UNIT - III

Analysis of Prestressed members for flexure & shear

Flexure :- Similar to members under axial load, the analysis of members under flexure refers to the Evaluation of the following:

1) Permissible Prestress based on allowable stresses at transfer.
2) Stresses under service loads. These are compared with allowable stresses under service conditions.
3) Ultimate strength. This is compared with the demand under factored loads.
4) The entire load versus deformation behaviour.

Assumptions!-

The analysis of members under flexure considers the following.

1. Concrete is a homogeneous elastic material.
2. With in the range of working stress, both Concrete & steel behave elastically, not with standing the small amount of creep, which occurs in both the materials under the sustained loading.
3. A plane section before bending is assumed to remain plane even after bending, which implies a linear strain

distribution across the depth of the member.

4. Prestress Concrete is one in which there have been introduced internal stresses of such magnitude and distribution the stresses resulting from given external loading is counter balanced to a desired ~~degree~~ degree.

5. Plane sections remain plane till failure (known as Bernoulli's hypothesis).

6. Perfect bond between concrete and prestressing steel for bonded tendons.

Principles of mechanics:-

The analysis involves 3 principles of mechanics

1) Equillibrium of internal forces with the external loads. The compression in concrete (C) is equal to the tension in the tendon (T). The couple of C and T are equal to the moment due to external loads.

2) Compatibility of the strains in concrete and in steel for bonded tendons. The formulation also involves the first assumption of plane section remaining plane after bending. For unbonded tendons, the compatibility is in terms of deformation.

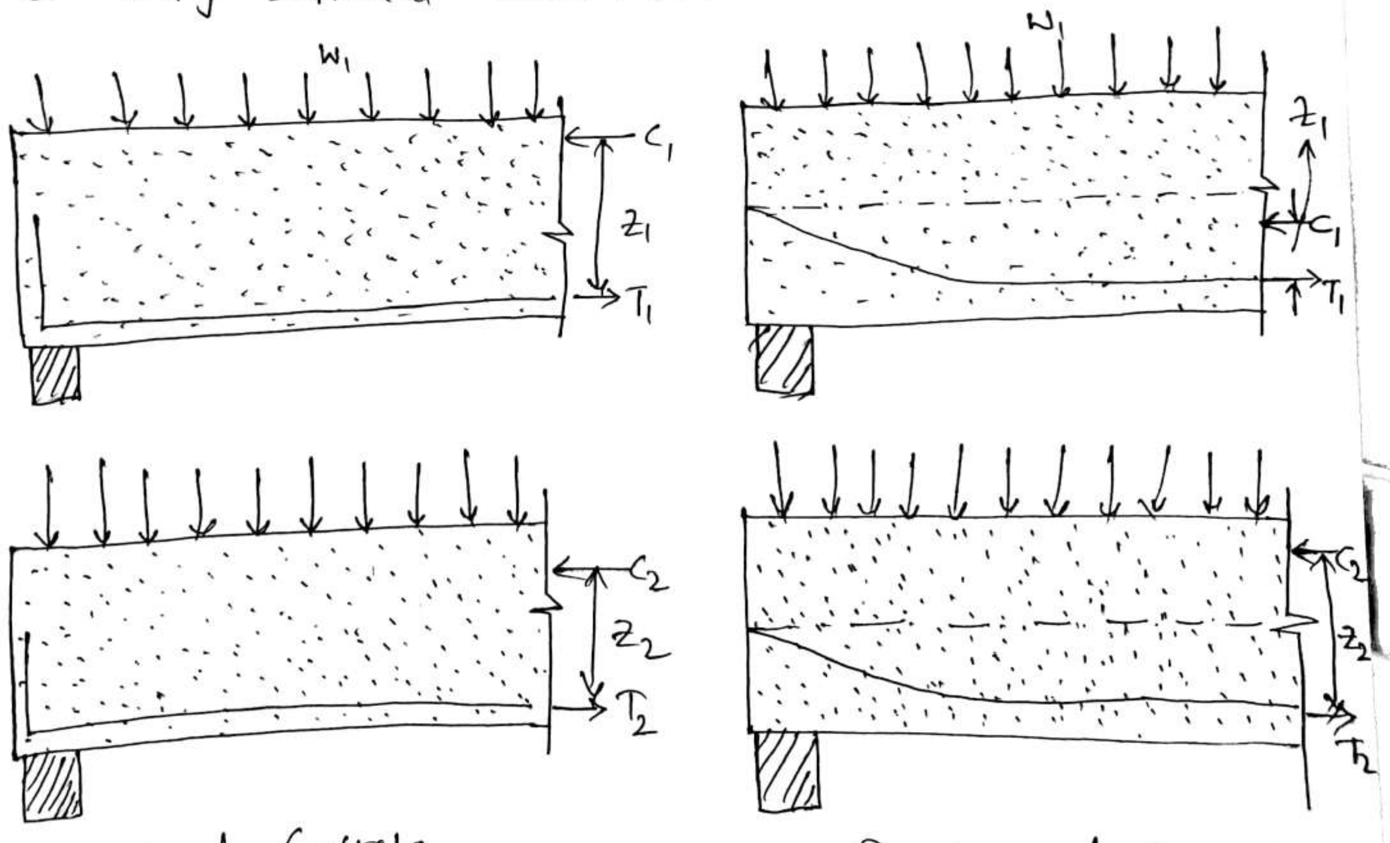
3) Constitutive relationships relating the stresses and the strains in the materials.

Variation of Internal forces

In reinforced concrete members under flexure, the values of compression in concrete (C) and tension in the steel (T) increase with increasing external load. The change in the lever arm (z) is not large.

In prestressed concrete members under flexure, at transfer of prestress C is located close to T. The couple of C and T balance only the self weight. At service loads, C shifts up and the lever arm (z) gets large. The variation of C and T is not appreciable.

The following figures explains the difference schematically for a simply supported beam under uniform load.



Reinforced Concrete

$C_2 > C_1$; $z_2 \simeq z_1$

Prestressed Concrete

$C_2 \approx C_1$, $z_2 > z_1$

C_1, T_1 = Compression & tension at transfer due to self weight.

C_2, T_2 = Compression & tension at service due to service loads.

W_1 = self weight

W_2 = service loads

z_1 = lever arm at transfer

z_2 = lever arm under service loads.

Analysis at Transfer and at service :-

The analysis at transfer and under service loads are similar. Hence, they are presented together. A prestressed member usually remains uncracked under service loads. The concrete and steel are treated as elastic materials. The principle of superposition is applied. The increase in stress in the prestressing steel due to bending is neglected.

There are 3 approaches to analyse a prestressed member at transfer and under service loads. These approaches are based on the following concepts

a) Based on stress concept

b) Based on force concept

c) Based on load balancing concept

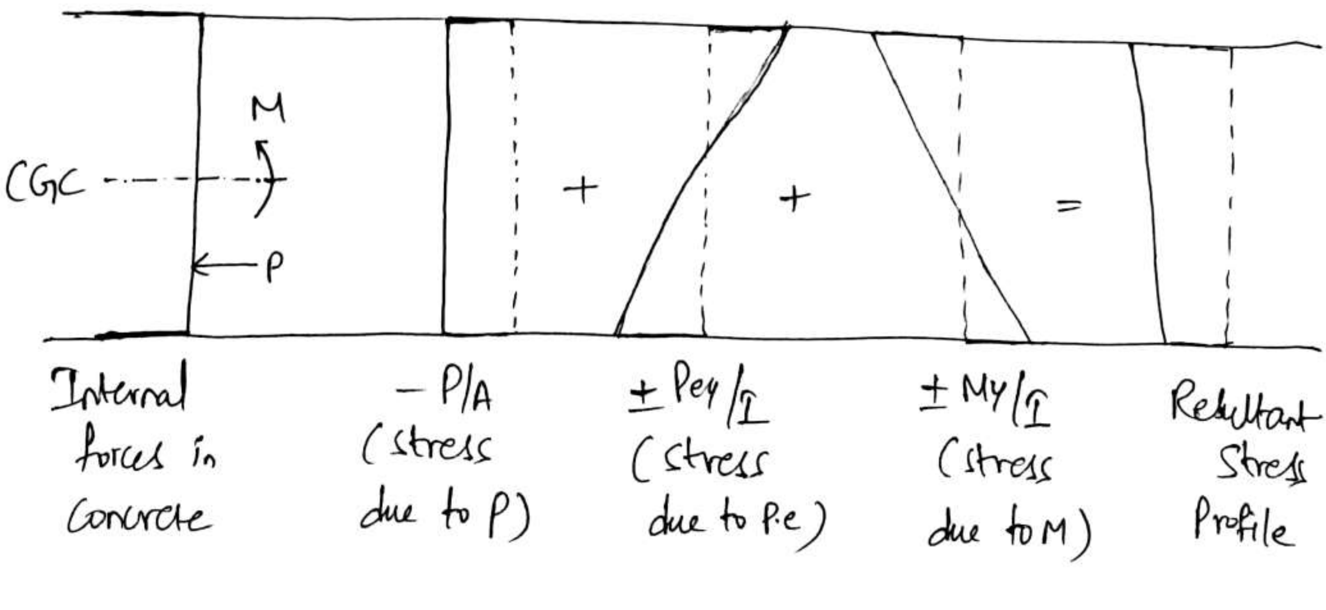
The following material explains the 3 Concepts.

Based on stress Concept :-

In the approach based on stress concept, the stresses at the edges of the section under the internal forces in Concrete are calculated. The stress concept is used to Compare the Calculated stresses with the allowable stresses.

The following fig. shows a simply supported beam under a uniformly distributed load (UDL) and Prestressed with Constant eccentricity (e) along its length.

The first stress Profile is due to the Compression P. The second Profile is due to the eccentricity of the Compression. The third Profile is due to the moment. At transfer, the Moment is due to self weight. At service the moment is due to service loads.



Stress Profiles at a section due to Internal forces

The resultant stress at a distance y from the CGC is given by the Principle of superPosition as follows.

$$f = -\frac{P}{A} \pm \frac{Pey}{I} + \frac{My}{I}$$

Based on force Concept :-

The approach based on force concept is analogous to the study of reinforced Concrete. The tension in Prestressing steel (T) and the resultant Compression in Concrete (C) are considered to balance the external loads. This approach is used to determine the dimensions of a section and to check the service load capacity. Of course, the stresses in concrete calculated by this approach are same as those calculated based on stress concept. The stresses at the extreme edges are compared with the allowable stresses.

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The Equillibrium Equations are as follows:

$$C = T \quad , M = C \cdot z \quad , M = C(e+e)$$

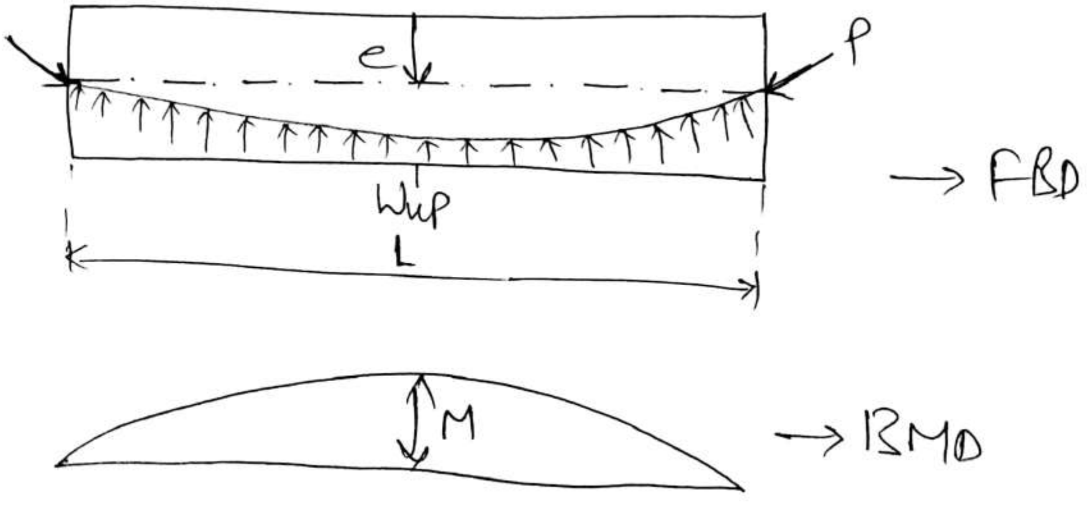
The resultant stress in concrete at distance y from the CGC is given as:

$$f = \frac{-P}{A} \pm \frac{Pey}{I} \pm \frac{My}{I}$$

Based on load balancing Concept :-

The approach based on load balancing concept is Used for a member with curved (or) harped tendons and in the analysis of indeterminate Continuous beams. The moment, upward thrust and upward deflection (Camber) due to the Prestress in the tendons are calculated. The upward thrust ~~is~~ balances Part of the Superimposed load. The expressions for three Profiles of tendons in Simply Supported beams are given.

a) For a Parabolic tendon.

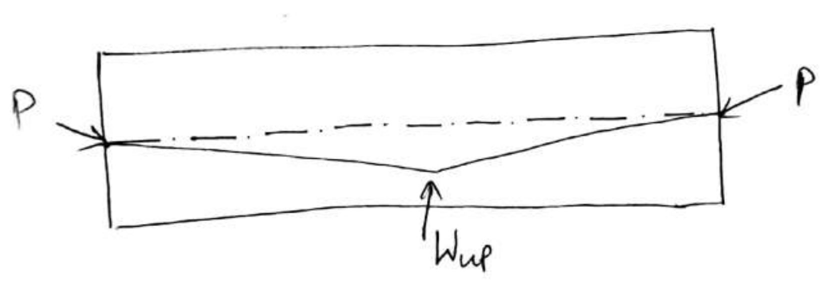


The moment at the centre due to uniform upward thrust (w_{up}) is given by the following equation.

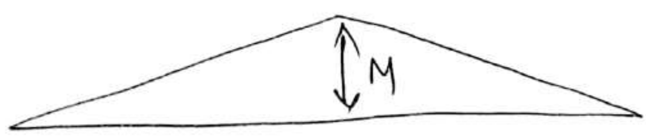
$$M = \frac{w_{up} L^2}{8}$$

The moment at the centre from the prestressing force is given as $M = Pe$. The expression of w_{up} is calculated by equating the two expressions of M. The upward deflection can be calculated from w_{up} based on elastic analysis.

b) For singly harped tendon



FBD



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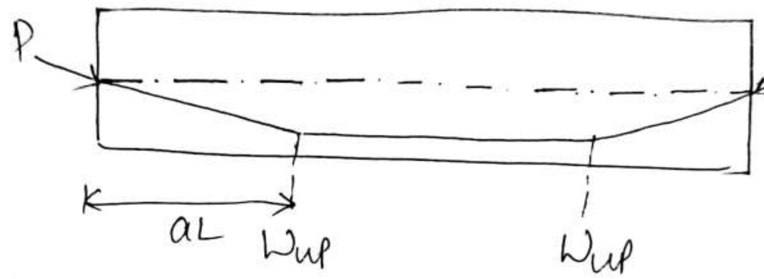
The moment at the centre due to the upward thrust (W_{up}) is given by the following equation. It is equated to the moment due to eccentricity of the tendon. As before, the upward thrust and the deflection can be calculated.

$$M = \frac{W_{up}L}{4} = Pe$$

$$W_{up} = \frac{4Pe}{L}$$

$$\Delta = \frac{W_{up}L^3}{48EI}$$

c) For Doubly harped tendon :



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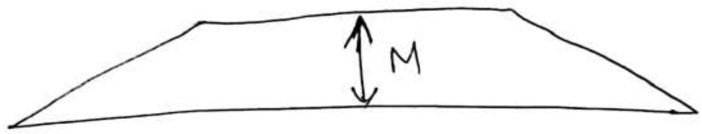

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The moment at the centre due to the upward thrusts (W_{up}) is given by the following equation. It is equated to the moment due to the eccentricity of the tendon. As before, the upward thrust and the deflection can be calculated.

$$M = W_{up} aL = Pe$$

$$W_{up} = \frac{Pe}{aL}$$

$$\Delta = \frac{a(3-4a^2)W_{up}L^3}{24EI}$$