta = 2 \times 0.04 = 0.08$

$\mu = 0.35$; $k = 0.0015$

at ends, $l = x = 10$

at centre, $x = \frac{l}{2}$

$P_x = (\mu\alpha + kx) \frac{P_0}{A}$

$= \left[(0.35 \times (2 \times 0.04)) + (0.0015 \times 10) \right] \times P_0$

$P_x = 0.043\, P_0$

% loss of stress $= \frac{0.044\, P_0}{P_0} \times 100$

$= 4.4\%$

⇒ A Pretensioned beam of 250mm x 300mm is prestressed by 12 wires each 7mm ϕ initially stressed to 1200 N/mm^2 with their centroids located 100mm from the soffit. Estimate the final % loss of stress due to elastic deformation. Creep, shrinkage, elastic shortening (relaxation) using following data.

Relaxation of steel = 90 N/mm^2

E_s = 210 kN/mm^2

E_c = 35 kN/mm^2

Creep coefficient (ϕ) = 1.6 ; Residual shrinkage strain = 3×10^{-4}

Section dimensions = 250 mm × 300 mm

12 wires of 7mm ϕ

Initial stress = 1200 N/mm^2

$$I = \frac{bd^3}{12} = \frac{250 \times 300^3}{12} = 562.5 \times 10^6 \text{ mm}^4$$

Eccentricity (e) = 150 − 100 = 50 mm

Relaxation stress = 90 N/mm^2

$\phi = 1.6$

Residual shrinkage strain = 3×10^{-4} (ε_{cs})

1) Loss of stress due to elastic shortening = $m \times f_c$

$$f_c = \frac{P_o}{A} + \frac{P_o \times e^2}{I}$$

$$= \frac{554.16 \times 10^3}{7.5 \times 10^4} + \frac{554.16 \times 10^3 \times 50^2}{562.5 \times 10^6}$$

$$f_c = 7.38 + 2.46$$

$$= 9.84 \text{ N/mm}^2$$

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Loss of stress = $m \times f_c$

$= 6 \times 9.84 = 59.04$ N/mm²

(ii) Loss of stress due to Creep

$= \phi \times m \times f_c$

$= 1.6 \times 6 \times 9.84$

$= 94.46$ N/mm²

(iii) Loss of stress due to shrinkage $= \varepsilon_{cs} \times E_s$

$= 3 \times 10^{-4} \times 210 \times 10^3$

$= 63$ N/mm²

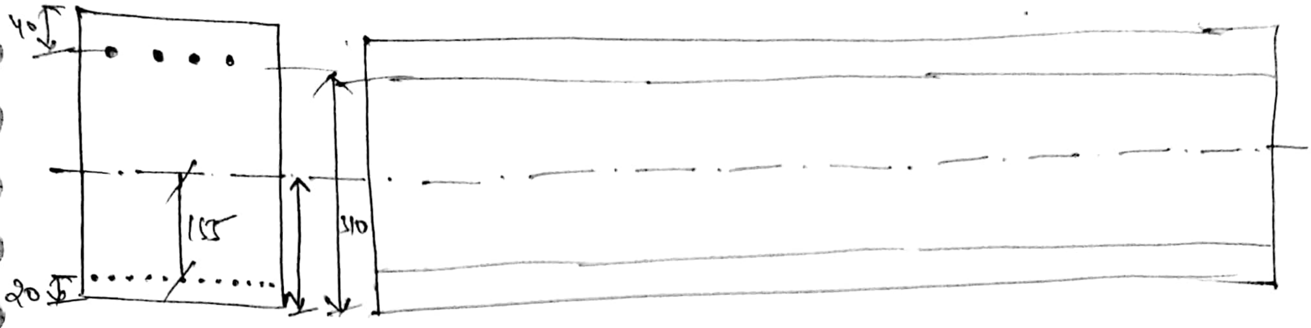
(iv) Loss of stress due to relaxation is steel $= 90$ N/mm²

Total loss of stress $= 306.596$ N/mm²

% loss $= \frac{306.596}{1200} \times 100$

$= 22.54$ %

⟹ A Prestressed Concrete beam of □lar section 250 x 350mm is Provided with 12 high tension wires of 6mm ϕ located 20mm from the bottom of the beam and 4 Similar 6mm wires at the top located at 40mm from the top of beam. The wires are Initially Stretched to a stress of 900 N/mm². Determine the % loss of stress in steel wires due to elastic shortening.



Pre stressing force = stress × a_s

$= 900 \times \frac{\pi}{4} 6^2 \times 16$

$= 40715 7.36$ N

$= 407.15$ kN

$\bar{y} = \frac{4 \times (350 - 40) + 12 \times 20}{12+4}$

$\bar{y} = 92.5$ m

$m = \frac{E_s}{E_c}$

$e = 175 - 92.5$

$e = 82.5$ mm

$y_t = 175 - 40 = 135$ mm

f_c at top steel

$= \frac{P_o}{A} - \frac{P_o e y_t}{I}$

$= \frac{407.15 \times 10^3}{(250 \times 350)} - \frac{407.15 \times 10^3 \times 82.5 \times 135}{\frac{250 \times 350^3}{12}}$

$= 4.653 - 5.076 = -0.42$ N/mm^2

f_c at bottom steel

$= \frac{P_o}{A} + \frac{P_o e y_b}{I}$

$= 4.652 + 5.82$

$= 10.48\ N/mm^2$

$y_c = 175 - 20$

$= 155$

$e = 82.5\ mm$

→ Loss due to elastic shortening at top $= 6 \times -0.42$

$= -2.52\ N/mm^2$

→ loss due to elastic shortening at bottom $= 6 \times 10.48$

$= 62.88\ N/mm^2$

% loss of stress at top $= \frac{-2.52}{900} \times 100 = -0.28\%$

% loss of stress at bottom $= \frac{62.88}{900} \times 100 = 6.9\%$

⇒ Rectangular concrete beam 350 X 400 mm is Prestressed by a force of 1000 KN at an constant eccentricity of 75mm. The span of beam is 4m. Draw the final stress distribution across the c/s due to dead and imposed load of 4.5 KN/m at support and centre of span.

Prestressed force (P_0) = 1000 KN

$e = 75\ mm$

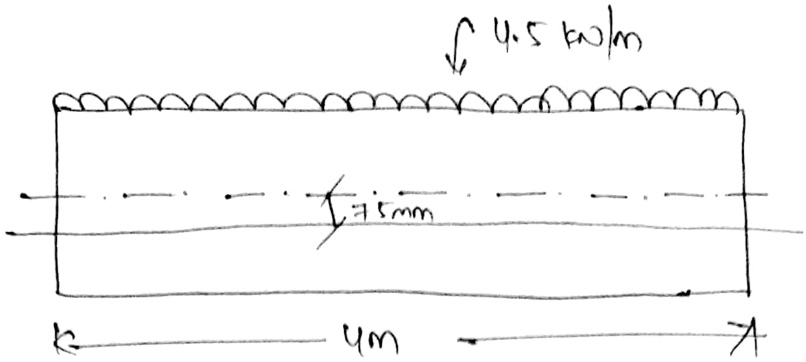
$l = 4m = 4000\ mm$

Dead and imposed load $= 4.5\ KN/m$

$1\ KN/m^2 = 1 \times 10^3\ N/mm^2$

$= 1 \times 10^3 \times 10^{-6}$

$= 10^{-3}$



Direct stress due to Prestressing $= \frac{P_o}{A} = \frac{1000 \times 10^3}{350 \times 400} = 7.14\ N/mm^2$

Stress due to eccentricity $= \frac{Pe}{Z} = \frac{1000 \times 10^3 \times 75}{\left(\frac{350 \times 400^2}{6}\right)}$

$= 8.03\ N/mm^2$

Bending moment due to dead load and imposed load $= \frac{M}{Z} = \frac{\left(\frac{4.5 \times 4000^2}{8}\right)}{9.33 \times 10^6}$

$= 0.96\ N/mm^2$

Stress at top fibres $= \frac{P}{A} - \frac{Pe}{Z} + \frac{M}{Z}$

$= 7.14 - 8.03 + 0.96$

$= 0.07\ N/mm^2$

Stress at bottom fibres $= \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$

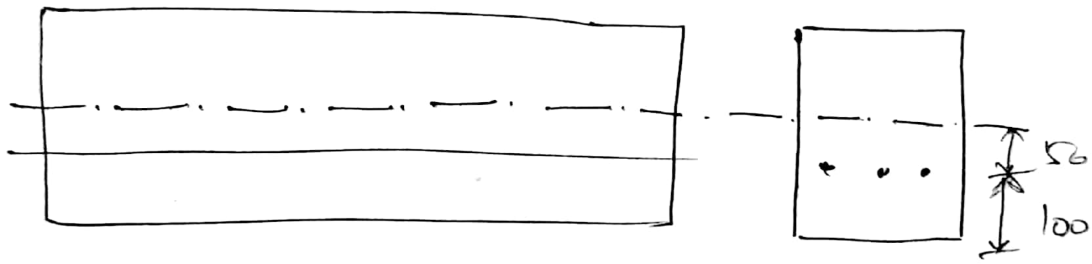
$= 7.14 + 8.03 - 0.96$

$= 14.21\ N/mm^2$

⇒ A Post tensioned Concrete Member 100 x 300 mm is Prestressed by 3 Cables each with a c/s area of 50 mm^2 and with an initial stress of 1200 N/mm^2 all the 3 Cables are straight are located 100 mm from the soffit of the beam. If the Modular ratio (m) is 6. Calculate the loss of stress in 3 Cables due to elastic deformation of Concrete for the following cases:

a) Simultaneously tensioning and anchoring of all 3 Cables.

b) Successive tensioning of 3 Cables one at a time



$e = 50$ mm $(150-100)$

$m = 6$

P_o = stress x area

= 1200×50

$P_o = 60000$ N

$$f_c = \frac{P_o}{A} + \frac{P_o \times e^2}{I}$$

$$= 2 + \frac{60000 \times 50^2}{\frac{100 \times 300^3}{12}}$$

$f_c = 2.66$ N/mm^2

Case 1 :- Simultaneously tensioning and anchoring all 3 cables

Loss of stress due to elastic shortening $= \frac{1}{2} \times m \times 3 \times f_c$ (3 = No. of cables)

$= \frac{1}{2} \times 6 \times 3 \times 2.66$

$= 23.94 \ N/mm^2$

Case 2 :- Wires are tensioned successively

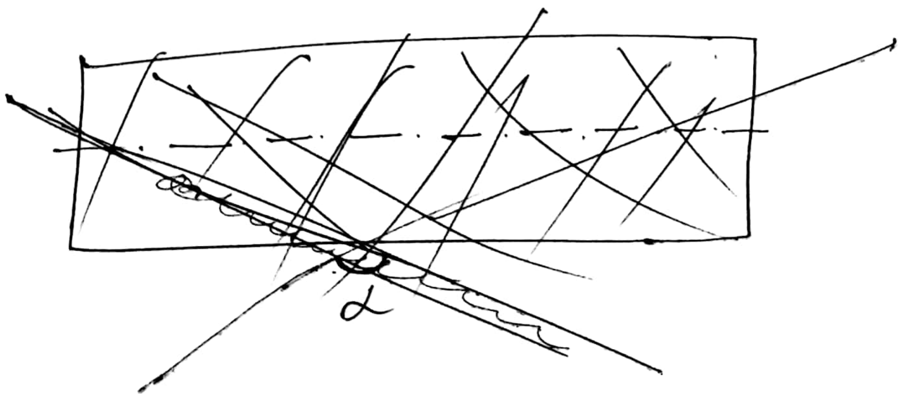
When cable (1) is tensioned & anchored = No loss

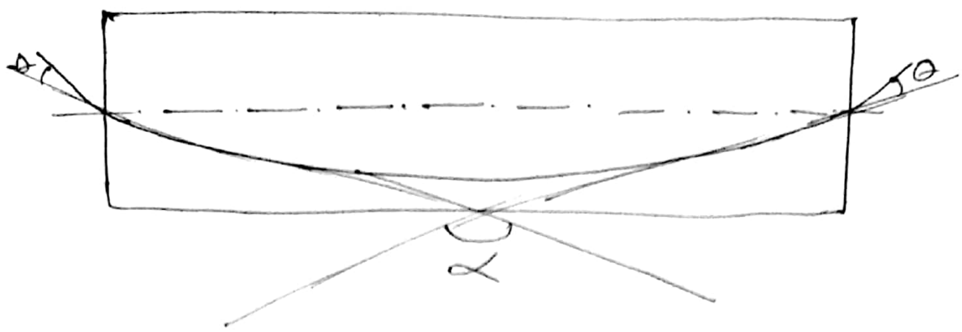
" Cable (2) " " " = Loss due to cable (1)

Loss in cable (1) $= m \times f_c$

$= 6 \times 2.6 = 15.96 \ N/mm^2$

⇒ A S.S Post tensioned concrete beam of span 15 m has a rectangular c/s 300 x 800 mm. The prestress at ends in 1300 kN with '0' eccentricity at supports and an eccentricity of 250 mm at the centre. The cable profile is parabolic. Assume k = 0.15/100 m ; μ = 0.35. Determine the % loss of stress due to friction at the centre of the beam.





Parabolic Equation $(y) = \frac{4h}{l^2} x(l-x)$

Slope at ends $= \frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$

$\theta_A = \theta_B = 4x$

Slope at ends $= \frac{1}{15}$

Slope at Centre $= 0$

Change of slope $= \frac{1}{15}$

at centre $\alpha = \theta$; $x = \frac{l}{2} = \frac{15}{2}$; $k = 0.15/100$

$k = 0.0015$

$P_x = P_0(1 - \mu\alpha - kx)$

$= 1300(1 - 0.35 \times \cdot \quad - 0.0015 \times 7)$

$= 1276.7$ kN

loss of force (or) effective force = Initial force − loss of force

$= 1300 - 1276.7$

$= 23.3$ kN

$$\% \text{ loss of stress} = \frac{\text{Effective force}}{\text{Initial force}} \times 100$$

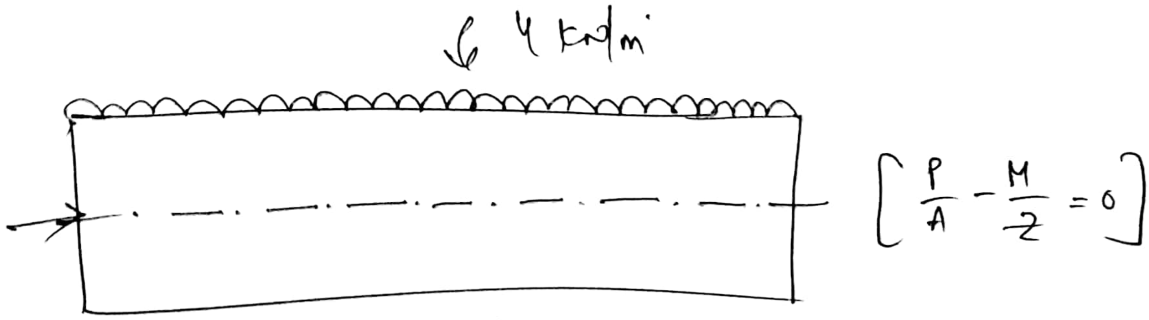
$$= \frac{22.3}{1200} \times 100$$

$$= 1.8\ \%$$

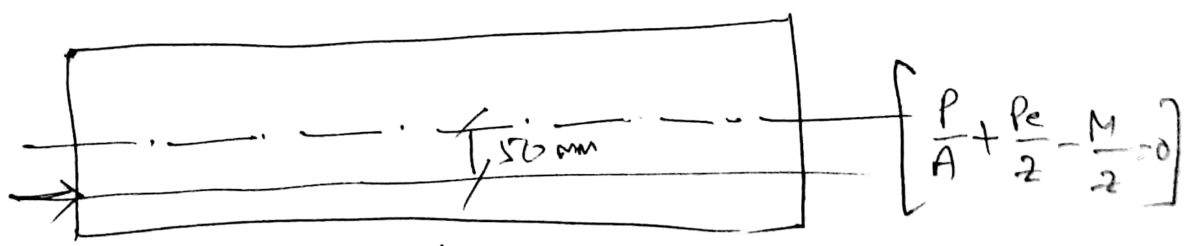
⟹ A Prestressed Concrete member (beam) of 200 mm x 300 mm is used over an effective span of 6m to support an imposed load of 4 kN/m. The density of concrete is 24 kN/m³ at the centre of span section of the beam. Find the magnitude of

a) The concentric prestressing force necessary for zero fibre stress at the soffit when the beam is fully loaded.

b) The eccentric prestressing force located 100 mm from the bottom of the beam, which would nullify the bottom fibre stress due to loading.



Concentric load



eccentric loading

Self wt of beam = $0.2 \times 0.3 \times 24 \times 1$

$= 1.44$ kN/m

Imposed load = 4 kN/m

$$M = \frac{wl^2}{8} = \frac{5.44 \times 6^2}{8} = 24.48 \times 10^6 \text{ N-mm}$$

Case 1:- For bottom stress $= \frac{P}{A} - \frac{M}{Z}$

$$\frac{P}{200 \times 300} - \frac{24.48 \times 10^6}{\left(\frac{200 \times 300^2}{6}\right)} = 0$$

$$\frac{P}{6 \times 10^4} = 8.16$$

$$P = 489.6 \text{ kN}$$

Case 2:- For bottom stress $= \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$

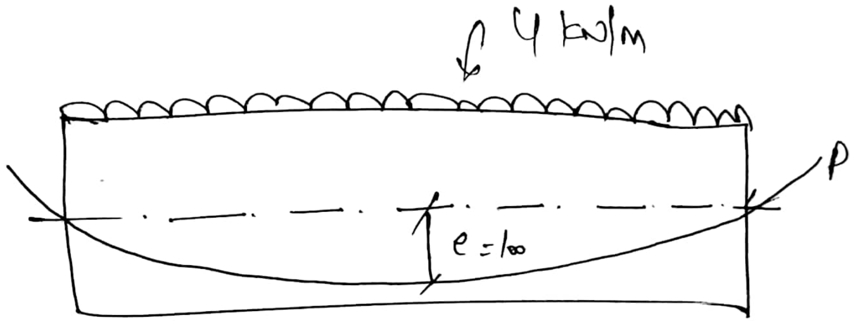
$$0 = \frac{P}{6 \times 10^4} + \frac{P \times 50}{3 \times 10^6} - \frac{24.48 \times 10^6}{3 \times 10^6}$$

$$0 = 1.66 \times 10^{-5} P + 1.66 \times 10^{-5} P - 8.16$$

$$0 = 3.33 \times 10^{-5} P - 8.16$$

$$P = 244.8 \text{ kN}$$

⇒ A Concrete beam of rectangular section 200 X 600 mm is Prestressed by a Parabolic Cable located at an eccentricity of 100mm at mid span and zero at supports. If the beam has a span of 10m and carries an U.D.L of 4 kN/m. Find the effective force necessary in the Cable for '0' shear stress in the beam. For this Condition, Calculate the stress at Mid-section. Find Prestressed force = ?



$$W_c = \frac{8Ph}{l^2} \quad \Big\} \text{ load balancing Concept}$$

$$P = \frac{wl^2}{8h}$$

$$W_d = 0.2 \times 0.6 \times 24 = 2.8 \text{ kN/m} \; ; \; W_L = 4 \text{ kN/m}$$

$$W = W_c = 4 + 2.8 = 6.8 \text{ kN/m}$$

$$P = \frac{6.8 \times (10000)^2}{8 \times 100} = 860 \text{ kN}$$

$$\text{Stress at Mid section} = \frac{P}{A} = \frac{.860}{200 \times 600} = 7.29 \text{ N/mm}^2$$