UNIT - I

Equilibrium and compatibility Equations :-

Equilibrium :-

Internal force equilibriates the external force is called as equilibrium.

Mechanisam :-

If no resistance against the deformation the structure is called as unstable structure.

Types of structure :-

Basically these are three types.

(1) skeletal (1-D)

(2) surface (2-D)

(3) Solid (3-D).

Skeletal :-

These are the structure member's are idealised to be serious of lines and curves.

② **Surface Structure:**

These are idealised to be series of planes straight and curved.

Examples: Roof truss, Building frames.

→ These are two dimensional structure.

Ex: R.C.C slab, folded plates and shells.

③ **Solid Structure:-**

which can't be idealised into neither a skeleton nor a space (or) surface.

Ex: massive foundation, machine foundation.

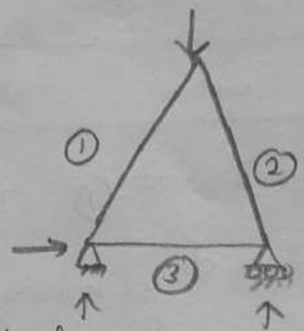
Types of Supports:-

Notation	Type of support	DOF.
	Fixed	0
	Hinge	1
	Roller	2
	free	3

DOF = degree of freedom of supports.

Statically determinate :-

If unknown reaction forces equal to no. of equation of equilibrium it is called as statically determinate structure.



3 no. of unknow reaction.

3 no. of unknow reaction member forces

6 unknown forces.

Equation of equilibrium we have in this structure is

$$\Sigma f_y = 0$$
$$\Sigma f_x = 0$$

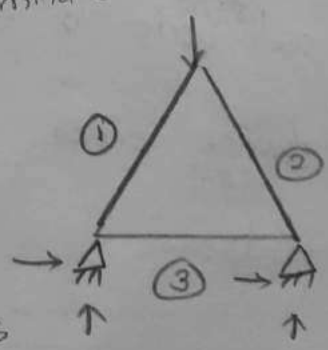
$2 \times 3 = 6$ eq of equilibrium.

for every joint's.

$\boxed{6 = 6}$

Statically indeterminate :

If unknown reaction forces are not equal to no. of equation of equilibrium it is called as indeterminate structure



3 no. of unknown reaction.

4 no. of member forces

7

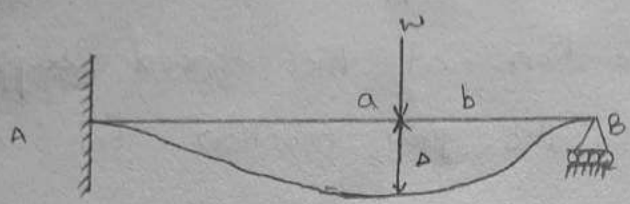
equation of equilibrium we have in this structure is '6' $6 \neq 7$ not satisfied.

Hence it is a statically indeterminate.

→ Differences between the statically determinate and statically indeterminate structure.

statically determinate structure	statically indeterminate structure
① The determination of internal forces in the structures do not require the cross-sectional areas and moment of inertia of beams and frame members.	① The cross-sectional area and moment of inertia are required as first step of analysis. This quanties has effect on the values of internal forces and moments.
② The internal forces and moments are not effected by settlement of supports of lack of fit.	② The settlements and lack of fit are the important factors in determination of internal forces and moments.
③ The equilibrium equations are adequete to determine the internal forces and moments.	③ The compatatiblity conditions of displacements are required over and above the equilibrium conditions to determine the internal forces and moments.

Propped Cantilever Beams :-



Consider a straight beam with the end 'A'. is clamped and 'B' is supported by a prop.

→ It is a statically indeterminate to one degree as the number of unknowns are four.

→ ① vertical ② horizontal ③ fixed end moment and ④ prop reaction.

→ The avaible equations of equilibrium are three viz ΣH, $\Sigma V = 0$, and $\Sigma M = 0$.

→ As three equation are not sufficient to determine four unknowns, one more equation can be formulated by knowing the final position of B relative to A' is known.

→ The props are classified as.

(i) Rigid props

(ii) Elastic props.

(1) The total number of support reaction are four. i.e three unknown reaction at the fixed support and the fourth one is the prop reaction. the avaible equilibrium equation are three.

(2). As an initial step draw the elastic load diagram (M/EI). Due to the prop reaction.

The bending moment is positive if the moment is acting clockwise to the left of the section. and if it is acting anticlockwise to the right of the section.

(3) Draw the free body diagram and use the equilibrium to calculate the other reaction and moment.

(4) Draw the shear force diagram. Bending moment diagram and the elastic curve.

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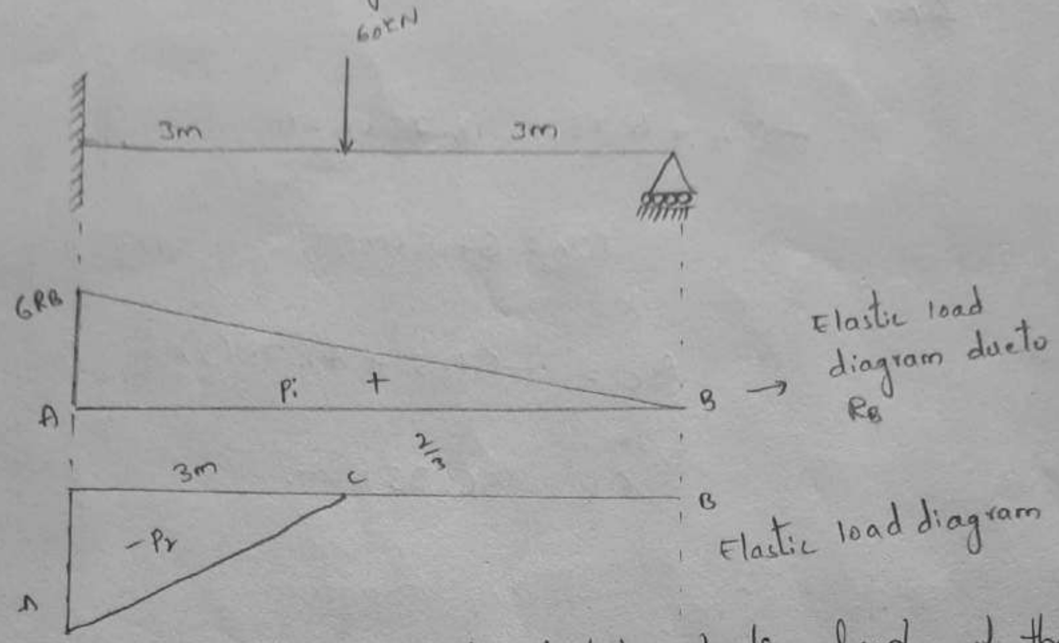
① Draw SFD and BMD for the propped cantilever beam loaded as shown in figure use consistent deformation method.



Thus. $\sum_{i=1}^{n} P_i x_i$ is zero 'P_i' is the total elastic load of the respective diagram and 'x_i' is the centroidal distance of the elastic load diagram.

$x_1 = \frac{2}{3} \times 6 = 4m.$

$x_2 = 3 \times \frac{2}{3} + 3 = 5m.$

$\Sigma M_B = 0.$

$P_1 x_1 + P_2 x_2 = 0.$

$\left(\frac{1}{2}\right)(6) \times 6R_B \times 4 - \left(\frac{1}{2} \times 3 \times 180\right) \times 5 = 0.$

$R_B = 18.75kN.$

$R_A = 60 - 18.75$ $\quad R_A = 41.25.kN.$

$\Sigma V = 0$

$\Sigma M_A = 0$

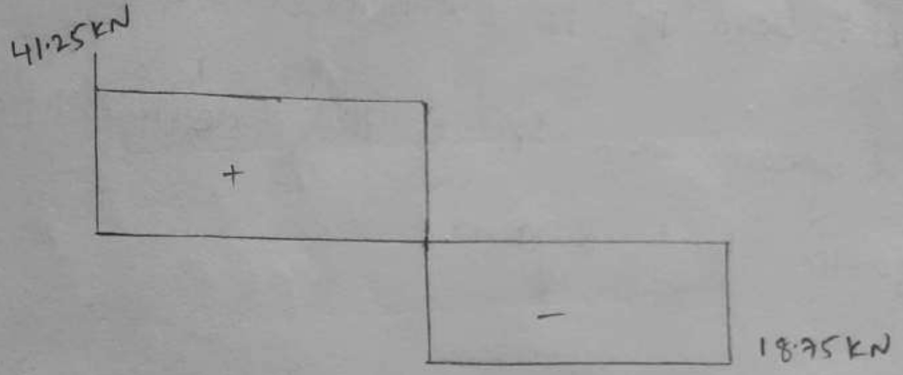
$$-M_A + 60 \times 3 - 18.75 \times 6 = 0$$

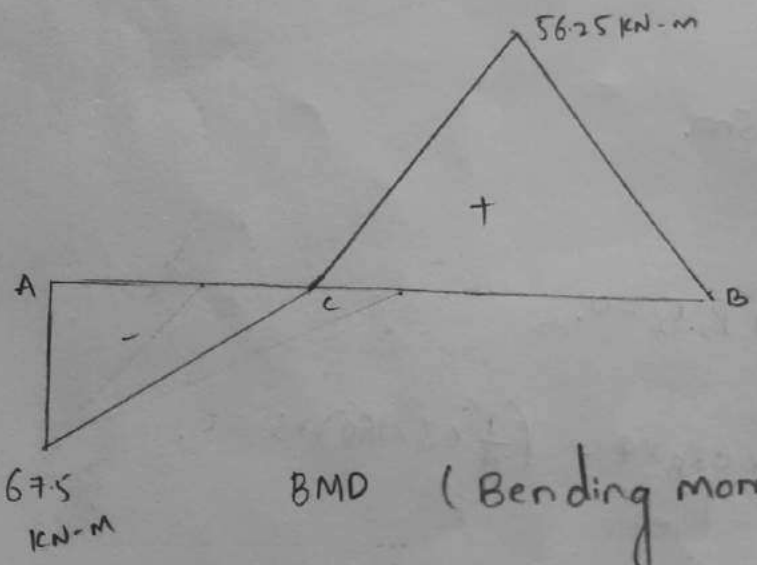
$$M_A = 67.5 \text{ kNm.}$$

$$M_C = -67.5 + 41.25(3)$$

$$M_C = 56.25 \text{ kNm.}$$

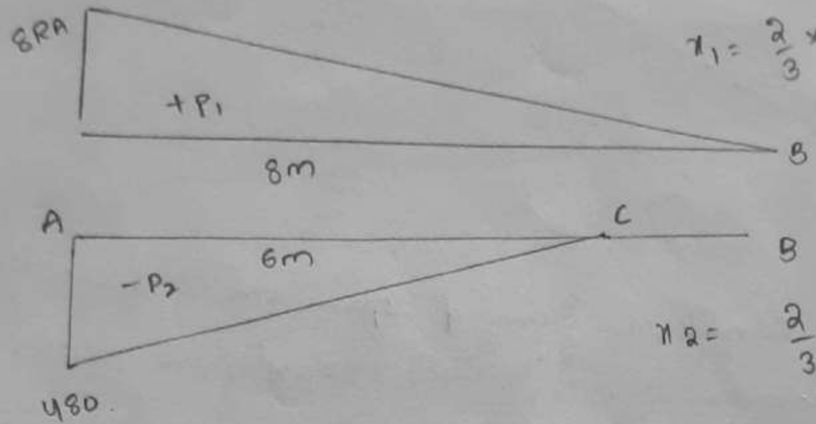
SFD: Shear force diagram





BMD (Bending moment diagram)

② Analyse the propped cantilever beam shown in sketch BMD and SFD.



$x_1 = \frac{2}{3} \times 8 = 5.33m$

$x_2 = \frac{2}{3} \times 6 + 2$
$= 6m$

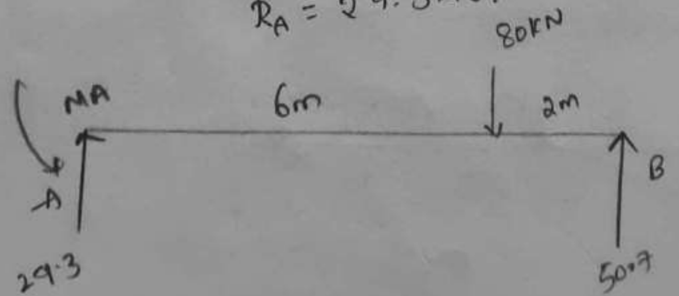
Soln Taking EI as unity the above elastic load diagrams were drawn. As the deflection at 'B' is zero.

$M_B = 0.$ $\quad P_1 x_1 + P_2 x_2 = 0$

$\Rightarrow (\frac{1}{2} \times 8) \times 8 R_B * 5.33 - (\frac{1}{2} \times 6 \times 480) \times 6 = 0.$

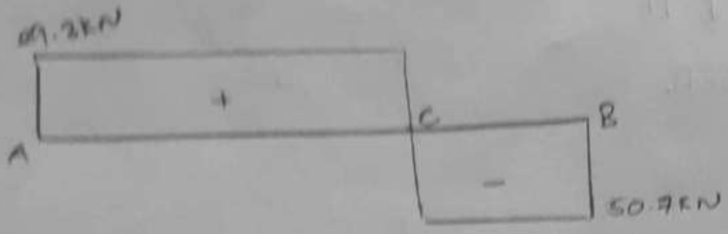
$R_B = 50.66\ kN.$

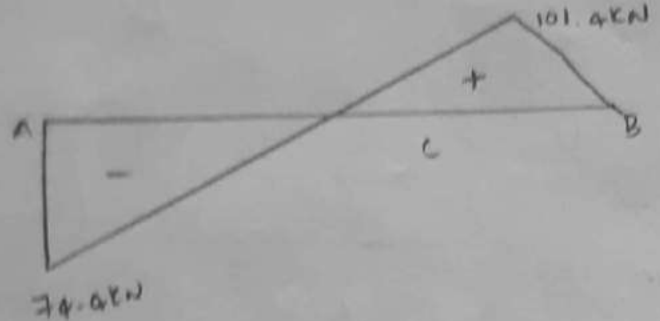
$R_A = 29.3\ kN.$



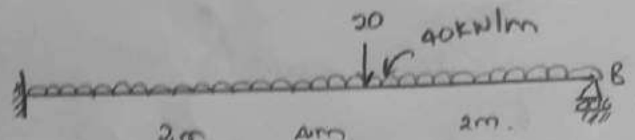
$80 \times 6 - M_A - 50.7 \times 8 = 0 \quad \Rightarrow M_A = 74.4\ kNm.$

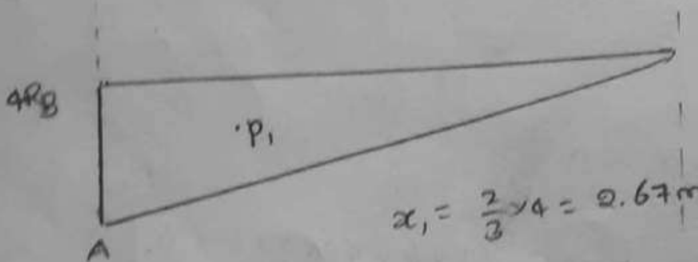
SFD:-



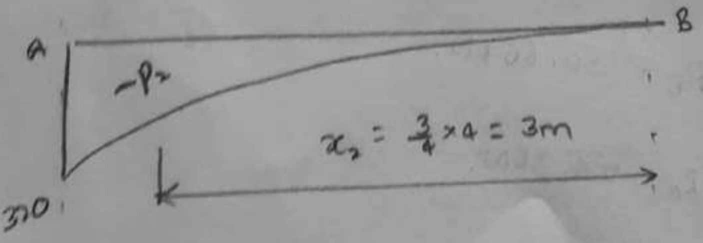


③ a propped cantilever beam of span 4m is subjected to UDL of intensity 40kN/m throughout the span draw SFD and BMD.

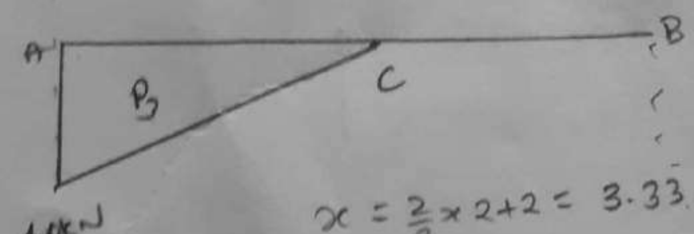




$x_1 = \frac{2}{3} \times 4 = 2.67m$



$x_2 = \frac{3}{4} \times 4 = 3m$



$x = \frac{2}{3} \times 2 + 2 = 3.3\bar{3}$

Sol. The deflection at the prop 'B'. is zero, Hence

$$P_1x_1 + P_2x_2 = 0.$$

$$(1/2)\,4(4R_B) \times 2.67 + (\tfrac{1}{3} \times 4 \times 320) \times 3 = 0.$$

$$R_B = 59.9\,kN$$

$$R_A = 160 - 59.9 = 100.1\,kN \Rightarrow R_A = 100.1\,kN.$$

Taking moment about A:

$$-M_A + (40 \times 4 \times \tfrac{4}{2}) - 59.9 \times 4 = 0.$$

$$M_A = 80.4\,kNm.$$

To find the maximum positive bending moment equate the shear force equation to zero

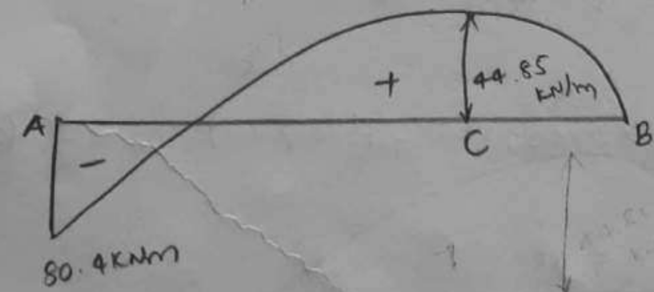
$$100.1 - 40x = 0$$

$$x = 2.5\,m.$$

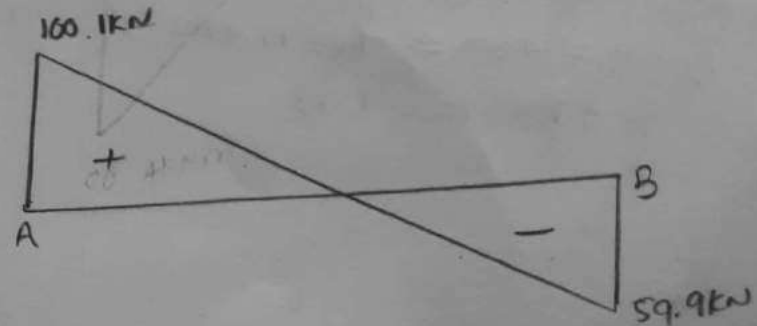
Maximum positive BM $= -80.4 + 100.1(2.5) - 40 \times \frac{2.5^2}{2}$

$$M_C = 44.85\,kNm.$$

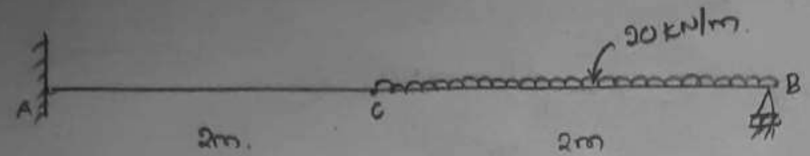
BMD



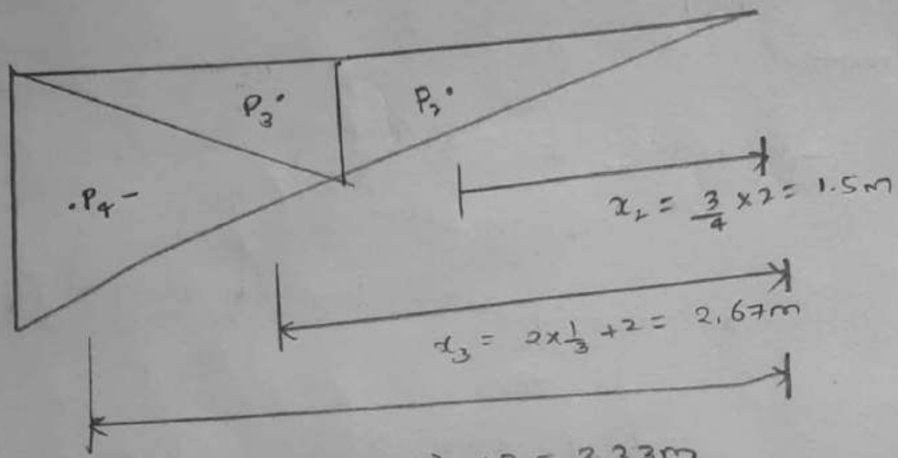
SFD



(1) Analyse the propped cantilevers shown in fig and draw SFD and BMD.



$x_1 = \frac{2}{3} \times 4 = 2.67m$



$x_4 = 2 \times \frac{2}{3} + 2 = 3.33m.$

Elastic load diagram due to UDL

The deflection at B is zero.

$$P_1 x_1 + P_2 x_2 + P_3 x_3 + P_4 x_4 = 0.$$

$$\left(\frac{1}{2}\right)4(4R_B)2.67 - \left(\frac{1}{3} \times 2 \times 40\right) \times 1.5 - \left(\frac{1}{2} \times 2 \times 40\right)2.67 - \frac{1}{2} \times (2 \times 20) \times (3.33$$

$$R_B = 25.6kN$$

$\Sigma V = 0.$

$$R_A + R_B = 40 \Rightarrow R_A = 14.4kN.$$

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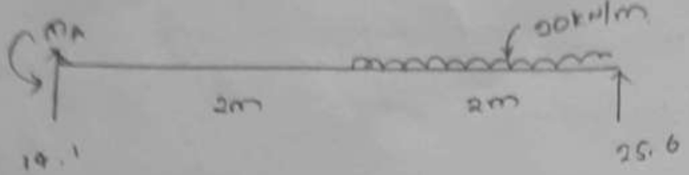
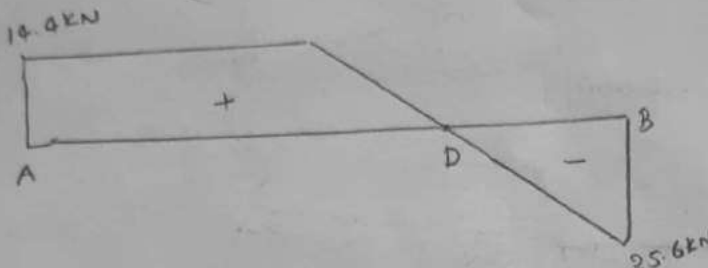
Taking moment about A;

$$-M_A + 20\times2\times(2+1) - 25.6\times4 = 0$$

$$M_A = 17.6\ kN/m$$



<u>SFD</u>



To obtain the maximum positive bending moment the location of zero shear is formed out from 'B'.

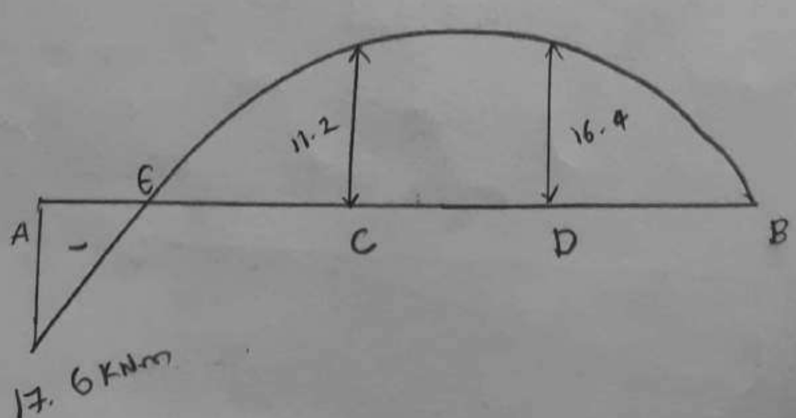
$$R_B - 20(x) = 0$$

$$25.6 - 20(x) = 0$$

$$x = 1.28m.$$

$$\text{Max +ve B.M} = 25.6(1.28) - 20\times\frac{1.28^2}{2}$$

$$M_D = 16.4\ kNm.$$



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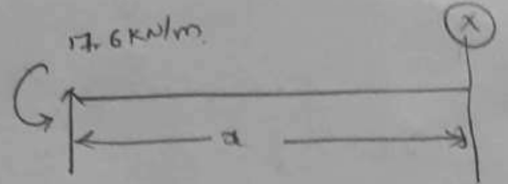
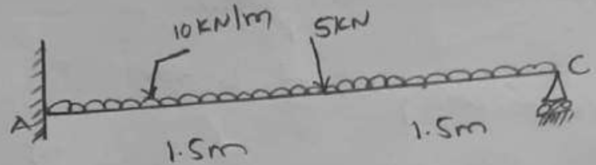


$-17.6 + 14.4x = 0$

$x = 1.22m.$

The point of contraflexure is found out by equation to bending moments is zero.

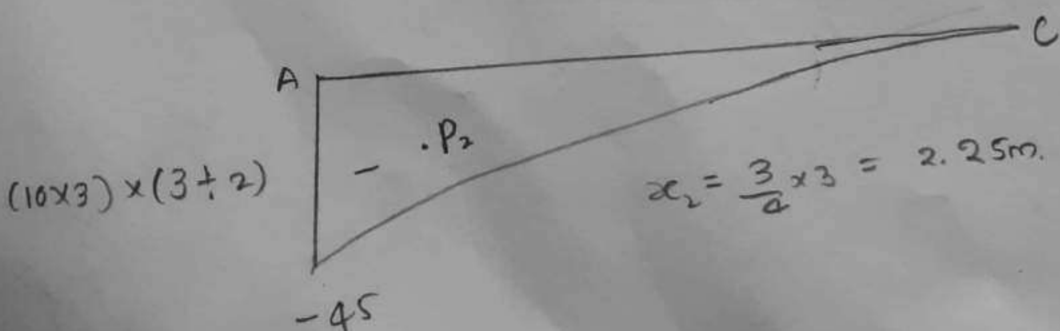
(5) Analyse the propped cantilever loaded as shown in figure. Draw BMD and SFD. Assume constant EI throughout the span.



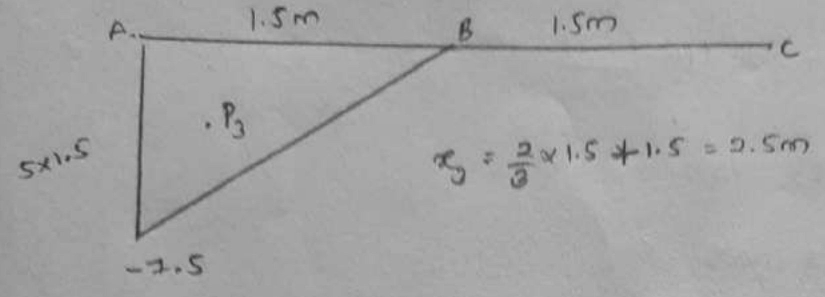
Soln The elastic load diagram due to prop reaction applied udl and concentrated load are given below as EI is constant it is taken as unity.

$x_1 = \frac{2}{3} \times 3 = 2m$

Elastic load diagram for UDL



$x_2 = \frac{3}{4} \times 3 = 2.25m.$



$x_3 = \frac{2}{3} \times 1.5 + 1.5 = 2.5m$

as the deflection at

$$P_1 x_1 + P_2 x_2 + P_3 x_3 = 0.$$

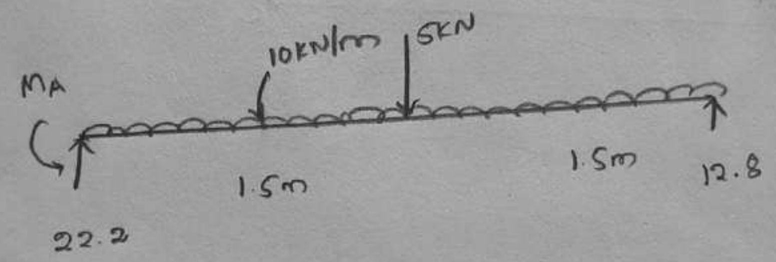
$$= \frac{1}{2}(3)(3R_c)2 - \left(\frac{1}{3}\right)3 \times 45 (2.25) - \left(\frac{1}{2} \times 1.5 \times 7.5\right) \times 2.5 = 0$$

$$9R_c - 101.25 - 14.0625 = 0$$

$$R_c = 12.8 \, kN$$

$$R_A = 10(3) + 5 - 12.8 = 22.2 \, kN$$

$$R_A = 22.2 \, kN.$$



Taking moment about A:

$$-M_A + 5(1.5) + 10 \times \frac{3^2}{2} - 12.8 \times 3 = 0$$

$$M_A = 14.1 \, kN/m.$$

The point of contraflexure is located as.

$$-14.1 + 22.2x - 10 \cdot \frac{x^2}{2} = 0.$$

$$5x^2 - 22.2x + 14.1 = 0$$

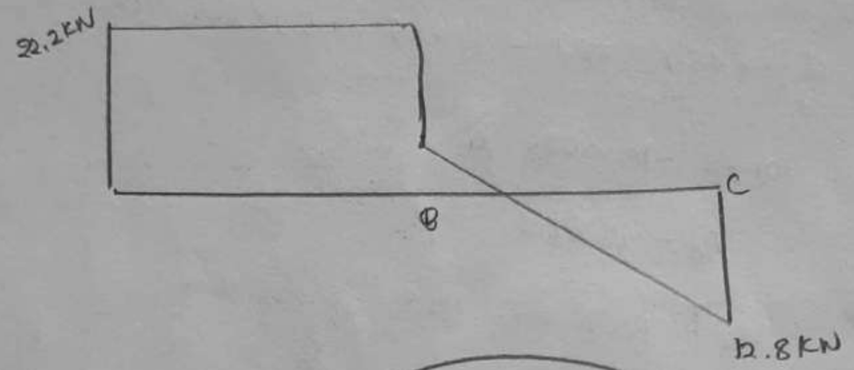
$$x = 0.77m.$$

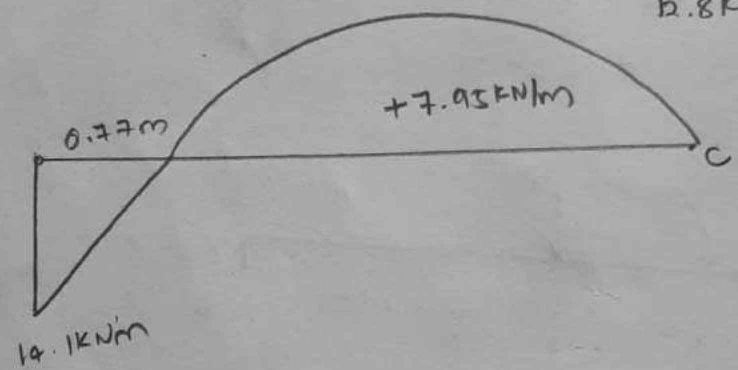
$$-14.1 + 22.2(2.2) - (10)\frac{(2.2)^2}{2}$$

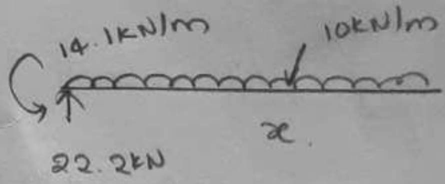
$$22.2 - 5 \times (1.5) - 10x = 0$$

$$14.7 - 10x = 0$$

$$x = 1.47$$







$$-14.1 + 22.2x - \frac{10x^2}{2} = 0$$

$$5x^2 - 22.2x + 14.1 = 0$$

$$x = 0.77m.$$

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Fixed Beams :-

The fixed beam can be considered as a simply supported beam with end moments.

→ The restraint moments at the ends reduce the slope of the beam and therefore they are of opposite sign to the moments due to applied load.

→ The free body diagram of entire beam contains four unknowns namely R_A, R_B and M_A, M_B.

→ The fixed beam is statically inderminate to second degree as only two equilibrium equation can be used.

The analysis is done as follows :-

(1) Remove the statically indeterminate forces and moments at one end make the beams as a cantilever beam. which is statically determinate.

(2) Draw the B.M.D due to the external loading.

(3). Divide each ordinate of the bending moments diagram by EI and draw elastic load (M/EI) diagram.

(4) Draw the (M/EI) diagram for the unknown forces and the moment at the end support

(5) The elastic curve of the fixed beam infers that the slope at each end is horizontal due to the built in supports.

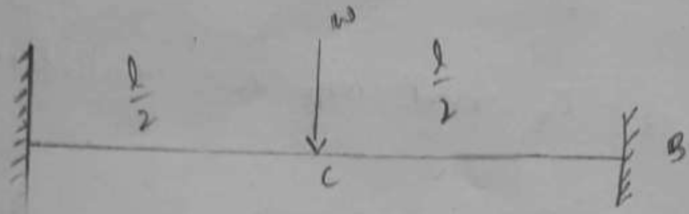
Hence the change of slope b/w the end supports is zero.

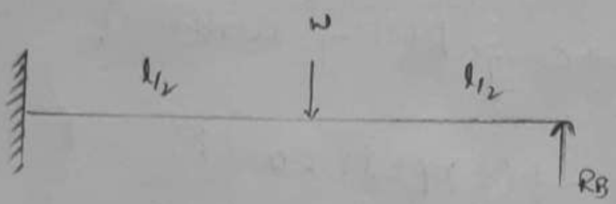
(6) ed second compatibility condition is obtained by nothing that tangential deviation of the support with respect to other support is zero.

(7) using the above two compatibility conditions the unknown reaction and moment are calculated.

(8) Draw the free body diagram and determine the other reaction and moments sketch the shear force diagram. B.M.D and the elastic curve.

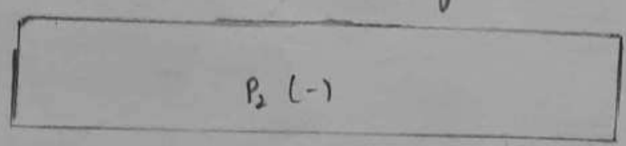
① Analyse the pand calculate BM values and SF values and draw BMD and SFD has the fixed beams as shown below





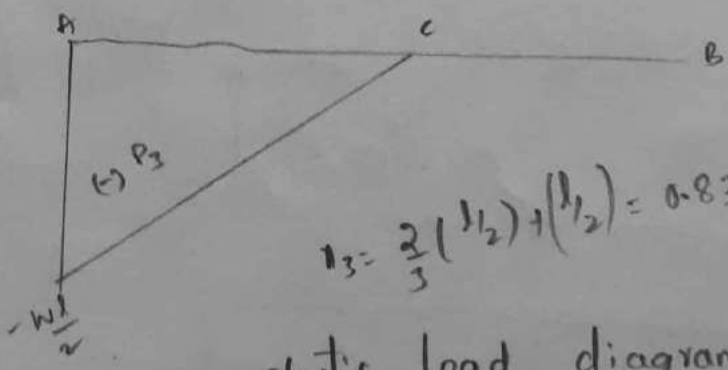
$x_1 = \frac{2}{3}l$

Elastic Load diagram dueto R_B



$x_2 = l/2$

elastic Load diagram due to M_B



$x_3 = \frac{2}{3}(l/2) + (l/2) = 0.833l$

elastic load diagram due to application of point load "w".

Soln.

The elastic load diagrams were drawn due to R_B, M_B and due to applied load are given in the above diagram.

The values of R_B and M_B are calculated the compatibility equation.

$\Sigma p_i = 0$ (first compatibility condition)

$$= \left(\frac{l}{2}\right)(l)(R_B l) - M_B l - \frac{1}{2}\left(\frac{l}{2}\right)\left(\frac{wl}{2}\right) = 0$$

$$R_B \frac{l^2}{2} - M_B l = \frac{wl^2}{8} \rightarrow (1)$$

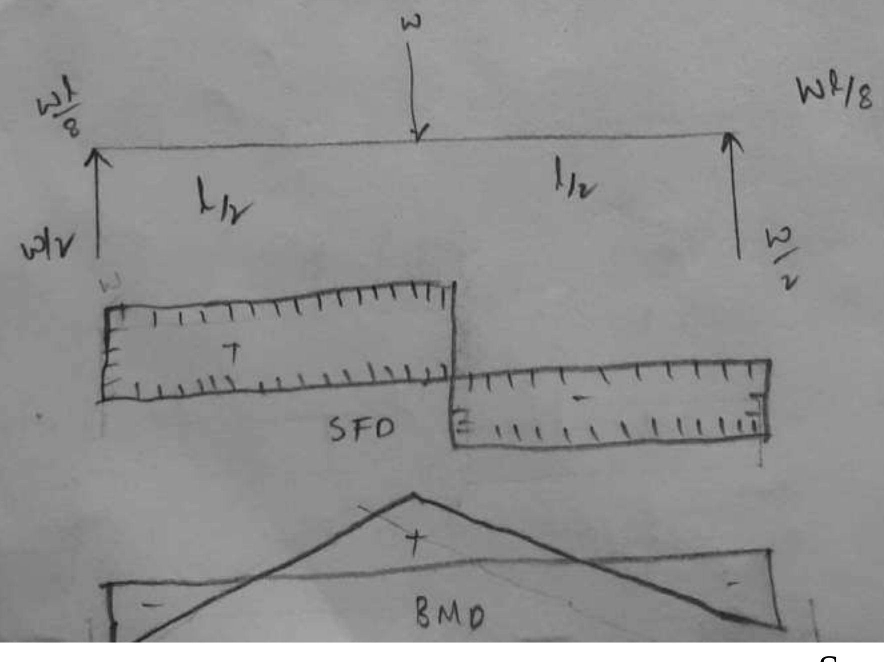
$\Sigma p_i x_i = 0$ (second compatibility condition)

$$R_B \frac{l^2}{2} \times \left(\frac{2l}{3}\right) - M_B l \left(\frac{l}{2}\right) = \frac{wl^2}{8}\left(\frac{5l}{8}\right)(0.83)$$

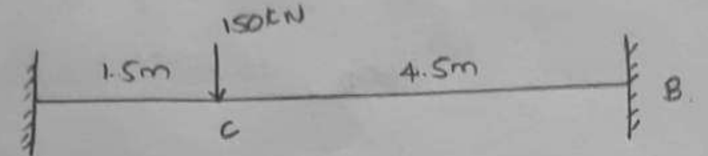
Solving (1) & (2)

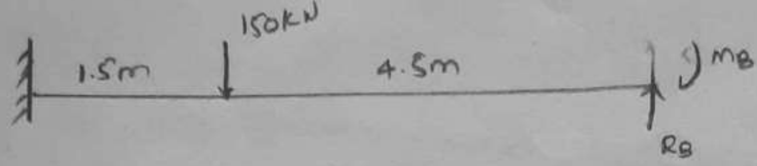
$$R_B = w/2$$

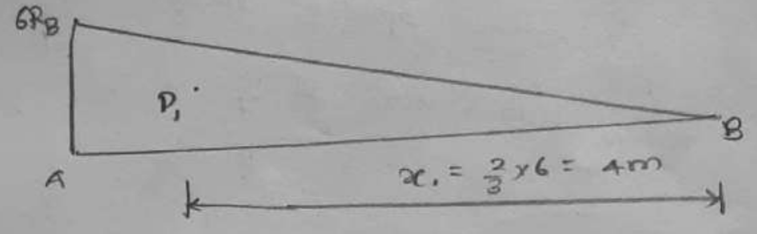
$$M_B = \frac{wl}{8}.$$



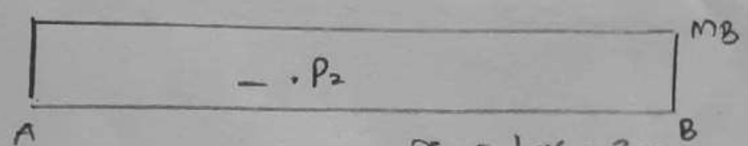
(2) A fixed beam of 6m span carries a concentrated load of 150kN at a distance of 1.5m from the left support calculate the bending moment at mid span.



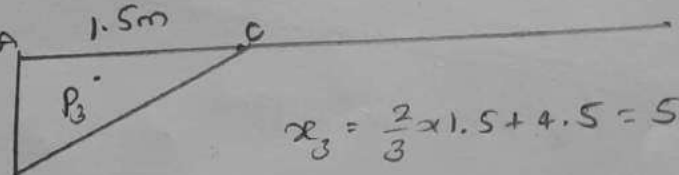




$x_1 = \frac{2}{3} \times 6 = 4m$



$x_2 = \frac{1}{2} \times 6 = 3m$



$x_3 = \frac{2}{3} \times 1.5 + 4.5 = 5.5m$

$\Sigma p_i = 0$ (first Compatibility).

$\left(\frac{1}{2} \times 6 \times 6R_B\right) - 6M_B - \frac{1}{2} \times 1.5 \times 225 = 0.$

$18R_B - 6M_B = 168.75 \rightarrow (1)$

$\Sigma p_i x_i = 0$ (second compatibility condition)

$18R_B(4) - 6(M_B)(3) = 168.75(5.5)$

$72R_B - 18M_B = 928.13 \rightarrow (2)$

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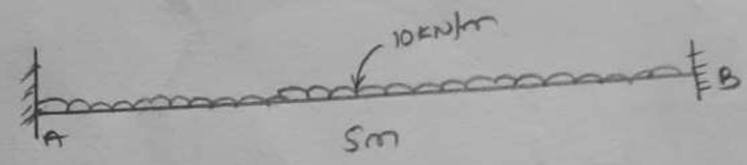
Solving $R_B = 23.44\,kN$

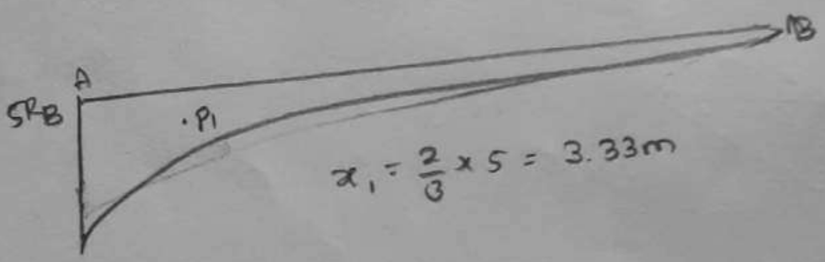
$M_B = 42.2\,kNm$.

Bending moment at midspan $= 23.44(3) - 42.2$

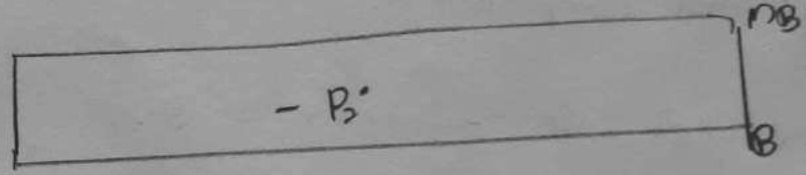
$M_D = 28.12\,kN$.

(3) For a rigidly fixed beam AB of spam 5m carrying a udl of 10kN/m over a entire span, locate the point of contraflexure and draw B.M.D and SFD.

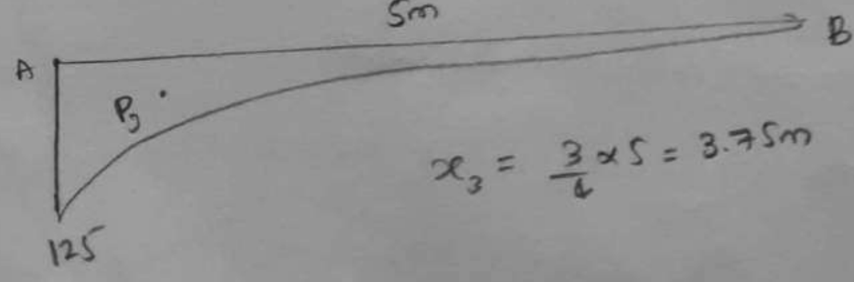




$x_1 = \frac{2}{3} \times 5 = 3.33m$



$x_2 = 2.5m$



$x_3 = \frac{3}{4} \times 5 = 3.75m$

$\Sigma P_i = 0$ (first compatibility equation)

$$\left(\frac{1}{2}\right)(5)(5R_B) - 5M_B - \frac{1}{3} \times 5 \times 125 = 0$$

$$12.5R_B - 5M_B = 208.33 \rightarrow (1)$$

$\Sigma P_i x_i = 0$ (second compatibility equation)

$$12.5R_B(3.33) - 5M_B(2.5) - 208.33 \times 3.75 = 0$$

$$41.625R_B - 12.5 = 781.25 \rightarrow (2)$$

By solving (1) & (2)

$R_B = 25\ kN$ and $M_B = 21\ kN/m$.

$\Sigma V = 0$ $\quad R_A + 25 = 5 \times 10$

$$R_A = 25\ kN$$

$\Sigma M = 0$ $\quad -M_A + \frac{10 \times 5^2}{2} + 21 - 25 \times 5 = 0$

$$M_A = 20.83\ kN/m.$$

Max +ve B.M at

$$M_C = -20.83 + 25 \times 2.5 - 10 \times \frac{2.5^2}{2}$$

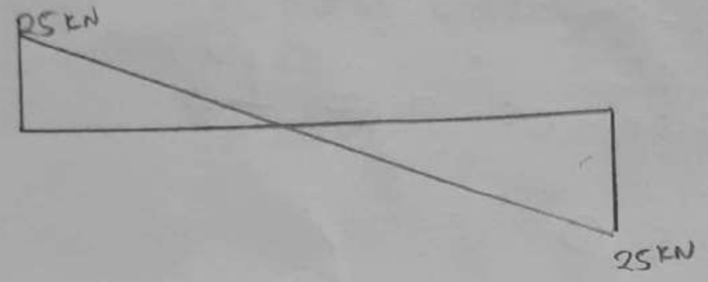
$$M_C = 10.42\ kN/m.$$

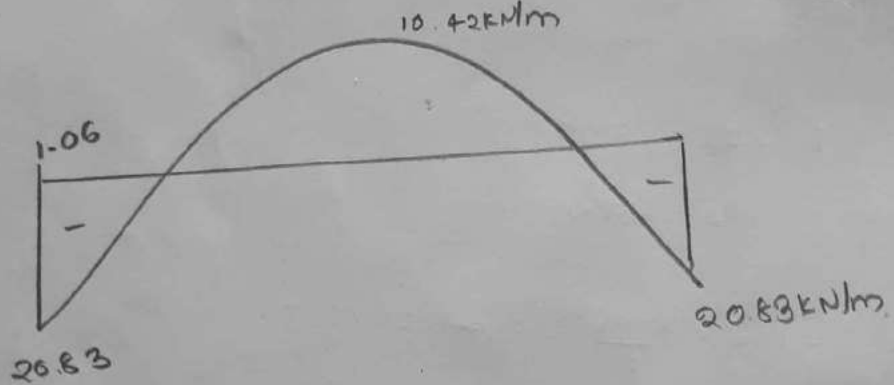
Contraflexure :-

$$25x - 20.83 - 10\frac{x^2}{2} = 0$$

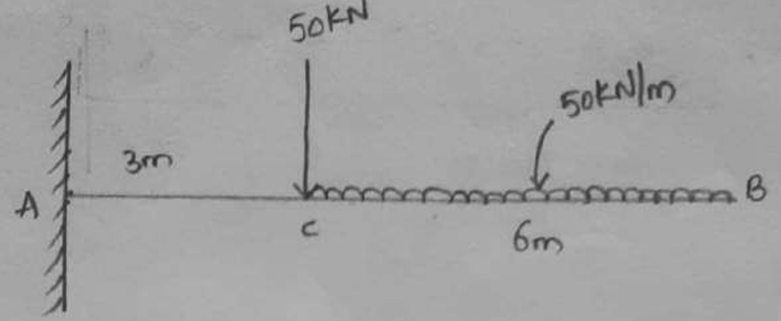
$$5x^2 - 25x + 20.83 = 0$$

$$x = 3.94m, \quad x = 1.06m$$





(4) Determine the moments at the supports of a fixed beam ACB of span. AC = 3m, CB = 6m fixed at A and B it carries a concentrated load of 50kN at C and UDL of 50kN/m over the entire span.



Soln

$$\Sigma p_i = \left(\frac{1}{2}\right) \times 9 \times (9R_B) - 9M_B - \frac{1}{3} \times 9 \times 2025 - \frac{1}{3} \times 3 \times 150 = 0$$

$$40.5R_B - 9M_B = 6075 + 225$$

$$40.5R_B - 9M_B = 6300 \rightarrow (1)$$

$\Sigma p_i x_i = 0$

$$40.5R_B(6) - (9M_B \times 4.5) = 6075 \times 6.75 + 225 \times 8$$

$$243R_B - 40.5M_B = 42806.425.$$

By solving ① & ②

$R_B = 237.96 kN$

$M_B = 370.8 kNm.$

$\Sigma V = 0 \quad R_A + R_B = 50 + 50 \times 9 \Rightarrow R_A = 500 - 237.96$

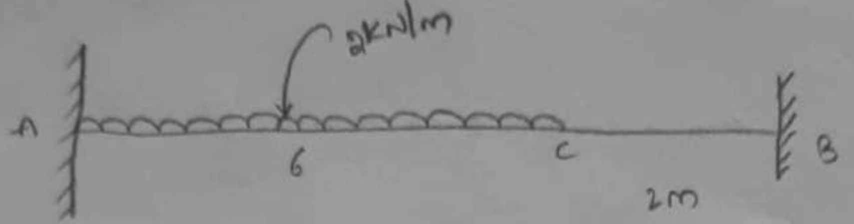
$R_A = 262 kN$

$\Sigma M = 0$

$- M_A + 50 \times 3 + 370.8 - 237.96 \times 9 + 50 \times \frac{9^2}{2} = 0$

$M_A = 404.2 kN/m.$

(5) Determine the fixed end moments for the beam shown in figure.



Soln

$\Sigma p_i = 0$

$(\frac{1}{2})(8)(8R_B) - 8M_B - \frac{1}{3} \times 6 \times 36 = 0$

$32R_B - 8M_B = 72 \rightarrow (1)$

$\Sigma p_i x_i = 0$

$32R_B(5.33) - 8M_B(4) = 72 \times 6.5$

$170.56R_B - 32M_B = 468 \rightarrow (2)$

By solving (1) & (2).

$R_B = 4.23 kN$

$M_B = 7.29 kNm.$

$\Sigma v = R_A + R_B \Rightarrow 6 \times 2$

$R_A + 4.23 = 12$

$R_A = 7.78 kN.$

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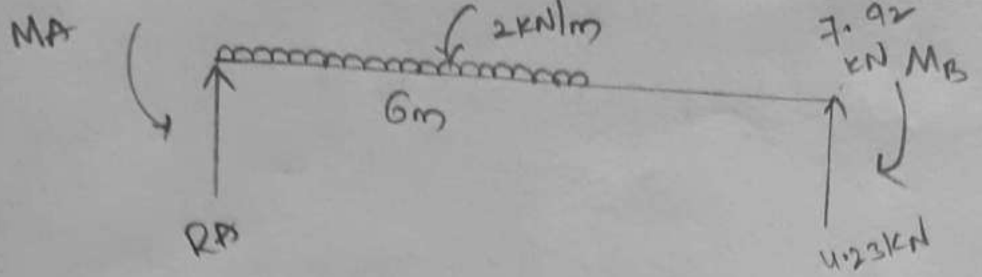


$\Sigma M = 0$

$$-M_A + 7.92 - 8 \times 4.23 + 2 \times \frac{6^2}{2} = 0$$

$$M_A = 10.1 \text{ kN/m.}$$

Maximum positive bending moment.

$$M_D = -10.1 + 7.78(x) - 2 \times \frac{x^2}{2}$$

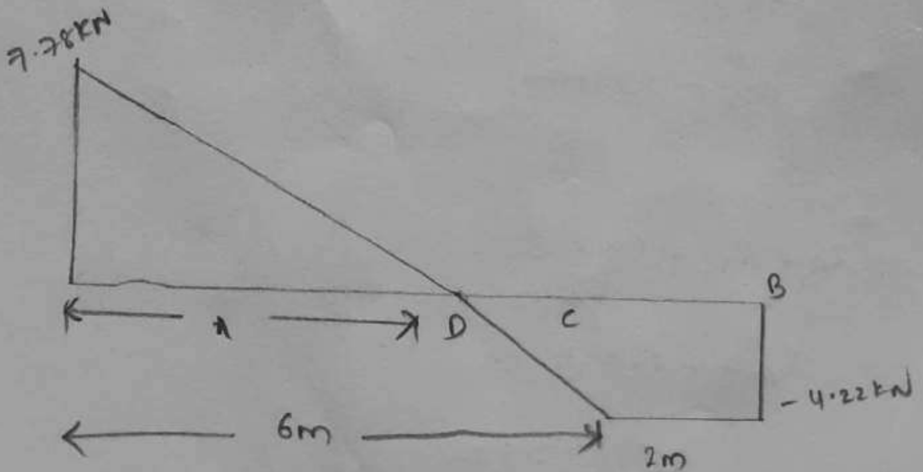
$$M_D = -10.1 + 7.78(3.89) - 2 \times \frac{3.89^2}{2}$$

$$M_D = 5$$

$$\frac{x}{6-x} = \frac{7.78}{4.22}$$

$$4.22x = 46668 - 7.78x$$

$$x = 3.89\text{m.}$$



S.F.D

To locate x_1

$$-10.1 + 7.78(x_1) - \frac{2x_1^2}{2} = 0$$

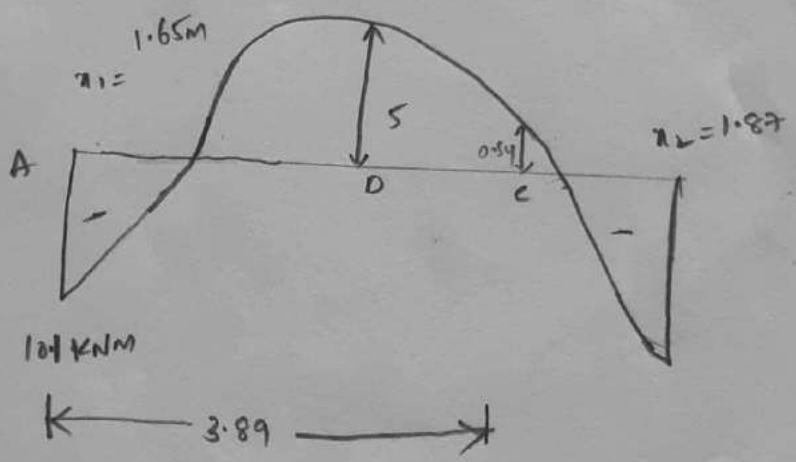
$$x_1^2 - 7.78x_1 + 10.1 = 0$$

$$x_1 = 1.65m$$

To locate $x_2 = 4.23x_2 - 7.92 = 0.$

$$x_2 = 1.87m.$$

BMD :-



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(UNIT - 2)

Analysis of perfect Frames:

Plane frames:

→ The plane trusses are here by termed as plane frame

→ A statically determinate frame can be completely analysed by using statics.

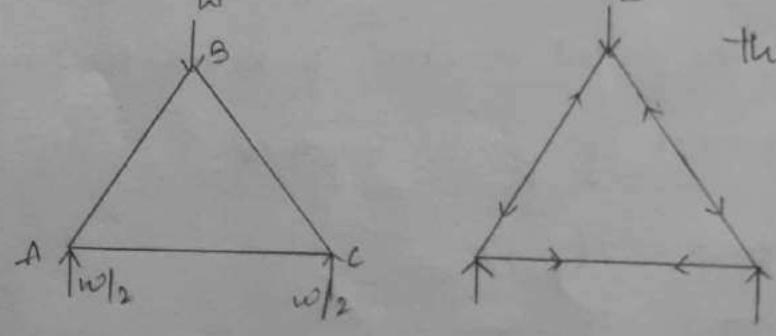
→ The no.of unknown forces is the same as the no.of equation of equilibrium.

→ The main methods in analysing statically determined pin jointed plane frames.

i) Graphical Solution
ii) Method of resolution at joints
iii) Method of section

} 3 are used in plane frames (trusses)

iv) Tension coefficient method → For analysis of the space frame.



→ A truss is an assemblage of three (or) more members which are hinged (or) pinned.

→ A load applied on the truss is transmitted to all joints so that the members are in pure compression (or) pure tension.

Types of frames:

Basically frames are divided into 3 types:

i) Perfect frame

ii) Imperfect frame.

iii) Redundant frame.

Perfect Frame: The simplest perfect frame is in the form of a triangle.

In essence the first three joints (or) pins require three bars to connect them while the remaining $(j-3)$ joints require $2(j-3)$ extra bars.

If "m" is the total number of bars (or) members required to connect "j" joints together

$$m-3 = 2(j-3)$$

$$m-3 = 2j-6$$

$$\boxed{m = 2j-3}$$

i.e.,

number of members to form a perfect frame

= Twice the number of joints − 3

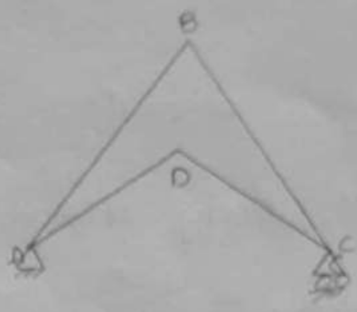
Imperfect frame:

An imperfect frame is one where the number of members is <u>less than</u> $(2j-3)$ then frame is called as imperfect frame.

$m < 2j-3$

$4 < (2\times4)-3$

$4 < 5$



Redundant frame:

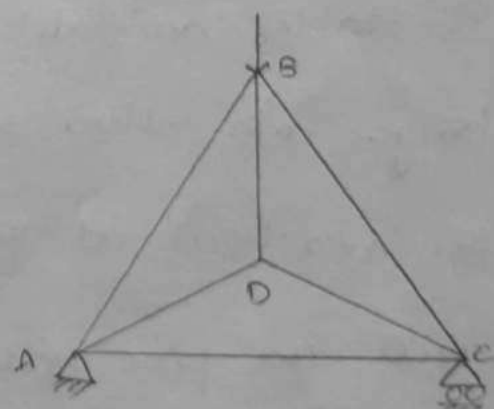
A redundant frame is one where the number of members are <u>more than</u> $(2j-3)$ then the frame is called as redundant frame.

$m > (2j-3)$

$6 > (2\times4)-3$

$6 > (8)-3$

$6 > 5$



i.e., Redundant frame is having more members necessary to produce stability.

Method of Joints:

→ In this method we are determined the member forces by using equilibrium conditions at particular joints.

→ In this resolution of forces at the joint as well as free body diagram at the joint is considered

Procedure:

1) Check the stability and assess its determinancy of the truss.

2) If the truss is of cantilever type, the reactions need not be computed in general if the truss is stable and determinate where one support is hinge and other support is on rollers. compute the reactions at the supports.

3) Draw the free body diagram at each joint and analyse the member forces at a joint where only two members meet.

4) Then consider the adjacent joint where only two unknown forces to be determined. This process is

repeated till the analyses of all joints are completed.

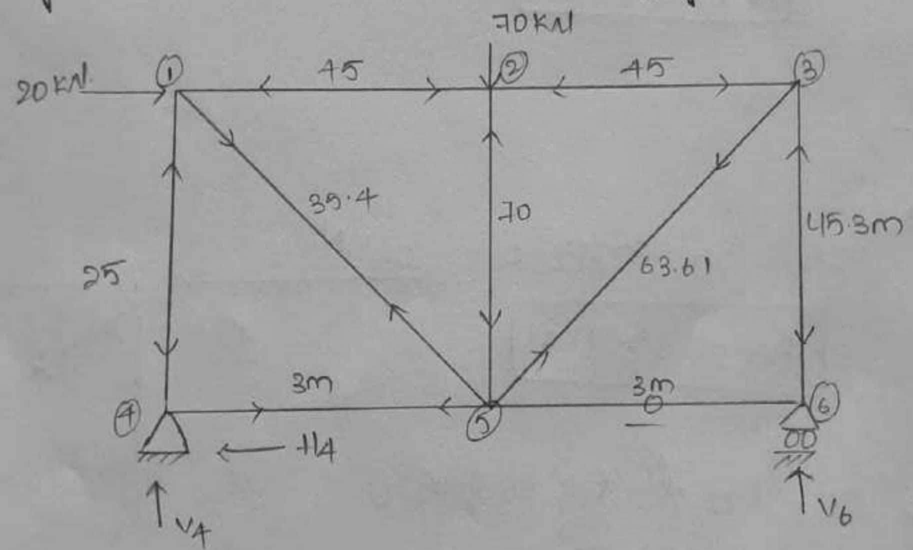
5) The results are tabulated along with magnitude of member forces and the nature of forces.

6) The forces are tensile if they are pulling (acting away) the joints.

7) The forces are compressive in nature if they are pushing (acting towards) the joints.

Example problems

① Analyse the truss as shown in fig by method of joints.



$\Sigma H = 0$ $\quad 20 - H4 = 0$

$\boxed{H4 = 20KN}$

(i) Determinacy:

NM + NR = 2× Joints

9 + 3 = 2×6

12 = 12

$\Sigma V = 0$ $\quad V_4 + V_6 = 70KN$

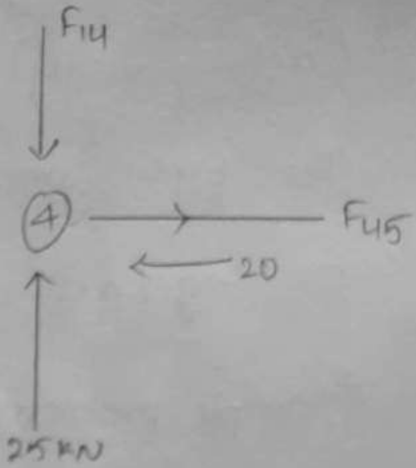
$\Sigma M_4 = 0$ $\quad 20 \times 3 + 70 \times 3 - 6V_6 = 0$

$$\boxed{V_6 = 45 \text{ kN}}$$

$V_4 + 45 = 70 \text{kN}$

$V_4 = 70 - 45 \implies \boxed{V_4 = 25 \text{kN}}$

Joint 4:



$\Sigma H = 0$

$F_{45} - 20 = 0$

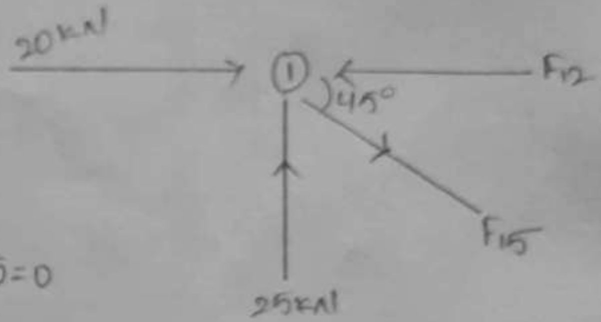
$$\boxed{F_{45} = 20 \text{ kN}}$$

$\Sigma V = 0$

$-F_{14} + 25 = 0$

$$\boxed{F_{14} = 25 \text{ kN}}$$

Joint 1:



$\Sigma V = 0$

$25 - F_{15} \sin 45 = 0$

$$\boxed{F_{15} = 35.4 \text{kN}}$$

$\Sigma H = 0$

$20 - F_{12} + F_{15} \cos 45 = 0$

$20 - F_{12} + 35.4 \cos 45 = 0$

$$\boxed{F_{12} = 45.03 \text{ kN}}$$

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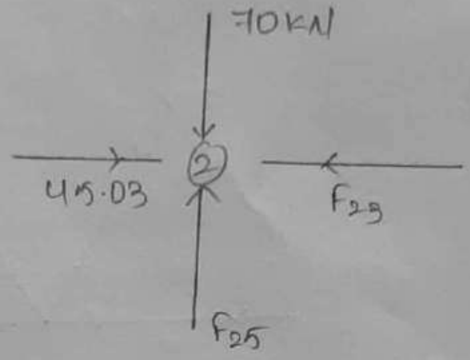
Joint-2 :



$\Sigma H = 0$

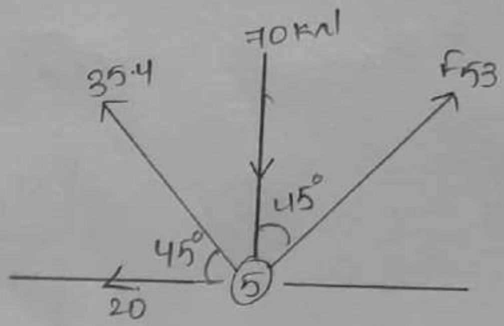
$45.03 - F_{23} = 0$

$\boxed{F_{23} = 45.03}$

$\Sigma V = 0 \quad F_{25} - 70 = 0$

$\boxed{F_{25} = 70.0\,KN}$

Joint-5 :

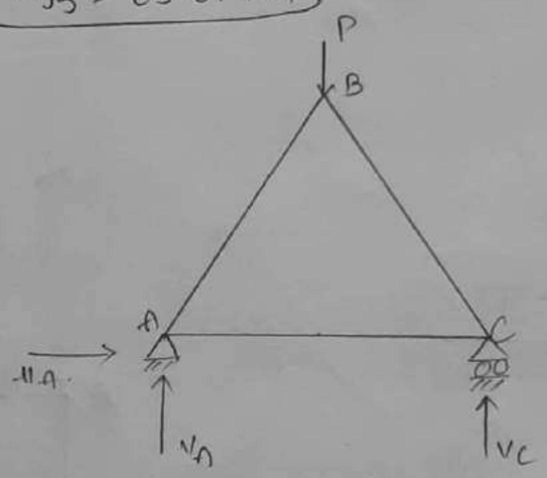


$\Sigma V = 0$

$F_{53}\cos 45 + 35.4\sin 25 - 70 = 0$

$\boxed{F_{53} = 63.61\,KN}$

method of joints



i) Check the determinacy → Statically determintacy.

ii) Isolate joins draw FBD

(2 unknown forces)

iii) Apply equation of equation.

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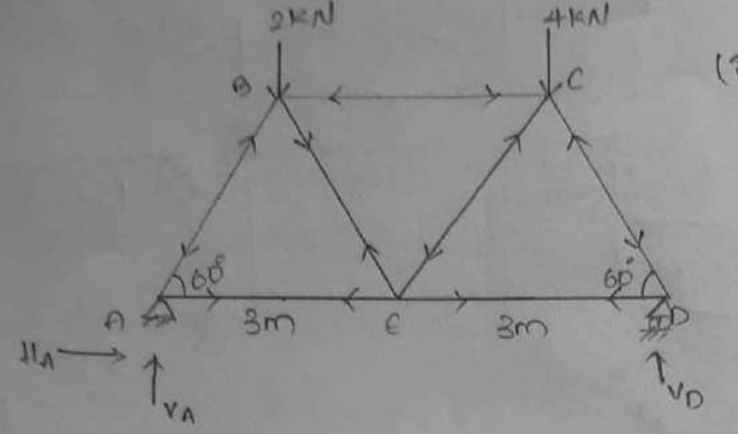
Eg:



(i) Determinacy

$NM + NR = 2 \times J$

$7 + 3 = 2 \times 5$

$10 = 10$

$\Sigma V = 0$ $\quad V_A + V_D = 2 + 4 = 6\,kN$

$\Sigma M_A = 0$ $\quad 2(1.5) + 4(4.5) - 6\,V_D = 0$

$\boxed{V_D = 3.5\,kN}$

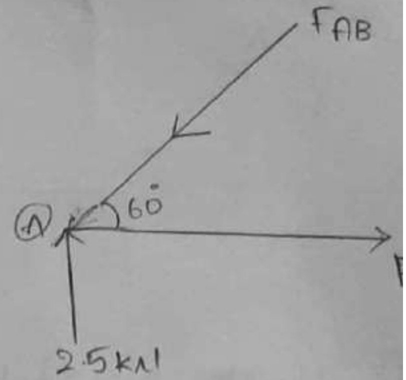
$\boxed{V_A = 2.5\,kN}$

Joint A :

$\Sigma V = 0$

$F_{AB} \sin 60 = 2.5$

$\boxed{F_{AB} = 2.89\,kN}$



$\Sigma H = 0$

$F_{AE} = F_{AB} \cos 60°$

$F_{AE} = 2.89 \cos 60°$

$\boxed{F_{AE} = 1.45\,kN}$

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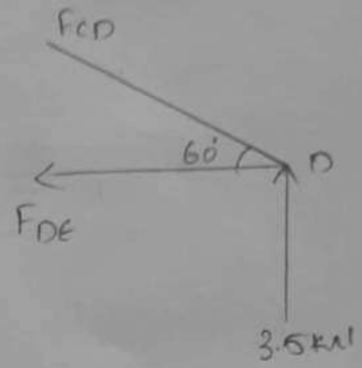
Joint-D:

$\Sigma V = 0$

$F_{CD} \sin 60° = 3.5$

$\boxed{F_{CD} = 4.04\ kN}$



$\Sigma H = 0$

$F_{CD} \cos 60° - F_{DE} = 0$

$F_{DE} = 4.04 \cos 60°$

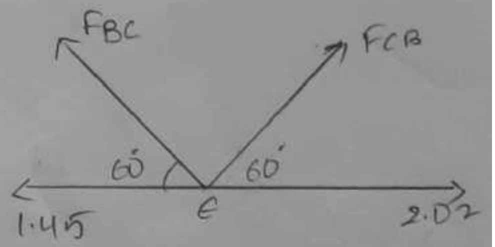
$\boxed{F_{DE} = 2.02\ kN}$

Joint- E:

$\Sigma V = 0$

$F_{EC} \sin 60 + F_{EB} \sin 60 = 0$

$F_{EC} = -F_{EB}$



$\Sigma H = 0$

$2.02 + F_{EC} \cos 60° - 1.45\ kN - F_{EB} \cos 60° = 0$

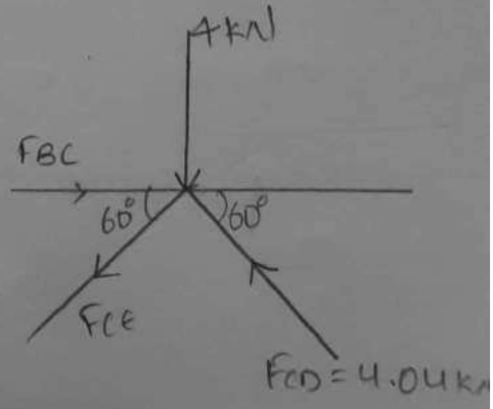
$\therefore F_{EC} = -F_{EB}$

$2.02 - F_{EB} \cos 60 - 1.45 - F_{EB} \cos 60 = 0$

$0.57 = 2 F_{EB} \cos 60°$

$$\boxed{\begin{array}{c} F_{EB} = 0.57 \\ \hline F_{EC} = -0.57 \end{array}}$$

Joint C:



$\Sigma H = 0$

$F_{BC} - F_{CE} \cos 60° - F_{CD} \cos 60° = 0$

$F_{BC} - F_{CE} \cos 60° = 4.04 \cos 60°$

$\Sigma V = 0$

$-4 - F_{CE}\sin 60° + F_{CD}\sin 60° = 0$

$-4 - F_{CE}\sin 60° + 4.04\sin 60° = 0$

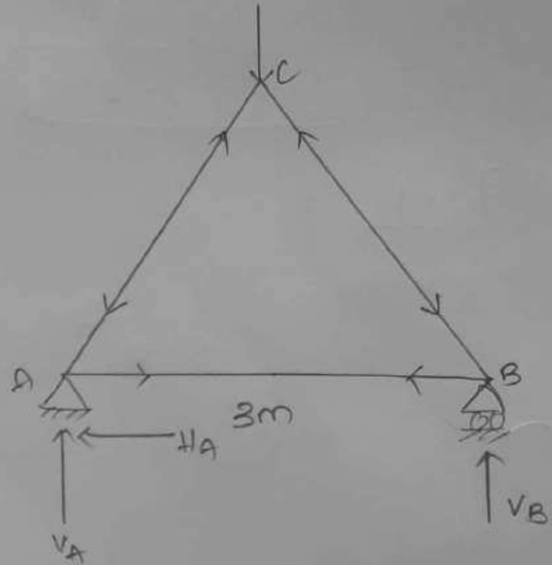
$\boxed{F_{CE} = -0.57\ kN}$

Substituting F_{CE} in the above equation

$\boxed{F_{BC} = 1.73\ kN}$

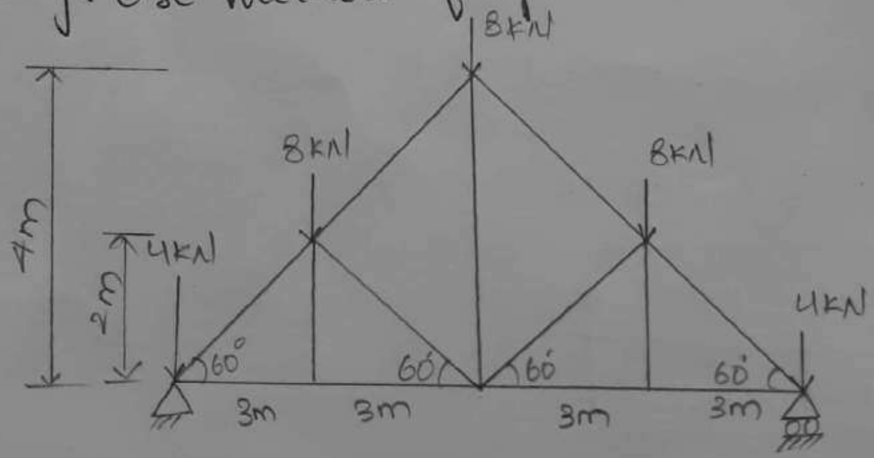
$V_A + V_B = 10\ kN$

$\Sigma M_A = 10(1.5) - 3V_B = 0$



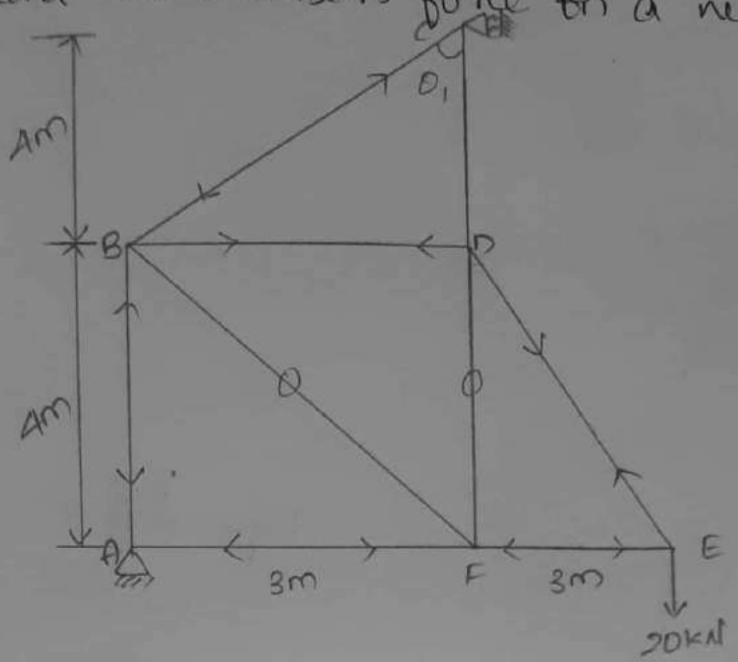
Assignment

Ex: Determine the forces in members and tabulate neatly. Use method of joints.



Ex: Determine the forces in members AC, BC, BD by using method of section.

Eg: Analyse the truss shown in fig., by method of joints Indicate the members force on a neat sketch of truss.



$\Sigma H = 0$

$H_A - H_C = 0$

$H_A = H_C$

$\boxed{\therefore H_A = 15 \text{ KN}}$

$\Sigma V = 0$

$V_A - 20 = 0$

$\boxed{V_A = 20 \text{ KN}}$

$\Sigma M_A = 0;$

$20(6) - 8H_C = 0$

$120 - 8H_C = 0$

$8H_C = 120$

$H_C = \frac{\cancel{120}^{15}}{\cancel{8}}$

$\boxed{H_C = 15 \text{ KN}}$

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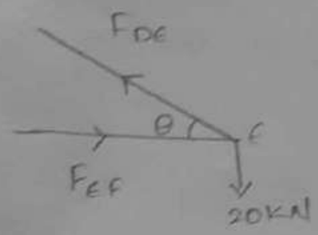
Joint - E:



$\Sigma V = 0$

$F_{DE} \sin\theta = 20$

$F_{DE} = \frac{20}{\sin\theta} = \frac{20}{0.8} = 25\,kN$

$F_{DE} = 25\,kN$

$\Sigma H = 0$

$F_{EF} - F_{DE}\cos\theta = 0$

$F_{EF} = F_{DE}\cos\theta$

$F_{EF} = 25 \times 3/5$

$F_{EF} = 15\,kN$

$hyp = \sqrt{4^2+3^2}$

$= \sqrt{16+9}$

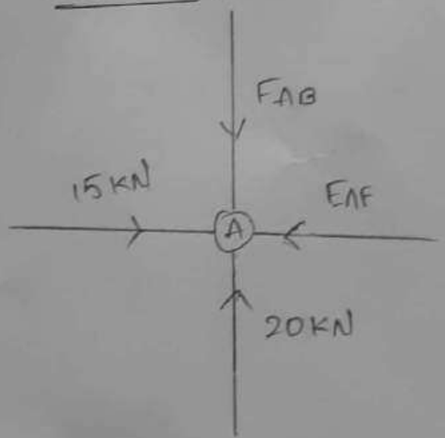
$= 5$

$\tan\theta = 4/3 = 1.33$

$\sin\theta = \frac{opp}{hyp} = \frac{4}{5} = 0.8$

$\cos\theta = \frac{adj}{hyp} = \frac{3}{5} = 0.6$

Joint - A:



$\Sigma H = 0$

$F_{AF} - 15 = 0$

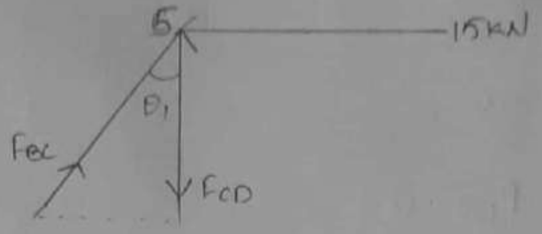
$F_{AF} = 15\,kN$

$\Sigma V = 0$

$F_{AB} - 20 = 0$

$F_{AB} = 20\,kN$

Joint-C:



$\Sigma H = 0$

$F_{BC}\sin\theta_1 = 15$

$F_{BC} = \dfrac{15}{0.6}$

$F_{BC} = \underline{\underline{25KN}}$

$\Sigma V = 0$

$-F_{CD} + F_{BC}\cos\theta_1 = 0$

$F_{CD} = \cancel{25} F_{BC} \times 0.8$

$F_{CD} = 25 \times 0.8$

$F_{CD} = \underline{\underline{20\ KN}}$

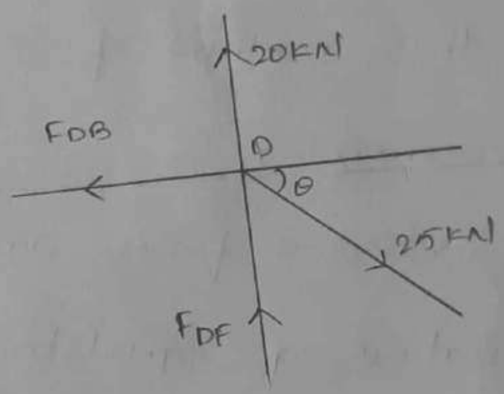
Joint-D:

$\Sigma H = 0$

$25\cos\theta - F_{DB} = 0$

F_{DC}

$F_{DB} = 25 \times 0.6 = \underline{\underline{15KN}}$



$\Sigma V = 0$

$20 + F_{DF} - 25\sin\theta = 0$

$20 + F_{DF} - 25 \times 0.8 = 0$

$\cancel{20} + F_{DF} = \cancel{20}$

$F_{DF} = \underline{\underline{0}}$

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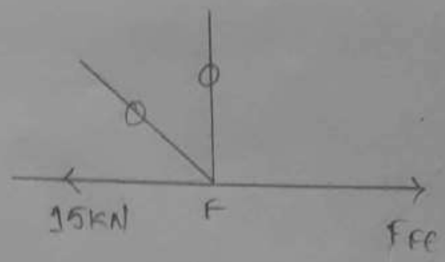
Joint - F:

$\Sigma V = 0$

$F_{DF} = 0$.



② Method of Section:

→ A statically determinate frame can be completely analysed by static method.

→ No. of unknows is the same as the number of equations obtained from static equilibrium conditions

→ In this method of sections we isolate a portion of a member.

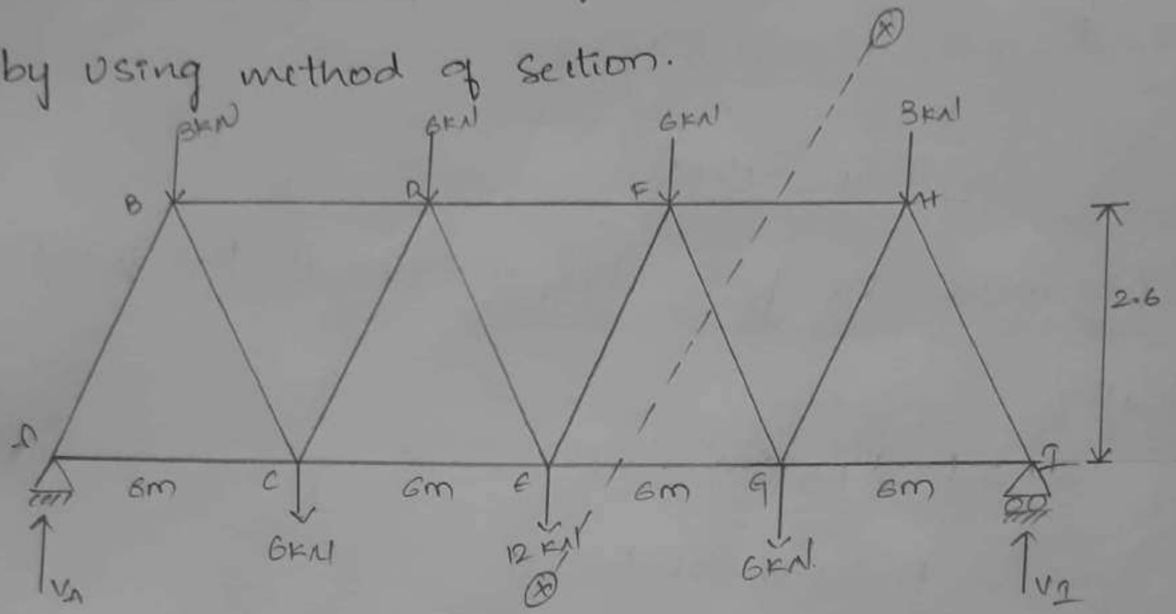
→ The unknown forces are evaluated by using equating of equilibrium.

→ The three equations of equilibrium are used to evaluate the unknowns.

→ Thus the section should cut only three members which include the one whose force members intersect

Example Problems:

Determine the nature and magnitude of forces in members FH, FG, EG of the truss as shown in figure by using method of Section.



Sol:

$\Sigma V = 0$

$$V_A + V_I = 42$$

$\Sigma M_A = 0$

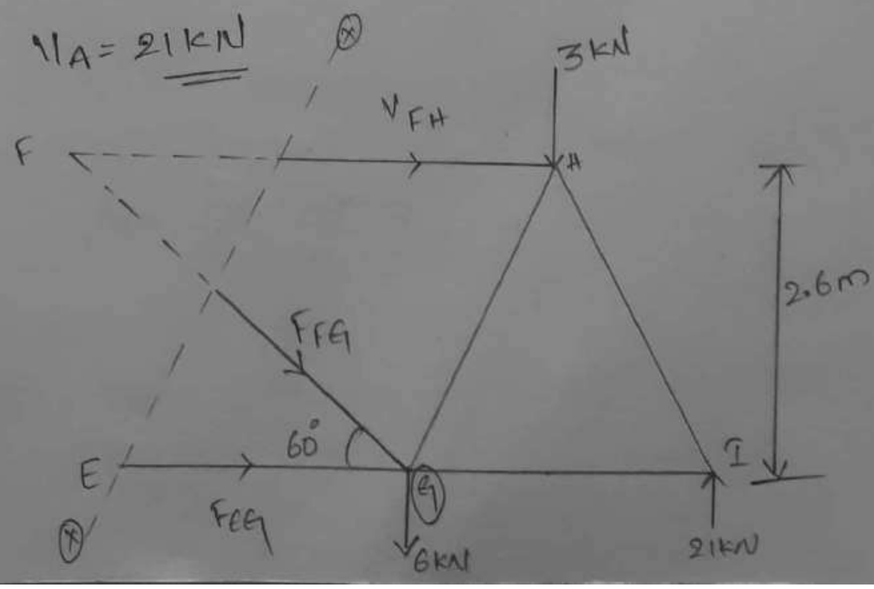
$$3(3) + 6(9) + 6(15) + 3(21) + 6(6) + 12(12)$$

$$+ 6(18) - 24 V_I = 0$$

$$V_I = \underline{\underline{21 KN}}$$

$$V_A + V_I = 42$$

$$V_A = \underline{\underline{21 KN}}$$



To determine the force. F_{FH} take moments about (G)

where other two members of the cut section meet.

$$F_{FH} \times 2.6 + 3(3) - 21 \times 6 = 0$$

$$F_{FH} = 45 kN$$

To determine the force F_{FG} resolve all the forces

vertically.

$$21 - 3 - 6 - F_{FG} \sin 60^\circ = 0$$

$$F_{FG} = 13.85 kN$$

To determine the force F_{EG} take moment about where

other two members of the section meet.

$$6 \times 3 + 3 \times 6 - 9 \times 21 - 2.6 F_{EG} = 0$$

$$F_{EG} = -58.85 kN$$

-ve sign indicates that the nature of F_{HI} is

tensile.

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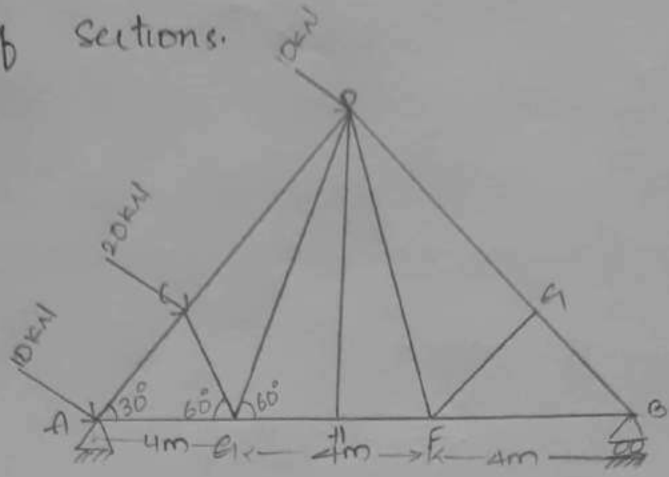
Ex: Find the forces in the members CD, EF, FG use method of sections.



Sol: The length of panel AC, CD are found using

$$\cos 30 = \frac{6}{AD}$$

$$AD = \frac{6}{\cos(30)}$$

$$AD = \underline{6.93m}$$

$$AC = CD = \frac{AD}{2} = 3.46m.$$

The reaction V_A, H_A and V_B are determined using the equilibrium equations

$$\Sigma H = 0$$

$$10\cos(60) + 20\cos(60) + 10\cos(60) - H_A = 0$$

$$H_A = \underline{20kN}$$

$$\Sigma V = 0$$

$$V_A + V_B = 10\sin 60 + 20\sin 60 + 10\sin 60 + 10$$

$$V_A + V_B = 44.64\underline{kN}$$

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$\Sigma M_A = 0$

$20(3.46) + 10(6.92) + 10(4) - 12 V_B = 0$

$V_B = 14.87\ \text{kN}$

The force in the member CD can be found by taking moment about E.

$10 \times 3.46 - 14.87 \times 8 - F_{CD} \times CE$

$$F_{CD} = \frac{-84.36}{CE} = \frac{-84.36}{4 \sin 30}$$

$F_{CD} = -42.18$

-ve sign indicates that F_{CD} is compressive

To determine the force F_{FE} take moment about 'D'. where other two member of the cut section CD and CE meet.

$-14.87 \times 6 + F_{FE} \times \underset{DH}{3.46} = 0$

$\tan 30 = \frac{D_H}{6}$

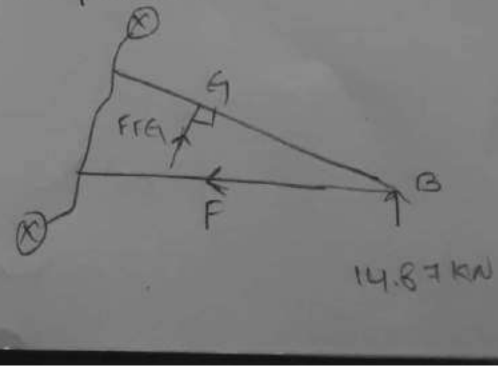
$F_{FE} = 25.8\ \text{kN}$

$D_H = 3.46\ \text{m}$

To compute the force in FG cut a section through the member DG, FG, FB.

$\Sigma M_B = 0 \quad F_{FG} \times GB = 0$

$\boxed{F_{FG} = 0}$



UNIT-III

Strain Energy Method

UNIT – III

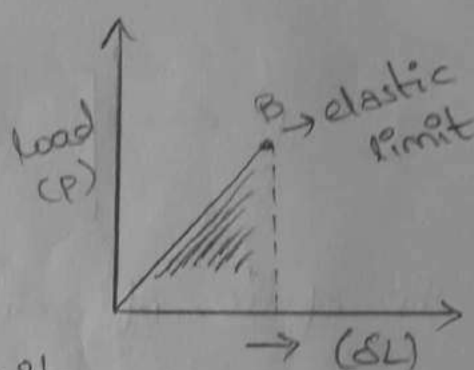
Strain energy :-

When a body is Subjected to load, their points of a application are displaced and the energy due to the displacements are imparted to the body

$$U = \tfrac{1}{2}\,\delta L \times P$$

$$= \tfrac{1}{2} \times \frac{\Delta L}{L} \times \frac{P}{A} \times (AL)$$

$$= \tfrac{1}{2}\,\sigma \times e \times V$$



Resilience → up to elastic limit

proof Resilience → Max Energy up to elastic limit

Energy principles are extensively used for determining the displacement in structures

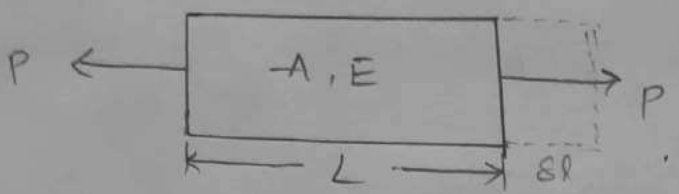
Strain energy

It is an amount of workdone by an internal resistance against the deformation.

It Means when the loads are removed, the energy stored is used to restore the body to its original position.

If the imposed strains due to the applied loads is greter than the elastic strains then a part of the strain energy is used in permarently deforming the body which cannot be recovered.

Strain energy due to axial load :-



E : The ratio of direct stress (σ) to direct strain (ε) with in proportionality young's Modulus.

$$\delta L = \frac{PL}{AE} \quad \text{—} \textcircled{1}$$

$$U = \tfrac{1}{2} \times P \times \delta L$$

$$= \tfrac{1}{2} P \times \left(\frac{PL}{AE}\right)$$

$$\boxed{U = \frac{P^2 L}{2AE}}$$

During the application of the load no kinetic energy is created.

The workdone is stored in the form of strain energy only.

(3) In essence the energy stored due to strains of an elastic type is known as strain energy.

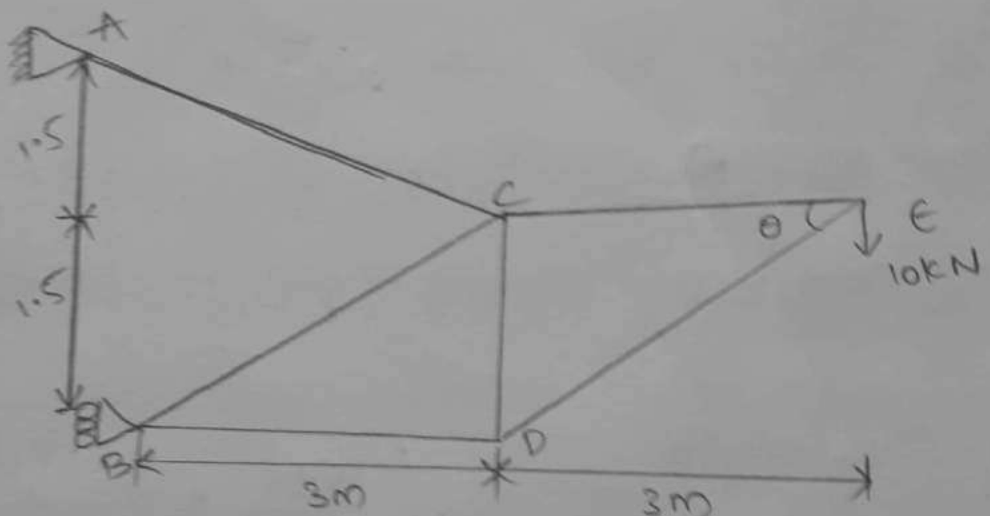
Where

L = Length of the bar

A = cross-section of the bar

E = young's Modulus of the bar

Example.



Determine the vertical deflection at E of the loaded frame show in the figure cross-sectional areas are 1000mm² E = 200 KN/mm²

Joint E

F_{DE} = 22.37 KN $\sin\theta$ = 0.447

F_{CE} = 20KN $\cos\theta$ = 0.894

Deflection at the point of Application of single load

$$\boxed{\Delta = \frac{2U}{\omega}} \quad \text{--- (2)}$$

$$\boxed{U = \frac{\omega \Delta}{2}} \quad \text{--- (1)}$$

$\Delta \rightarrow$ its displacement in the line of action of W.

$\omega \rightarrow$ is the external load

Point - D

$F_{DB} = 20\,kN$

$F_{DC} = 10\,kN$

Point - C

$F_{AC} = 22.37$

$F_{BC} = 0.$

$$\boxed{U = \frac{P^2 l}{2AE}}$$

Member	length	Areal	P/20	$P^2 l / AE$
BD	3000	1000	20	1200.0
DE	3354	1000	+ 22.37	1678.4
CE	3000	1000	+ 20.00	1200.0
CA	3354	1000	-22.37	1678.4
CB	3354	1000	00.00	-
CD	1500	1000	+10.00	1500

Σ 5906.8

In a framed structure $U = \Sigma \frac{P^2 L}{2AE}$

Hence from

$$\Delta = \frac{2U}{W}$$

$$= \frac{\cancel{2}}{WE} \Sigma \frac{P^2 L}{\cancel{2}A}$$

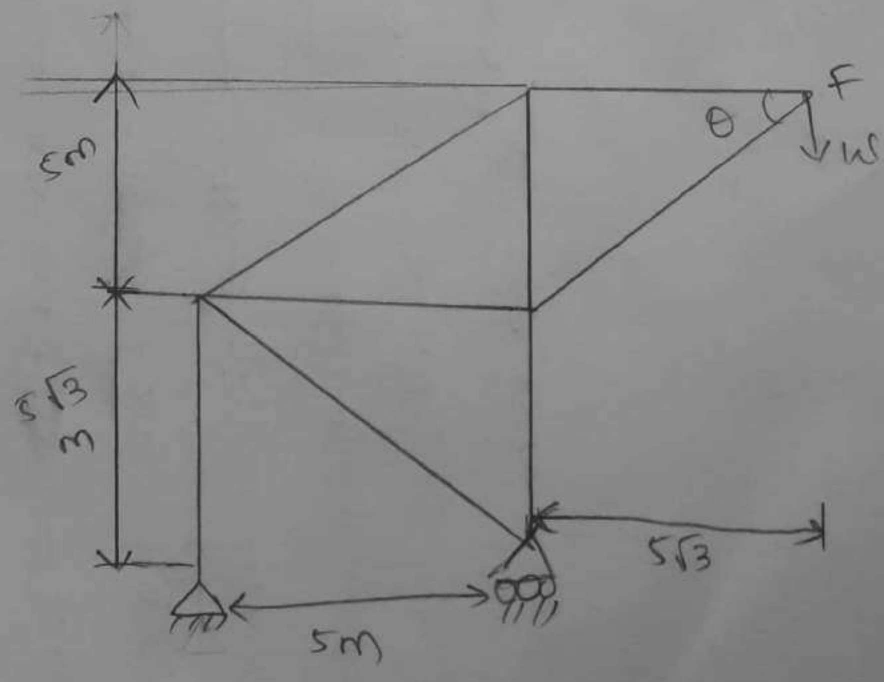
$$= \frac{\Sigma P^2 L/A}{WE}$$

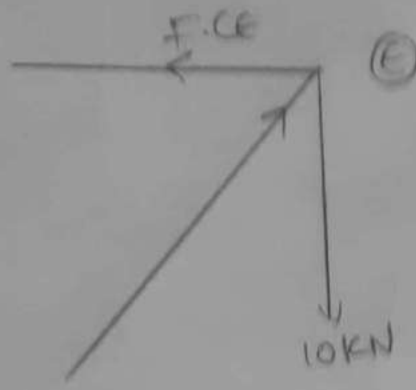
$$\boxed{\Delta = \frac{5906.8}{10 \times 200} = 2.95\,mm}$$

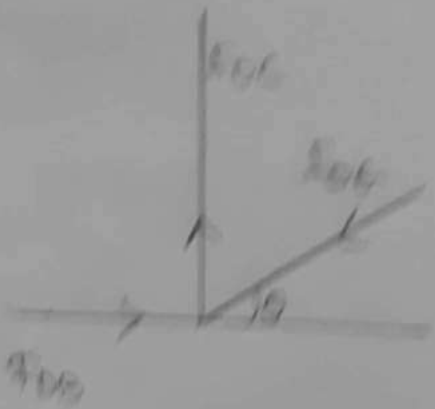
Example:

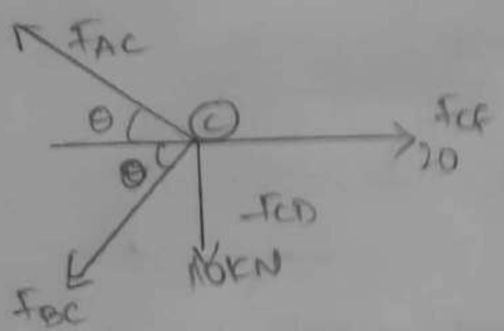
Determine the vertical deflection of the frame shown in figure. The load is such that the tension member of the frame are stressed to 150 N/mm² and the compression Members to 75 N/mm

Take $E = 200$ kN/mm²









$F_{DF} = 2w$

$F_{EF} = 1.732w$

$F_{EC} = 2.45w$

$F_{ED} = 1.732w$

$F_{DC} = 1.732w$

$F_{DB} = 2.73w$

$F_{BC} = 0$

$F_{AC} = 1.732w$

Member	length	P/A (N/mm^2)	$P(kN)$	$\frac{PL}{A} \times 10^3 (w)$
AC →	8.66 →	150 →	1.732w →	2.25
CB →	7.07 →	150 →	2.115w →	2.66
CF →	8.66 →	150 →	1.732w →	2.25
FD →	10.00 →	-75 →	-200 →	1.50
DB →	8.66 →	-75 →	-2.732w →	1.77
DC →	5.00 →	-75 →	-1.732w →	0.65
DE →	5.00 →	-75 →	-1.732w →	0.65
BC →	10.00 →	- →	0.00 →	0

$$U = 11.67 \times 10^3 w$$

$$\Delta = \frac{1}{wE} \Sigma \frac{P^2 l}{A}$$

$$= \frac{1}{wE} \times \frac{P^2 l}{A}$$

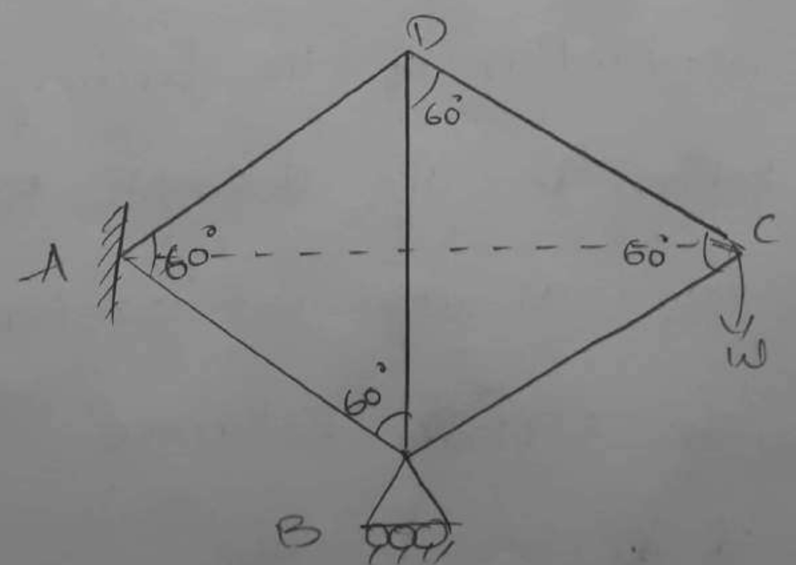
$$= \frac{11.67 \times 10^3 \times 40}{40 \times 200}$$

$$\boxed{\Delta_F = 58.33 \text{ mm}}$$

problem

A frame consists of the Member as shown in fig All the tension members are 125 mm^2 and the compression are 250 mm^2 in area find the vertical deflection of the loaded point.

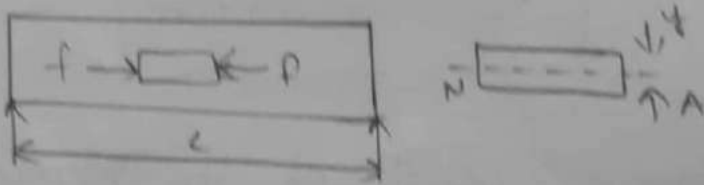
Take $E = 200$ kN/mm^2 ; $W = 25$ kN.



Strain energy due to Bending.

* In the previous Section, we have derived the expression for strain energy due to the action of direct force either tension (or) compression.

* There are other actions, by bending, shear and torsion also produces strain energy

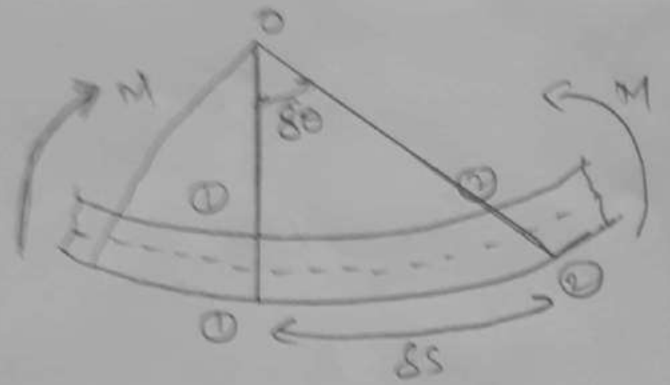


consider a beam subjected to pure bending.
Let area of the element be δa and its distance axis be 'y' An initially straight beam is Subjected to a uniform bending Moment a shown in fig.
Due to the application of the bending Moment Two Normal section in the straight Beam are deformed as (1)—(1) and (2)-(2)

Let δs be the curved distance

δθ be its subtended angle.

External work is expressed in terms of the forces



Strain energy due to external loads can be expressed as

$$\boxed{U = \tfrac{1}{2} \Sigma W \Delta}$$

$$\delta U_b = \tfrac{1}{2} M \delta\theta \quad \text{—— ①}$$

Where δU_b is the strain energy due to Bending

If R is the radius of curvature of the element due to the action of M

$$R\delta\theta = \delta s \quad \text{—— ②}$$

$$\delta\theta = \frac{\delta s}{R}$$

$$\delta U_b = \tfrac{1}{2} M \delta\theta$$

$$\delta U_b = \tfrac{1}{2} M \frac{\delta s}{R} \quad \text{—— ③}$$

$$\frac{M}{I} = \frac{f}{Y} = \frac{E}{R}$$

from theory of bending the curvature is expressed as

$$\frac{M}{I} = \frac{E}{R}$$

$$1/R = \frac{M}{EI} \quad \text{—} (6)$$

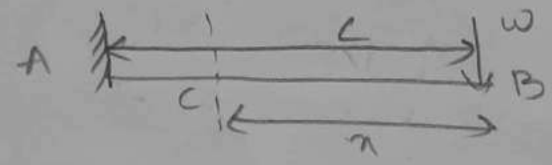
$$\delta U_b = \tfrac{1}{2} M^2 \frac{\delta s}{EI}$$

On Integration, we obtain the total Strain energy due to the bending as

$$\boxed{U_b = \int \frac{M^2 ds}{2EI}}$$

case - I

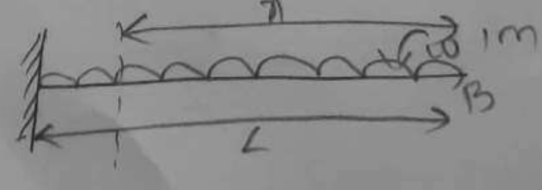
cantilever beam of spam 'L' sub, to point load



$$\boxed{U = \frac{W^2 L^3}{6EI}}$$

case - 2.

Cantilever beam of spam 'L' subjected to uniformly distributed load



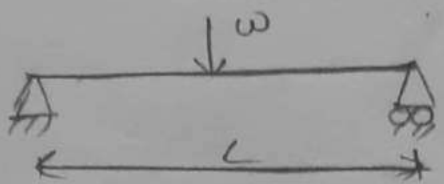
$$M_x = -wx \cdot \frac{x}{2} = -\frac{wx^2}{2}$$

$$U = \frac{1}{2EI}\int_0^L \left(\frac{w^2x^4}{4}\right) dx$$

$$U = \frac{w^2L^5}{40EI}$$

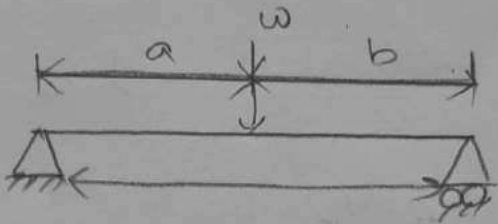
case-3

A Simply Supported beam with concentrated load



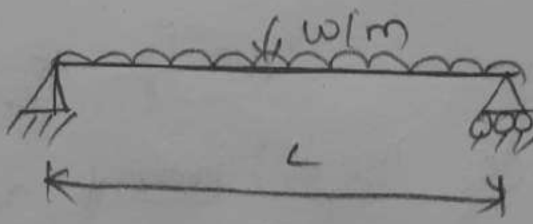
$$\boxed{U = \frac{w^2L^3}{96EI}}$$

case-4



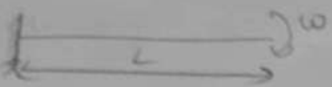
$$\boxed{U = \frac{w^2a^2b^2}{6EIL}}$$

case-5.



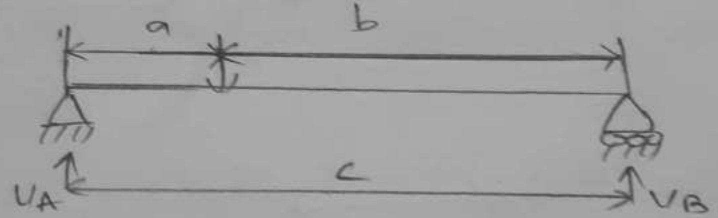
$$\boxed{U = \frac{w^2L^5}{240EI}}$$

case-6



$$\boxed{U = \frac{M^2 L}{2AE}}$$

Example A simply supported beam of span l carries a concentrated load p at a distance of 'a' & 'b' from two ends find the strain energy stored in the beam and the deflection under the load by castigliano's therom.



solu:-

$\Sigma V = 0 \qquad V_A + V_B = P$

$\Sigma M = 0$

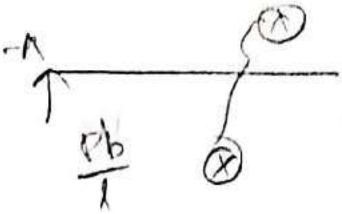
$Pa - V_B l = 0$

$V_B = \frac{Pa}{l}$

$V_A = \frac{Pb}{l}$

in the length of Ac the Moment at a distance from A is $\frac{Pbx}{l}$ Hence the strain energy for this portion is

$$U_{AC} = \int_0^a \frac{M^2 dx}{2EI}$$



$$U_{AC} = \frac{1}{2EI}\int_0^a M^2 dx$$

$$= \frac{1}{2EI}\int_0^a \left(\frac{Pbx}{l}\right)^2 dx$$

$$= \frac{P^2b^2}{2l^2EI}\int_0^a x^2 dx$$

$$= \frac{P^2b^2}{2l^2EI}\left[\frac{x^3}{3}\right]_0^a$$

$$U_{AC} = \frac{P^2a^3b^2}{6l^2EI\cancel{l^2}}$$

Simillarly for the length BC with B as the origin the strain energy for the portion BC can be obtained.

$$U_{BC} = \frac{1}{2EI}\int_0^b \left(\frac{Pax}{l}\right)^2 dx$$

$$\boxed{U_{BC} = \frac{P^2a^2b^3}{6EIl^2}}$$

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Hence the total strain energy

$$U = U_{AC} + U_{BC}$$

$$= \frac{P^2 a^3 b^2}{6EI l^2} + \frac{P^2 a^2 b^3}{6EI l^2}$$

$$\boxed{U = \frac{P^2 \cdot a^2 \cdot b^2}{6EI l}}$$

$\Delta = \frac{2U}{P}$

$$\boxed{\Delta_C = \frac{P a^2 b^2}{3EI l}}$$

Example. Find the deflection under concentrated load for the beam shown in figure using strain energy method

$E = 2 \times 10^8 \ kN/m^2$

$I = 14 \times 10^{-5} \ m^4$.

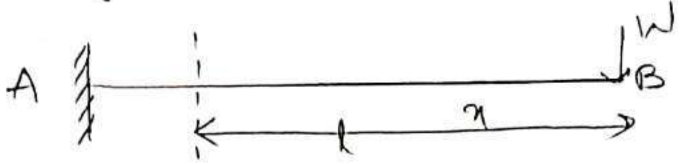
$$\Delta = \frac{p a^2 b^2}{3EI l}$$

$$= \frac{50 \times 3^2 \times 4^2}{3 \times 2 \times 10^8 \times 14 \times 10^{-5} \times 7}$$

$\Delta = 0.012 m$

$= 12 mm$

→ calculate the deflection at the free end of a cantilever subjected to a vertical load.



The Bending moment at a distance x from B is

$$M = -wx$$

The total strain Energy can be obtained as

$$U_{AB} = \int_0^l \frac{M^2 dx}{2EI}$$

$$= \frac{w^2}{2EI} \int_0^l x^2 dx$$

$$U_{AB} = \frac{w^2 l^3}{2EI}$$

Deflection at the free end is

$$\boxed{\Delta = \frac{wl^3}{3EI}}$$

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Castigliano's Theorems :-

First theorm is applied to determine the linear (or) angular deflection at a point in a structure caused by the applied load.

It is applicable for

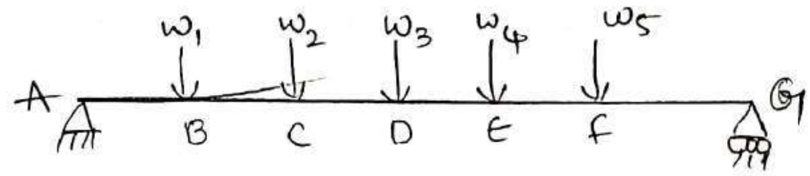
i) when the structure is stressed with in elastic limit.

ii) The structure may oby Hook's law but the deformation need not be liner.

Statement of Theorems - 1.

The displacement of the point of application of the load in its own line of action will be obtained by ~~eta~~ evaluating (or) calculating the first partial derivative of the total internal strain energy of the structure with respect to the applied load $\frac{\partial U}{\partial W} = \Delta$

Proof :- consider a beam AG under the action of loads w_1 ; w_2, w_3 etc at B, C and let displace -ments be Δ_1, Δ_2, Δ_3 respectively.



Internal strain energy will be

$$U = w_1 \frac{\Delta_1}{2} + w_2 \frac{\Delta_2}{2} + w_3 \cdot \frac{\Delta_3}{2} + --- \quad (1)$$

Let there be additional load δw_1 at B in the direction of w_1.

Then additional displacements will be $\delta\Delta_1, \delta\Delta_2, \delta\Delta_3$, in the direction w_1 ; w_2 ; w_3

$$\delta U = w_1 \delta\Delta_1 + \frac{\delta w_1 \delta\Delta_1}{2} + w_2 \delta\Delta_1 + w_3 \delta\Delta_3 \quad (2)$$

Now the loads are replaced with another system of forces $(w_1 + \delta w_1)$, w_2, w_3 at B, C, D.

Hence

Now system of displacements are

$$(\Delta_1 + \delta\Delta_1) \; ; - (\Delta_2 + \delta\Delta_2) \; , + (\Delta_3 + \delta\Delta_3)$$

$$U_1 = \frac{(w_1+\delta w_1).(\Delta_1+\delta\Delta_1)}{2} + \frac{w_2(\Delta_2+\delta\Delta_2)}{2} + \frac{w_3(\Delta_3+\delta\Delta_3)}{2} \quad \text{--- (3)}$$

eq (1) - (3)

$$U_1 - U = \partial U = w_1\frac{\delta\Delta_1}{2} + \frac{\delta w_1\Delta_1}{2} + \frac{\delta w_1\delta\Delta_1}{2} + w_2\frac{\delta\Delta_2}{2} + w_3\frac{\delta\Delta_3}{2} + ---$$

$$2\partial U = w_1\delta\Delta_1 + \delta w_1\Delta_1 + \delta w_1\delta\Delta_1 + w_2\delta\Delta_2 + w_3\delta\Delta_3 \quad \text{--- (4)}$$

From eq^n (4) - (2)

$$2\partial U - \partial U = \delta w_1\Delta_1 + \frac{\delta w_1\delta\Delta_1}{2}$$

Hence $\partial U = \delta w_1 \Delta_1$

$$\boxed{\Delta_1 = \frac{\partial U}{\partial w_1}}$$

(or)

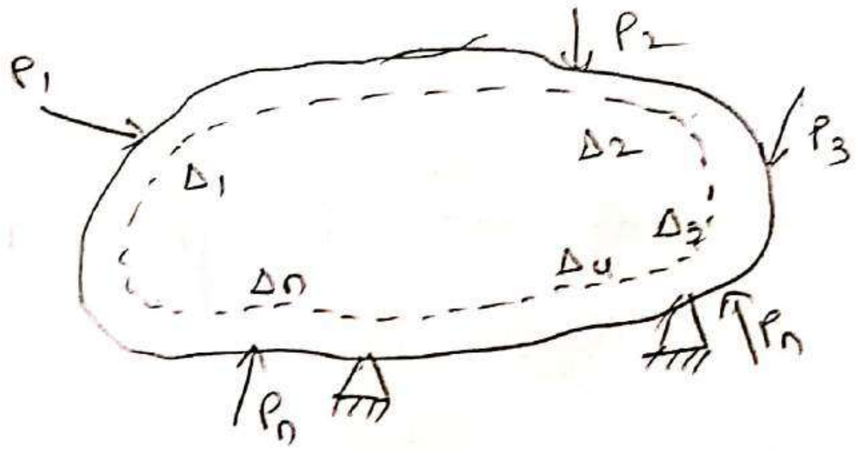
$$\boxed{\Delta = \frac{\partial U}{\partial w}}$$

Unit Load Method :-

consider the body shown in figure which is Subjected to force $P_1, P_2, P_3, P_4 ------ P_n$ applied gradually. Let the displacement under load at point be $\Delta_1, \Delta_2, \Delta_3 --- \Delta_n$ and at point c be Δ. Then

$$\text{External work done} = \tfrac{1}{2}\Delta_1 P_1 + \tfrac{1}{2}\Delta_2 P_2 + \tfrac{1}{2}\Delta_3 P_3 + ------ + \tfrac{1}{2}\Delta_n P_n$$

and strain energy stored $= \int \frac{1}{2} pe\,dv$.



Where p is stress

e is the strain in the element

consider

$$\tfrac{1}{2}\Delta_1 P_1 + \tfrac{1}{2}\Delta_2 P_2 + ---- + \tfrac{1}{2}\Delta_n P_n = \int \tfrac{1}{2} pe\,dv \quad (1)$$

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Now consider the same body subjected to a unit load applied gradually at c when it is free of system of p forces. let the displacement at 1, 2, 3 ------ n be δ_1 ; δ_2, δ_3 -----, δ_n respectively and displacement at 'c' and 'δ'.

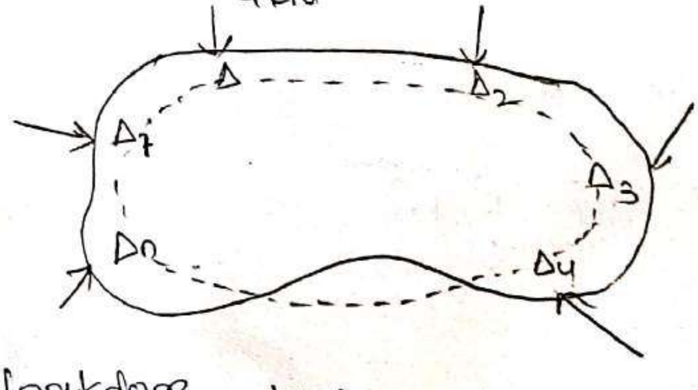
Let the stress produced in the element be 'p'' and the strain be 'e''. Then.

External work done $= \frac{1}{2} \times 1 \times \delta$

Internal work done $= \int \frac{1}{2} p' e' dv$

$$\frac{1}{2} \times 1 \times \delta = \int \frac{1}{2} p' e' dv \quad \text{(2)}$$

Now if p system of force is applied to the body shown in figure.



External Workdone $= \frac{1}{2} \Delta_1 P_1 + \frac{1}{2} \Delta_1 P_2 + \text{------}$

$$+ \frac{1}{2} \Delta_n P_n + 1 \times \Delta .$$

Since unit load is already action internal

work done $= \int \frac{1}{2} Pedv + \int P'edv$.

since the stress p is acting throught the deformation · equating internal work to external work.

$$\frac{1}{2}\Delta_1 P_1 + \frac{1}{2}\Delta_2 P_2 + \cdots \frac{1}{2}\Delta_n P_n + 1 \times \Delta$$

$$= \int \frac{1}{2} P \cdot e\, dv + \int p' e\, dv \quad \text{—— } (3)$$

Sub eq. (1) in (3)

$$1 \times \Delta = \int p' e\, dv$$

$$\boxed{\Delta = \int p' e\, dv}$$

Δ = is the deflection at the point where unit load is applied and is Measured in the direction of unit load.

p' – is stress in an element due to unit load and

e' = is strain element due to given load system

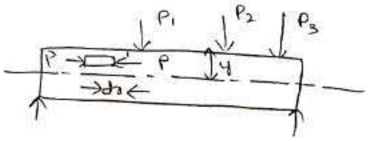
$\boxed{\Delta = \int p' e \, dv}$ → it is the basic for the unit load Method.

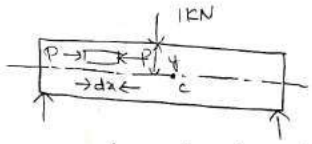
Deflection of simple beams :-

considers the beam as shown in figure subjected to a system of load 'p' forces

The stress in the element at distance 'y' from neutral axis is

$p = \frac{M}{I} Y$, where M is the Moment acting at the section.





∴ Strain in the element due to given system of forces

$e = \frac{M}{EI} Y$.

Let m be the moment at the section where the element is considered due to unit load acting at 'c'

stress $p' = \frac{mY}{I}$

$\Delta = \int p' e \, dv$

$= \int \frac{mY}{I} \times Y \cdot \frac{M}{EI} \times dv$

$\Delta = \int \frac{Mm}{EI^2} y^2 \, dv$

$\boxed{\int_0^A y^2 dA = I}$

$= \int_0^L \frac{Mm}{EI^2} \left[\int_0^A y^2 dA \right] dx$

$= \int_0^L \frac{Mm}{EI^2} \cdot I \, dx$

$\boxed{\Delta = \int_0^L \frac{Mm}{EI} \, dx}$

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Deflection of pin jointed frames :-

Castigliano's first theorem can be applicable for the estimating the deflection of pin jointed frames.

If the deflection of only a few joints are required.

Let P_i be the force developed due to loading in any member 'i' of the truss.

The strain energy stored in the Member is

$$\frac{P_i^2 l_i}{2A_i E}$$

where $l_i \rightarrow$ is length of the Member

$A_i \rightarrow$ is area of cross-section of the Member.

$\therefore$ Total strain energy will be

$$u = \frac{\Sigma P_i l_i}{2A_i E}$$

Differentiating with respect to w and External load on the structure.

We obtain the deflection due to the force 'W'.

$$\Delta = \frac{\partial u}{\partial w} = \frac{\Sigma P_i l_i}{A_i E}\left[\frac{\partial P_i}{\partial w}\right]$$

$\therefore \left[\frac{\partial P_i}{\partial w}\right]$ is rate of change of force P_i in any Member 'i' with respect to External load 'w'.

If 'k' denotes the force produced in any bar 'i' by a unit load in the direction of 'w'

then $\frac{\partial P_i}{\partial w} = k$.

$\therefore$ The Expression for the deflection of a pin joint frame is written in the form of

$$\Delta = \frac{\Sigma P_i l_i k}{A_i E}$$

(or) $\boxed{\Delta = \Sigma \frac{Pkl}{AE}}$

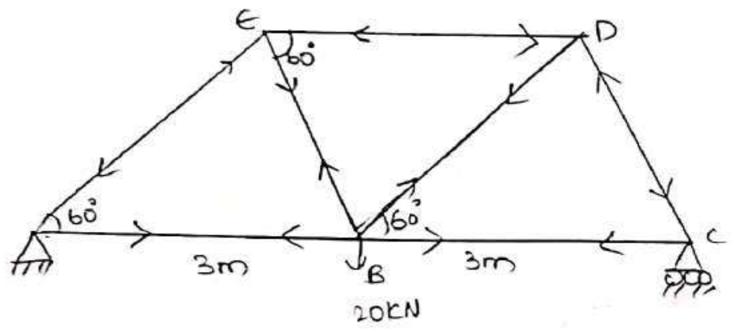
$P \rightarrow$ force in the member

$k \rightarrow$ denotes the force produced in any bel of the Member

$A \rightarrow$ Area of cross-section.

$E \rightarrow$ young's Modulus.

Ex: calculate the vertical deflection components of joint B of the truss loaded as shown in figure by strain energy Method cross-sectional area of each Member is $500 \times 10^{-6}\ m^2$ $E = 200 \times 10^6\ kN/m^2$



Sol:-

$$\Delta_B = \frac{\Sigma PKl}{AE}$$

The forces in the Members are obtained by the Method of joints and taking the advantage of system meters.

$$V_A = V_C = \frac{20}{2} = 10KN$$

Joint A

$F_{AB} = 5.78\ KN$

$F_{AE} = 11.55\ KN$

$$\boxed{\Delta = \Sigma \frac{PKl}{AE}}$$

$$\boxed{K = \frac{\partial P}{\partial W}}$$

Joint E

$F_{BE} = 11.55\ KN$

$F_{ED} = 11.55\ KN.$

Members	length	Area (mm^2)	P(KN)	K	$\frac{PKl}{A}$
AB	3000	500	5.78	0.289	10.002
BC	3000	500	5.78	0.289	10.002
BCD	3000	500	-11.55	-0.578	40.055
DE	3000	500	-11.55	-0.578	40.055
BE	3000	500	+11.55	0.578	40.55
BD	3000	500	11.55	0.578	40.55
AE	3000	500	-11.55	-0.578	8.4055

$$\Sigma \frac{PKl}{A} = 220.319$$

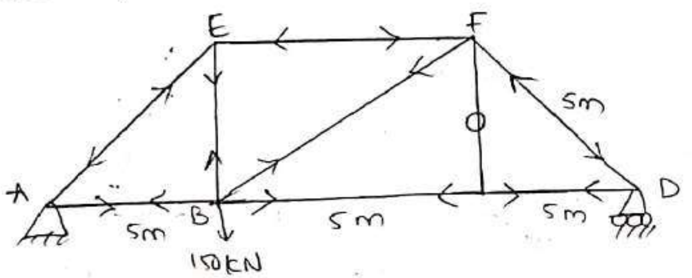
$$\Delta = \frac{220.319}{200}$$

$$\boxed{\Delta = 1.1mm}$$

Assignment.

A steel truss of span 15m is loaded as shown in figure. the cross section of each members is such that it is subjected to a stress of 100N/mm². Find the vertical deflection of the joint c.

Take $E = 200\ kN/mm^2$.



$V_A + V_D = 150$

$150(5) - 15V_D = 0$

$V_D = 50\ kN$

$V_A = 100\ kN$

Joint - A

$F_{AE} = 141.42\ kN$

$F_{AB} = 141.42\ kN$

Joint - E

$F_{EB} = 100\ kN$

$F_{FE} = 100\ kN$

Joint - C

$F_{CF} = 0$

$F_{CD} = 50\ kN$

Joint - D

$F_{DF} = 70.71\ kN.$

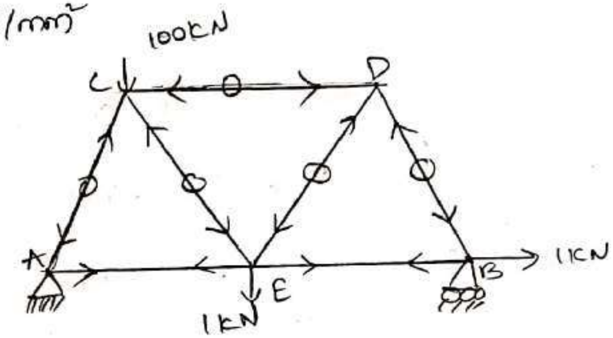
Joint - B

$F_{BF} = 70.72\ kN$

$F_{BC} = 50.0\ kN.$

Ex: Determine the horizontal movement of the roller's and the vertical deflection of joint 'c' of the truss shown in figure. Areas of chord members $1000mm^2$ and the area of diagonals are $1300 mm^2$

$E = 200\ kN/mm^2$



Sol:

$\Sigma V = 0 \quad V_A + V_B = 100$

$\Sigma M_A = 0 \quad 100(5) = 20V_B$

$V_B = 25\ kN$

$V_A = 75\ kN$

Joint - A

$F_{AC} = 75\sqrt{2}$

$F_{AE} = 75\ kN$

Joint - B

$F_{DB} = 35.355\ kN$

$F_{BE} = 25\ kN$

Joint - E

$F_{CE} = -35.355$

$F_{DE} = -33.35\ kN$

Joint - D.

$F_{DB} = 35.355$ KN

$F_{CD} = 50$ KN

A unit vertical load is applied at E where the deflection is required.

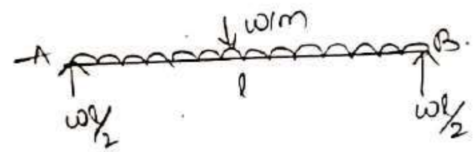
A unit horizontal load is applied at B to calculate the movement of the rolley

Member	length	Area	P	K	PKl/A	k'	$PK'l/A$
AE	10000	1000	75	0.5	375	1	750
EB	10000	1000	25	0.5	125	1	250
BD	$5000\sqrt{2}$	13000	-35.35	-0.707	135.96	0	0
DC	10000	1000	-50.00	-1.000	500	0	0
CA	$5000\sqrt{2}$	1300	-106.05	-0.707	407.8	0	0
CE	"	"	-35.35	+0.707	-135.94	0	0
DE	"	"	+35.355	0.707	+135.94	0	0
					1543.78		1000

$$\Delta_{VE} = \Sigma \frac{PKl}{AE} = \frac{1543.78}{200} = 7.72\,mm$$

$$\Delta_{HB} = \frac{1000}{200} = 5mm$$

Ex: using strain Energy method compute the deflection at midspan of a simply supported beam carrying a uniformly distributed load w KN/m Assume an uniform flexural rigidity



Members	origin	limits	M	m	I
AC	A	$0-l/2$	$\frac{wl}{2}x-\frac{wx^2}{2}$	$x/2$	I
CB	B	$0-l/2$	$\frac{wl}{2}x-\frac{wx^2}{2}$	$x/2$	I.

$$\Delta_C = \int_0^{l/2} 2\times\left(\frac{wl}{2}x - \frac{wx^2}{2}\right)\left(\frac{x}{2}\right)\frac{dx}{EI}$$

$$= \frac{1}{2}\int_0^{l/2} (wlx - wx^2)\,x \cdot \frac{dx}{EI}$$

$$= \frac{1}{2}\left[\frac{wlx^3}{3} - \frac{wx^4}{4}\right]_0^{l/2}$$

$$= \frac{1}{2}\left[\frac{wl}{3}\left(\frac{l}{2}\right)^3 - \frac{w}{4}\left(\frac{l}{2}\right)^4\right]$$

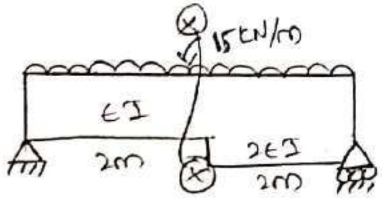
$$= \frac{1}{2}\left[\frac{wl^4}{24} - \frac{wl^4}{64}\right] \cdot \frac{1}{EI}$$

$$= \frac{1}{2}\left(\frac{(8-3)\, wl^4}{192}\right) \cdot \frac{1}{EI}$$

$$\boxed{\Delta_C = \frac{5}{384} \cdot \frac{wl^4}{EI}}$$

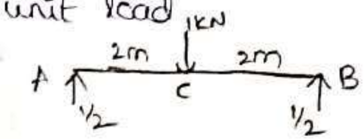
Ex: Determine the deflection for the beam shown in figure at midspan using strain energy Method

$EI = 3 \times 10^4\ kNm^2$



$$V_A = V_B = \frac{\text{total load}}{2} = \frac{wl}{2} = \frac{15 \times 4}{2} = 30\,kN$$

The relevent bending moment expression due to applied loading and due to unit load



Members	origin	limits	M	m	I
AC	A	0-2	$30x - \frac{15x^2}{2}$ $\frac{wl}{2}$	$x/2$	I
CB	B	0-2	$30x - \frac{15x^2}{2}$ $\frac{wl}{2}$	$x/2$	2I

$$\Delta_C = \int_0^2 (30x - 7.5x^2)(0.5)x \cdot \frac{dx}{EI} + \int_0^2 \frac{(30x - 7.5x^2)(0.5)x}{2EI}\, dx$$

$$= \frac{3}{2EI}\int_0^2 (15x^2 - 3.75x^3)\, dx$$

$$= \frac{3}{2EI}\left[\frac{15x^3}{3} - 3.75\frac{x^4}{4}\right]_0^2$$

$$= \frac{3}{2EI}\left[\frac{15(2)^3}{3} - 3.75\frac{(2)^4}{4}\right]$$

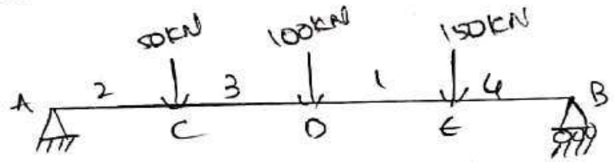
$$= \frac{3}{2EI} \times 25$$

$$= \frac{75}{2EI}$$

$$= \frac{75}{2 \times 3 \times 10^4} \times 1000$$

$$\boxed{\Delta_C = 1.25\ mm}$$

Ex: A steel beam of UDL is simply supported on a span of 10m and caries concentrated loads of 50, 100 and 150 kN at a distance of 2, 5, 6 meter from the left support compute the deflection under 150 kN load $EI = 400 \times 10^4$ kNm^2



Sol: The reaction at the supports are found using equation of equilibrium

$\Sigma V = 0$ $\quad V_A + V_B = 50 + 100 + 150$

$V_A + V_B = 300$

$\Sigma M_A = 0$

$50(2) + 100(5) + 150 \times 6 - 10 V_B = 0$

$\dfrac{100 + 500 + 900}{10} = V_B$

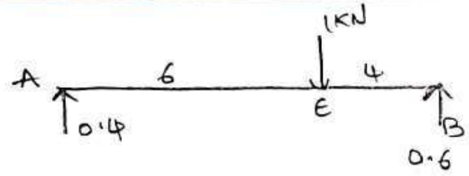
$\dfrac{1500}{10} = V_B$

$V_B = 150$ kN

$V_A + V_B = 300$

$V_A = 300 - 150$

$V_A = 150$ kN



$R_A + R_B = 1$ kN

$R_B \times 10 - 1 \times 6 = 0$

$R_B = \dfrac{6}{10} = 0.6$

$R_B = 0.6$

$R_A = 1 - 0.6$

$R_A = 0.4$

The required expression for bending moment due to applied loads and due to unit load is

Members	origin	M	m	limit
AC	A	$1.50x$	$0.4x$	0–2
CD	A	$150x - 50(x-2)$	$0.4x$	2–5
DE	B	$150x - 50(x-2)$	$0.6x - 1(x-4)$	4–5
EB	B	$150x$	$0.6x$	0–4

$$\Delta_E = \int_0^2 (150x)(0.4)x \frac{dx}{EI} + \int_0^5 (150x - 50(x-2))$$

$$+ \int_4^5 \{150x - 150(x-4)\}\{0.6x - (x-4)\} \cdot \frac{dx}{EI}$$

Section	origin	M	m'	limits
AC	A	$25.8x - 5x^2$	$0.5x$	0 - 3
CD	B	$14.2x - 10(x-2)$	$0.5x$	2 - 3
DB	B	$14.2x$	$0.5x$	0 - 2

$$\Delta_C = \int_0^3 (25.8x - 5x^2)(0.5x)\frac{dx}{EI} + \int_2^3 (14.2x - 10(x-2))(0.5x)\frac{dx}{EI}$$

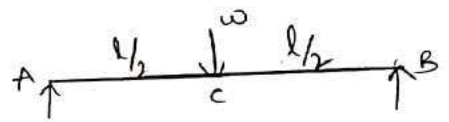
$$+\int_0^2 14.2x(0.5x).\frac{dx}{EI}$$

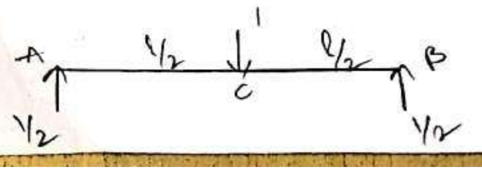
$$\Delta_C = \int_0^3 (12.9x^2 - 2.5x^3)\frac{dx}{EI} + \int_2^3 7.1x^2 - 5x^2 + 10\frac{dx}{EI}$$

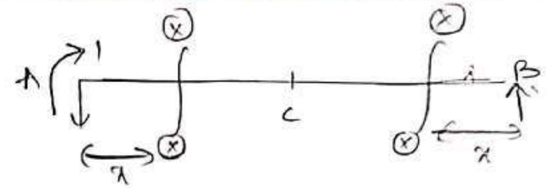
$$+\int_0^2 7.1x^2\frac{dx}{EI}$$

$$\boxed{\Delta_C = \frac{122.7}{EI}}$$

Ex: Determine the maximum slope and deflection of a simply supported beam subjected to a central concentrated load, using unit load Method.







Max deflection occurs at the centre and to obtain the same apply a unit load at 'c'

Section	origin	M	m_1	$\frac{\partial M}{\partial M_1}$	limits
AC	A	$\frac{w}{2}x$	$x/_2$	$(1-(x/l))$	$0 - l/_2$
BC	B	$\frac{w}{2}x$	$x/_2$	(x/l)	$0 - l/_2$

$$\Delta_C = \int_0^{l/2} \frac{Mm'}{EI}dx + \int_0^{l/2}\frac{Mm'}{EI}dx$$

$$= \int_0^{l/2} \left(\frac{w}{2}x\right)\left(x/_2\right)\frac{dx}{EI} + \int_0^{l/2}\left(\frac{wx}{2}\right)\left(x/_2\right)\frac{dx}{EI}$$

$$= 2\int_0^{l/2}\left(\frac{wx}{2}\right)\left(\frac{x}{2}\right)\frac{dx}{EI}$$

$$= 2\left(\frac{w}{4EI}\right)\int_0^{l/2} x^2.dx$$

$$= \cancel{2}\left(\frac{w}{\cancel{4}_2EI}\right)\left[\frac{x^3}{3}\right]_0^{l/2}$$

$$= \frac{w}{2EI}\left[\frac{(l/_2)^3}{3}\right]$$

$$\boxed{\Delta_C = \frac{wl^3}{48EI}}$$

— A similar procedure is adopted to determine maximum slope at the ends. A unit Moment is applied at 'A' and the bending moment Expression due to unit moment

$$\theta_A = \int_0^{l/2} \frac{wx}{2}\left(1-\frac{x}{l}\right)\frac{dx}{EI} + \int_0^{l/2} \frac{wx}{2}\left(\frac{x}{l}\right)\cdot\frac{dx}{EI}$$

$$= \frac{w}{2EIl}\left[\int_0^{l/2}(lx - x^2)\,dx + \int_0^{l/2} x^2\,dx\right]$$

$$\boxed{\theta = \frac{wl^2}{16EI}}$$

4th Unit Slope Deflection & Moment distribution

Moment

Slope deflection method:

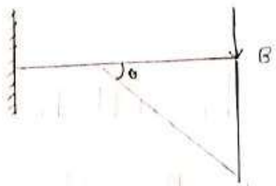
In the slope deflection, the relationship is established between moments at the ends of members and corresponding ~~and~~ rotations and displacements.

* The slope deflection method can be used to analyze statically determinant and indeterminant beams
* when a structure can be solved by using the equations of static equilibrium it is known as "determinant structures."

* when a structure cannot be solved by using the equations of static equilibrium it is known as "In determinant structure".

Ex: continous beam, proped cantilever etc....

* 

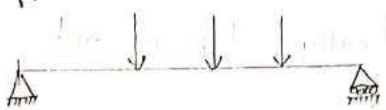


$\left(\frac{dy}{dx}\right)$ slope at $A = 0$

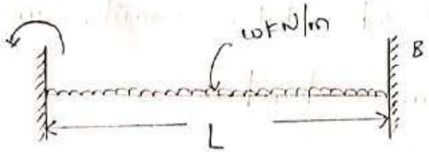
y deflection at $A = 0$

* Simply Supported beam

"For Simply Supported at the middle slope is zero"



* Case (i)

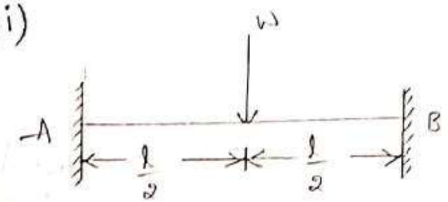


M'_{AB} = fixed end moment at $A = \frac{-wL^2}{12}$

m'_{BA} = fixed end moment at $B = + \frac{wL^2}{12}$

Bending moment at middle $= \frac{wL^2}{8}$

* case(ii)

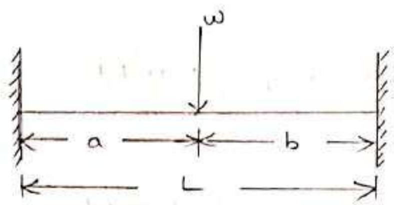


$M'_{AB} = \frac{-WL}{8}$

$M'_{BA} = \frac{WL}{8}$

B.m at the load $= \frac{WL}{4}$

* case(iii)



$M'_{AB} = \frac{-Wab^2}{L^2}$

$m'_{BA} = \frac{Wba^2}{L^2}$

B.m at Load $= \frac{Wab}{L}$

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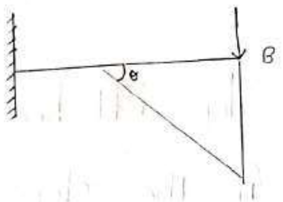
Ex: continous beam, proped cantilever etc....

* 

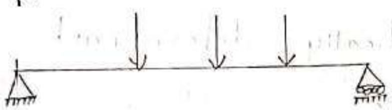


$\left(\frac{dy}{dx}\right)$ slope at $A = 0$

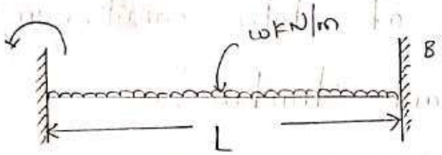
y deflection at $A = 0$

* Simply Supported beam

"For Simply Supported at the middle slope is Zero"



* Case (i)

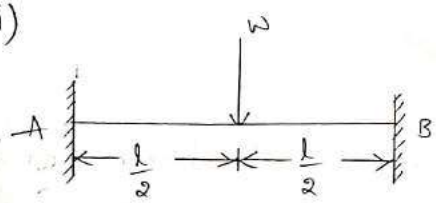


$M'AB$ = fixed end moment at $A = \frac{-wL^2}{12}$

$m'BA$ = fixed end moment at $B = + \frac{wL^2}{12}$

Bending moment at middle $= \frac{wL^2}{8}$

* case(ii)

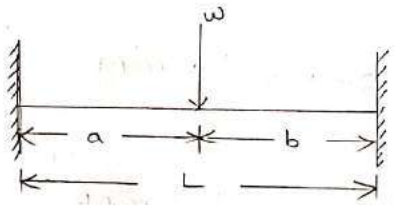


$M'AB = \frac{-WL}{8}$

$M'BA = \frac{WL}{8}$

B.m at the load $= \frac{WL}{4}$

* case(iii)



$M'AB = \frac{-Wab^2}{L^2}$

$m'BA = \frac{Wba^2}{L^2}$

B.m at Load $= \frac{Wab}{L}$

Step: 1

To find fixed end moments:

$$m'_{AB} = \frac{-wL^2}{12}$$

$$= \frac{-2\times6^2}{12}$$

$$= -6\,KN\text{-}m$$

$$m'_{BA} = \frac{wL^2}{12}$$

$$= \frac{2\times6^2}{12}$$

$$= 6\,KN\text{-}m$$

from B to C it is point load which is acting center to the distance given.

$$m'_{BC} = \frac{-wL}{8}$$

$$= \frac{-12\times6}{8}$$

$$= -9\,KN\text{-}m$$

$$m'_{CB} = \frac{wL}{8}$$

$$= \frac{12\times6}{8}$$

$$= 9\,KN\text{-}m$$

step: 2

To find the Support moments

Slope at A $\Rightarrow$ $i_A = 0$

Slope at C $\Rightarrow$ $i_C = 0$

$$m_{AB} = \frac{2EI}{L}(2i_A + i_B) + m'_{AB}$$

$$= \frac{\cancel{2}EI}{\cancel{6}\,3}(2(0) + i_B) + (-6)$$

$$m_{AB} = \frac{EI}{3}\,i_B - 6$$

$$m_{BA} = \frac{2EI}{L}(2i_B + i_A) + m'_{BA}$$

$$= \frac{2EI}{6}[2i_B + 0] + 6$$

$$= \frac{2EI}{3}[i_B + 6]$$

$$m_{BC} = \frac{2EI}{L}\left[2i_B + i_C\right] + m'_{BC}$$

$$= \frac{2EI}{6}\left[2i_B + 0\right] - 9$$

$$= \frac{2EI}{3} i_B - 9$$

$$m_{CB} = \frac{2EI}{L}\left(2i_C + i_B\right) + m'_{CB}$$

$$= \frac{2EI}{6} i_B + 9$$

$$= \frac{EI}{3} i_B + 9$$

Step: 3:

To find the equations of equilibrium in the diagram B

$$m_{BA} + m_{BC} = 0$$

$$\frac{2EI}{3} i_B + 6 + \frac{2EI}{3} i_B - 9 = 0$$

$$\frac{4EI}{3} i_B = 3$$

$$\boxed{i_B = \frac{9}{4EI}} \quad \text{—} \quad (1)$$

Substitute equation (1) in m_{AB}, m_{BA}, m_{BC}, m_{CB}.

$$m_{AB} = \frac{EI}{3}\left(\frac{9}{4EI}\right) - 6$$

$$= -5.25$$

$$m_{BA} = \frac{2EI}{3}\left[\frac{9}{4EI}\right] + 6 =$$

$$= 7.5$$

$$m_{BC} = \frac{2EI}{L}\left[\frac{9}{4EI}\right] - 9$$

$$= -7.5$$

$$m_{CB} = \frac{EI}{3}\left[\frac{9}{4EI}\right] + 9$$

$$= 9.75$$

Step: 4

To find the Bending moment at middle fat load:

$$B.m \text{ at } \cancel{AB} = \frac{wL^2}{8} = \frac{2 \times 6^2}{8} = 9\,kN\text{-}m$$

$$B.m \text{ at } BC = \frac{wL}{4} = \frac{12 \times 6}{4} = 18\,kN\text{-}m$$

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Step: 5

To find all the reactions and consider

$\Sigma M_B = 0$ on AB span

$R_B \times 0 + M_{BA} + R_A \times 6 - M_{AB} - 2 \times 6 \times \frac{6}{2} = 0$

$7.5 + 6R_A + 5.25 - 36 = 0$

$\boxed{R_A = \cancel{8.6.85\ KN}\ 3.87\ KN.}$

consider $\Sigma M_B = 0$ on the BC span

$R_B \times 0 - M_{BC} - R_C \times 6 + M_{CB} + 12 \times 3 = 0$

$-7.5 - 6R_C \times \cancel{+}\ 9.75 + 36 = 0$

$\boxed{R_C = 6.375\ KN}$

$R_A + R_B + R_C = (2 \times 6) + 12$ (total load)

$\boxed{R_B = 12\ KN}$

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Moment Distribution Method

Working process for analysis of continous beam by using moment distribution method :

step: 1

To find relative stiffness(K) and modified stiffness factor (K')

$$\text{Relative stiffness (K)} = \frac{\text{moment of inertia}}{\text{Length}}$$

$$\text{modified stiffness factor (K')} = \frac{3}{4} K$$

(either in hing, (or) roller support)

step: 2 To find distribution factor

$$\text{Distribution factor} = \frac{\text{Relative stiffness factor}}{\text{R.S.F}}$$

ΣK = Sum of K_i for joint

$$DF = \frac{K}{\Sigma K}$$

Note: Distribution factor for fixed Support is equal to zero

* Distribution factor for hinged (or) Roller support is equal to 1.

step: 3
To find the fixed end moments (m')

According to table data we can find fixed end moments

step: 4 To find distribution of moment and carry over of moment

D.o.m = Sum of fixed end moment at middle joint) × DF

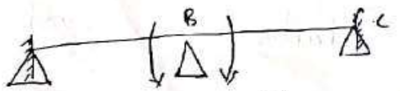
$$com = \frac{Dom}{2}$$

step: 5 To find end moments (M)

End moments = m^F + DOM + COM

step: 6
cheaking

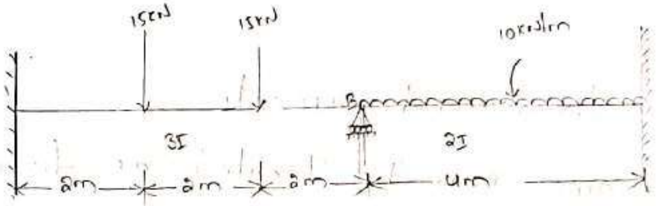
1) fixed Support slope = 0
2) End moment at middle joint is eaual and opposite direction



$$3)\ (\theta)\ slope = \frac{m_{AB} - m^F_{AB} - \frac{1}{2}(m_{BA} - m^F_{BA})}{\frac{3EI}{L}}$$

Problem:

1) Analyse the two span continous beam by moment distribution method.



step: 1
To find K & K' (Relative stiffness & modified stiffness)

$$K_{BA} = \frac{M.O.I}{L_{AB}} = \frac{3I}{6} = 0.5I$$

$$K_{BC} = \frac{2I}{4} = 0.5I$$

To find Distribution factor

$DF_{AB} = 0$

$$DF_{BA} = K_{BA} = \frac{0.5I}{0.5I + 0.5I} = 0.5$$

$D.F_{BC} = \frac{K_{BC}}{\Sigma K}$

$= \frac{0.5I}{0.5I+0.5I}$

$= 0.5$

$D.F_{CB} = 0$

step: 3

To find fixed end moments

Every beam can be treated as fixed beam

$m_{AB} = -\frac{2wL}{9} = -20$ $m_{BA} = +\frac{2wL}{9} ; -20$ $m_{BC} = -\frac{wL^2}{12} = +13.33$

$m_{CB} = +\frac{wL^2}{12} = +13.33$

step: 4 To find D.o.m & c.o.m

member	A		B	C
	AB	BA	BC	CB
DF	0	0.5	0.5	0
FEM	−20	+20	−13.33	+13.33
DoM	0	+3.335	+3.335	0
CoM	1.6675	-	-	1.6675
	21.66	−16.66	16.665	11.6625

cheaking step: 5

$$\frac{(m_{AB} - m_{AB}^F) - \frac{1}{2}(m_{BA} - m_{BA}^F)}{\frac{3EI}{L}}$$

$$(21.6675 - 20) - \frac{1}{2} \quad \frac{16.6675 - 5.385}{0.5I}$$

$= 1.6675$

$$= \frac{(m_{BC} - m_{BC}^F) - \frac{1}{2}(m_{CB} - m_{CB}^F)}{\frac{2I}{4} = 0.5I}$$

step:

$\theta_A =$

$$\frac{[m_{AB} - m_{AB}^F] - \frac{1}{2}[m_{BA} - m_{BA}^F]}{\frac{3EI}{L}}$$

$$= \frac{[21.6675 - 20] - \frac{1}{2}[-16.665 + 20]}{\frac{3E(3I)}{6}}$$

$= 0$

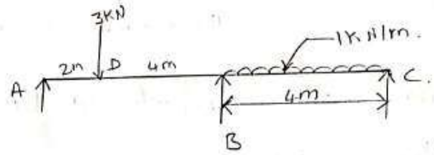
It is correct because "A" is fixed support fixed Support:

$$\theta_C = \frac{(m_{CB} - m^F_{CB}) - \frac{1}{2}(m_{BC} - m^F_{BC})}{\frac{3EI}{2}}$$

$$= \frac{(-11.6625 + 13.33) - \frac{1}{2}(10.665 - 13.33)}{3E(2I)/4} = 0$$

It is correct be cause c is fixed support

Ex: A continuous beam ABC 10m long rests on three supports A, B and C at the same level is loaded as shown in figure.



Sol

Assume the continuous beam ABC to split up into two fixed beams AB and BC.

Step - I

$$M'_{AB} = -\frac{wab^2}{l^2} = -\frac{3 \times 2 \times (4)^2}{(6)^2} = -\frac{8}{3} = -2.67\,KN\text{-}m.$$

$$M'_{BA} = \frac{wa^2b}{l^2} = \frac{3 \times 2^2 \times 4}{6^2} = +1.33\,KN\text{-}m$$

$$M'_{BC} = \frac{-wl^2}{12} = -\frac{1 \times (4)^2}{12} = \frac{-4}{3} = -1.33\,KN\text{-}m.$$

$$M'_{CB} = \frac{wl^2}{12} = \frac{1 \times 4^2}{12} = \frac{4}{3} = 1.33\,KN\text{-}m$$

Caluclating Distribution factor at B.

Stiffness factor for BA

$$K_{BA} = \frac{3EI}{l} = \frac{3EI}{6} = \frac{EI}{2}$$

(The beam is hinged at A)

Similarly, Stiffness factor for BC

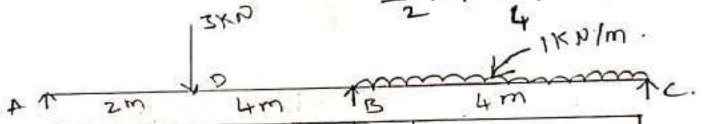
$$K_{BC} = \frac{3EI}{l} = \frac{3EI}{4}$$

($\because$ The beam is hinged at C)

⇒ Calculating Distribution factor for BA and BC

$$D.F_{BA} = \frac{\frac{EI}{2}}{\frac{EI}{2} + \frac{3EI}{4}} = \frac{2}{5}$$

$$D.F_{BC} = \frac{\frac{3EI}{4}}{\frac{EI}{2} + \frac{3EI}{4}} = \frac{3}{5}$$



A	B		C	
	2/5	3/5		
-2.67	+1.33	-1.33	+1.33	F.E.M
+2.67			-1.33	Release A & C.
0	0 +1.33	0 -0.67	0	Carry over.
0	+2.66	-2.00	0	Initial moment
	-0.26	-0.40		Distribution.
0	+2.40	-2.40	0	Final moments.

⇒ Calculating B.M and S.F

B.M For the Span AB $= \frac{wab}{l} = \frac{3 \times 2 \times 4}{6}$

$= 4\,KN\text{-}m.$

B.M For the Span BC $= \frac{wl^2}{8} = \frac{1 \times 4^2}{8} = 2\,KN\text{-}m$

S.F.

$R_A + R_B + R_C = 3 + 1(4) = 7\,KN.$

Taking moment about - B.

$\Sigma M_B = 0.$

$R_A \times 6 - 3 \times 4 = -2.4$

$R_A = \frac{-2.4 + 12.0}{6} = \frac{9.6}{6} = 1.6\,KN.$

Similarly

~~Rc x 4 - 4 x 2~~

$R_C \times 4 - 4 \times 2 = -2.4$

$R_C = \frac{-2.4 + 8}{4} = \frac{5.6}{4}$

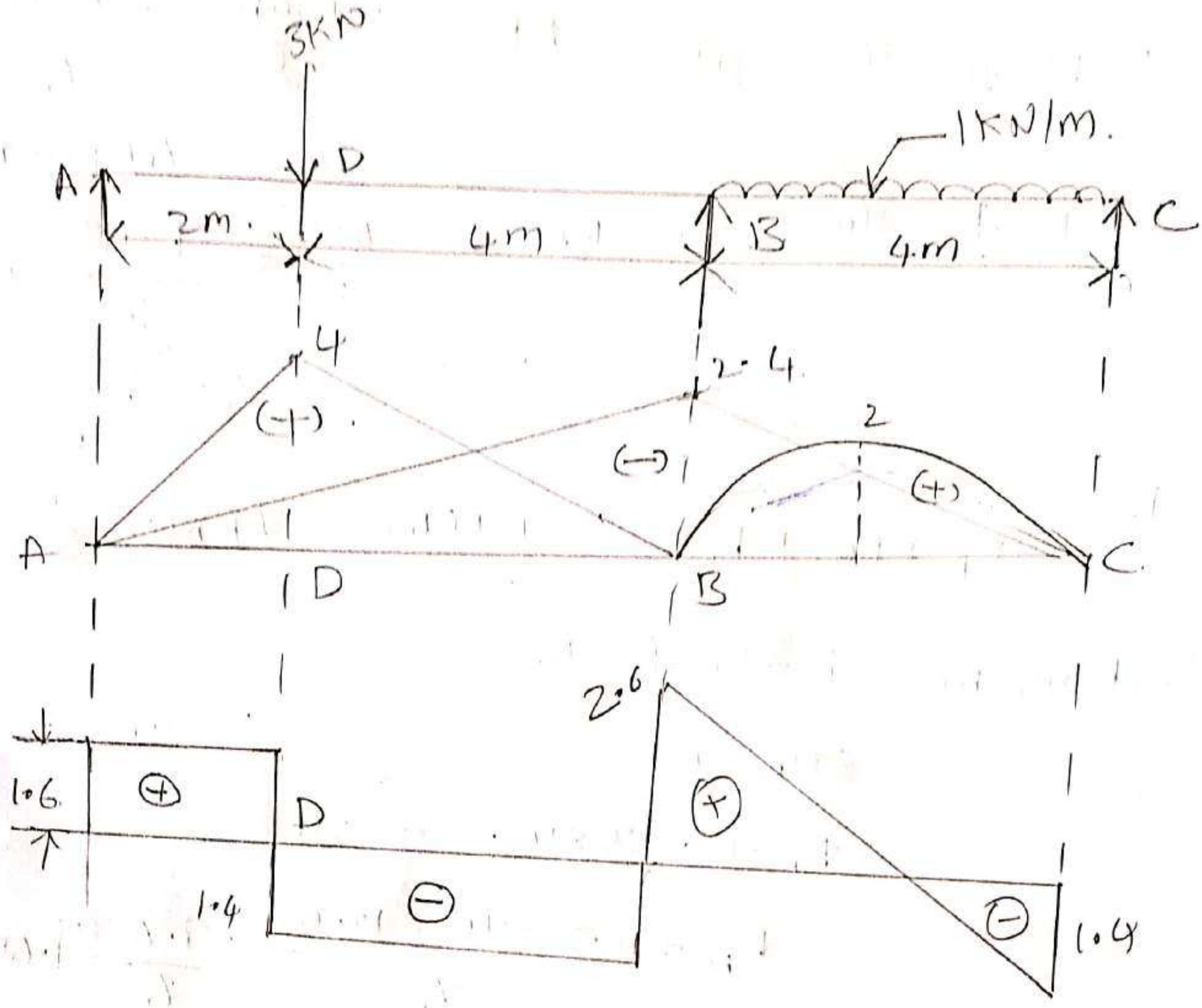
$= 1.4\,KN.$

~~R_B =~~

$R_A + R_B + R_C = 7.$

$R_B = 7 - (1.6 + 1.4)$

$R_B = 4\,KN.$


3KN
D
1KN/m.
A
2m.
4m
B
4m
C
4
2.4
(+)
(−)
2
(+)
A
D
B
C
2.6
1.6
(+)
D
(+)
1.4
(−)
(−)
1.4