DESIGN OF MACHINE MEMBERS - II

UNIT - I

-: SLIDING CONTACT BEARINGS:-

29/12/18

INTRODUCTION

Bearing is a Mechanical Element that permits relative motion between two parts, such as the shaft and the housing, with minimum friction.

The functions of the bearing are as follows:

(i) The bearing Ensures free rotation of the shaft or the axle with minimum friction

(ii) The bearing supports the shaft or axle and holds it in the correct position

(iii) The bearing takes up the forces that act on the shaft or the axle and transmits them to the frame or the foundation.

Classification of Bearings:

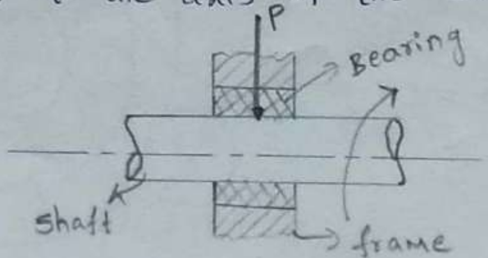
Bearings are classified in different ways.

a) Depending upon the direction of force that acts on them, Bearings are classified into two catagories

(i) Radial Bearing.

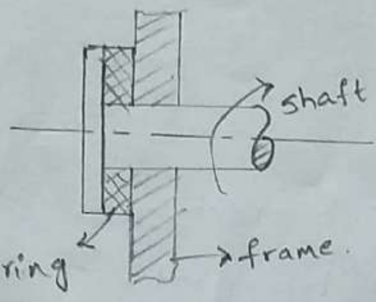
(ii) Thrust Bearing.

(i) Radial Bearing: A Radial Bearing supports the load, which is Perpendicular to the axis of the shaft.



: Radial Bearing:

Thrust Bearing: A Thrust Bearing supports the load, which acts along the axis of shaft.



:Thrust Bearing:

b) Depending upon the type of friction.

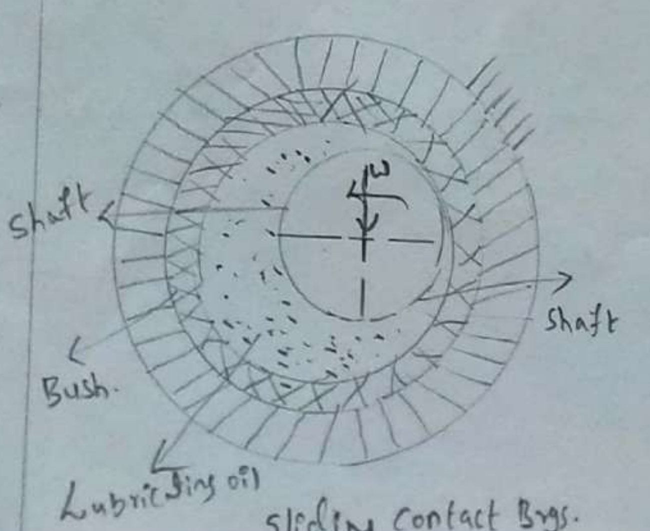
(i) Sliding contact Bearings.

(ii) Rolling Contact Bearings.

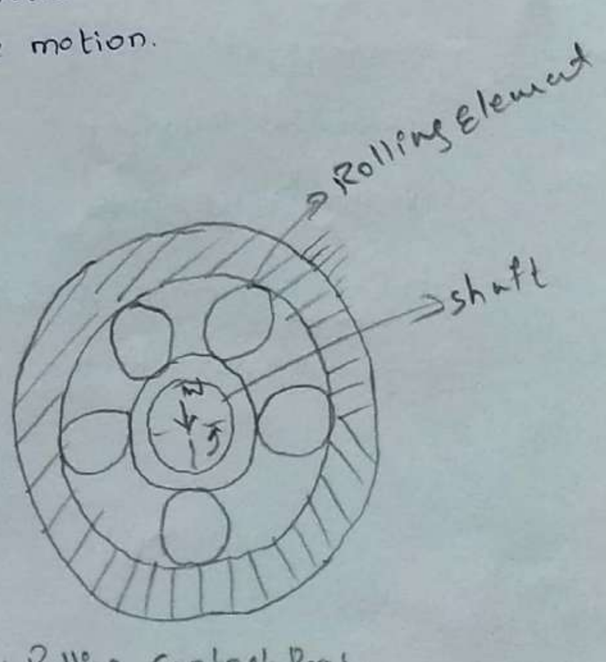
(i) Sliding contact Bearings: (plain Bearings / Journal Bearings / sleeve Brgs)

In this, the surface of the shaft slides over the surface of Bush resulting in friction and wear. In order to reduce the friction, these two surfaces are seperated by a film of lubricating oil. The bush is made of special bearing Materials like white metal or Bronze.

(ii) Rolling contact Bearings: (Antifriction Bearings) (Ball Bearings):- Rolling Elements, such as balls or rollers, are introduced b/w the surfaces that are in relative motion.



Sliding Contact Brgs.



Rolling Contact Brgs.

Sliding contact Bearings are used in following applications:

(i) Crankshaft Bearings in petrol and diesel Engines

(ii) Centrifugal pumps.

(iii) Large size electric motors.

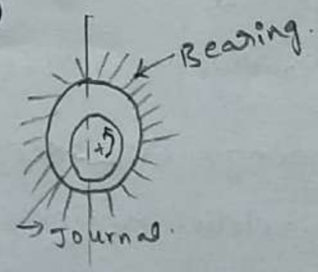
(iv) Steam and gas turbines

(v) Concrete Mixers, rope conveyors and Marine installations.

Types of sliding Contact Bearings.

Slipper (or) Guide Bearings: In which the sliding action is guided in a straight line and carrying radial loads. called as slipper (or) Guide Bearings. (In Cross-Head of steam Engines)

Full journal Bearing:- The sliding contact Bearings in which the sliding action is along the circumference of a circle (or) an arc of circle and carrying radial loads are known as journal (or) sleeve Bearings. When the angle of contact of Bearing with the journal is 360° then the Bearing is called full journal Bearing (Industrial Machinery)



Full JOURNAL BEARING.

PARTIAL JOURNAL BEARING:

When the angle of contact of the Bearing with the journal is 120° then the Bearing is said to be partial journal Bearing. (rail road car axle)

: partial journal Bearing:

Clearance Bearing: The full and partial journal Bearings may be called as clearance Bearings. Because the dia of the journal is less than that of bearing.

fitted Bearing:- When the partial Bearings has no clearance i.e the diameters of the journal and Bearings are equal, then the Bearing is called a fitted Bearing.

The sliding contact Bearings, according to the thickness of layer of the lubricant b/w the Bearing and journal

(i) Thick film bearing: In these the working surfaces are Completely seperated from each other by the lubricant. (or Hydrodynamic lubricated bearings)

(ii) Thin film bearing: The thin film bearings are those in which the lubricant is present, the working surfaces are partially contact each other atleast part of the time. (Boundary lubricated Bearings)

(iii) Zero film bearing:- The Zero film Bearings are those which operate without any lubricant present.

(iv) Hydrostatic (or) Externally pressurized lubricated Bearings:

The Hydrostatic bearings are those which can support steady loads without any relative Motion between the journal and the Bearing

This is achived by forcing Externally pressurized lubricant between the Members.

Properties of Sliding Contact Bearing Materials:

1. Compressive strength: The Bearing Material should have high Compressive strength to withstand this max. pressure so as to prevent Extrusion (or) other permanent deformation of the Bearing.

2. Fatigue strength: The Bearing Material should have sufficient fatigue Strength so that it can withstand repeated loads w/o developing surface fatigue cracks.
   It is Major importance in Aircraft & Automotive Engines.

3. Comformability: It is ability of Bearing Material to accommodate shaft deflections and bearing inaccuracies by plastic deformation w/o Excessive wear and tear.

4. Embeddability: It is the ability of Bearing Material to accommodate small particles of dust, grit etc. w/o scoring the material of Journal

5. Bondability: Many high capacity bearings are made by bonding one or more thin layers of a bearing Material to high strength steel shell.

6. Corrosion resistance: The Bearing Material should not corrode away under the action of lubricating oil.

7. Thermal conductivity: The Bearing Material should have high thermal-conductivity so as to permit the rapid removal of heat generated by friction.

8. Thermal Expansion: The Bearing Material should have low thermal Expansion so that when the bearing operates over a wide range of temp. there is no undue change in the clearance.

Advantages and disadvantages of sliding Contact Bearing:

ADVANTAGES:

(i) The design of bearing and housing is simple.

(2) They occupy less radial space & more compact.

(3) They cost is less.

(4) The design of shaf is simple

(5) They operate more silently

(6) They have good shock and capacity.

(7) They are ideally suited for medium and high speed operation.

DISADVANTAGES:

(1) The frictional power is more

(2) They required good attention to lubrication

(3) They are normally designed to carry radial load (or) Axial load only

Materials used for sliding contact Bearings:

1) Babbit Metal: Tin base babbitts & lead base babbitts are widely used as a bearing material. The babbitt are recommended where the maximum bearing pressure is not over 7 to 14 N/mm^2.

Tin base babbits: Tin 90% ; copper 4.5% , Antimony 5%, Lead 0.5%.

Lead base babbits: Lead 84% ; Tin 6% ; Antimony 9.5% ; Copper 0.5%

2) Bronze: The Bronzes (Alloys of cu, ti & Zn) are generaly used in the form of machined bushes pressed into the shell. The bush may be in one or two pieces. The Bronzes Commonly used for Bearing material are gun metal & phosphor Bronzes.

Gun metal (Cu 88% ; Tin 10% , Zn 2%) is used for high grade bearings subjected to high pressure & high speeds.

phosphor Bronze: (Cu 80% ; Tn 10% ; Ld 9%).

3) Cast iron: The castiron bearings are usually used with steel journal. Such type of bearings are fairly successful where lubrication is adequate & the pressure is limited to 3.5 N/mm^2 & speed to 40 m per minute

4) Silver:- The silver and silver lead bearings are mostly used in Aircraft bearings Engines where the fatigue strength is most important consideration.

5) Non-Metallic:- The various non-metallic bearings are made of carbon graphite, rubber, wood and plastics.
The carbon graphite bearings are self lubricating, dimension-ally stable over a wide range of operating conditions, chemically inert & can operate at higher temperatures than other bearings. Such type of bearings are used in food processing and other equipment where contamination by oil or grease must be prohibited.

→ Soft Rubber bearings are used with water or other low viscosity lubricants. particularly where sand or other large particles are present. In addition to high degree of Embeddability and comformability, the rubber bearings are excellent for absorbing shock loads and vibrations. The rubber bearings are used mainly on marine propeller shafts, hydrallic turbines and pumps.

→ The wood bearings are used in many applications where low cost, cleanliness, inattention to lubrication & anti-seizing are important.

⇒ The commonly used plastic material for bearing is Nylon & Teflon. These materials have many characteristics desirable in bearing Materials & both can be used dry i.e as a zero film bearing. It is used for applications in which these proporties are important: Eg. Elevator bearings, Cams in telphone dials etc.

Lubrication is the science of reducing friction by application of a suitable substance called lubricant, b/w the rubbing surfaces of bodies having relative motion.

The lubricants are classified into following three groups.

(i) Liquid lubricants

(ii) Semi- solid lubricants.

(iii) Solid lubricants. — Graphite (or) molybdenum

The objectives of lubricant are follows.

a) Improve performance of bearing.

b) Increase life of bearing.

c) to reduce the friction.

d) to reduce (or) prevent wear

e) to carry away heat generated due to friction.

f) to protect the journal and bearing from corrosion.

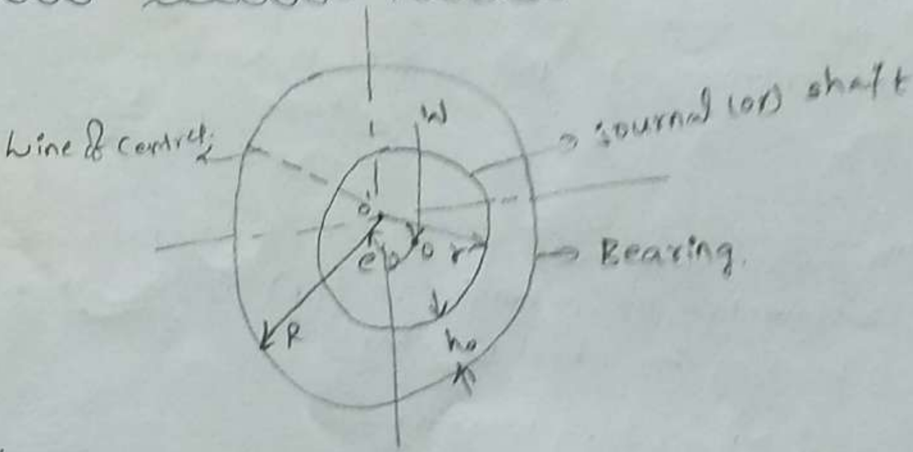
Liquid Lubricants: usually used in bearings are mineral oil (or) vegetable oils. The mineral oils are most commonly used because of their cheapness and stability. The liquid lubricants are usually preferred where they may be retained.

Semi-solid lubricants: A Grease is a semi- Solid lubricant having higher viscosity than oils. The greases are employed where slow speed and heavy pressure exist and where oil drip from bearing is undesirable.

Solid lubricants: The solid lubricants are useful in reducing friction where oil films cannot be maintained. because of pressure and temp. They should be soft than materials being lubricated.

A Graphite is the most common of solid lubricants either alone or mixed with oil (or) grease.

Terms used in Hydrodynamic journal Bearing:



A Hydrodynamic journal Bearing is shown in fig. in which O is the centre of the journal & O' is the Centre of Bearing

D = Dia of Bearing

d = Dia of journal

l = length of Bearing.

1. **Diametral Clearance:** It is the differance between the dias of bearing and journal.

$$c = D - d$$

2. **Radial clearance:** It is the differance b/w radii of the bearing and the journal.

$$c_1 = R - r = \frac{D-d}{2} = \frac{c}{2}$$

3. **Diametral clearance Ratio:** It is the ratio of diametral clearance to the diameter of the journal.

$$= \frac{c}{d} = \frac{D-d}{d}$$

4. **Eccentricity:** It is the radial distance b/w the centre (o) of the bearing and displaced centre (O') of the bearing under load. It is denoted by (e)

5. **Minimum oil film thickness:** It is the minimum distance b/w bearing and the journal, under complete lubrication condition

It is denoted by h_o and it's value $h_o = \frac{c}{4}$

5. Attitude (or) Eccentricity ratio:- It is the ratio of the Eccentricity to the radial clearance

$$\varepsilon = \frac{e}{c_1} = \frac{c_1 - h_o}{c_1} = 1 - \frac{h_o}{c_1} = 1 - \frac{2h_o}{c} \quad \left[\because c_1 = c/2\right]$$

6. short and long Bearing: It is the ratio of length to the diameter of journal (i.e l/d) is lessthan 1, then the bearing is short bearing

$\frac{l}{d} > 1 \rightarrow$ long Bearing

$l = d \rightarrow$ square Bearing.

Bearing Characteristic Number and Bearing Modulus for Journal Bearing:-

The co-efficient of friction in design of bearings is of great importantance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal Bearing is a function of three variables. i.e

(i) $\frac{ZN}{P}$ (ii) $\frac{d}{c}$ and (iii) $\frac{l}{d}$

$$\mu = \phi\left(\frac{ZN}{P}, \frac{d}{c}, \frac{l}{d}\right)$$

where μ = Co-efficient of friction

ϕ = A functional relation ship

Z = Absolute viscosity of the lubricant kg/m-s

N = Speed of the journal (r.p.m)

P = Bearing pressure on the projected bearing area N/mm²

= load on the journal / $l \times d$

d = Diameter of the journal

l = length of the Bearing &

c = Diametral clearance.

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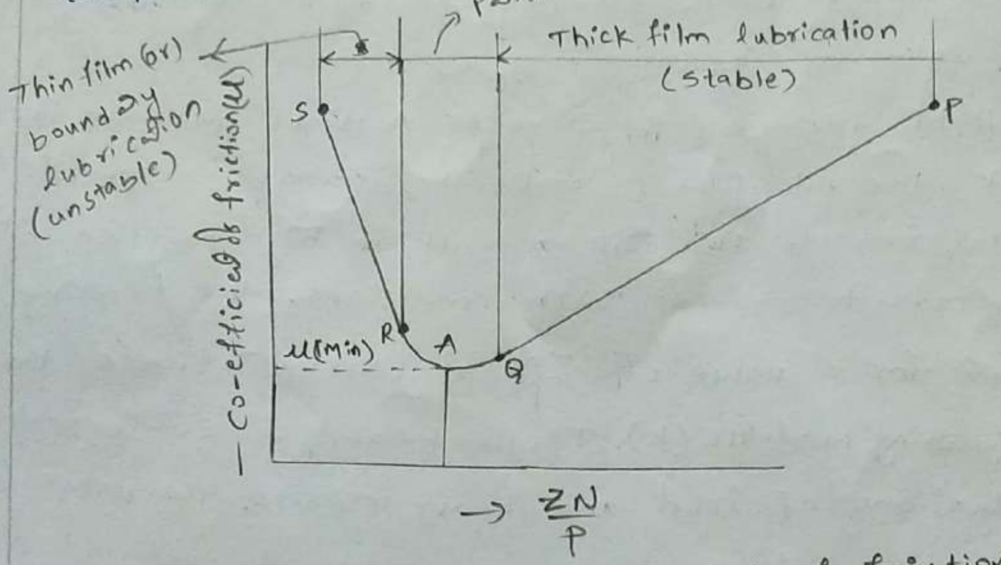
The factor $\frac{ZN}{P}$ is termed as bearing characteristic Number & is dimensionless number. The factor $\frac{ZN}{P}$ helps to predict the performance of a bearing



Variation of coefficient of friction with $\frac{ZN}{P}$

The part of the curve PQ represents the region of thick film lubrication. Between Q and R, The viscosity (Z) or the speed (N) are so low, or the pressure (P) is so great that their combination $\frac{ZN}{P}$ will reduce the film thickness so that partial Metal to Metal contact will result. The thin film (or) boundary lubrication (or) imperfect lubrication. Exists b/w R and S on the curve.

This is the region where the viscosity of the lubricant ceases to be a measure of friction characteristics but the oiliness of the lubricant is effective in preventing complete Metal to metal contact and seizure of the parts.

It May be noted that the part PQ of the curve represent stable operating conditions, since from any point of stability a decrease in viscosity (Z) will reduce ZN/P. This will result in a decrease in coefficient of friction (μ) followed by a lowering of Bearing temp. that will raise the viscosity (Z).

We see that the min. of amount of friction occurs at A and at this point the value of ZN/p is known as bearing Modulus. which is denoted by k The bearing should not be operated at these value of bearing Modulus. because.

A slight decrease in speed or A slight increase in pressure will break the oil film & make the journal to operate with Metal-Metal contact. This will result in high friction, where heating. In order to prevent such conditions, the bearing should be designed for a value of $\frac{ZN}{p}$ at least 3 times the min. value of bearing Modulus (k). If the bearing is subjected to large fluctuations of load and heavy impacts, the value of $\frac{ZN}{p}$ = 15k May be used.

From the above it is concluded that when the value of $\frac{ZN}{p}$ is greater than k, then the bearing will operate with thick film lubrication (or) under Hydrodynamic conditions. On the other hand, when the value of $\frac{ZN}{p}$ is less than k. Then the oil film will rupture & there is a metal-metal contact.

Co-effecient of friction

In order to determine the coefficient of friction for well lubricated full Journal Bearings. the following Empirical relation established by Mc Kee based on the Experimental data, may be used.

∴ Co-effecient of friction

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{P}\right)\left(\frac{d}{c}\right) + K$$

$$(\because Z - kg/m\text{-}s \ \& \ P - N/mm^2)$$

K = Factor to correct for End leakage. It depends upon the ratio of length to dia. of the bearing (l/d)

= 0.002 for l/d ratio 0.75 to 2.8

Critical pressure:

The pressure at which oil film breakes down so that Metal to Metal contact begins is known as critical pressure (or) the Min. operating pressure of the Bearing. It may be obtained by the following Empirical relation i.e

Critical pressure (or) Min. operating pressure,

$$P = \frac{ZN}{4.75\times10^6}\left(\frac{d}{c}\right)^2\left(\frac{l}{d+l}\right) N/mm^2 \qquad (Z = kg/m\text{-}s)$$

Sommerfeld Number:

The sommerfeld number is also a dimensionless parameter used Extensively in the design of journal Bearing

$$\text{Sommerfeld number} = \frac{ZN}{P}\left(d/c\right)^2$$

For design purpose, its value is taken as follows;

$$\frac{ZN}{P}\left(d/c\right)^2 = 14.3\times10^6$$

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Heat generated in Bearing:-

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion.

Mathematically, heat generated in a bearing,

$$Q_g = \mu . W . V \quad N\text{-}m/s \ (or) \ J/s \ (or) \ Watts$$

μ = coefficient of friction

W = Load on the Bearing in N.

= Pressure on the bearing in N/mm^2 X projected area of the Bearing in mm^2

$= p(l \times d)$

V = Rubbing velocity in m/s $= \frac{\pi d N}{60}$

d in meters.

N = Speed of the journal in r.p.m

After the thermal Equilibrium has been reached, heat will be dissipated at the outer Surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat dissipated will depend upon the temp difference, size and Mass of the radiating surface and on the amount of air flowing around the bearing. However, for the convenience in bearing design, the actual heat dissipating area may be Expressed in terms of projected area of the journal.

(1 J/s = W)

Heat dissipated by the bearing.

$$Q_d = CA(t_b - t_a) \quad J/s \ (or) \ W$$

C = Heat dissipation Coefficient in $W/m^2/°C$

A = Projected area of the bearing in $m^2 = l \times d$

t_b = Temperature of Bearing surface in °c &

t_a = Temperature of surrounding air in °C

The value of 'C' depend upon the type of bearing, its ventillation and temp differance.

The Avg value of c (in $W/m^2/°c$) for journal bearing is

For unventilated bearing (still air) = 140 to 420 $W/m^2/°c$

For well ventilated Bearing = 490 to 1400 $W/m^2/°c$

The temp of the Bearing (t_b) is approximately Mid-Way b/w temp of oil film (t_o) & temp of outside air (t_a).

$$t_b - t_a = \frac{1}{2}(t_o - t_a)$$

Note: (i) For well designed bearing, the temp of the oil film should not be more than 60°c, otherwise viscosity of the oil decreases rapidly and the operation of the bearing is found to suffer. The temp of the oil film is often called as the 'operating temp' of the Bearing.

(ii) In case the temp of oil film is higher, then the Bearing is cooled by circulating water through coils buit in the bearing.

(iii) The mass of the oil to remove the heat generated at the bearin may be obtained by equating the heat generated to the heat taken away by the oil

$$Q_t = m.S.t \quad J/s \text{ (or) } W$$

m = Mass of the oil in kg/s

S = Specific heat of the oil. Its value may be taken as 1840 to 2100 J/kg/°c

t = Differance b/w outlet & Inlet temp of oil in °C

Design procedure for Journal Bearing:-

In designing journal Bearings. When the Bearing load, the dia & speed of shaft are known.

1. Determine the Bearing length by choosing a ratio of $\frac{l}{d}$
2. check the Bearing pressure, $P = \frac{W}{ld}$
3. Assume a lubricant & it's operating temp (t_o). This temp should be b/w 26.5°c & 60°c with 82°C as a maximum for high temp. installations such as steam turbines.
4. Determine the operating value of $\frac{ZN}{P}$ for the assumed bearing-temp & check this value with corresponding values. to determine the possibility of maintaing fluid film operation.
5. Assume a clearance ratio $\frac{c}{d}$
6. Determine the Co-effecient of friction
7. Determine the heat generated
8. Determine the Heat dissipated
9. Determine thermal Equillibrium to see that the heat generated dissipated becomes atleast equal to the heat generated. In case of the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially Cooled by Water.

$$\mu = \left[\frac{33.25}{10^8} \times \frac{ZN}{P} \times \frac{d}{c}\right] + k$$

$Z = kg/_{m\text{-}s}$ $\quad P = N/_{mm^2}$

$$\mu = \left[\frac{33.25}{10^{10}} \times \frac{ZN}{P} \times \frac{d}{c}\right] + k$$

$P = kgf/_{cm^2}$, Z = centipoise

k = correction factor for end leakage.

It depends upon l/d

(i) Heat generated

$H_g = \mu W V$ $\quad N\text{-}m/_{sec}$ (or) $J/_s$ (or) Watts

μ = coeffecient of friction. calculated using the relation.

W = load on the Bearing in Newtons.

V = sliding velocity of the journal m/sec.

$= \frac{\pi d n}{60}$ $\quad d \Rightarrow$ meters, $n \Rightarrow$ r.p.m

(ii) According to M.K.S.

$H_g = \frac{\mu W V}{J}$ kcal/min. (or) $\mu W V$ kgf-m/min

W = load in (kgf)

v = velocity in m/min.

J = Joule's coffeciest = 427 kgf-m/kcal

μ = coefficient of friction.

Heat dissipated by the Bearing is given by:

$$H_d = CA(t_b - t_a) \text{ Watts.}$$

$$= CA(t_b - t_a) \text{ Kcal/min.}$$

C = Heat dissipation coeffecient.

$= W/m^2/°c$ (or) $(Kcal/min/cm^2/°c)$

$= 140$ to 240 $W/m^2/°c$ (or) $[2 \times 10^{-4}$ to 6×10^{-4} $Kcal/min/cm^2/°c]$

for unventilated bearings (still air)

$= 490$ to 1400 $W/m^2/°c$ (or) $[7 \times 10^{-4}$ to 20×10^{-4} $Kcal/min/cm^2/°c]$

for ventilated bearings.

A = projected area of the Bearing in $m^2 = l \times d$

t_b = temp of the Bearing surface °C

t_a = temp of the Surrounding Air °C

Note

$$1 \, Kal/min/cm^2/°c = 427 \; kgf-m/min/cm^2/°c$$

$$= 427 \times 9.81 \; N-m/min/cm^2/°c$$

$$= \frac{42700}{60} \times 10^4 \; N-m/s/m^2/°c$$

$$= 7 \times 10^5 \; W/m^2/°c$$

The mass of oil

$$H_g = H_t = m c_p \Delta t \quad J/s \text{ (or) Watts} \Rightarrow m = \frac{H_g}{c_p \Delta t}$$

m = kg/sec

c_p = Specific Heat of the oil

1840 to 2100 $J/kg/°c$

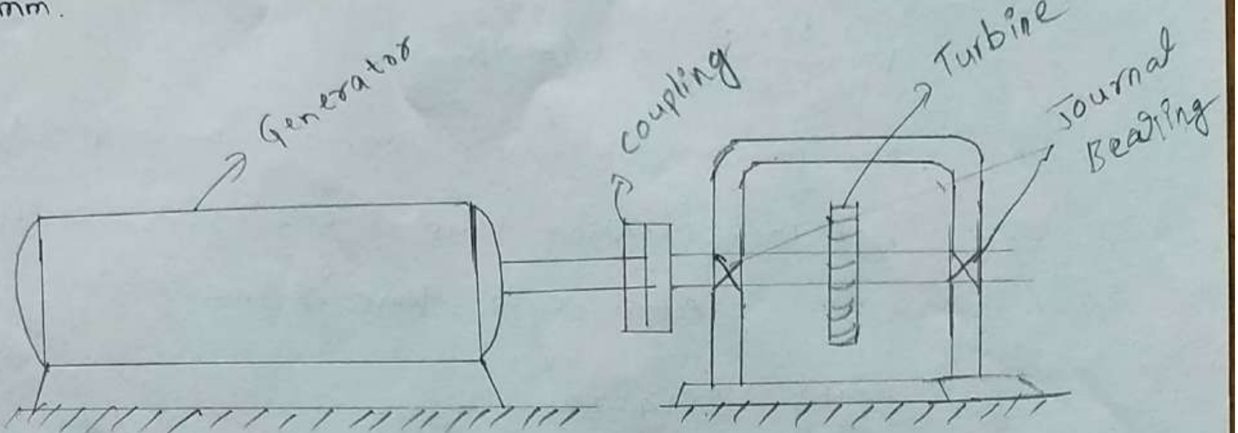
Δt = Diff b/w outlet & Inlet Temp of oil in °c

MKS

$H_t = m c_p \Delta t$ Kcal/min

m = kg/min $\quad c_p$ = Kcal/kg/°c.

Design a journal Bearing for a steam turbine, whose shaft is supported on two bearings one at each side of the turbine, and is coupled with a generator for power production. The weight of turbine with shaft is measured as 40 kN and the shaft rotates at 1500 rpm. Diameter of the shaft is 100 mm.



Given

$W = 40 kN = 40 \times 10^3 N$

Load acting on one bearing $W = 20 \times 10^3 N$

$N = 1500$ r.p.m

$d = 100$ mm

$H_g = \mu W V$ J/s (or) Watts

$$\mu = \left[\frac{33.25}{10^8} \times \frac{ZN}{P} \times \frac{d}{c}\right] + k$$

Z = Absolut viscosity in kg/m-s

P = pressure developed in the bearing over the projected area in N/mm²

d = Dia of the journal

c = Diametral clearance $= 0.001 d$ (or) $\frac{c}{d} = 1000$

k = Correction factor for leakage

l = length of journal

d = dia of Journal = 100mm

$P = \frac{W}{l.d}$

$\frac{l}{d} = 1.2 \Rightarrow l = 1.2 \times d = 1.2 \times 100 = 120\ mm$

$P = \frac{20 \times 10^3}{120 \times 100} = 1.67\ N/mm^2$

The pressure is within the safe range [0.7 to 2 N/mm^2]

Let the operating temp of Bearing, $t_b = 75°C$

Atmospheric temp ⇒ $t_a = 30°C$

$\left(\frac{ZN}{P}\right)_{min} = 14.22$

$Z_{min} = 14.22 \times \frac{P}{N} = \frac{14.22 \times 1.67}{1500} = 0.016\ kg/m\text{-}s$

The min viscosity required for the lubricant is 0.016 kg/m-s the next standard lubricant i.e SAE 30 oil will be selected whose Actual viscosity is 0.017 kg/m-s

$\mu = \left[\frac{33.25}{10^8} \times \left(\frac{ZN}{P}\right) \times \left(\frac{d}{c}\right)\right] + k$ $\left[\frac{l}{d} = 1.2,\ k = 0.002\right]$

$= \left[\frac{33.25}{10^8} \times \frac{0.017 \times 1500}{1.67} \times 1000\right] + 0.002$

$= [0.005 + 0.002] + 0.002$

$\mu = 0.007$

$V = \frac{\pi d N}{60} = \frac{\pi \times 100 \times 1500}{60} = 7.85\ m/sec$

$H_g = \mu W V$ Watts.

$= 0.007 \times 20 \times 10^3 \times 7.85$

$= 1099$ Watts.

$H_d = CA(t_b - t_a)$ Watts.

$C = 1000\ W/m^2/°c$ (Assumed)

$A =$ projected Area $m^2 = l \times d$

$= \dfrac{100 \times 120}{1000 \times 1000} = (0.1 \times 0.12)\,m^2$

$H_d = 1000 \times 0.1 \times 0.12 \times (75 - 30)$

$= 540$ Watts.

Dia of Bearing $D = d + c = d + \dfrac{d}{1000} = d(1 + 0.001)$

$= 100(1.001)$

$D = 100.1$ mm

Specifications for Hydro-dynamic full journal Bearing.

(i) Dia of the journal (d) = 100 mm

(ii) Length " " (l) = 120 mm

(iii) Dia of the Bearing (D) = 100.1 mm

(iv) Dia of clearance (c) = 100 microns (= 0.1 mm)

(v) Lubricant selected = SAE 30 oil

(vi) Operating temp = 75°c

(vii) Atmospheric temp = 30°c

(viii) Material selected = Bronze bushing (or) Heavy babbit liner on steel (or) CI

Design a journal Bearing for generator to the following specifications.

Load on the journal = 1200 kgf

Diameter of the journal = 75mm.

Speed of the journal = 1400 r.p.m.

Given $W = 1200$ kgf (or) $1200 \times 9.81N = 11772$ N

$d = 75mm = 7.5cm$

speed $N = 1400$ r.p.m

For generator $\frac{l}{d} = 1$ to 2

$\frac{l}{d} = 1.5$

$l = 1.5 \times 7.5 = 11.25$ cm

$P = \frac{W}{l \times d} = \frac{1200}{7.5 \times 11.25} = 14.22$ kgf/cm^2

Since the safe pressure range is 7 to 14 kgf/cm^2

$\frac{l}{d} = 1.73$

$l = 1.73 \times 7.5 = 13$ cm

$P = \frac{1200}{13 \times 7.5} = 12.3$ kgf/cm^2

Bearing pressure is within the safe range

To find out the value of Z. (the viscosity), the min value of

$\frac{ZN}{P} = 2845$

$Z_{min} = \frac{2845 \times 12.3}{1400} = 25$ centipoise

∴ Mini viscosity required = 25 cp

Let Assume operating temp = 70°c

Atmospheric temp = 25°c

Hence the lubricant oil selected is SAE 40 oil, which is having Viscosity of 27 cp at 70°c.

$$H_g = \frac{\mu W V}{J} \text{ Kcal/min.}$$

$$\mu = \left[\frac{33.25}{10^{10}} \times \frac{ZN}{P} \times \frac{d}{c}\right] + K$$

$$c = 0.001d$$

$$\mu = \left[\frac{33.25}{10^{10}} \times \frac{27 \times 1400}{12.3} \times 1000\right] + 0.0025$$

$$= 0.013$$

$\left[\frac{l}{d} = 1.73 \text{ for } k = 0.0025\right]$

$$V = \pi d n \text{ m/Min}$$

$$= \pi \times \frac{75}{100} \times 1400$$

$$V = 330 \text{ m/Min}$$

$$J = 427 \text{ kgf-m/kcal}$$

$$H_g = \frac{\mu W V}{J} = \frac{0.013 \times 1200 \times 330}{427}$$

$$= 12 \text{ kcal/Min}$$

$$H_d = CA(t_b - t_a) \text{ Kcal/min}$$

$C = 15 \times 10^{-4}$ kcal/min/°c for ventilated condition

A = Projected Area cm^2 = 0.7.5 × 13 = 97.5 cm^2

$$t_b - t_a = 70 - 25 = 45^\circ c$$

$$H_d = 15 \times 10^{-4} \times 97.5 \times 45$$

$$= 6.58 \text{ kcal/min}$$

$H_g > H_d$ Artificial cooling must be arranged

$c = D - d \Rightarrow D = d + c = 75 + (0.001 \times 75)$ $[c = 0.001d]$

$$= 75.075 \text{ mm}$$

Specifications:

(i) Type of Bearing = Hydro dynamic full journal Bearing.

$l = 130$ mm

$d = 75$ mm

$D = 75.075$ mm

Lubricant = SAE 40 oil

$t_b = 70^\circ c$

$t_a = 25^\circ c$

Material - Bronze Bushing

① The load on the Journal bearing is 150 kN due to turbine shaft of 300 mm dia. running at 1800 r.p.m. Determine the following:

1. Length of the Bearing if the allowable bearing pressure is. 1.6 N/mm^2 &

2. Amount of heat to be removed by the lubricant per minute if the Bearing temp. is 60°C and viscosity of the oil at 60°C is 0.02 kg/m-s & the Bearing clearance is 0.25 mm.

Given

$W = 150\ kN = 150 \times 10^3\ N$

$d = 300\ mm = 0.3\ m$

$N = 1800\ r.p.m$

$p = 1.6\ N/mm^2$

$Z = 0.02\ kg/m\text{-}s$

$c = 0.25\ mm.$

1) Length of the Bearing:

l = length of Bearing in mm

projected area $A = l \times d = l \times 300 = 300\, l\ mm^2$

Allowable Bearing pressure $p = \frac{W}{A} = \frac{W}{l \times d}$

$$1.6 = \frac{150 \times 10^3}{300\, l}$$

$$l = 312.5\ mm$$

2. Amount of Heat to be removed by the lubricant:

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{P}\right)\left(\frac{d}{c}\right) + k$$

$$= \frac{33}{10^8}\left(\frac{0.02 \times 1800}{1.6}\right)\left(\frac{300}{0.25}\right) + 0.002$$

$$= 0.009 + 0.002$$

$$= 0.011$$

Rubbing Velocity $v = \frac{\pi d N}{60} = \frac{\pi \times 0.3 \times 1800}{6} = 28.3\ m/s$

$Q_g = \mu W V$

$= 0.011 \times 150 \times 10^3 \times 28.3$

$= 46,695$ J/s (or) W

$= 46.695$ kW

② Design a journal Bearing for a Centrifugal pump from the following data; Load on the journal = 20000 N; speed of the journal = 900 rpm. Type of oil is SAE 10, for which absolute viscosity at 55°C = 0.017 kg/m-s; Ambient temp of oil = 15.5°C, Max. Bearing pressure for the pump = 1.5 N/mm².

Calculate also mass of the lubricated oil required for artifical cooling, if rise of temp of oil be limited to 10°C. Heat dissipation Coefficient = 1232 W/m²/°C.

Given.

$W = 20000$ N

$N = 900$ r.p.m

$t_o = 55°C$

$Z = 0.017$ kg/m-s

$t_a = 15.5°C$

$p = 1.5$ N/mm²

$t = 10°C$

$C = 1232$ W/m²/°C

(i) Let us find length of the journal (l). Assume the dia. of journal (d) as 100 mm

$\frac{l}{d}$ ratio for centrifugal pump varies from 1 to 2

let take $\frac{l}{d} = 1.6$

$l = 1.6 \times d = 1.6 \times 100 = 160$ mm

$$p = \frac{W}{ld} = \frac{20000}{160 \times 100} = 1.25$$

Since the bearing pressure for the pump is 1.5 N/mm²

∴ 'p' value is safe hence the dimensions of l & d are safe

(iii) $\frac{ZN}{P} = \frac{0.017 \times 900}{1.25} = 12.24$

operating value of $\frac{ZN}{P} = 28$

that the min. value of the bearing Modulus at which the oil film will break is given by.

$$3k = \frac{ZN}{P}$$

∴ Bearing Modulus at the Mini point of friction

$$k = \frac{1}{3}\left(\frac{ZN}{P}\right) = \frac{1}{3} \times 28 = 9.33$$

Since the calculated value of Bearing characteristic $\left[\frac{ZN}{P} = 12.24\right]$ is more than 9.33

∴ The Bearing will operate under Hydrodynamic conditions

(iv) For centrifugal pumps. the clearance ratio $[c/d] = 0.0013$

(v) The co-effecient of friction

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{P}\right)\left(d/c\right) + k$$

$$= \frac{33}{10^8} \times 12.24 \times \frac{1}{0.0013} + 0.002$$

$$= 0.0031 + 0.002$$

$$= 0.0051$$

(vi) Heat Generated

$$Q_g = \mu W V = \mu W\left(\frac{\pi d N}{60}\right)$$

$$= 0.0051 \times 20000\left(\frac{\pi \times 0.1 \times 900}{60}\right)$$

$$= 480.7\ W$$

(vii) Heat dissipated

$$Q_d = CA\,(t_b - t_a) = C l d\,(t_b - t_a) \text{ Watts.}$$

$$(t_b - t_a) = \frac{1}{2}(t_o - t_a)$$

$$= \frac{1}{2}(55^\circ - 15.5^\circ)$$

$$= 19.75^\circ C$$

$$Q_d = 1232 \times 0.16 \times 0.1 \times 19.75$$

$$Q_d = 389.3 \text{ W}$$

The Heat generated is greater than the heat dissipated which indicates that the bearing is warming up.

Either the Bearing should be redesigned by taking $t_o = 63^\circ C$ the Bearing should be cooled artificially.

Amount of Artifical cooling required

= Heat Generated − Heat dissipated

$= Q_g - Q_d$

$= 480.7 - 389.3$

$= 91.4$ kW

Mass of lubricating oil required for artificial cooling

m = Mass of the lubricating oil required for artificial cooling in kg/s

Q_t = Heat taken away by the oil

$Q_t = m.S.t$

$= m \times 1900 \times 10$

$Q_t = 19000\, m$

[∵ Specific Heat of oil ⇒ S = 1840 to 2100 J/kg/°c]

Equating this to the amount of artificial cooling required.

$$19000m = 91.4$$

$$m = \frac{91.4}{19000} = 0.0048 \text{ kg/sec}$$

$$m = 0.288 \text{ kg/min}$$

A 80 mm long journal Bearing support a load of 2800N on a 50 mm dia shaft. The Bearing has radial clearance of 0.05 mm the viscosity of oil is 0.021 kg/m-s at the operating temp. If the Bearing is capable of dissipating 80 J/s, determine the Max. safe speed

Given $l = 80\text{mm}$

$W = 2800\text{ N}$

$d = 50\text{mm} = 0.05\text{m}$

$\frac{c}{2} = 0.05\text{ mm}$

$c = 0.1\text{ mm}$

$Z = 0.021\text{ kg/m-s}$

$Q_d = 80\text{ J/s}$

$N =$ Max. safe speed in r.p.m

We know that the Bearing pressure

$$p = \frac{W}{l \times d} = \frac{2800}{80 \times 50} = 0.7\text{ N/mm}^2$$

Co-effecient of friction.

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + 0.002$$

$$= \frac{33}{10^8}\left(\frac{0.021 \times N}{0.7}\right)\left(\frac{50}{0.1}\right) + 0.002$$

$$= \frac{495N}{10^8} + 0.002$$

$\therefore$ Heat Generated, $Q_g = \mu W V = \mu W\left(\frac{\pi d N}{60}\right)$

$$= \left(\frac{495N}{10^8} + 0.002\right) 2800 \left(\frac{\pi \times 0.05N}{60}\right)$$

$$= \frac{3628N^2}{10^8} + 0.01466N$$

Equating the Heat Generated to the Heat dissipated.

$$\frac{3628\, N^2}{10^8} + 0.01466\, N = 80$$

$$N^2 + 404 N - 2.2 \times 10^6 = 0$$

$$N = \frac{-404 \pm \sqrt{(404)^2 + 4 \times 2.2 \times 10^6}}{2}$$

$$= \frac{-404 \pm 2994}{2}$$

$$N = 1295 \text{ r.p.m}$$

A 150mm diameter shaft supporting a load of 10 kN has a speed of 1500r.p.m. The shaft runs in a Bearing whose length is 1.5 times the shaft diameter. If the diametral clearance of the Bearing is 0.15mm and the absolute viscosity of the oil at the operating temp. is 0.011 kg/m-s, find the power wasted in frict

Given $d = 150\text{mm} = 0.15\text{m}$

$W = 10\text{kN} = 10 \times 10^3 \text{ N}$

$N = 1500 \text{ r.p.m}$

$l = 1.5 d$

$c = 0.15 \text{ mm}$

$Z = 0.011 \text{ kg/m-s}$

$$l = 1.5 d = 1.5 \times 150 = 225 \text{ mm}$$

$$p = \frac{W}{A} = \frac{W}{ld} = \frac{10 \times 10^3}{225 \times 150} = 0.296 \text{ N/mm}^2$$

Coeffecient of friction

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k$$

$$= \frac{33}{10^8}\left(\frac{0.011 \times 1500}{0.296}\right)\left(\frac{150}{0.15}\right) + 0.002$$

$$= 0.018 + 0.002$$

$$= 0.02$$

Rubbing velocity $V = \frac{\pi d N}{60}$

$$= \frac{\pi \times 0.15 \times 1500}{60}$$

$$= 11.78 \text{ m/s}$$

Heat Generated due to friction,

$$Q_g = \mu W V$$

$$= 0.02 \times 10000 \times 11.78$$

$$Q_g = 2356 \text{ W}$$

Power Wated in friction $= Q_g = 2356 \text{ W} = 2.356 \text{ kW}$

The load on a 100 mm full hydro-dynamic journal Bearing is 9000 N Speed of the journal is 320 r.p.m

Let $\frac{l}{d} = 1$, $\frac{c}{d} = 0.0011$. The operating temp = 65°c and

Min oil film thickness = 0.022 mm

(i) Select an oil that will closely accord with the stated conditions. For the selected oil Determine

(ii) The friction loss.

(iii) The Hydro dynamic oil flow through the Bearing.

(iv) Amount of leakage.

(v) Temp. rise of oil passes through the Bearing

(vi) Max. oil pressure.

Given $W = 9000 N$

$d = 100 mm$, $N = 320 rpm$

$\frac{l}{d} = 1$, $\frac{c}{d} = 0.0011$

Min oil film thickness $h_o = 0.022$ mm

Operating temp = 65°c

$l = d = 100 mm$

$c = 0.0011 d = 0.0011 \times 100 = 0.11$ mm

Sommerfeld number, $S = \frac{Zn'}{p}\left(\frac{d}{c}\right)^2$

$Z = \frac{N-S}{m}$ (or) $\frac{kg}{m\text{-}s}$

$n' = rps$

$P = N/m^2$

$$P = \frac{W}{l \times d} = \frac{9000}{\frac{100}{1000} \times \frac{100}{1000}}$$

$P = 9 \times 10^5 \; N/m^2$

$$n' = \frac{320}{60} = 5.33 \; rps$$

$$\frac{2h_o}{c} = \frac{2 \times 0.022}{0.11} = \frac{0.044}{0.11} = 0.4$$

$\frac{2h_o}{c} = 0.4$, $S = 0.121$ (JDB 19.16) Table

$$S = \frac{Zn'}{P}\left(\frac{d}{c}\right)^2$$

$$0.121 = \frac{Z \times 5.33}{9 \times 10^5}\left(\frac{1}{0.0011}\right)^2$$

$$Z = 0.0247 \frac{N\text{-}s}{m^2} = 0.025 \frac{kg}{m\text{-}s}$$

(i) The next standard oil, SAE30 oil will closely accord with the stated condition, whose viscosity is 0.0255 kg/m-s

$S = 0.121$

$\mu\frac{d}{c} = 3.22$

$\mu = 3.22 \times \frac{c}{d} = 3.22 \times 0.0011 = 0.0035$

frictional force: $\mu W = 0.0035 \times 9000 = 31.5$ N

$$V = \frac{\pi d n}{60} = \frac{\pi \times 100 \times 320}{60 \times 100} = 1.68 \text{ m/s}$$

(ii) power loss due to friction $P_f = \mu W V$

$= 31.5 \times 1.68$

$= 52.92$ N-m/s

$= 53$ Watts

(iii) Total oil flow

$$\frac{4q}{dcn'l} = 4.33 \Rightarrow q = \frac{4.33 \times dcn'l}{4}$$

$$q = \frac{4.33 \times 0.1 \times 0.11 \times 10^{-3} \times 5.33 \times 0.1}{4}$$

$$q = 6.35 \times 10^{-6} \text{ m}^3/\text{sec}$$

(iv) Side leakage:

$$\frac{q_s}{q} = 0.68$$

$$q_s = q \times 0.68 = 0.68 \times 6.35 \times 10^{-6} = 4.32 \times 10^{-6} \ m^3/s$$

(v) Temperature rise:

$$\frac{\rho c' \Delta t_o}{P} = 14.2$$

$$\Delta t_o = \frac{14.2 \times P}{\rho c'} = \frac{14.2 \times 9 \times 10^5}{142 \times 10^4} = 9^\circ c$$

Rise in temp $= 5^\circ c$

(vi) Max. pressure:

$$\frac{P}{P_{max}} = 0.415$$

$$P_{max} = \frac{P}{0.415} = \frac{9 \times 10^5}{0.415} = 21.7 \times 10^5 \ N/m^2$$

Maxi. oil pressure $= \underline{21.7 \times 10^5} \ N/m^2$

PETROFF'S EQUATION:

Petroff's Equation is used to determine the coeffecient of friction in journal Bearings. It is Based on the following Assumptions:

(i) The shaft is concentric with the Bearing.

(ii) The Bearing is subjected to light load.

A vertical shaft rotating in the Bearing is shown in fig.

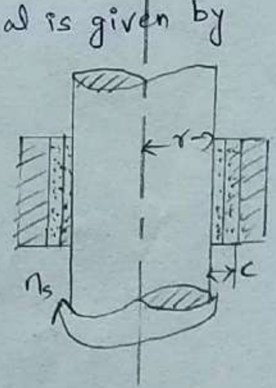
r = radius of journal (mm)

l = length of the Bearing (mm)

c = Radial clearance

n_s = journal speed (rev/sec)

The velocity at the surface of the journal is given by



$$U = (2\pi r)\, n_s$$

$$F = Z A \left(\frac{U}{h}\right)$$

F = Tangential frictional force

A = Area of journal Surface $= (2\pi r)\, l$

U = Surface velocity $= (2\pi r)\, n_s$

h = distance b/w journal and Bearing surfaces = c

$$F = Z(2\pi r l)(2\pi r n_s)\tfrac{1}{c} = \frac{4\pi^2 r^2 l Z n_s}{c}$$

Torque $T = F \times r$

$$T = \frac{4\pi r^3 l Z n_s}{c}$$

Let us consider a radial force (W), acting on the Bearing

$$P = \frac{W}{\text{projected area of bearing}} = \frac{W}{2rl}$$

$$W = 2prl$$

The frictional force will be μW

frictional torque $T = \mu W r$

$= \mu (2pl) r$

$= \mu \, 2pr^2 l$

$$\frac{4\pi^2 r^3 l Z n_s}{c} = \mu (2pr^2 l)$$

$$\mu = 2\pi^2 \left(\frac{r}{c}\right)\left(\frac{Z n_s}{P}\right)$$ (Petroff's Equation).

$\therefore \quad F \propto \frac{AV}{h}$

$F = \frac{Z A V}{h}$

S.I. $Z = \frac{Fh}{AV} = \frac{Nm}{m^2 \times m/s} = \frac{N\text{-}s}{m^2}$ (or) $= \frac{kg\, m/s^2 \times s}{m^2} = \frac{kg}{m\text{-}sec}$

$\begin{bmatrix} F = ma \\ F = kg\, m/s^2 \end{bmatrix}$

MKS = $Z = 1\, kgf\text{-}sec/m^2$

CGS Z = Poise

1 Poise $= 1$ dyne-sec/cm²

$= 1\, gm \frac{cm}{sec^2} \times \frac{sec}{cm^2}$

1 Poise. $= 1 \frac{gm}{cm\text{-}sec}$

[1 centi pois = $\frac{1}{100}$ poise]

$1 \frac{kg}{m\text{-}sec} = \frac{1000\, gm}{100\, cm\text{-}se}$

$= 10 \frac{gm}{cm\text{-}sec}$

$\boxed{1 \frac{kg}{m\text{-}sec} = 10 \text{ poise.}}$

Kinematic viscosity $\nu = \frac{Z}{\rho}$

C.G.S = cm²/sec. as stokes. $= \frac{1\, gm}{cm\text{-}sec} \times \frac{cm^3}{gm} = 1\, cm^2/sec$

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ROLLING CONTACT BEARINGS

Introduction to rolling contact Bearings:

Rolling contact Bearings are also called antifriction Bearings (or) simply Ball Bearings. Rolling Elements such as balls (or) Rollers, are introduced b/w the surfaces that are in relative motion.

Rolling contact Bearings are used in the following applications.

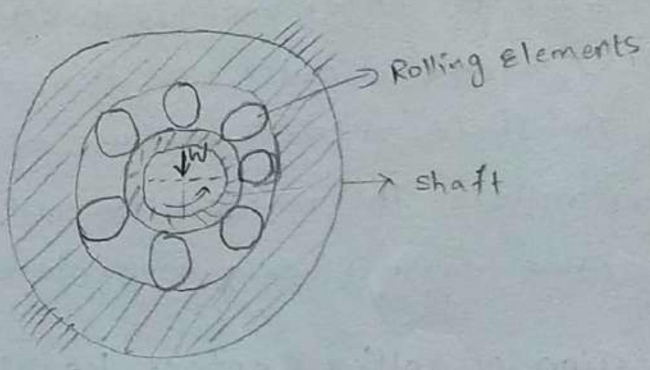
(i) Machine tool spindle.

(ii) Automobile front and rear axle

(iii) Gear Boxes.

(iv) Small size Electric motors, and

(v) Rope sheaves, crane Hooks and Hoisting drums.



Rolling Contact Bearings

Advantages and disadvantages of rolling contact Bearings over Sliding contact Bearings:

Advantages:

1. Low starting and running friction Except at very high speeds.
2. Ability to withstand momentry shock loads.
3. Accuracy of shaft alignment.
4. Low cost of maintance, as no lubrication is required while in service.
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and Erect.

8. cleanliness.

Disadvantages:

1. More noisy at very high speeds.

2. Low resistance to shock loading.

3. More initial cost.

4. Design of Bearing housing complicated

Components of Rolling Bearings:-

The rolling Bearing consits of four main components.
1) Inner ring, 2) outer ring, 3) the Balls (or) rollers and 4) retainers-
-(or) Seperators

The Inner ring (or) Inner race is force fitted with machine shaft and outer ring (or) outer race is force fitted with machine-
-housing. The shaft rotates because of relative rotations of Balls (or) rollers. The retainer is used to prevent the Balls (or) rollers Exiting from Bearing rings during operation.

classification of rolling Contact Bearings:

classified into two major groups with respect to their structure.

(i) Ball Bearings

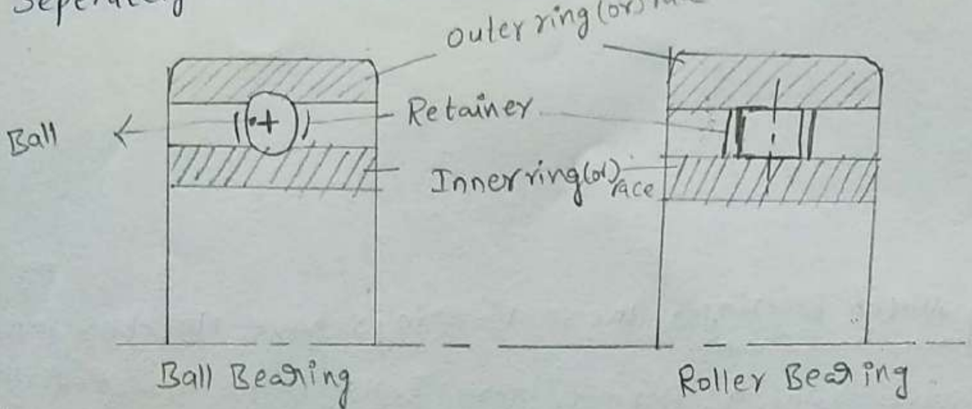
(ii) Roller Bearings.

Basically the structure of Ball and roller Bearings are Similar except that whether the rolling Elements b/w the inner ring & outer ring are balls (or) rollers.

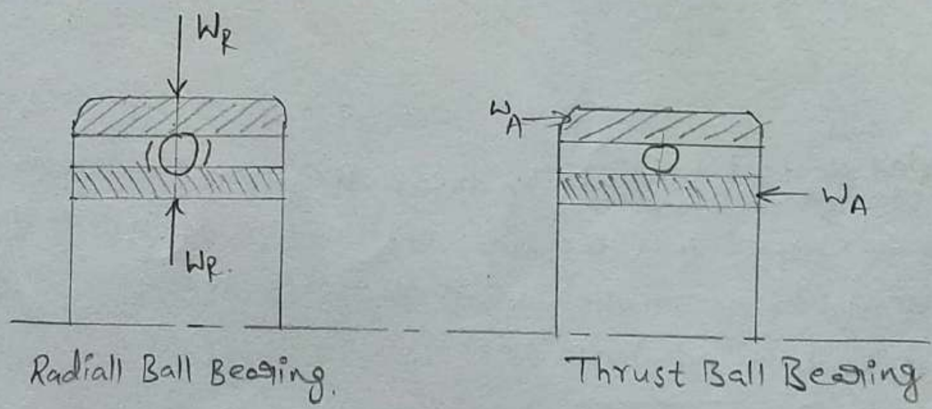
∴ Generally the Ball Bearings are used for light loads an the roller Bearing for heavier loads.

In case of Ball Bearings Nature of Contact is point cont So friction produced is very low.

* In roller Bearing where Nature of contact is the line contact which produce more friction.
* Both type of Bearings can carry radia loads & Axial loads seperately or in combined form.



Depend with respect to load the
(i) Radial Ball Bearings and Radial roller Bearings.
(ii) Thrust Ball Bearing & Thrust roller Bearings.



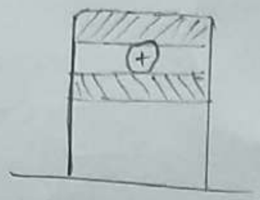
Types of Radial Ball Bearings:

1) Deep Groove Bearing :- The most frequently used Bearing is deep groove Bearing. It is found in almost all kinds of products in general Mechanical Engg.
(i) Deep Groove Ball Bearing has high load carying capacity.
(ii) It takes the load in radial as well as Axial
(iii) Generates less noise due to point contact.
(iv) Excellent performance in High speed also.

Disadvantages:

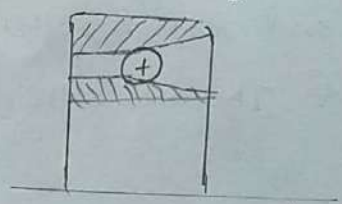
(i) It is not self-aligning. Accurate alignment b/w axes of the shaft and housing bore is required.

(ii) It is poor rigidity.



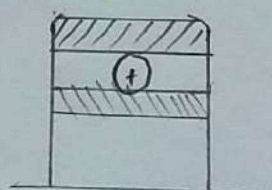
Deep groove Ball Bearing..

Filling Notch Bearing:- These Bearings have Notches inner and outer race which permit more Balls to be inserted than deep groove Ball Bearing. large Bearing load capacity..



Filling Notch. Bearing:

Angular contact Bearing:- In these, the grooves in inner and outer race are so shaped that the line of reaction at the contact b/w balls and races makes an angle with the axis of the Bearing. can take Both radial and Axial load..



Angular contact Bearings.

Self-Aligning Bearings:- These Bearing permit shaft deflections within 2°-3°. It may be noted that normal clearance in a Ball Bearing are too small to accommodate any appreciable missalignment of the shaft relative to the housing. If the unit is assembled with shaft misalignment present, then the bearing will be subjected to a load that may be in excess of the design value and premature failure may occur.

Thrust Ball Bearing:- Thrust ball Bearing carries thrust load in only one direction and cannot carry any radial load. The use of a large number of balls results in high thrust load carrying capacity in smaller space. This is major advantage in thrust Bearing.

Disadvantages: (i) Thrust Ball Bearing cannot take radial loads.

(ii) It is not self-aligning and cannot tolerate misalignment.

(iii) Their performance is satisfactory at low & medium speeds. At high speeds, such Bearings give poor service Because the Balls are subjected to centrifugal forces and Gyroscopic couple.

(iv) Thrust Ball Bearings donot operate as well on horizental shafts as they do on vertical shafts.

Types of Roller Bearings:-

(i) cylindrical roller Bearings: When Maximum load carrying capacity is required going to use cylindrical Bearings.

Advantages:

(i) Dueto line contact b/w rollers and races, the radial load carrying capacity of the cylindrical roller Bearing is very high.

(ii) cylindrical roller Bearing is more rigid than Ball Bearing.

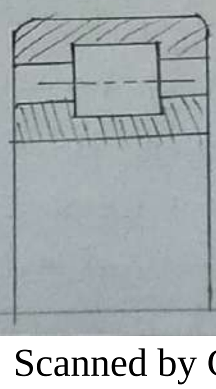
(iii) The co-effecient of friction is low and frictional loss is less in high-speed Applications.

Disadvantages:

(i) cylindrical roller Bearings cannot take thrust load.

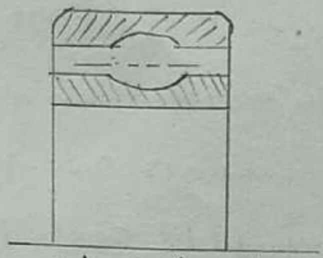
(ii) These are not self-aligning.

(iii) These Generates more noise.



cylindrical roller Bearings

2) Spherical Roller Bearings:- These Bearings are self-aligning Bearings. The self-aligning feature is achived by grinding one of the races in the form of sphere. Thes Bearings can normally tolerate angular misalignment in order of $\pm 1\frac{1}{2}°$ and when used with a double row of rollers, these can carry thrust loads in Either direction.

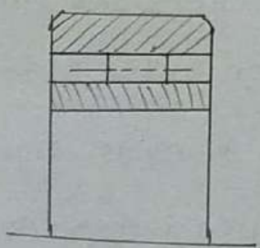


Spherical roller.

3) Needle roller Bearing:- These Bearings are relatively slender and completely fill the space so that neither a cage nor a retainer is needed.

These Bearings are used when heavy loads are to be carried with an oscillatory motion.

E.g:- Piston pin, Bearings in Heavy duty diesel Engines, Where the reversal of motion tends to keep the rollers in correct alignment



-: Needle roller :-

4) Taper roller Bearing:- It consits of rolling Elements in the form of a frustum of cone. They are arranged in such a way that that the axes of individual rolling Elements intersect in a common apex point on the axes of the Bearing. In these the line of resultant reaction through the rolling Elements makes an angle with the the axis of Bearing. Therfore taper roller Bearing can carry Both radial and Axial loads.

Advantages:

(i) Taper roller Bearing can take heavy radial and thrust loads.

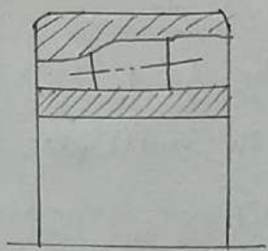
(ii) Taper roller Bearings has more rigidity.

(iii) It can be easily assembled and

Disadvantages:

i) These cannot tolerate misalignment b/w the axes of the shaft and the housing bore.

(ii) Taper roller Bearings are costly.



:- Tapered roller Bearing :-

Materials used for rolling Bearing:-

(i) Ball Bearings (i.e the inner ring, outer ring and balls) are commonly made of High carbon chromium steel that is through Hardened to rockwell C58-65.

(ii) Roller Bearings are usually fabricated of case Hardened (i.e carburizing grade) steel. The steels are Surface Hardened to Rockwell C58-63.

(iii) Ball Seperator (i.e cages) are normally made of low carbon Steel.

(iv) Seals are made of low carbon steel. sheilds that are used to retain grease and to prevent chips, dirt, etc also.

DESIGNATING THE ROLLING BEARINGS:

1) Depend upon load carrying capacities.

According to SAE (Society of Automotive Engineerings), Bearings are grouped into

(i) Very light series whose number start with 100
(ii) light " " " " " 200
(iii) Medium " " " " " 300
(iv) Heavy " " " " " 400

In any Bearing number, the last two digits are used to denote the Bore diameter in multiples of 5mm.

Thus the Bearing 315 signifies a medium series Bearing of 75mm Bore dia.

415 signifies the Heavy series Bearing of 75mm Bore.

Medium series Bearings have load carrying capacity upto 40% greater than light series. Heavy series Bearings have a load carrying capacity of about 20 to 30% greater than Medius series Bearings.

2) According to Anti-Friction Bearing Manufactures Association (AFBM and International standard organisation (ISO), designed Based on Bore diameter followed by type of Bearing and type of duty.

Eg: 30 BC 02

BC - Represents type of Bearing
30 - Bore diameter in mm
02 - light duty.

3) According to SKF (Svenska Kullager Fabriken) Ball Bearings company, Bearings are designated by different groups of Bearings.

For Example, Series started with following numbers are

60, 62, 63, 64 — For deep groove Ball Bearings.

12, 13, 22, 23 — For self-aligning Bearings.

72, 73, 32, 33 — For Angular contact Ball Bearing.

512, 513, 522, 523, 532 — For Thrust Ball Bearings.

222, 230, 231, 240, 241 — For Spherical roller Bearings.

302, 322, 323 — For taper roller Bearings.

NU2 NU22, NN30 K, NNU — for cylindrical roller Bearings.

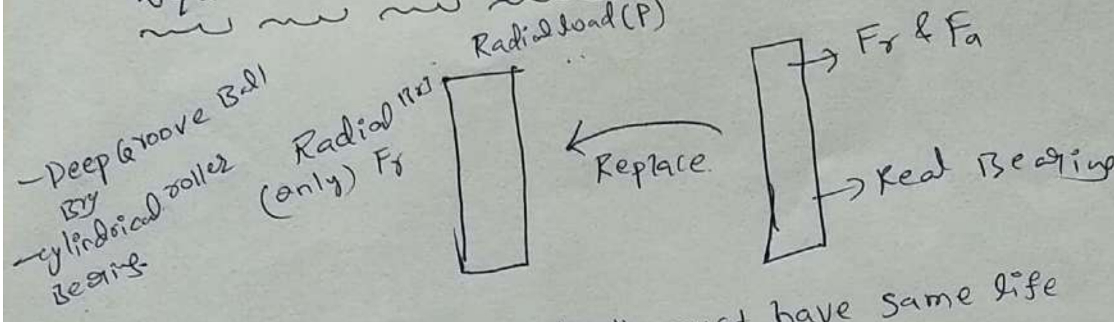
NA49, NA69, RNA49, NKXR — For needle Bearing.

Design of rolling contact Bearing:

(i) Design for static load (C_o)

(ii) Design for Dynamic load (C).

Equivalent Bearing load (P):



Both must have same life

No. of cycle sustaining is same.

Equivalent radial load acting on radial Bearing. so that both the real-
- Bearing & Radial Bearing will have same life.

$$P = XVF_r + YF_a$$

X = Radial factor

Y = Thrust factor

F_r = radial load

F_a = Thrust (Axial) load

V = Race rotation factor.

$V = 1$ Inner race rotates

$V = 1.2$ outer " "

Rating lif (L_{10}) Expected life:

The rating life of Group of similar bearing is defined as the no. of revolutions that 90% of Bearings will complete before the first evidence of fatigue crack.

(or)

The no. of cycle upto which 10% of Bearing will fail

$$Life \propto \frac{1}{(load)^k}$$

$(L_{10})_c \longrightarrow C$

$(L_{10})_p \longrightarrow P$

$$\frac{(L_{10})_c}{(L_{10})_p} = \left(\frac{P}{C}\right)^k \Rightarrow \frac{(L_{10})_p}{(L_{10})_c} = \left(\frac{C}{P}\right)^k$$

L_{10} = Number of Million revolutions.

$(L_{10})_c = 1$ m.r.

$$L_{10} = \left(\frac{C}{P}\right)^k$$

$k = 3$ Ball Brgs
$k = 10/3$ roller Brgs.

$L_{10} \times 10^6$ revolutions $= 60 N \times (L_{10})_h$ — hours

N rev per Min

60N rev per hours.

$60 N \times (L_{10})_h$ rev per hour.

$$L_{10} = \frac{60 \times N \times L_{10h}}{10^6}$$

Basic load Rating / Dynamic load capacity (C) :-

The radial load at which 90% of Group of apparently identical Bearings run for 1 million rev. before the evidence of first crack.

Selection of Bearing size:

The size of a Bearing for any application may be decided on the basis of its load carrying capacity in relation to the loads to be carried and the requirments regarding life and reliability. Generally the Basic static load rating and Basic dynamic load rating are used to express the load carrying capacities of any Bearing.

(i) Basic static load rating: (C_o):

It is used in calculations when the Bearing are rotate at, very low speeds, or to be stationary under load during certain period. It must also taken into account when heavy shockloads at short duration acting on a rotating (dynamically-stressed) Bearing. The value of C_o depend upon the Bearing Material, the number of rows of rolling Elements, the number of rolling Elements per row, the Bearing contact angle, & Ball (or) roller diameter. Normally the Basic static load rating has little influence in the selection of rolling Bearing.

(ii) Basic dynamic load rating (C):

It is also known as specific dynamic capacity is used for calculations involving dynamically stressed Bearings. i.e a Bearing which is rotating under load. It expresses a Bearing load which will give a Basic rating life of 1 million (i.e 10^6) revolutions. The Basic dynamic load rating for different Materials, depends on the same factors (C_o). Except for additional Parameter concerning the load Geometry. The dynamic load rating C Enters directly into the process of selecting a Bearing.

(iii) The life :- The life of rolling Bearings is defined as the number of revoluations (or the number of operating hours at a given constant speed) which the Bearing is capable of Enduring before the first Evidence of fatigue, that is developed in the material of Either rings or rolling Elements. It is, However, Evident from Both laboratry tests and practical Experience that seemingly identical bearings operating under identical conditions have different lives (i.e. number of hours (or) Revolutions)

All the information presented on the dynamic load rating is Based on the life of that 90% of sufficiently large group of apparently identical Bearings can be Expected to attain or exceed. This is called as the Basic rating life (or nominal life) It may also be referred as L_{10} life and this is the min. life. The majority of the Bearings attain a much longer life than this nominal life which may be known as the "median life" & it is approximately Equal to five times the "nominal life". (i.e median-life = $5 L_{10}$).

The Avg life of the Bearing is defined as the summation of all Bearings lives in series of life tests divided by the number of life tests. This Avg life is different from median-life.

Some manufactures define Avg live of a Bearing is Approximately Equal to median life i.e 5 times the nominal life

Influence of operating temp on Bearing Material:-

An Elevated temp, the [~~operating~~] Hardness of the Bearing materials is reduced and thus the dynamic load carrying capacity also reduced.

Equivalent load (P):-

Defined as that constant radial load, which, if applied to a bearing with rotating inner ring and stationary outer ring, would give the same life as that which the Bearing would attain under the actual condition of load and rotation.

In many applications, Bearings have to carry a load composed of axial and radial components. In addition, they are sometimes required to operate with a rotating outer and stationary inner ring. It then becomes necessary to express these conditions by an Equivalent load satisfying the above defination.

The Expression used to define the equivalent load is given By.

$$P = [V X F_r + Y F_a] S$$

P = Equivalent load.

F_r = Radial load

F_a = Axial load

X = Radial load factor

Y = Axial load factor

V = Race rotation factor

= 1.0 for inner ring rotation & outer ring stationary

= 1.2 for inner ring stationary & outer ring rotation

= 1.0 for inner ring or outer ring rotation in the case of Self-aligning Ball Bearing.

S = Service factor.

(or)

Equivalent radial load acting on the radial Bearing, so that Both the real Bearing & radial Bearing will have same life.

Selection of Bearings for steady loading:-

The size of Bearing required is judged by the magnitude and nature of applied load, life and reliability. The Bearing load is composed of weights involved, forces derived from Power transmitted and additional forces Based on method of operation etc.

According to AFBMA, the Bearings are selected Based on dynamic load rating 'c'. By conducting number of tests, the dynamic load capacity can be determined as,

$$C = \left[\frac{L}{L_{10}}\right]^{\frac{1}{k}} \times P$$

C = Basic dynamic load rating

L = Life of the Bearing required in million revolutions

L_{10} = Life of Bearing for 90% Survival at 1 million revolutions (or 10% failure)

P = Equivalent load

k = Exponent

= 3 for Ball Bearings.

= $\frac{10}{3}$ for roller Bearings.

The relationship b/w the life of Bearing & load capacity as

$$L = \left[\frac{C}{P}\right]^{K} \text{ million revolutions}$$

If a Bearing is required to operate for a particular life and the selected Bearing is having more life than requirement, then, the probability of selected Bearing surviving the required life is given By Expression as

$$\frac{L}{L'_{10}} = \left[\frac{\ln(1/p)}{\ln\left(\frac{1}{P_{10}}\right)}\right]^{1/b}$$

L = Required life in million revolutions

L'_{10} = Calculated life of selected Bearing for given load at 90% Survival in million rev

$$\ln\left(\frac{1}{P_{10}}\right) = \left(\frac{1}{0.9}\right) = 0.1053$$

$b = 1.17$ for a median life $= 5L_{10}$

$= 1.34$ for a median life $= 4.08\,L_{10}$

(for Deep Groove Ball Bearing)

P = Required probability of selected Bearing

For system having 'x' Bearings, each having the probability as P, then the probability of survival of $P_{system} = p^x$

Scanned by CamScanner

Selection procedure for Rolling Bearings:-

1. Determine the Radial and Axial forces (i.e loads) Accurately from the working conditions.
2. Select the type of Bearing such as ball or Roller Bearings etc from load conditions.
3. Calculate the working life of Bearing.
4. Find out the dynamic capacity 'C' using

$$C = \left(\frac{L}{L_{10}}\right)^{1/k} \times P$$

L = Life required in million revolutions

$$= \frac{\text{Life in Hours} \times \text{r.p.m} \times 60}{10^6}$$

L_{10} = 1 million revolutions

$K = 3$ for Ball Bearings.

$\frac{10}{3}$ for roller Bearings

$$P = (V \times F_r + Y F_a) S$$

Select the radial factor (X), Axial factor (Y) Based on the ratio $\frac{F_a}{F_r}$, Whether it is greater than or less than the limiting value 'e'

Assume other factors like V and S Based on the requirement.

5. If the Bearing is to be operated at higher temperatures, then determine the dynamic capacity suitably.
6. Pick out required Bearing from the manufacture's catalogue which should have better or atleast Equal properties

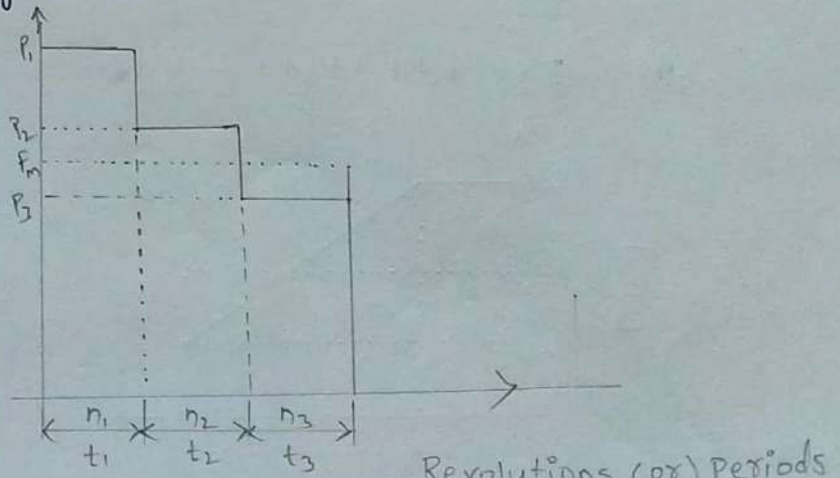
with the requirement, i.e, it should have higher dynamic load rating (or) more life than assumed and so on.

7. Evaluate the other proportions of Bearing & its housing and give tolerance.

Selection of Bearing for Variable loading:-

selection of Bearings under constant loading at given period. i.e the Bearings acted by same load in their entire life. But practically, the rolling element Bearings are frequently operated under variable load and speed conditions. This is due to many causes like power fluctuations as in electrical machineries or requirement of different cutting forces for different kinds of materials as in machine tools, or running with loading & unloading conditions as in Automobiles & so on.

Such a variable loaded Bearings are designed by considering all these different loaded conditions of work cycle and not solely upon the most severe operating conditions. The work cycle may be divided into a number of portions in each of which the operating conditions may be taken as constant



Let P_1 = constant load during n_1 revolutions (or) during the period of time t_1.

P_2 = constant load n_2 revolutions (or) during the period of time t_2.

P_3 = constant load at n_3 (or) t_3

⋮

P_n = constant load at n_n (or) t_n

Then the approximate value of the equivalent mean load (i.e cubic mean load) for the whole cycle of operation, which will have the same influence on the life of Bearing as the fluctuating load is obtained as

For variable speed, the cubic mean load

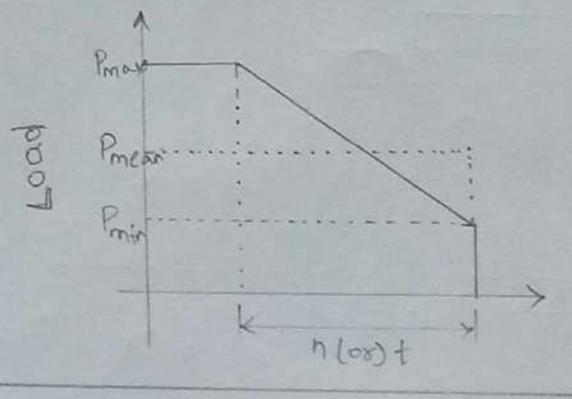
$$P_m = \left[\frac{P_1^3 n_1 + P_2^3 n_2 + P_3^3 n_3 + \cdots + P_n^3 n_n}{\Sigma n}\right]^{1/3}$$

Where $\Sigma n = n_1 + n_2 + n_3 + \cdots + n_n$

And for variable time

$$P_m = \left[\frac{P_1^3 t_1 + P_2^3 t_2 + P_3^3 t_3 + \cdots + P_n^3 t_n}{\Sigma t}\right]^{1/3}$$

where $\Sigma t = t_1 + t_2 + t_3 + \cdots + t_n$



If the load is fluctuating linearly over a certain period.

The mean load for this period may be obtained as.

$$P_{Mean} = \frac{2 P_{max} + P_{min}}{3}$$

Selection procedure for the Bearings under variable loading:-

1. Note the different loads acting at a different speeds or at different periods accurately from the working conditions.
2. Calculate the number of revolutions per unit time or portion of period per unit revolution.
3. Find out the equivalent mean load Assuming time or speed constant.
4. If the load is acting linearly over a certain period, then calculate the mean load for that period before calculating Equivalent mean load.
5. Then determine the Basic dynamic capacity, & the life of Bearings as mentioned.

A Bearing for an axial flow compressor is used to support a radial load 2500N and thrust load 1500 N. The service required for the Bearing is 5 years at the rate of 40 hours per week. The shaft speed is 1000 rpm. Select suitable Ball Bearing for the purpose. Diameter of the shaft is 50mm.

Given Radial load $F_r = 2500$ N

Axial load $F_a = 1500$ N

Life = 5 years at the rate of 40 hours/week.

Speed, N = 1000 rpm

Dia. of shaft = 50 mm.

Since the Bearing is subjected to radial and axial loads, Angular contact Ball Bearings is preferred than deep groove Ball Bearings. Let us select single row angular contact ball Bearing.

Dynamic capacity of the Bearing

$$C = \left(\frac{L}{L_{10}}\right)^{1/k} P = \left(\frac{L}{L_{10}}\right)^{1/k} \times \left(V X F_r + Y F_a\right) S$$

K = 3 for Ball Bearings.

S = Service factor = 1.5 (for Compressor)

To find out the values of X and Y, compare the ratio of $\left(\frac{F_a}{F_r}\right)$ to the values of e.

For the available single row angular contact Ball Bearings 72B & 73B the e = 1.14

$\therefore \frac{F_a}{F_r} = \frac{1500}{2500} = 0.6 \leq e$

Hence $X = 1,\ Y = 0.$

$P = (1 \times 1 \times 2500 + 0 \times 1500)\, 1.5 = 3750$ N.

L = Life in million revolutions

$= \frac{\text{Life in hours} \times n \times 60}{10^6} = \frac{(5 \times 52 \times 40) \times 1000 \times 60}{10^6}$

$= \frac{10400 \times 1000 \times 60}{10^6}$

= 624 million revolutions.

L_{10} = 1 million revolutions.

$C = \left(\frac{624}{1}\right)^{1/3} \times 3750$

C = 32050 N

7310B SKF Ball Bearing is selected

[OR]

$P = (V \times F_r + YF_a)S = 3750$ N.

Life in Hours = $5 \times 52 \times 40$ = 10400 hours.

$\therefore \frac{C}{P} = 8.56$ (for 10400 hr & 1000 rpm)

$C = 8.56 \times 3750$

C = 32100 N

Hence SKF 7310 B Angular contact Ball Bearing selected

Select a suitable deep groove Ball Bearing for Supporting a radial load of 10KN and an Axial load of 3KN for a life of 4000 hours at 800 rpm. Select from series 63. Calculate the Expected life of the selected Bearing.

Given $F_r = 10\,KN = 10000\,N$

$F_a = 3KN = 3000\,N.$

Life = 4000 hours

Speed N = 800 rpm

Required Bearing should be of SKF 63 series. Since the desired life is 4000 hours, the selected bearing should have more life than or atleast equal to 4000 hours.

Let us select SKF 6312 Bearing, For this Bearing the static capacity, $C_o = 47070\,N.$ & $C = 62270\,N.$

$$\frac{F_a}{C_o} = \frac{3000}{47070} = 0.06$$ and corresponding $e = 0.26$

$$\frac{F_a}{F_r} = \frac{3000}{10000} = 0.3$$

$\frac{F_a}{F_r} > e$, the radia & Axial load factors are

$X = 0.56,\ Y = 1.65$

$$P = (VXF_r + YF_a)\,S$$

V = 1 (Assume inner ring rotation)

S = 1.5

$$P = (1 \times 0.56 \times 10000 + 1.65 \times 3000)\,1.5$$

$$P = 15825\,N$$

Life of this Bearing

$$L = \left(\frac{C}{P}\right)^k = \left(\frac{62270}{15825}\right)^3 = 61 \text{ million Revolutions}$$

$$= \frac{61\times10^6}{60\times n} = \frac{61\times10^6}{60\times800}$$

$$= 1270 \text{ hours.}$$

Since the life of this Bearing is than our Requirement, let us select a higher capacity Bearing

Now let the Bearing SKF 6316 Bearing.

$C_o = 78450\,N$, $C = 94140\,N$

$\frac{F_a}{C_o} = \frac{3000}{78450} = 0.04$ and $C = 94140N.$

the corresponding $e = 0.24$

$\frac{F_a}{F_r} = 0.3 > e \quad X = 0.56, \; \& \; Y = 1.8$

$P = (0.56\times10000 + 1.8\times3000)1.5$

$P = 16500\,N$

$$\text{Life } L = \left(\frac{C}{P}\right)^k = \left(\frac{94140}{16500}\right)^3 = 185.73 \text{ mr}$$

$$= \frac{185.73\times10^6}{60\times800}$$

$$= 3870 \text{ hours}$$

the life of selected bearing is little bit lower.

Let select SKF 6317

$C_o = 85810\,N$ & $C = 101500\,N$

$$\frac{F_a}{C_o} = \frac{3000}{85810} = 0.035 \; \& \; e = 0.235$$

$$\frac{F_a}{F_r} = 0.3 > e \quad \& \quad X = 0.56 \; \& \; Y = 1.85$$

$$\therefore P = (0.56 \times 10000 + 1.85 \times 3000) 1.5$$

$$P = 16725 N$$

$$L = \left(\frac{C}{P}\right)^k = \left(\frac{101500}{16725}\right)^3 = 233.5 \text{ m.r}$$

$$= \frac{233.5 \times 10^6}{60 \times 800}$$

$$= 4056 \text{ hours.}$$

Since the life of SKF 6317 Bearing is more than required life, this Bearing is selected for the purpose.

In a practicular application, the radial load acting on a Ball Bearing is 5 KN and the expected life for 90% of the Bearings is 8000 h. Calculate the dynamic load carrying capacity of the Bearing, when the shaft rotates at 1450 rpm.

Given

$F_r = 5\,KN$

$L = 8000$ hrs

$N = 1450$ rpm

Bearing life:

$$L_{10} = \frac{60 N L_{10h}}{10^6} = \frac{60 \times 1450 \times 8000}{10^6}$$

$= 696$ million revolutions

Dynamic load capacity:

$P = F_r = 5000\,N$

Since the Bearing is subjected to purely Radial load.

$$C = P(L)^{1/3}$$

$$C = 5000(696)^{1/3}$$

$$C = 44310.\cancel{4}8\,N$$

$$C = 44.31\,KN$$

A taper roller Bearing has a dynamic load capacity of 26kN. The desired life for 90% of the Bearings is 8000h and the speed is 300 rpm. Calculate the Equivalent radial load that the Bearing can carry.

Given,

$$C = 26kN = 26 \times 10^3 N$$

Bearing life

$$L = \frac{60 N (L_{10})_h}{10^6}$$

$$L = \frac{60 \times 300 \times 8000}{10^6}$$

$$L = 144 \text{ million rev}$$

Equivalent radial load

$$C = P(L)^{0.3}$$

$$P = \frac{C}{(L_{10})^{0.3}}$$

$$P = \frac{26000}{(144)^{0.3}}$$

$$P = 5854.16 N$$

Since the Bearing is subjected to purely radial load

$$F_r = P = 5854.16 N$$

A single-row deep groove Ball Bearing is subjected to a Pure radial force of 3 kN from a shaft that rotates at 600 rpm. The expected life of the Bearing is 30000 hrs. The min. acceptable dia. of the shaft is 40mm. Select a suitable Ball bearing for this application.

Given

$$F_r = 3\,kN = 3000\,N$$

$$L_h = 30000\ hrs$$

$$N = 600\ rpm$$

$$d = 40\,mm$$

Dynamic load capacity.

The Bearing is subjected to purely radial load

$$P = F_r = 3000\,N$$

$$L = \frac{60\,N\,L_h}{10^6}$$

$$L = \frac{60 \times 600 \times 30000}{10^6}$$

$$L = 1080 \text{ million rev.}$$

$$C = P(L)^{1/3}$$

$$= 3000\,(1080)^{1/3}$$

$$= 30779.57\,N$$

Selection of Bearing.

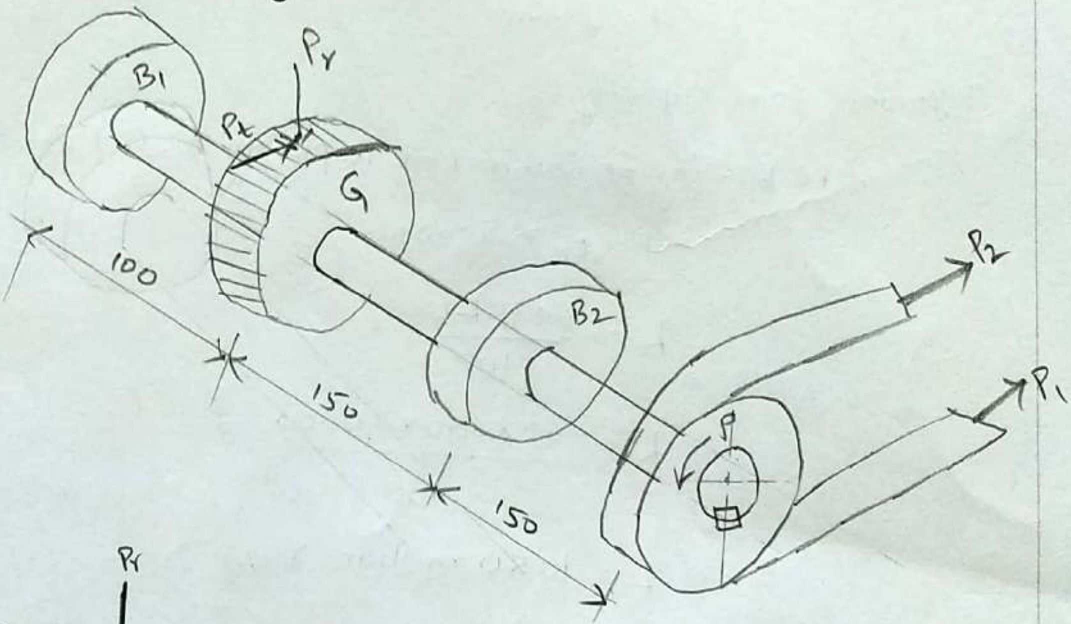
The selected Bearing is 6308 ($C = 31380\,N$)

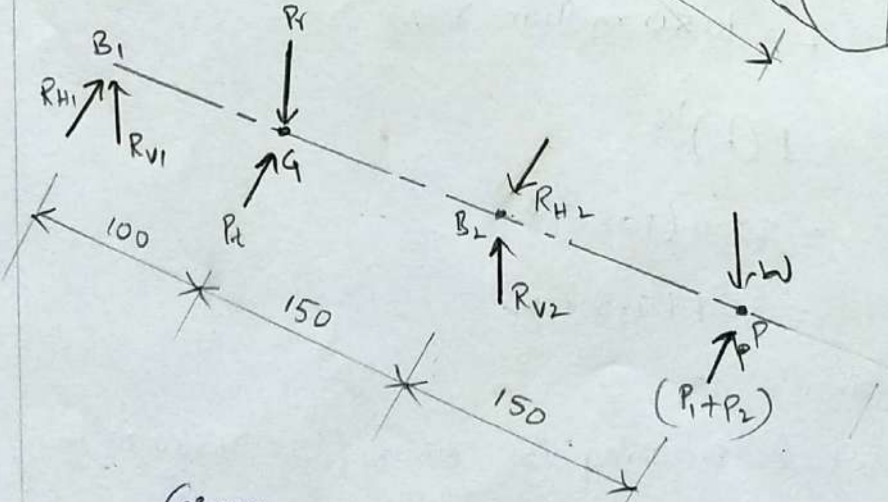
Based on ISI 40BC03

A transmission of shaft rotating at 720 rpm and transmitting power from the pulley 'P' to the spur Gear 'G' is shown in fig. The Belt tensions & the gear tooth forces are as follows:

$$P_1 = 480\,N, \quad P_2 = 166\,N, \quad P_t = 497\,N, \quad P_r = 181\,N.$$

The weight of the pulley is 100 N. The dia of the shaft at Bearings B_1 & B_2 is 10 mm & 20 mm respectively. The load-factor 2.5 & the expected life for 90% of the Bearings is 8000h. Select Single row deep groove Bearings at B_1 & B_2





Given

$N = 720$ rpm.

$d_1 = 10$ mm

$d_2 = 20$ mm , $L = 8000$ hrs.

Radial and Axial forces:

Considering forces in vertical plane and taking moment of forces about the Bearing B_1, We have.

$$P_r(100) + W(400) - R_{V2}(250) = 0$$

$$181(100) + 100(400) - R_{V2}(250) = 0$$

$$R_{V2} = 232.4\ N$$

$$\Sigma y = 0$$

$$R_{V1} + R_{V2} = P_r + W$$

$$R_{V1} + 232.4 = 181 + 100$$

$$R_{V1} = 48.6\,N$$

Considering forces in Horizontal plane & taking moment of forces about the Bearing B_1

$$P_t(100) + (P_1 + P_2)(400) - R_{H2}(250) = 0$$

$$497(100) + (498 + 166)(400) - R_{H2}(250) = 0$$

$$R_{H2} = 1261.2\ N$$

$$R_{H2} = R_{H1} + P_t + (P_1 + P_2)$$

$$1261.2 = R_{H1} + 497 + (498 + 166)$$

$$R_{H1} = 100.2\ N$$

$$R_1 = \sqrt{R_{V1}^2 + R_{H1}^2} = \sqrt{(48.6)^2 + (100.2)^2}$$

$$R_1 = 111.36\,N$$

$$R_2 = \sqrt{R_{V2}^2 + R_{H2}^2} = \sqrt{(232.4)^2 + (1261.2)^2}$$

$$R_2 = 1282.43\ N$$

∴ The Bearing reactions are in the radial direction

$F_{r1} = R_1 = 111.36\,N$

$F_{r2} = R_2 = 1282.43\,N$

Ther Bearing is no Axial thrust on these Bearins,

$F_{a1} = F_{a2} = 0$.

Dynamic load capacities:-

$P_1 = F_{r1} = 111.36\,N$

$P_2 = F_{r2} = 1282.43\,N$

$$L = \frac{60\,N\,L_h}{10^6} = \frac{60 \times 720 \times 8000}{10^6}$$

$L = 345.6$ million rev.

$C_1 = P_1 (L)^{1/3} \times$ Load factor

$= (111.36)\,(345.6)^{1/3}\,(2.5)$

$C_1 = 1953.71\,N$

$C_2 = P_2 (L)^{1/3} \times$ Load factor

$= 1282.43\,(345.6)^{1/3} \times 2.5$

$C_2 = 22499.09\,N$

Select a suitable contrad - type ball bearing for the following data. The radial load is 7500 N. and axial-load 4500 N. The shaft speed is 2000 rpm and the L_{10} life required is 4.9×10^8 revolutions. The inner ring of the Bearing rotates.

Given

$F_r = 7500$ N

$F_a = 4500$ N

$N = 2000$ rpm

$L = 4.9 \times 10^8$ revolutions

Let us select the Bearing from SKF 63 series, consider SKF 6318 Bearing.

For this Bearing

$C_o = 96110$ N, $C = 107870$ N

How $\frac{F_a}{C_o} = \frac{4500}{107870} = 0.05$ & corresponding $e = 0.25$

& $\frac{F_a}{F_r} = \frac{4500}{7500} = 0.6 > e \; (=0.25)$

Hence $X = 0.56$ & $Y = 1.74$

$P = (V X F_r + Y F_a) S.$

$V = 1.$ (Inner ring rotates)

$S =$ service factor $= 1.2$

$P = (1 \times 0.56 \times 7500 + 1.74 \times 4500) 1.2$

$P = 14436$ N

$$L = \left(\frac{C}{P}\right)^k$$

$$L = \left(\frac{107870}{14436}\right)^3$$

$$L = 417.2 \text{ m.r}$$

$$L = 4.17 \times 10^8 \text{ rev}$$

Since this Bearing has life less than requirement (4.9×10^8 revolutions)

Let us select higher capacity of Bearing.

Consider SKF 6319 Bearing

$$C_o = 107870\,N, \quad C = 117680\,N$$

$$\frac{F_a}{C_o} = \frac{4500}{107870} = 0.04 \quad \& \quad e = 0.24$$

$$\frac{F_a}{F_r} = 0.6 > e$$

$$X = 0.56 \quad Y = 1.8$$

$$P = (1 \times 0.56 \times 7500 + 1.8 \times 4500)1.2$$

$$P = 14760\,N$$

$$L = \left(\frac{C}{P}\right)^k = \left(\frac{117680}{14760}\right)^3 = 506.8 \text{ m.r}$$

$$L = 5.068 \times 10^8 \text{ m.r}$$

Since this Bearing has more life than required life SKF 6319 Ball Bearing may be selected..

A single-row deep groove Ball Bearing Number 6002 is subjected to an axial thrust of 1000 N & a radial load of 2200 N. Find the Expected life that 50% of the Bearings will completed under this condition.

Given

$F_a = 1000\,N$

$F_r = 2200\,N$

The capacities of Bearing Number 6002 are.

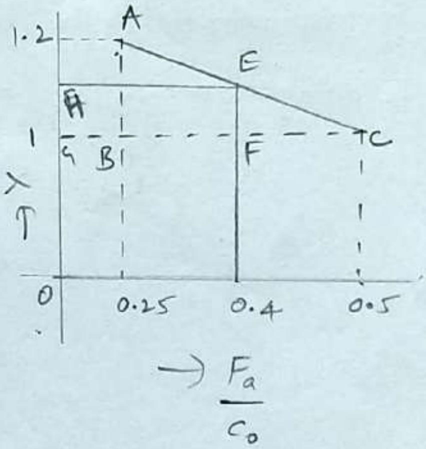
$C_o = 2500\,N$ & $C = 5590\,N$

$F_a = 1000\,N$, $F_r = 2200\,N$

$\frac{F_a}{F_r} = \frac{1000}{2200} = 0.455$

$\frac{F_a}{C_o} = \frac{1000}{2500} = 0.4$

$\left(\frac{F_a}{F_r}\right) > e.$



$\boxed{GH = EF}$

∴ ∆le ABC & EFC

$\frac{BC}{FC} = \frac{AB}{EF}$

$\frac{0.5-0.25}{0.5-0.4} = \frac{1.2-1}{EF}$

$2.5 = \frac{0.2}{EF} \Rightarrow EF = \frac{0.2}{2.5}$

$EF = 0.08$

$OH = OG + GH$

$OH = 1 + 0.08$

$OH = 1.08$

$\therefore X = 0.56$

$P = XF_r + YF_a = 0.56 \times (2200) + 1.08(1000)$

$= 2312\ N$

$C = P(L)^{1/3}$

$5590 = 2312\ (L)^{1/3}$

$L = 14.13$ million revolutions.

Bearing life (L_{50})

∴ which 50% of the Bearings will complete or Exceed.
is approximately five time the life L_{10} which 90% of the Bearings will complete or Exceed

$L_{50} = 5L_{10} = 5 \times 14.13$

$L_{50} = 70.65$ million revolutions.

A single-row deep groove Ball Bearing is subjected to Axial a radial force of 8 kN and thrust force of 3 kN. The shaft rotates at 1200 rpm. The Expected life of the Bearing is 20000h. The minimum acceptable dia. of the shaft is 75 mm. Select a suitable Ball Bearing for this application.

Given

$F_r = 8\,kN = 8 \times 10^3\,N$

$F_a = 3\,kN = 3 \times 10^3\,N$

$L_H = 20000$ hrs

$N = 1200$ rpm

$d = 75\,mm$

When the Bearing is subjected to radial as well as Axial load, the values of X ~~& Y~~ are constant & the values of Y vary only in case when

$$\frac{F_a}{F_r} > e$$

Y varies from 1 to 2

∴ Assume $Y = 1.5$

$X = 0.56$, $Y = 1.5$, $F_r = 8000\,N$, $F_a = 3000\,N$

$$P = XF_r + YF_a = 0.56(8000) + 1.5(3000)$$

$$= 8980\,N$$

$$L = \frac{60 \times N \times L_H}{10^6} = \frac{60 \times 1200\,(20000)}{10^6}$$

$L = 1440$ million revolutions.

$C = P(L)^{1/3} = 8980\,(1440)^{1/3} = 101460.04\ N$

For 75mm shaft dia. Bearing No. 6315 ($C = 112000$) is suitable for above data.

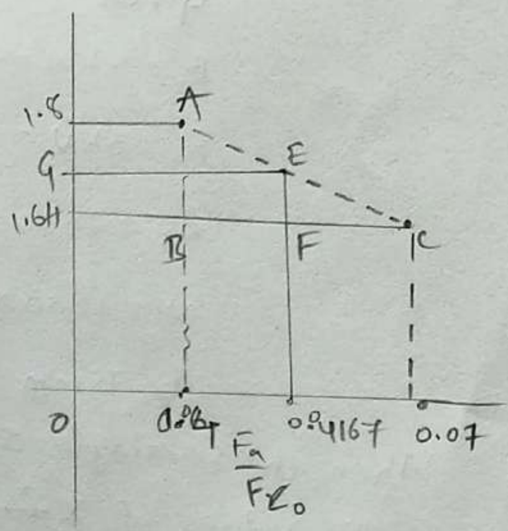
$C_0 = 72000\ N$

$\frac{F_a}{F_r} = \frac{3000}{8000} = 0.375$

$\frac{F_a}{C_0} = \frac{3000}{72000} = 0.04167$

$e = 0.24$

$\frac{F_a}{F_r} > e$



Δle.

$\frac{0.07 - 0.04}{0.07 - 0.04167} = \frac{1.8 - 1.6}{EF}$

$1.0589 = \frac{0.2}{EF}$

$EF = 0.188 = GH$

$OG = OH + GH$

$= 1.6 + 0.188$

$= 1.788$

$OG = 1.79$

$P = XF_r + YF_a = 0.56(8000) + 1.79\,(3000) = 9850N$

$C = P(L)^{1/3} = 9850\,(1440)^{1/3} = 111230.46N$

Bearing Number 6315 ($C = 112000$) is suitable for above application.

A 6207 radial Bearing is to operate in the following work cycle.

Radial load of 4500N at 150 rpm for 30% of time

Radial load of 6750N at 600 rpm for 10% of time

Radial load of 2250N at 300 rpm for 60% of time

The inner ring rotates, loads are steady, what is the Expected Avg life of the Bearing.

Given

6207 Radial Bearing

$C = 19615$ N

Load (P) in N (1)	cycle time ratio (2)	Speed in rpm (3)	Number of revolutions (4) = (2)x(3)
4500 (P_1)	0.3 (30%)	150	45 (n_1)
6750 (P_2)	0.1 (10%)	600	60 (n_2)
2250 (P_3)	0.6 (60%)	300	180 (n_3)

$\Sigma N = 285$

Cubic mean load

$$P_m = \left[\frac{P_1^3 n_1 + P_2^3 n_2 + P_3^3 n_3}{n_1+n_2+n_3}\right]^{1/3}$$

$$= \left[\frac{[(4500)^3 \times 45] + (6750^3 \times 60) + (2250^3 \times 180)}{45+60+180}\right]^{1/3}$$

$= 4420 \text{ N} \equiv P$

$$L = \left(\frac{C}{P}\right)^k$$

$$L = \left(\frac{19615}{4420}\right)^3 = 87.4 \text{ million revolutions}$$

$$L_H = \frac{L \times 10^6}{60 \times N} = \frac{87.4 \times 10^6}{60 \times 285}$$

$$L_H = 5110 \text{ hours}$$

Expected Avg life of Bearing = 5 × Rated life

= 5 × 5110

= 25550 hours

A Ball Bearing is operating on a work cycle consisting of three parts – a Radial load of 3000 N at 1440 rpm for one quarter cycle,
Radial load of 5000 N at 720 rpm for one half cycle,
Radial load of 2500 N at 1440 rpm for the remaining cycle. The Expected life of the Bearing is 10000 hours
Calculate the dynamic load carrying capacity of the Bearing.

Given $L_H = 10000 \text{ hours}$

$$n_1 = \tfrac{1}{4} \times 1440 \text{ rpm} = 360 \text{ rev}$$

$$n_2 = \tfrac{1}{2} \times 720 \text{ rpm} = 360 \text{ rev}$$

$$n_3 = \tfrac{1}{4} \times 1440 \text{ rpm} = 360 \text{ rev}$$

$$\Sigma N = n_1 + n_2 + n_3 = 360 + 360 + 360 = 1080 \text{ rpm.}$$

$$P_m = \left[\frac{P_1^3 n_1 + P_2^3 n_2 + P_3^3 n_3}{n_1 + n_2 + n_3}\right]^{1/3}$$

$$= \left[\frac{360(3000)^3 + 360(5000)^3 + 360(2500)^3}{1080}\right]^{1/3}$$

$$P_m = 3823 \text{ N}$$

$$L = \frac{60 \times N \times L_H}{10^6} = \frac{60 \times 1080 \times 10000}{10^6}$$

$L = 648$ million rev

$$C = P(L)^{1/3} = 3823(648)^{1/3}$$

$$C = 33082 \text{ N}$$

A single-row deep groove Ball Bearing is subjected to 30 sec work cycle that consists of the following two parts.

	Part I	Part II
Duration (s)	10	20
Radial load (kN)	45	15
Axial load (kN)	12.5	6.25
Speed rpm.	720	1440

The static and dynamic load capacities of the Ball Bearing are 50 and 68 kN respectively. Calculate the Expected life of the Bearing in Hours.

Given

$C_0 = 50$ kN

$C = 68$ kN.

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for part I

$$\frac{F_a}{F_r} = \frac{12.5}{45} = 0.278$$

$$\frac{F_a}{C_o} = \frac{12.5}{50} = 0.25$$

$$e = 0.37$$

$$\frac{F_a}{F_r} < e$$

$$X = 1 \;,\; Y = 0$$

$$P_1 = F_r = 45000\,N$$

$$N_1 = \frac{10}{60} \times 720 = 120\,N$$

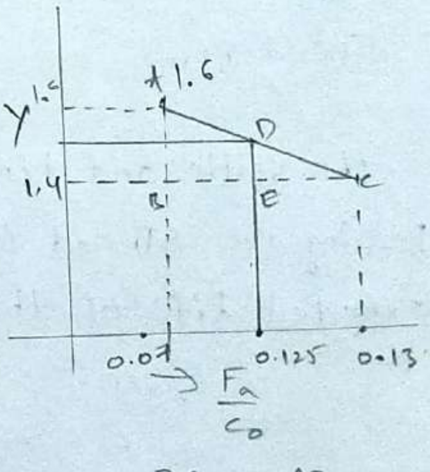
For part II

$$\left(\frac{F_a}{F_r}\right) = \frac{6.25}{15} = 0.417$$

$$\left(\frac{F_a}{C_o}\right) = \frac{6.25}{50} = 0.125$$

($e = 0.31$ Approximately)

$$\left(\frac{F_a}{F_r}\right) > e$$



$$\frac{0.13 - 0.07}{0.13 - 0.125} = \frac{1.6 - 1.4}{DE}$$

$$12 = \frac{0.2}{DE}$$

$$DE = 0.0166$$

$$Y = 1.4 + 0.0166$$

$$Y = 1.416 \cong \underline{1.42}$$

$$\frac{BC}{EC} = \frac{AB}{DE}.$$

$X = 0.56$ & $Y = 1.42$

$P_2 = X F_r + Y F_a$

$= 0.56(15000) + 1.42(6250)$

$P_2 = 17275 N$

$N_2 = \frac{20}{60} \times 1440 = 480$ rev

$N_1 + N_2 = 120 + 480 = 600$ rev

$$P_m = \left[\frac{P_1^3 n_1 + P_2^3 n_2}{n_1 + n_2}\right]^{3}$$

$$= \left[\frac{120(45000)^3 + 480(17275)^3}{600}\right]^{1/3}$$

$P_m = 28167.89 N$

Bearing life:

$$L = \left(\frac{C}{P}\right)^3 = \left(\frac{68000}{28167.89}\right)^3 = 14.069 \text{ million rev.}$$

$$L_H = \frac{L \times 10^6}{60 \times N} = \frac{14.069 \times 10^6}{60 \times 1200}$$

$L_H = 195.4$ hours

For 6307 Ball Bearing, the load varies as follows.

S.No	Radial load N	Axial load N	Cycle time ratio	Speed in rpm
1.	6000	3000	0.5	400
2.	7500	—	0.3	650
3.	4000	1000	0.2	900

The inner-ring rotates, loads are ~~steady~~ Medium shock. Find the.

Expected Avg life of the Bearing.

Given

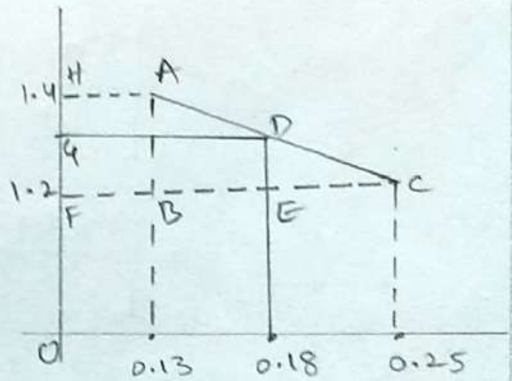
For 6307 Ball Bearing

$C_o = 16970N$ and $C = 25300N$.

Equivalent load for first operating period (P_1):

$\frac{F_{a1}}{C_o} = \frac{3000}{16970} = 0.18$ & corresponding $e = 0.33$

$\frac{F_{a1}}{F_{r1}} = \frac{3000}{6000} = 0.5 > e$



$\frac{0.25-0.18}{0.25-0.18} = \frac{1.4-1.2}{DE}$

$FG = DE = 0.1166$

$Y = OG = 1.2 + 0.1166$

$OG = OF + FG$

$Y = 1.3$ & $X = 0.56$

$P_1 = (V \times F_{r1} + YF_{a1})S$

$V = 1$ & $S = 1.2$

$P_1 = [(0.56 \times 6000) + (1.3 \times 3000)] 1.5$

$P_1 = 10890N$

Equivalent load for second operating period (P_2):

$F_{a2} = 0$.

$P_2 = F_{r2} \times S = 7500 \times 1.5 = 11250N$

Equivalent load for third operating period (P_3):

$\frac{F_{a3}}{F_{r3}} = \frac{1000}{4000} = 0.25$ & $\frac{F_{a3}}{C_o} = \frac{1000}{0.06}$ & $e = 0.26$

$\frac{F_{a3}}{F_{r2}} < e$, $X=1$, & $Y=0$

$P_3 = [1 \times 4000 + (0 \times 1000)]\, 1.5$

$P_3 = 6000N$

Load P (N)	Cycle ratio time	Speed in rpm	No. of rev for 1 min. cycle
10890 (P_1)	0.5	400	200 (n_1)
11250 (P_2)	0.3	650	195 (n_2)
6000 (P_3)	0.2	900	180 (n_3)

$\Sigma N = 575$

$$P_m = \left[\frac{P_1^3 n_1 + P_2^3 n_2 + P_3^3 n_3}{n_1 + n_2 + n_3}\right]^{1/3}$$

$$P_m = \left[\frac{(10890)^3 \times 200 + (11250)^3 \times 195 + (6000)^3 \times 180}{575}\right]^{1/3}$$

$P_m = 9998.95N$

$$L = \left(\frac{C}{P}\right)^k = \left(\frac{25300}{9998.95}\right)^3 = 16.19 \text{ m.r}$$

$$L_H = \frac{L \times 10^6}{60 \times N} = \frac{16.19 \times 10^6}{60 \times 575} = 469.27 \text{ hours.}$$

Expected Avg life of Bearing. $= 5 \times$ Rated life

$= 5 \times 469.27$

$= 2346.3$ hours

BEARING WITH A PROBABILITY OF SURVIVAL OTHER THAN 90 PERCENT:

The rating life, it is mentioned that the rating life
~~it is mentioned that the rating life~~ is the life 90% of group of identical bearings will complete or exceed before fatigue failure.

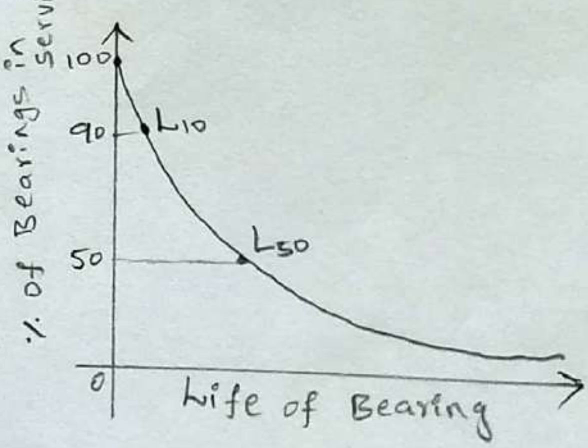
The relability R is defined as.

$$R = \frac{\text{Number of Bearings which have succesfully completed 'L' million revolutions}}{\text{Total number of Bearings under test.}}$$

Therfore, the reliability of Bearings selected from the manufactaer's catalogue is 0.9 (or) 90%

In certain applications, where there is risk to human life, it becomes necessary to select a bearing having a relabilty of more than 90%.

The relation ship b/w bearing life & reliablity is given by staticale curve known as Wiebull distribution.



For Wiebull distribution

$$R = e^{-(L/a)^b}$$

R = Reliability

L = Corresponding life

∴ a and b are constants.

$$\left(\frac{1}{R}\right) = e^{\left(\frac{L}{a}\right)^b}$$

$$\log_e\left(\frac{1}{R}\right) = \left(\frac{L}{a}\right)^b \quad \text{—— (1)}$$

If L_{10} is the life corresponding to a reliability of 90% (or) R_{90} then

$$\log_e\left(\frac{1}{R_{90}}\right) = \left(\frac{L_{10}}{a}\right)^b \quad \text{—— (2)}$$

∴ Dividing (1) & (2)

$$\left(\frac{L}{L_{10}}\right) = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b}$$

Where $R_{90} = 0.9$

$a = 6.84$ and $b = 1.17$

The above Eqn is used for selecting the bearing when the reliability is other than 90%.

A single-row deep Groove Ball Bearing is subjected to a radial force of 8 KN and a thrust force of 3 KN. The values of X and Y factors are 0.56 and 1.5 respectively. The shaft rotates at 1200 rpm. The dia of the shaft is 75 mm and Bearing No. 6315 (C = 89280 N) is selected for this application.

(i) Estimate the life of this bearing, with 90% reliability.

(ii) Estimate the reliability for 20 000 hours life.

Given

$F_r = 8\,kN = 8\times10^3\,N$

$F_a = 3\,kN = 3\times10^3\,N$

$X = 0.56$

$Y = 1.5$

$N = 1200\,rpm$, $d = 75\,mm$,

$C = 89280\,N$.

1> Bearing life with 90% reliability:

$$P = XF_r + YF_a = 0.56(8000) + 1.5(3000)$$

$$P = 8980\,N$$

$$L_{10} = \left(\frac{C}{P}\right)^3 = \left(\frac{89280}{8980}\right)^3 = 982.72 \text{ million rev}$$

$$L_{10h} = \frac{L_{10}\times10^6}{60\times N} = \frac{982.72\times10^6}{60\times1200} = 13648.88\,hr.$$

2) Reliability for 20000 hrs. :-

$$\left(\frac{L}{L_{10}}\right) = \left(\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right)^{1/b}.$$

$$\left(\frac{L}{L_{10}}\right)^b = \left[\frac{\log_e(1/R)}{\log_e(1/R_{90})}\right]$$

$L = 20000$ hrs. $L_{10} = 13648.88$ hrs.

$R_{90} = 0.90$, $b = 1.17$

$$\left[\frac{20000}{13648.88}\right]^{1.17} = \frac{\log_e(1/R)}{\log_e\left(\frac{1}{0.90}\right)}$$

$1.5636 =$

$$1.5636 = \frac{\log_e(1/R)}{0.10536}$$

$$0.1647 = \log_e(1/R)$$

$$e^{0.1647} = 1/R$$

$$1.1790 = 1/R$$

$$R = \underline{84\%}$$

A Ball Bearing Subjected to a radial load of 5 kN. is expected to have a life of 8000 hrs at 1450 rpm with a Reliability of 99%. Calculate the dynamic load capacity of the Bearing. so that it can be selected from the manufactur's catalogue based on a reliability of 90%.

Given $F_r = 5$ kN

$N = 1450$ rpm

$L_{99h} = 8000$ hrs.

Bearing life with 99% reliability:

$$L_{99} = \frac{60 \times N \times L_{99h}}{10^6}$$

$$L_{99} = \frac{60 \times 1450 \times 8000}{10^6}$$

$$L_{99} = 696 \text{ m.r}$$

2) Bearing life with 90% reliability:-

$$\left(\frac{L_{99}}{L_{10}}\right) = \left[\frac{\log_e\left(\frac{1}{R_{99}}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/1.17}$$

$$= \left[\frac{\log_e\left(\frac{1}{0.99}\right)}{\log_e\left(\frac{1}{0.9}\right)}\right]^{1/1.17}$$

$$= 0.1342$$

$$L_{10} = \frac{L_{99}}{0.1342} = \frac{696}{0.1342} = 5186.29 \text{ m.r}$$

3) $C = P(L_{10})^{1/3}$

$$= 5000(5186.29)^{1/3}$$

$$C = 86547.7 \text{ N}$$

A shaft rotating at constant speed is subjected to variable load. The bearings supporting the shaft are subjected to stationary equivalent radial load of 3 KN for 10% of time, 2KN for 20% time, 1 KN for 30% time and no load for remaining time of cycle. If the total life expected for the Bearing is 20×10^6 rev at 95% reliability. Calculate dynamic load rating of the Ball Bearing.

Given

$P_1 = W_1 = 3\text{kN}, \; n_1 = 0.1\,n$

$P_2 = W_2 = 2\text{kN}, \; n_2 = 0.2\,n$

$P_3 = W_3 = 1\ kN$; $n_3 = 0.3n$

$P_4 = W_4 = 0$; $n_4 = (1 - 0.1 - 0.2 - 0.3)n = 0.4n$

$L_{95} = 20 \times 10^6\ rev = 20\ m.r$

L_{90} = Life of the Bearing corresponding to reliability 90%.

L_{95} = Life of the Bearing corresponding to reliability 95% $= 20 \times 10^6$ rev

$$\frac{L_{95}}{L_{90}} = \left[\frac{\log_e\left(\frac{1}{R_{95}}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/b} = \left[\frac{\log_e\left(\frac{1}{0.95}\right)}{\log_e\left(\frac{1}{0.9}\right)}\right]^{1/1.17}$$

$$= \left[\frac{0.0513}{0.1054}\right]^{0.8547}$$

$$= 0.54$$

$$L_{90} = \frac{L_{95}}{0.54} = \frac{20 \times 10^6}{0.54} = 37 \times 10^6\ rev$$

$$P = \left[\frac{n_1 P_1^3 + n_2 P_2^3 + n_3 P_3^3 + n_4 P_4^3}{n_1 + n_2 + n_3 + n_4}\right]^{1/3}$$

$$= \left[\frac{0.1n \times 3^3 + 0.2n \times 2^3 + 0.3n \times 1^3 + 0.4n \times 0^3}{0.1n + 0.2n + 0.3n + 0.4n}\right]^{1/3}$$

$$= (2.7 + 1.6 + 0.3 + 0)^{1/3}$$

$$P = 1.663\ kN$$

$$C = P(L_{90})^{1/k} = 1.663(37)^{1/3}$$

$$C = 5.54\ kN$$

Design of connecting rod:-

The connecting rod is employed as the intermediate element b/w the piston and crankshaft in I.C Engine (or) b/w the piston rod and crank shaft in the case of steam Engine. It consists of an eye at the small End (so called due to its small size) for connecting piston through piston pin, a long shank usually of I-section, and a big End, which is connected to the crankshaft pin, of split type and has a separate cap. The cap is secured to the body of the rod by means of two (or) four Bolts.

The motion of the connecting rod is a combination of the rotation of the crank-end (big End) and the reciprocating motion of the piston End (or) small end. Therfore the rod should be as light as possible to keep down the inertia forces. It must have suffecient strength to with stand the thrust of the piston.

The connecting rods for petrol engines are made of steel containing carbon 0.4 to 0.45% and manganese 1.4 to 1.8%. Material for diesel Engine rods must have higher strength and Hence steels containing chromium, Nickel, Molybdenum with 0.4% carbon are used

When the fuel is transmitted from piston rod to crank shaft through connecting rod, the connecting rod is also subjected to alternate tensile and compressive forces. The failure of connecting rod is more during compression than during tension. Hence the compressive

Load, Especially the Buckling load may be taken as the design load for the connecting rod.

Generally, a connecting rod, subjected to axial Compressive load may buckle in two directions i.e in the plane of Motion with X-axis as neutral axis (or) in the plane ⊥er to the plane of motion with y-axis as neutral axis. To overcome the buckling failure in these two directions, the connecting rod should be Strong in Both sections i.e about x-axis and y-axis.

Determination of shank thickness:-

For design of connecting rod mostly Rankine's formula may be followed. Let us consider a I-section Connecting rod with following parameters.

Let A = Cross-sectional area of Connecting rod.

l = Actual length of the connecting rod

L = Equivalent length of the connecting rod.

σ_c = Compressive yield stress (crushing stress)

F_{cr} = Crippling load (or) Buckling load.

I_{xx} & I_{yy} = Area Moment of Inertia about X & Y-axis.

K_{xx} & K_{yy} = Radius of Gyration of the section about X & Y axis.

In General, the connecting rod is considered in Such a way that its both Ends are Hinged for Buckling about X-axis due to the relative Motion of the Connecting rod with the piston pin and crank shaft and at the same time the Both Ends are assumed as fixed Ends for Buckling about Y-axis Because of no relative Movement with these Assumptions.

$$\text{i.e } F_{crx} = \frac{\sigma_c \cdot A}{1+a\left[\frac{L}{K_{xx}}\right]^2} = \frac{\sigma_c \cdot A}{1+a\left[\frac{l}{K_{xx}}\right]^2}$$

$\{\because L = l$ for Both Ends Hinged$\}$

$$F_{cry} = \frac{\sigma_c \cdot A}{1+a\left[\frac{L}{K_{yy}}\right]^2} = \frac{\sigma_c \cdot A}{1+a\left[\frac{l}{2K_{yy}}\right]^2}$$

$\{\because L = \frac{l}{2}$ for both Ends fixed$\}$

In order to obtain the Connecting rod equally Strong in Buckling about both the axes, its resisting Strengths in Both Sections to the applied buckling load Should be Equal

$$\text{ie } \frac{F_{crx}}{A} = \frac{F_{cry}}{A}$$

$$\frac{\sigma_c}{1+a\left[\frac{l}{K_{xx}}\right]^2} = \frac{\sigma_c}{1+a\left[\frac{l}{2K_{yy}}\right]^2}$$

$$\therefore K_{xx}^2 = 4K_{yy}^2$$

$\therefore I_{xx} = A k_{xx}^2$ & $I_{yy} = A k_{yy}^2$ then

$$I_{xx} = 4 I_{yy}$$

From the above relation, we can understand that, if $I_{xx} = 4 I_{yy}$, Buckling will not occur about any axis.

If $I_{xx} > 4 I_{yy}$, Buckling may occur about Y-axis.

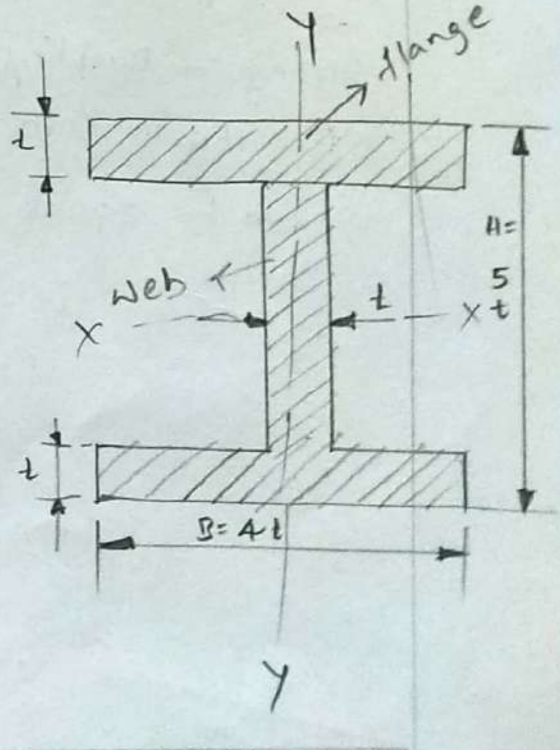
If $I_{xx} < 4 I_{yy}$, Buckling May occur about X-axis.

But in Actual practice I_{xx} is kept slightly less than $4 I_{yy}$ so that the connecting rod should not buckle about Y-axis at any occasion. Even though it may be allowed to buckle very slightly about X-axis.

The usual ratio of $\frac{I_{xx}}{I_{yy}}$ May be 3 to 3.5

the proportions of various parts of mostly used I-section are shown diagram. For this section

$$\frac{I_{xx}}{I_{yy}} = 3.2$$



Area of the section

$$A = 2(4t \times t) + t \times 3t$$

$$A = 11t^2$$

(or)

$$A = (4t \times 5t) - (3t \times 3t)$$

$$A = 11t^2$$

Moment of inertia of the section about x-axis

$$I_{xx} = \frac{1}{12}\left[4t(5t)^3 - 3t(3t)^3\right]$$

$$I_{xx} = \frac{419}{12}t^4$$

Moment of inertia of the section about y-axis

$$I_{yy} = \left[2 \times \frac{1}{12} t \times (4t)^3 + \frac{1}{12}(3t)t^3\right]$$

$$I_{yy} = \frac{131}{12}t^4$$

$$\therefore \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Buckling load $F_{cr} = \dfrac{\sigma_c \cdot A}{1 + a\left(\frac{L}{K_{xx}}\right)^2}$

F_{cr} = Max. Gas force × factor of safety (5 to 6).

Dimensions of the Crankpin at the big End and the piston pin at the Small End:-

The small end is made as solid eye without any split and is provided with Brass bushes inside the eye and the Big end is split and the top cap is Joined with the remaining parts of connecting rod by means of Bolts.

Let l_1, d_1 = length and dia. of piston pin (i.e Small end)

l_2, d_2 = " " Crank pin (i.e Big end)

P_{b1}, P_{b2} = Design Bearing pressure for the Small and Big Ends respectively

Bearing load applied on piston pin is given by

$$F_1 = P_{b1} \cdot l_1 \cdot d_1$$

∴ Bearing load applied on the Crank pin

$$F_2 = P_{b2} \cdot l_2 \cdot d_2$$

Usually, the design Bearing pressure for Small end & Big end

$P_{b_1} = 12.5$ to 15.4 N/mm²

$P_{b_2} = 10.8$ to 12.6 N/mm².

lly the ratio of $\frac{l}{d}$

$$\frac{l_1}{d_1} = 1.5 \text{ to } 2 \quad \& \quad \frac{l_2}{d_2} = 1 \text{ to } 1.25$$

∴ The Highest load to be carried by these Bearings Containing piston pin & Crank pin is the Max. Compressive load produced by the Gas pressure neglecting the inertia force due to its small value.

(iii) Size of Bolts for securing the Big end cap:

∴ At the same time, the Bolts are designed based on the inertia force Because the Maximum load applied on the Bolts is the inertia force of the reciprocating parts..

Inertia force

$$F_i = m r \omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n} \right]$$

$$n = \frac{l}{r} = \frac{\text{Length of connecting rod}}{\text{Crank radius}}$$

The Max. inertia force will be obtained when the crank shaft is at dead centre position.

i.e at $\theta = 0$

$$\therefore F_{im} = m r \omega^2 \left(1 + \frac{1}{n}\right).$$

By Equating this Max. inertia force to the tensile Strength of bolts at their Core dia; the size of Bolts May be determined.

For two Bolts

$$F_{im} = 2 \times \frac{\pi}{4} d_c^2 \times \sigma_t$$

Nominal dia. $d = \frac{d_c}{0.84}$

(iv) Thickness of the Big End Cap :-

The cap is usually treated as a beam freely supported at the Bolts centres & loaded in a manner intermediate b/w uniformly distributed load and centrally Concentrated load.

Max. Bending Moment at the centre of cap is

$$M = \frac{Wl'}{6}$$

W = Max. load equal to inertia force of reciprocating parts = F_{im}

$$M = \frac{F_{im} l'}{6}$$

l' = Distance b/w bolt centres

= Dia of crank pin + [2 x Wall thickness of Bush] + dia of Bolt + Some extra Marginal thickness.

Width of Cap May calculated as

b = length of crank pin - 2 x flange thickness of Bush

Bending stress induced in the cap

$$\sigma_{bc} = \frac{M}{Z}$$

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Where Z = Section Modulus of the cap

$$= \frac{1}{6} b t_c^2$$

t_c = Thickness of cap.

By comparing this induced Bending stress with the design stress, the thickness of cap may be evaluated.

(1) The connecting rod of a petrol engine is to be designed for the following data.

Piston diameter = 80 mm

Stroke = 120 mm

Weight of reciprocating parts = 15 N

Length of connecting rod = 240 mm

Speed (Maximum) = 2800 rpm

Explosion pressure corresponding to 10° of crank angle is 3 MPa

Factor of safety = 6

If the connecting rod is to be made of 40 Cr1 steel, find the dimensions of I-section connecting rod.

Given

Piston diameter, $D = 80$ mm

Stroke, $L = 2r = 120$ mm

$\therefore$ crank radius $r = 60$ mm

Length of connecting rod, $l = 240$ mm

Pressure of gas, $P = 3$ MPa $= 3 \times 10^6$ N/m² $= 3$ N/mm²

factor of safety $f_s = 6$

Maxi. speed $= 2800$ rpm

force acting on piston due to gas pressure

$$F_p = \frac{\pi}{4} D^2 \times P = \frac{\pi}{4} \times 80^2 \times 3 = 15080 \text{ N}$$

Design load acting on the piston, $F_d = F_p \times f_s$

$= 15080 \times 6$

$F_d = 90480$ N

(i) Design of Connecting rod shank:

t = Web thickness of connecting rod shank

$\sigma_c = 330\ N/mm^2$ (for 40 Cr 1 steel)

$A = 2(4t \times t) + 1(t \times 3t)$

$A = 11t^2$

$I_{xx} = \frac{1}{12}(4t)(5t)^3 - \frac{1}{12}(3t)(3t)^3$

$I_{xx} = \frac{419}{12} t^4$

$I_{xx} = 35\,t^4\ mm^4$

Least Radius of Gyration about x-axis $k_{xx} = \sqrt{\frac{I_{xx}}{A}}$

$$k_{xx} = \sqrt{\frac{35\,t^4}{11t^2}} = 1.78\,t$$

Inertia force due to reciprocating mass is having very little Effect on connecting rod.

We can Assume that the rod can be designed aganist gas force only

$$F_{cr} = 90480\ N$$

Acc. Rankine's formula, the buckling load (or) crippling load is

$$F_{cr} = \frac{\sigma_c \cdot A}{1 + a\left(\frac{L}{k_{xx}}\right)^2}$$

Buckling About x-axis, both Ends of Connecting rod are Hinged. Hence Equivalent length of rod $L = l = 240\ mm$

$$90480 = \frac{330 \times 11t^2}{1 + \frac{1}{7500}\left(\frac{240}{1.78t}\right)^2} \qquad \left[\because a = \frac{1}{7500} \text{ for mild steel}\right]$$

$$\therefore 90480 = \frac{3630\,t^2}{1+\frac{2.42}{t^2}} = \frac{3630\,t^4}{t^2+2.42}$$

$$3630\,t^4 - 90480\,t^2 - 218962 = 0$$

$$t^4 - 25t^2 - 60 = 0$$

$$\therefore t^2 = \frac{25+\sqrt{25^2+4(60)\times 1}}{2} = 27.2$$

$$\therefore t = 5.22 \text{ mm}$$

Thickness of Web $\Rightarrow t = 6$ mm

Height of Connecting rod $= 5t = 30$ mm

Width of Connecting rod $= 4t = 24$ mm

Length of connecting rod $= 240$ mm

$$M_{Max} = mr\omega^2 \times \frac{l}{9\sqrt{3}}$$

m = Mass of Connecting rod in kg

$= V \times \rho = A \times l \times \text{density}$

$= 11t^2 \times l \times \rho$

$= 11 \times 6^2 \times 240 \times \frac{7800}{10^9}$

$m = 0.75$ kg $\quad [\because \rho = 7800 \text{ kg/m}^3]$

ω = Angular Speed of crank

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2800}{60}$$

$\omega = 293$ rad/sec

r = Radius of the crank = 60 mm

$$\therefore M_{Max} = 0.75 \times 60 \times 293^2 \times \frac{240}{9\sqrt{3}} \times \frac{1}{10^3}$$

$$M_{Max} = 59.5 \times 10^3 \text{ N-mm}$$

Section Modulus, $Z_{xx} = \frac{I_{xx}}{\frac{5t}{2}} = \frac{419t^4}{12} \times \frac{2}{5}$

$= 14t^3$

$Z_{xx} = 14 \times 6^3 = 3\times10^3\ mm^3$

Max. Bending stress,

$\sigma_{bmax} = \frac{M_{max}}{Z_{xx}} = \frac{59.5\times10^3}{3\times10^3} = 19.8\ N/mm^2\ (< \sigma_{cr})$

Design is safe

(ii) Design of connecting rod Ends:

$l_1 d_1$ = length & dia of piston pin

$l_2 d_2$ = " " crank pin

P_{b1}, P_{b2} = Design Bearing pressure for piston pin & crank pin

F_1, F_2 = Bearing load applied on piston pin & crankpin $= F_p$

$F_1 = P_{b1} \times l_1 d_1$ & $F_2 = P_{b2} \times l_2 d_2$

$P_{b1} = 14\ N/mm^2$ & $P_{b2} = 11\ N/mm^2$

$l_1 = 1.5\,d_1$

$F_1 = P_{b1} \times 1.5 d_1 \times d_1 = F_p$

$d_1 = \left[\frac{F_p}{P_{b1}\times1.5}\right]^{1/2} = \left(\frac{15080}{14\times1.5}\right)^{1/2} = 26.8\,mm = 30\,mm$

$l_1 = 1.5\,d_1 = 1.5\times30 = 45\,mm$

$l_2 = d_2$

$F_2 = P_{b2} \times d_2 \times d_2 = F_p$

$d_2 = \left[\frac{F_p}{P_{b2}\times1}\right]^{1/2} = \left[\frac{15080}{11\times1}\right]^{1/2} = 37\,mm \cong \underline{40\,mm}$

$\therefore l_2 = d_2 = 40\,mm$

The inner dia of small end, $D_{is} = d_1 = 30\,mm$

Outer dia of " " $D_{os} = D_{is} + (2 \times \text{Bush thickness}) + (2 \times \text{Marginal thickness})$

$= 30 + (2\times5) + (2\times5)$

$D_{os} = 50\,mm$

For Big End

Inner dia, $D_{ib} = d_2 = 40\,mm$

Outer dia $D_{ob} = D_{ib} + 2 \times \text{Bush thickness} + (2 \times \text{Bolt dia}) + (2 \times \text{Marginal thickness})$

(iii) Design of Bolts:-

Bolts are subjected to inertia force by reciprocating parts

Inertia force $\Rightarrow F_i = m\omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

Max. Inertia force is obtained when $\theta = 0$

$$F_{im} = mr\omega^2\left(1 + \frac{1}{n}\right)$$

$$m = \frac{W_r}{g} = \frac{15}{9.81} = 1.53\,kg$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2800}{60} = 293.2\ rad/sec$$

$r = 60\,mm$

$$n = \frac{l}{r} = \frac{240}{60} = 4$$

$$F_{im} = 1.53 \times 293.2^2 \times \frac{60}{10^3}\left(1 + \frac{1}{4}\right) = 9865\,N$$

d_c = core dia of Bolt

σ_t = Allowable tensile stress for Bolt Material

n' = Number of Bolts.

$$\sigma_t = \frac{\text{Yield stress in tension}}{\text{factor of safety}} = \frac{\sigma_y}{FOS} = \frac{600}{6} = 100\ N/mm^2$$

$$F_{im} = 2 \times \frac{\pi}{4} d_c^2 \times \sigma_t$$

$$d_c = \left[\frac{4 \times F_{im}}{2\pi\ \sigma_t}\right]^{1/2} = \left[\frac{4 \times 9865}{2\pi \times 100}\right]^2 = 7.92\ mm$$

$$d_b = \frac{d_c}{0.84} = 9.4\ mm = 10\ mm$$

M10 Bolt may be selected

$$D_{ob} = D_{ib} + (2 \times \text{Bush thickness}) + (2\ \text{Bolt thick}) + (2\ \text{Marginal thick})$$

$$= 40 + (2 \times 5) + (2 \times 10) + (2 \times 10)$$

$$D_{ob} = 90\ mm$$

(iv) Design of cap:-

$$\sigma_{bc} = \frac{M}{Z} = \frac{Wl'}{6} \times \frac{6}{bt_c^2} = \frac{Wl'}{bt_c^2}$$

W = Max. load applied on the cap = Inertia force = 9865N

l' = Distance b/w Bolt centres

= Dia of crank pin + (2 × Bush thick) + dia of Bolt + (2 × Marginal thickness)

$$= 40 + (2 \times 5) + 10 + (2 \times 7.5)$$

$$= 75\ mm$$

b - width of cap = length of crank pin − (2 × flange thickness of Bush)

$$= 40 - (2 \times 5) = 30\ mm$$

$$t_c = \left(\frac{Wl'}{b\ \sigma_{bc}}\right)^{1/2} = \left[\frac{9865 \times 75}{30 \times 100}\right]^{1/2} = 15.7\ mm \simeq 16\ mm$$

$(\because \sigma_{bc} = 100\ N/mm^2)$.

2) Design a connecting rod for an I.C Engine running at 1800 rpm and developing a maximum pressure of 3.15 N/mm². The dia of piston is 100 mm: Mass of the reciprocating parts per cylinder 2.25 kg; Length of connecting rod 380 mm; stroke of piston 190 mm and compression ratio 6:1. Take a factor of safety of 6 for the design. Take length to dia ratio for big end bearing as 1.3 and small end bearing as 2 and the corresponding pressure as 10 N/mm² and 15 N/mm². The density of material of the rod may be taken as 8000 kg/m³ and the allowable stress in Bolts as 60 N/mm² and in cap as 80 N/mm². The rod is to be of I- section for which you can choose your own proportions.

Draw a neat dimensioned sketch showing provisions for lubrication. Use Rankine formula for which the numerator constant may be taken as 320 N/mm² and the denominator constant $\frac{1}{7500}$.

Given

$N = 1800$ rpm, $FOS = 6$

$p = 3.15$ N/mm², $D = 100$ mm;

$m_R = 2.25$ kg; $l = 380$ mm $= 0.38$ m

$L = 190$ mm; Compression Ratio $= 6:1$

$I_{xx} = 4 I_{yy}$

$A = 2(4t \times t) + 3t \times t = 11t^2$

$I_{xx} = \frac{1}{12}\left[4t(5t)^3 - 3t(3t)^3\right] = \frac{419}{12} t^4$

$I_{yy} = 2 \times \frac{1}{12} \times t(4t)^3 + \frac{1}{12} \times 3t \times t^3 = \frac{131}{12} t^4$

$\frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$

$$F_P = \frac{\pi}{4} D^2 \times P = \frac{\pi}{4}(100)^2 \times 3.15$$

$$F_P = 24740\,N$$

The connecting rod is designed for Buckling about X-axis

$$W_B = F_P \times FOS = 24740 \times 6$$

$$W_B = 148440\,N$$

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{419}{12} t^4 \times \frac{1}{11t^2}} = 1.78t$$

Length of crank

$$r = \frac{\text{Stroke of piston}}{2} = \frac{190}{2} = 95\,mm$$

length of Connecting rod $l = 380\,mm$

$\therefore$ Equivalent length of Connecting rod for Both Ends Hinged

$$L = l = 380\,mm$$

$$148440 = \frac{\sigma_c \cdot A}{1 + a\left(\frac{L}{k_{xx}}\right)^2} = \frac{320 \times 11t^2}{1 + \frac{1}{7500}\left(\frac{380}{1.78t}\right)^2}$$

$$\frac{148440}{320} = \frac{11t^2}{1 + \frac{6.1}{t^2}} = \frac{11t^4}{t^2 + 6.1}$$

$$464(t^2 + 6.1) = 11t^4$$

$$t^4 - 42.2t^2 - 257.3 = 0$$

$$t^2 = \frac{42.2 \pm \sqrt{(42.2)^2 + 4 \times 257.3}}{2}$$

$$t^2 = \frac{42.2 \pm 53}{2} = 47.6$$

Thickness of flange & Web Section $\quad t = 6.9 \cong 7\,mm$

∴ Width of the Section $B = 4t = 4\times7 = 28\,mm$

Depth (or) Height of the Section $H = 5t = 5\times7 = 35\,mm$

Depth Near the Big End $= 1.1H$ to $1.25H$

$= 1.2H = 1.2\times35 = 42\,mm$

Depth Near the Small End $= 0.75H$ to $0.9H$

$= 0.85H = 0.85\times35$

$= 29.75 \cong 30\,mm$

∴ Dimensions of the Section Near the Big End $= 42\times28\,mm^2$

" " Small End $= 30\times28\,mm^2$

(ii) Dimensions of the crankpin (or) the Big end Bearing & piston pin:

d_1, l_1, P_{b_1} = Belong to piston pin (Small End)

d_2, l_2, P_{b_2} = " " crank pin (Big End)

F_1, F_2 = Bearing load applied on piston pin & crankpin $= F_p$

$F_1 = P_{b_1}\times l_1\times d_1 = F_p$

$l_1 = \cancel{1.3d_1}\ 2d_1$

$P_{b_1} = 15\,N/mm^2$

$F_p = P_b\times l_1\times d_1 \Rightarrow 24740 = 15\times 2d_1\times d_1$

$d_1^2 = \frac{24740}{30} = 825$

$d_1 = 28.7 \cong 29\,mm.$

$l_1 = 2\times29 = 58\,mm$

The Small End Bearing is usually a phosphor Bronze Bush of about 3mm thickness

$F_2 = P_{b_2} \times l_2 \times d_2$ $\qquad$ $l_2 = 1.3\, d_2$

$24740 = 10 \times 1.3\, d_2 \times d_2$ $\qquad$ $P_{b_2} = 10\ N/mm^2$

$$d_2^2 = \frac{24740}{13} = 1903$$

$$d_2 = 43.6 \cong 44\,mm$$

$$l_2 = 1.3 \times 44 = \underline{\underline{58\ mm}}.$$

(iii) Size of Bolts for Securing the Big End Cap:-

$$F_{im} = mr\omega^2 \left(1 + \frac{1}{n}\right)$$

$$= 2.25 \times 0.095 \times \left(\frac{2\pi \times 1800}{60}\right)^2 \left[1 + \frac{0.095}{0.38}\right]$$

$$F_{im} = \underline{\underline{9490\ N}}$$

d_c = Core dia of Bolt

σ_t = Allowable tensile Stress for Bolt Material.

η = Number of Bolts.

$$F_{im} = 2 \times \frac{\pi}{4} d_c^2 \times \sigma_t$$

$$= 2 \times \frac{\pi}{4} (d_c)^2 \times 60 = 94.26 (d_c)^2$$

$$9490 = 94.26\, (d_c)^2$$

$$d_c = 10.03\,mm$$

$$d_b = \frac{d_c}{0.84} = \frac{10.03}{0.84} = 11.94 \cong 12\,mm.$$

(iv) Thickness of Big End Cap:-

$$\sigma_{bc} = \frac{M}{Z} = \frac{wl^1}{6} \times \frac{6}{bt_c^2} = \frac{wl^1}{bt_c^2}$$

$\therefore$ l' = Distance b/w Bolt centres

= Dia of crank pin + (2 x Bush thickness) + Dia of Bolt + (2 x Marginal thickness) (or) clearance.

= ~~40 + (2x8) +~~

= 44 + (2x3) + 12 + (2 x 1.5)

= 44 + 6 + 12 + 3

= 65 mm

$$t_c = \left(\frac{Wl'}{b\,\sigma_{bc}}\right)^2 \therefore = \left(\frac{9490 \times 65}{58 \times 80}\right)^2 = 11.5\text{ mm.}$$

$\therefore$ Max. Bending Moment $M_{Max} = m r \omega^2 \times \frac{l}{9\sqrt{3}}$

m = Mass of connecting rod in kg

$= V \times \rho = A \times l \times \rho$

$= 11t^2 \times l \times \rho$

$= 11(0.007)^2 (0.38)\, 8000$

$m = 1.64$ kg

$$= 1.64 \times \left(\frac{2\pi \times 1800}{60}\right)^2 (0.095) \frac{(0.38)^2}{9\sqrt{3}} = 51.3\text{ N-m}$$

= 51300 N-mm

$$Z_{xx} = \frac{I_{xx}}{\frac{5t}{2}} = \frac{419t^4}{12} \times \frac{2}{5t} = 13.97\,t^3$$

$$= 13.97 \times 7^3 = 4792\text{ mm}^3$$

$$\sigma_b = \frac{M_{Max}}{Z_{xx}} = \frac{51300}{4792} = 10.7\text{ N/mm}^2 < 80\text{ N/mm}^2$$

$\therefore$ Design safe

Design of Piston:

When designing a piston, the following points must be considered such as.

(i) Adequate strength to withstand high pressure produced by the gas.

(ii) Capacity of piston to withstand high temp.

(iii) Sealing of the working space aganist escape of gases.

(iv) Good dissipation of Heat to the cylinder wall.

(v) Suffecient projected area (i.e surface area) and rigidity of the barrel.

(vi) Minimum loss of power due to friction.

(vii) Minimum weight, to reduce inertia force and unbalance effects.

(viii) Suffecient length to have better guidance and so.on.

(i) piston Head:

The piston head is assumed to be a flat disc of uniform thickness fixed at edges and subjected to uniformly distributed load gas. Also the piston head has to withstad high thermal stress.

Hence the piston Head is designed based on its stregth to withstad gas load and also Heat dissipation.

Based on stregth Considration.

$$t_1 = \sqrt{\frac{3 P_m D^2}{16 \sigma_{tp}}} \text{ mm}$$

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When P_m = Maximum gas pressure (N/mm^2)

D = Dia of piston (or) cylinder Bore (mm)

σ_{tp} = 35 to 40 N/mm^2 for CI

= 60 to 100 N/mm^2 for steel

= 50 to 90 N/mm^2 for aluminium Alloy.

Based on Heat dissipation, the Head thickness is determined as.

$$t_1 = \frac{1000\,H}{12.56\,k\,(T_c - T_e)} \text{ mm}$$

H = Heat flowing through the Head (kW)

$= 0.05\,m \times C_v \times P_B$.

m = Mass of fuel used (i.e fuel consumption) kg/kw/sec

C_v = Higher Calorific value of the fuel (kJ/kg)

$= 44 \times 10^3$ kJ/kg for diesel fuel

$= 11 \times 10^3$ kJ/kg for petrol fuel

P_B = Brake power of the Engine per cylinder (kW)

$$= \frac{PLAn}{60 \times 10^6} \text{ kW}$$

P = Brake mean Effective pressure (N/mm^2)

L = stroke length (mm)

A = Area of piston at its top side $\left(\frac{\pi}{4}D^2\right)$ (mm^2)

n = Number of power stokes per minute.

$n = N$ for two stroke Engine

$n = N/2$ for four stroke Engine.

k = Heat conductivity factor (kw/m/°c)

$= 46.6 \times 10^{-3}$ for CI

$= 51 \times 10^{-3}$ for steel

$= 175 \times 10^{-3}$ for Aluminium Alloys.

T_c = Temp at the centre of piston Head (°c)

T_e = Temp at the edge of piston Head (°c)

= 75°c Al

$T_c - T_e$ = 200°c CI

$T_c - T_e$ = 110°c Al

Ribs:-

To make the piston rigid and to present distortion due to gas load and connecting rod thrust, four to six ribs are provided at the inner side of the piston. The thickness of rib is assumed as.

$$t_2 = (0.3 \text{ to } 0.5)\, t_1$$

Piston Rings:-

To maintain the seal b/w the piston and inner wall of the cylinder, some split-rings called piston rings are employed. By making some sealing the escape of gas through piston side wall to the connecting rod side can be prevented. The piston rings also serve to transfer the Heat from the piston head to cylinder walls.

With respect to the location of piston rings, they are called as top rings (or) Bottom rings. Rings inserted at the top of the piston side-wall are compression rings which may be 3 to 4 for Automobile & Aircraft Engines 5 to 7 for stationary compression Ignition Engines. Rings inserted at the Bottom of piston side wall are oil scraper rings, used to scrap the oil from the liner so as to minimize the flow of oil into Combustion-chamber. The number of oil scrapper rings may be taken as 1 to 3.

Usually the piston rings are made of Alloy CI with chromium plated to possess good wear-resisting qualities and spring characteristics even at high temp. When designing the piston rings the contact pressure exerted by the rings on the linear-Wall should be limited b/w 0.025 N/mm² and 0.042 N/mm²

Let t_3 = Radial thickness of piston rings

t_4 = Axial thickness of piston rings

P_c = Contact pressure (i.e wall pressure) N/mm².

$$t_3 = D\sqrt{\frac{3P_c}{\sigma_{br}}} \quad (mm)$$

$$t_4 = (0.7 \text{ to } 1)\, t_3 \quad (or)$$

$$t_4 = \frac{D}{10\,i}$$

D = Bore dia (mm)

σ_{br} = Allowable Bending stress of ring Material (N/mm²)

= 84 to 112 N/mm² for Alloy CI

i = Number of rings

Due to Some advantage like, Better sealing action, less wear of lands etc. usually thinner rings are preferred. The first ring groove is cut at a distance of t_1 to $1.2 t_1$ from top. The lands b/w the rings may be equal to or less than the axial thickness of ring t_4. The gap b/w the free ends of the ring is taken as $C = (3.5 \text{ to } 4) t_3$

Piston Barrel

The cylindrical portion of the piston is termed as piston-Barrel. The Barrel thickness may be varied (usually reduced) from top side to Bottom side of the piston.

The Max. thickness of Barrel nearer to piston head is given by

$$t_5 = 0.03D + b + 4.5 \text{ (mm)}$$

b = Radial depth of ring groove

$$b = t_3 + 0.4 \text{ (mm)}$$

The thickness of Barrel at the open end of the piston.

$$t_6 = (0.25 \text{ to } 0.35) t_5 \text{ (mm)}$$

Piston skrit:

The portion of piston barrel below the ring section upto the open End is called as piston-skrit. The piston-skrit takes up the side-thrust of the connecting rod The length of the piston skrit is selected in such a way that the Side thrust pressure should not exceed 0.28 N/mm² for slow speed engines and 0.5 N/mm² for High speed Engines

The side thrust force is given by

$$F_s = \mu F_g$$

μ = Coeffecient of friction b/w lines and skrit
= (0.03 to 0.1)

F_g = Gas force $= \frac{\pi}{4} D^2 \cdot P_m$

The side thrust pressure.

$$P_s = \frac{\text{Side thrust force}}{\text{Projected Area}} = \frac{F_s}{L_s \times D}$$

$$P_s = \frac{\mu F_g}{L_s \times D} = \frac{\mu \frac{\pi}{4} D^2 \cdot P_m}{L_s \times D} = \frac{\mu \pi D P_m}{4 L_s}$$

Length of piston:-

The length of piston, L_p

$L_p = L_s +$ length of ring section + Top land.

L_p = D to 1.5D

Gudgeon pin (or) Piston pin:-

The piston pin should be made of case Harded Alloy steel containing nickel, chromium, molybdenum etc. with An ultimate strength of 700 to 900 N/mm² in order to with stand High gas pressure. The piston pin is designed Based on the Bearing pressure Consideration.

l = length of piston pin

d = dia of piston pin

(P_b) = Allowable Bearing pressure for piston pin
= 15 to 30 N/mm²

Bearing strength of piston pin, F_b = Bearing Pressure × Projected Ara

$F_b = P_b \cdot l \times d$

By Equating this Bearing strength to gas force, F_g

We get $P_b \cdot l \cdot d = F_g$ $\left(F_g = \frac{\pi}{4} D^2 \cdot P_m\right)$

$$\frac{l}{d} = 1.5 \text{ to } 2$$

The piston pin is checked for Bending as. the induced Bending stress

$$\sigma_b = \frac{32 M}{\pi d^3} < (\sigma_b)$$

Where M = Bending Moment $= \frac{F_g D}{8}$

D = Bore dia.

F_g = Gas force

σ_b = Allowable Bending stress

$= 84\ N/mm^2$ for case Hardened steel

$= 140\ N/mm^2$ for Heat treated Alloy steel

The gudgeon pin is fitted at a distance of $\left(\frac{L_s}{2}\right)$

Piston clearance:-

Proper clearance must be provided b/w the piston and the liner to take care of thermal expansion and distortion under load. Usually the clearance may be b/w 0.04 mm & 0.20 mm, depending upon the Engine design & piston dia. Small clearance may be adopted for the pistons cooled by oil or water

① Design a cast-iron piston for a single acting four-stroke I.C Engine for the following specifications

Cylinder Bore = 100 mm

stroke length = 120 mm

Maximum Gas pressure = 6 MPa

Break Mean Effective pressure = 0.7 MPa

Fuel consumption = 0.24 kg/kW/hr

Speed = 2200 rpm

Given

Bore dia => $D = 100$ mm

stroke length => $L = 120$ mm

Max. gas pressure $P_m = 6\ MPa = 6\ N/mm^2 = 6\times10^6\ N/m^2$

Break Mean effective pressure $P = 0.7\ MPa = 0.7\times10^6\ N/m^2$
$= 0.7\ N/mm^2$

Fuel consumption = 0.24 kg/kW/hr

speed N = 2200 rpm

(i) Piston Head:

Based on strength, the thickness of piston head is

$$t_1 = \sqrt{\frac{3P_m D^2}{16\,\sigma_{tp}}} = \sqrt{\frac{3\times6\times100^2}{16\times40}} = 16.8\ mm$$

$\cong 18$ mm

$\sigma_{tp} = 40\ N/mm^2$

Based on Heat dissipation thickness of piston-Head is given by

$$t_1 = \frac{1000\,H}{12.56\,k\,(T_c - T_e)} \text{ mm}$$

Where H = Heat flowing through Head in kW

$H = C \times m \times C_v \times P_B$

$c = 0.05$

m = Mass of fuel (i.e fuel consumption)

$= 0.24$ kg/kW/hr

$= \frac{0.24}{3600}$ kg/kW/sec

C_v = Higher calorific value

$= 44 \times 10^3$ kJ/kg for diesel fuel

P_B = Brake Power

$$P_B = \frac{PLAN}{60 \times 10^6} \text{ kW}$$

P = Brake Mean Effective pressure $= 0.7$ N/mm²

L = Length of stroke = 120 mm

A = Area of piston $= \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 100^2$

$A = 7.85 \times 10^3$ mm²

n = Number of working strokes $= \frac{N}{2}$

$n = \frac{2200}{2} = 1100$

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$$P_B = \frac{0.7 \times 120 \times 7.85 \times 10^3 \times 1100}{60 \times 10^6} = 12.1 \text{ KW}$$

$$H = 0.05 \times \frac{0.24}{3600} \times 44 \times 10^3 \times 12.1$$

$H = 1.8$ KW

k = Heat conductivity factor = 46.6×10^{-3} KW/m/°c

$(T_c - T_e) = 220°c$ cast-iron

$$t_1 = \frac{1000 \times 1.8}{12.56 \times 46.6 \times 10^{-3} \times 220} = 14 \text{ mm}$$

Take the thickness of piston Head $t_1 = 18$mm

(ii) Ribs:

Rib thickness, $t_2 = (0.3 \text{ to } 0.5)\, t_1$

$t_2 = 0.5\, t_1 = 0.5 \times 18 = 9$ mm

Number of Rib = 4

(iii) Piston Rings:

Let us select 3 compression rings and 2 oil Rings.

Radial thickness of Ring, $t_3 = D \times \sqrt{\dfrac{3 P_c}{\sigma_{br}}}$

P_c = Contact pressure = 0.035 N/mm²

σ_{br} = Allowable Bending stress of ring Material

$\sigma_{br} = 100$ N/mm²

$$t_3 = 100 \times \sqrt{\frac{3 \times 0.035}{100}}$$

$t_3 = 3.24$ mm $= 3.5$ mm

Axial thickness of ring, $t_4 = (0.7 \text{ to } 1)\, t_3$

$= 0.8\, t_3$

$= 0.8 \times 3.5$

$t_4 = 3\,mm$

Depth of top land $x_1 = t_1 = 18\,mm$

Depth of other land $x_2 = 2.5\,mm$ $(\because x_2 \le t_4)$

Radial depth of piston ring groove $b = t_3 + 0.4\,mm$

$= 3.5 + 0.4$

$b = 3.9\,mm$

(iv) Piston Barrel:

Maximum thickness of Barrel $t_5 = 0.03D + b + 4.5\,mm$

$= (0.03 \times 100) + 3.9 + 4.5$

$= 11.4\,mm$

$\cong 11.5\,mm$

Thickness of Barrel at the open end of piston

$t_6 = (0.25 \text{ to } 0.35)\, t_5$

$t_6 = (0.3)\, t_5 = 0.3 \times 11.5 = 3.45\,mm$

$t_6 = 3.5\,mm$

(v) Piston Skirt:

Length of piston skirt $L_s = \dfrac{F_s}{(P_s \times D)}$

F_s = Side thrust force $= \mu \times F_g$

Now. $F_g = \frac{\pi}{4} D^2 \times P_m$, P_m $F_g = \frac{\pi}{4} \times 100^2 \times 6 = 47124\,N$

Assume $\mu = 0.1$

$\therefore F_S = \mu F_g = 0.1 \times 47124 = \underline{4712.4 \text{ N}}$

Assuming the side the thrust pressure, $P_S = 0.4 \text{ N/mm}^2$

for High speed Engine

$$L_S = \frac{F_S}{P_S \times D} = \frac{4712.4}{0.4 \times 100} = 118 \text{ mm}.$$

(vi) Length of piston:

Length of piston, $L_P = L_S +$ Length of ring section + Top land

length of ring section for 5 rings and 4 lands

$$L_r = (5 \times t_u) + (4 \times x_2)$$

$$= (5 \times 3) + (4 \times 2.5)$$

$$L_r = 25 \text{ mm}$$

$$\therefore L_P = L_S + L_r + X_1$$

$$= 118 + 25 + 18$$

$$= \underline{161 \text{ mm}}$$

(vii) Piston pin:

l = length of piston pin,

d = dia of piston pin

P_b = Allowable Bearing pressure = 25 N/mm^2

Bearing strength of pin. $f_b = P_b \times l \times d$

$$F_g = P_b \times l \times d$$

$$l = 1.5d$$

$$P_b \times 1.5d \times d = F_g$$

$$d = \left[\frac{F_g}{P_b \times 1.5}\right]^{1/2}$$

$$d = \frac{47124}{25 \times 1.5} = 36\ \text{mm}$$

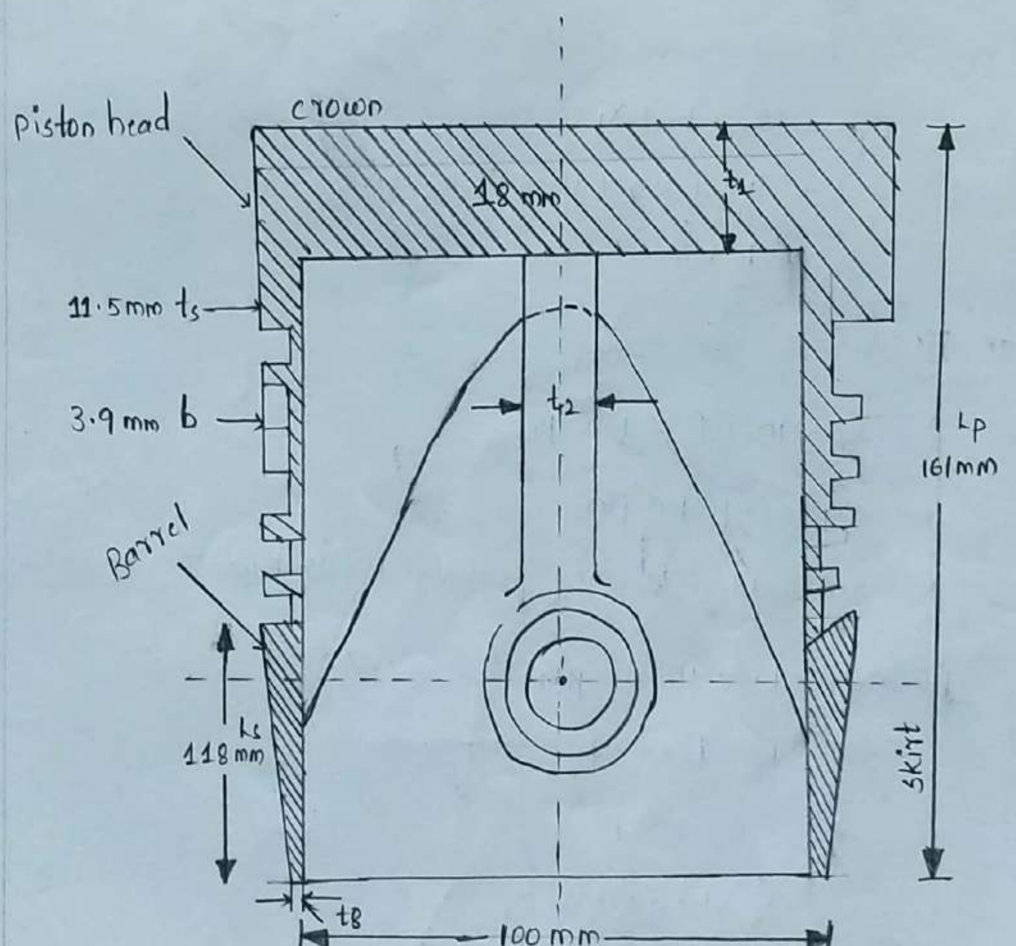
Length of piston pin $l = 1.5\,d = 1.5 \times 36 = 54$ mm

The piston pin is checked for Bending strength.

$$\sigma_b = \frac{32\,M}{\pi d^3}$$

$$M = \frac{F_g \times D}{8} = \frac{47124 \times 100}{8} = 589 \times 10^3\ \text{N-mm}$$

$$\sigma_b = \frac{32 \times 589 \times 10^3}{\pi \times 36^3} = 129\ \text{N/mm}^2 \ \left(\le \sigma_b = 140\ \text{N/mm}^2\right)$$



(2) Design a cast iron piston for single acting four-stroke Engine for the following data;

Cylinder Bore = 100 mm

stroke = 125 mm

Max. gas pressure = 5 N/mm²

Indicated Mean effective pressure = 0.75 N/mm²

Mechanical Effeciency = 80%

Fuel consumption = 0.15 kg Per Brake power per hour

Higher Calorific value of fuel = 42×10^3 KJ/kg

speed = 2000 rpm

Any other data required for the design may be Assumed.

Given

$D = 100\ mm,\ L = 125\ mm = 0.125\ m$

$P_m = 5\ N/mm^2$

$P = 0.75\ N/mm^2,\ \eta_m = 80\% = 0.8$

$m = 0.15\ kg/Bp/hr$

$m = \frac{0.15}{3600}\ kg/Bp/sec = 41.7 \times 10^{-6}\ kg/Bp/hr$

$C_V = 42 \times 10^3\ KJ/kg;$

$N = 2000\ rpm$

(i) Piston Head (or) Crown:

$$t_1 = \sqrt{\frac{3 P_m D^2}{16\ \sigma_t}} = \sqrt{\frac{3 \times 5 \times (100)^2}{16 \times 38}} = 15.7$$

$t_1 \cong 16\ mm$

Since the Engine is a four stroke,

∴ The Number of Working strokes per min.

$$n = \frac{N}{2} = \frac{2000}{2} = 1000$$

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (100)^2 = 7855 \text{ mm}^2$$

$$I_P = \frac{PLAn}{60} = \frac{0.75 \times 125 \times 7855 \times 1000}{60 \times 10^6}$$

$$I_P = 12.27 \text{ KW}$$

∴ Brake power $BP = IP \times \eta_m$

$$BP = 12.27 \times 0.8$$

$$P_B \text{ (or) } BP = \underline{9.8} \text{ KW}$$

$$H = 0.05 \times C_v \times m \times P_B$$

$$= 0.05 \times 42 \times 10^3 \times 41.7 \times 10^{-6} \times 9.8$$

$$H = 0.86 \text{ KW}$$

$$t_1 = \frac{1000\ H}{12.56\ k (T_c - T_e)}$$

$$= \frac{1000 \times 0.86}{12.56 \times 46.6 \times 220}$$

$$= 0.0067 \text{ m}$$

$$t_1 = 6.7 \text{ mm}$$

Take $t_1 = \underline{16} \text{ mm}$

(ii) Ribs:

Rib thickness, $t_2 = (0.3 \text{ to } 0.5)\, t_1$

$= 0.3 \times 16 \text{ to } 0.5 \times 16$

$t_2 = 4.8 \text{ to } 8 \text{ mm}$

(iii) Piston Rings:

Let us Assume 3 - compression rings & 1 oil ring;

$$t_3 = D \times \sqrt{\frac{3 P_c}{\sigma_{br}}}$$

$$= 100 \times \sqrt{\frac{3 \times 0.035}{90}}$$

$t_3 = 3.4 \text{ mm}$

Axial thickness of ring $t_4 = (0.7 \text{ to } 1)\, t_3$

$t_4 = 0.7 \times 3.4 \text{ to } 1 \times 3.4$

$t_4 = 2.38 \text{ to } 3.4 \text{ mm}$

$t_4 = 3 \text{ mm}$

Depth of top land $x_1 = t_1 = 16 \text{ mm}$

Depth of other land $x_2 = 2.5 \text{ mm}$ $(x_2 \leq t_4)$

Radial depth of piston ring groove

$b = t_3 + 0.4 \text{ mm} = 3.4 + 0.4$

$b = 3.8 \text{ mm}$

(iv) Piston Barrel:-

Max. thickness of Barrel $t_5 = 0.03D + b + 4.5$ mm

$= 0.03 \times 100 + 3.8 + 4.5$

$t_5 = 11.3$ mm

Thickness of Barrel at the open End of piston

$t_6 = (0.25 \text{ to } 0.35)\, t_5$

$= (0.25 \times 11.3 \text{ to } 0.35 \times 11.3)$

$t_6 = 2.8 \text{ to } 3.9$ mm

$t_6 = 3.4$ mm

(v) Piston skirt:-

$F_s = \mu \times F_g = \mu \times \frac{\pi}{4} D^2 \times P_m$

$= 0.1 \times \frac{\pi}{4} (100)^2 \times 5$

$F_s = 3928$ N

$L_s = \frac{F_s}{P_s \times D} = \frac{3928}{0.45 \times 100} = 87.2$ mm

$L_s \cong 90$ mm

(vi) Length of piston:

$L_p = L_s + L_r + L_t$

$L_p = 90 + (4 \times t_4 + 3 \times x_2) + 16$

$= 90 + (4 \times 3 + 3 \times 3) + 16$

$L_p = 128 \text{ mm} \cong 130 \text{ mm}$

(vi) piston Pin:-

d_o = Outside diameter of the pin in mm.

l = length of piston pin. mm

P_b = Bearing pressure at the small end of connecting rod. (N/mm²) $\cong$ 25 N/mm²

Load on the pin due to Bearing pressure

= Bearing pressure x Bearing Area

$= P_{b1} \times d_o \times l$ $\quad (\because l = 0.45D)$

$= 25 \times d_o \times 0.45 \times 100$

$= 1125\, d_o$ (N)

Max. Gas load $= \frac{\pi}{4} D^2 \times P_m = \frac{\pi}{4} \times (100)^2 \times 5$

$= 39275$ N

$1125\, d_o = 39275$

$d_o = 34.9 \cong \underline{35}$ mm

$d_i = 0.6 \times 35 = \underline{21}$ mm

$$M = \frac{F_g D}{8} = \frac{39275 \times 100}{8} = 491 \times 10^3 \text{ N-mm}$$

$$M = \frac{\pi}{32}\left[\frac{d_o^4 - d_i^4}{d_o}\right]\sigma_b$$

$$491 \times 10^3 = \frac{\pi}{32}\left[\frac{35^4 - 21^4}{35}\right]\sigma_b$$

$\sigma_b = \underline{134}$ N/mm² [< 140 N/mm²]

design is $\underline{\text{safe}}$.

* Belt drives (or) flexible machine elements: *

The belts (or) ropes are used to transmits power from one shaft to another by means of pulleys which rotate at the same (or) different speeds.

The Amount power transmitted depends, on

(i) The velocity of the belt

(ii) The tension under which the belt is placed on the pul

(iii) The Arc of contact b/w the belt & the smaller pulley.

(iv) The conditions under which belt is used.

Selection of a belt drive:

(i) speed of driving & driven shaft

(ii) speed reduction ratio

(iii) power to transmitted.

(iv) centre distance b/w shafts.

(v) positive drive requirements.

(vi) shaft layout

(vii) space Available. (viii) service conditions.

Types of Belt drive:

(i) light drives: transmit Small power at belt speeds upto about 10m/s as in agriculture machines &

(ii) Medium drives: transmit medium power at belt speeds over 10m/s upto 22 m/s

(iii) Heavy drives: large power at above 22 m/s Ex: compressor & Generator.

Types of belts:

(i) flat belt: moderate Amount of power is transmit. The pulleys are not more than 8 meters apart.

(ii) V-belt: Great Amount of power is transmitted. Where two pulleys are very near to each other.

(iii) Circular belt (or) Rope: Great, two pulleys are more than 8 meters apart.

Materials used for Belts:

i) Leather belts: The best leather belt made from 1.2 m to 1.5 m long strips cut from either side of a back bone of the top grade steer hide.

" oka-tanned (or) mineral salt-tanned eg. chrome-tanned.

(ii) Cotton (or) fabric belts: farm Machinery, Conveyors.

Most of the fabric belts are made by folding convass or cotton duck to three or more layers. & stitching together.

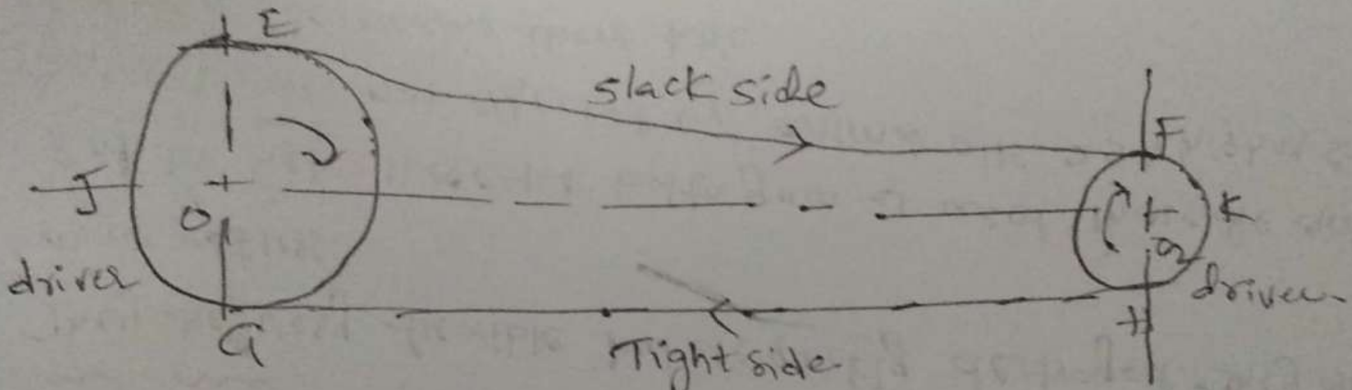
(iii) Rubber Belt: paper mills, saw mills

They are very flexible but are quickly destroyed. They are easily made endless.

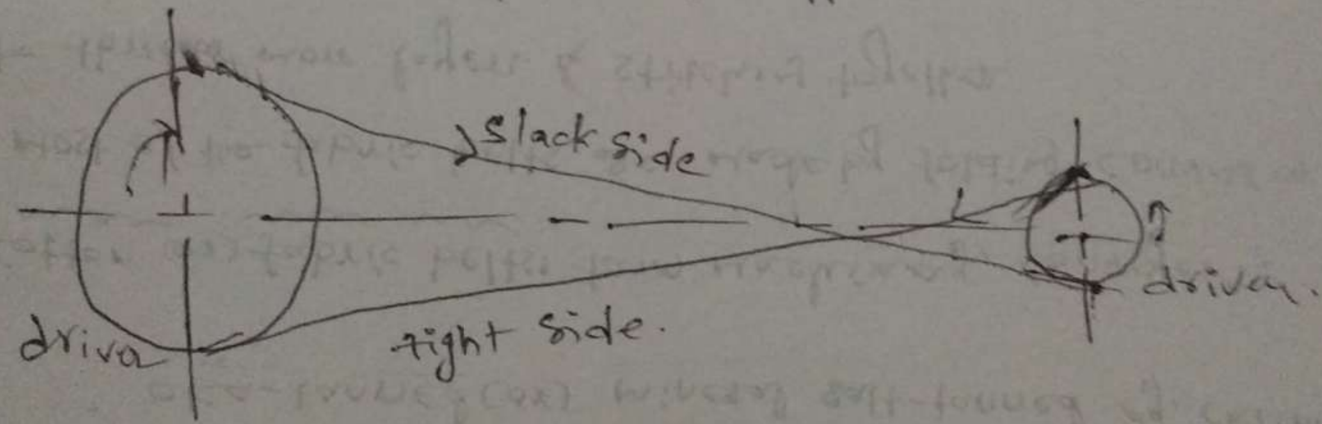
(iv) Batata bets: Here the balta gum is used. Those are acid proof & water proof not effected by animal oils.. 25% higher streth than rubber. Should not overcome above 40°C.

Types of flat belt drives:

(i) open belt drive: shafts are arranged llel & same direction.



ii) crossed Belt drive: llel & rotating in opposite direction.



The wear & tear at crossing place. The max. distance of 20b
width of belt.

Speed of the belt should not be less than 15m/s

Velocity ratio of Belt drive:

b/w driver & follower (or) driven.

Let d_1 = Dia of the driver

d_2 = Dia of driven

N_1 = Speed of driver r.p.m

N_2 = speed of driven r.p.m

∴ Length of the belt that passes over the driver, in 1 min
$= \pi d_1 N_1$

Similarly, " " passes " driven " $= \pi d_2 N_2$

$\therefore \pi d_1 N_1 = \pi d_2 N_2$

& $\frac{N_2}{N_1} = \frac{d_1}{d_2}$ & $\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}$

$V_1 = \frac{\pi d_1 N_1}{60}$, $V_2 = \frac{\pi d_2 N_2}{60}$

In case of Compound belt drive velocity ratio $= \frac{N_4}{N_1} = \frac{\text{Speed of last drive}}{\text{speed of 1st driv}}$

 $= \frac{d_1 \times d_3}{d_2 \times d_4} = \frac{\text{product of dia of dri}}{\text{" dia of driv}}$

Slip of the Belt:

But in sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This is called slip of the belt.

$S_1\% =$ % slip b/w driver & belt &

$S_2\% =$ slip b/w belt & follower.

velocity of belt passing over the driver per sec =

$$V = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{S_1}{100} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)$$

velocity of belt passing over follower

$$\frac{\pi d_2 N_2}{60} = V - V\left(\frac{S_2}{100}\right) = V\left(1 - \frac{S_2}{100}\right)$$

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_1}{100} - \frac{S_2}{100}\right) = \frac{d}{d_2}\left(1 - \left(\frac{S_1 + S_2}{100}\right)\right)$$

Oratil

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S}{100}\right)$$

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If thickness of belt

$$\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}\left(1-\frac{S}{100}\right)$$

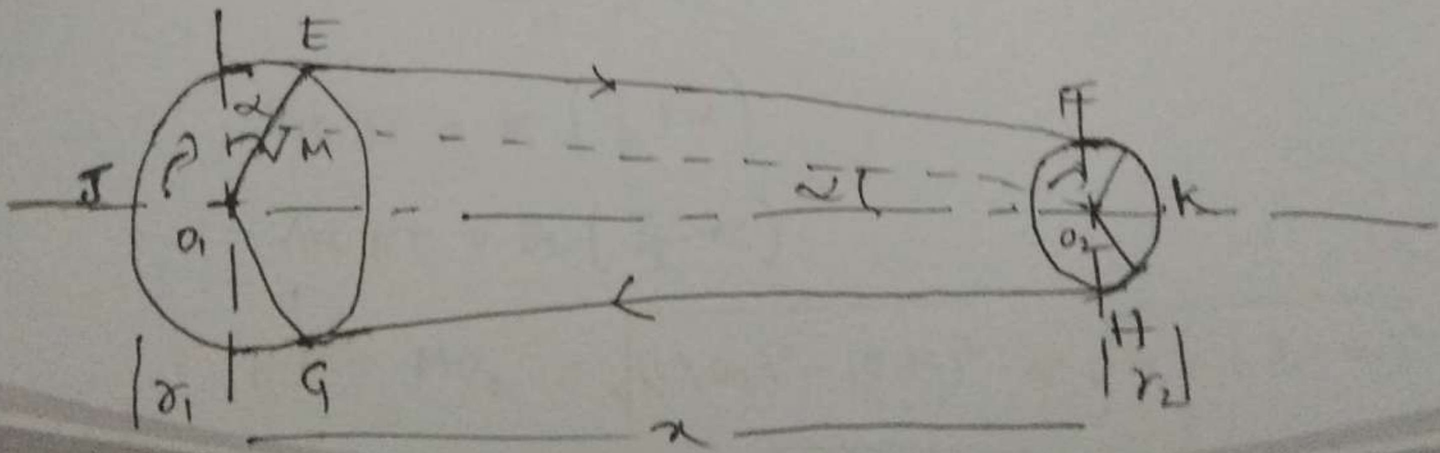
Creep of belt:

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E+\sqrt{\sigma_2}}{E+\sqrt{\sigma_1}}$$

σ_1 & σ_2 = stress in the belt on the tight side & slack side

E = Young's modulus.

Length of an open belt drive:



r_1 & r_2 = Radii of driver & driven pulleys

x = centre distance b/w two pulleys (i.e O_1O_2)

L = Total length of the belt

Let $MO_2O_1 = \alpha$ radians.

Length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\left[\text{Arc } JE + EF + \text{Arc } FH\right]$$

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}$$

$$\alpha = \frac{r_1 - r_2}{x}$$

$$\text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right)$$

$$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right)$$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem.

$$EF = x\left[1-\frac{1}{2}\left(\frac{r_1-r_2}{x}\right)^2+\dots\right] = x - \frac{(r_1-r_2)^2}{2x}.$$

$$L = 2\left[r_1\left[\frac{\pi}{2}+\alpha\right] + x - \frac{(r_1-r_2)^2}{2x} + r_2\left(\frac{\pi}{2}-\alpha\right)\right]$$

$$= 2\left(r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1-r_2)^2}{2x} + r_2\frac{\pi}{2} - r_2\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1+r_2) + \alpha(r_1-r_2) + x - \frac{(r_1-r_2)^2}{2x}\right]$$

$$= \pi(r_1+r_2) + 2\alpha(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

$$L = \pi(r_1+r_2) + 2\times\frac{r_1-r_2}{x}(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

$$= \pi(r_1+r_2) + \frac{2(r_1-r_2)^2}{x} + 2x - \frac{(r_1-r_2)^2}{x}$$

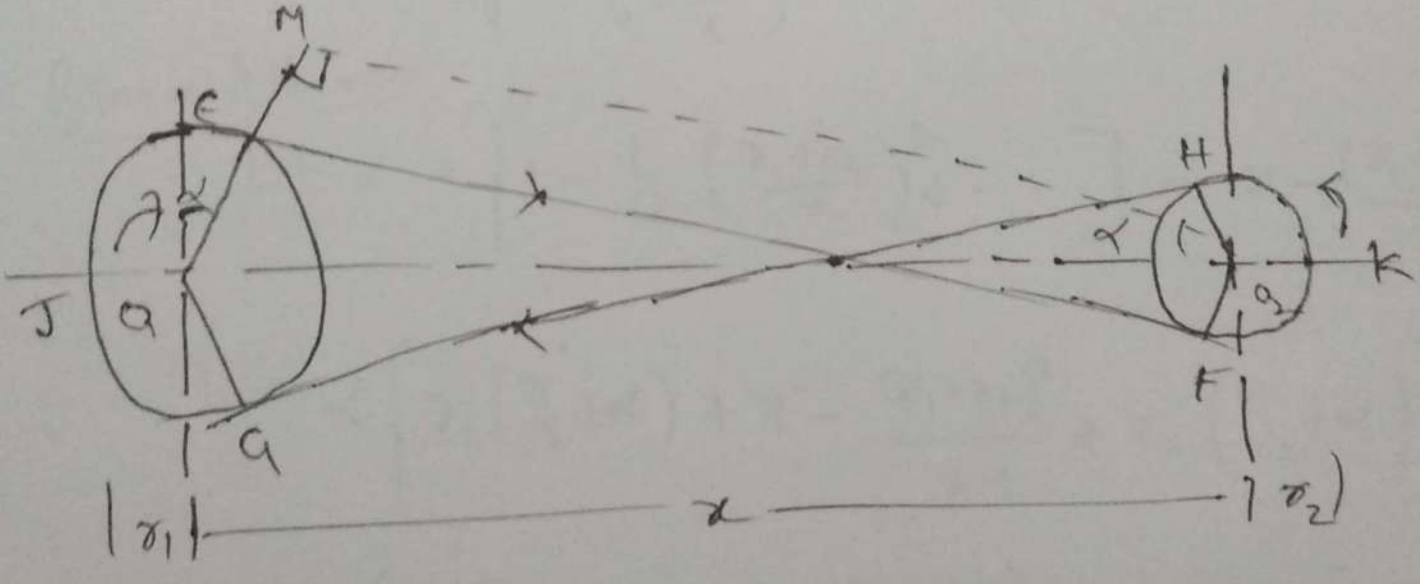
$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$

Length of a cross Belt drive:

Both the pulleys rotate in opposite direction.

r_1 & r_2 = Radii of driver & driven.



$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

Oratil

Arc JE = $r_1\left(\frac{\pi}{2}+\alpha\right)$

Similly. Arc FK = $r_2\left(\frac{\pi}{2}+\alpha\right)$

$L = \text{Arc}\,GJE + EF + \text{Arc}\,HKF + GH$

$EF = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1+r_2)^2}$

$= x\sqrt{1-\left(\frac{r_1+r_2}{x}\right)^2}$

Binomial theorm

$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1+r_2}{x}\right)^2 + \cdots\cdots\right] = x - \frac{(r_1+r_2)^2}{2x}$

$L = 2\left[r_1\left(\frac{\pi}{2}+\alpha\right) + x - \frac{(r_1+r_2)^2}{2x} + r_2\left(\frac{\pi}{2}+\alpha\right)\right]$

$= 2\left[r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1+r_2)^2}{2x} + r_2\frac{\pi}{2} + r_2\alpha\right]$

$= 2\left[\frac{\pi}{2}(r_1+r_2) + \alpha(r_1+r_2) + x - \frac{(r_1+r_2)^2}{2x}\right]$

Oratil

$\therefore$

$$L = \pi(r_1+r_2) + 2\times\frac{(r_1+r_2)}{x}(r_1+r_2) + 2x - \frac{(r_1+r_2)^2}{x}$$

$$= \pi(r_1+r_2) + \frac{2(r_1+r_2)^2}{x} + 2x - \frac{(r_1+r_2)^2}{x}$$

$$= \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}$$

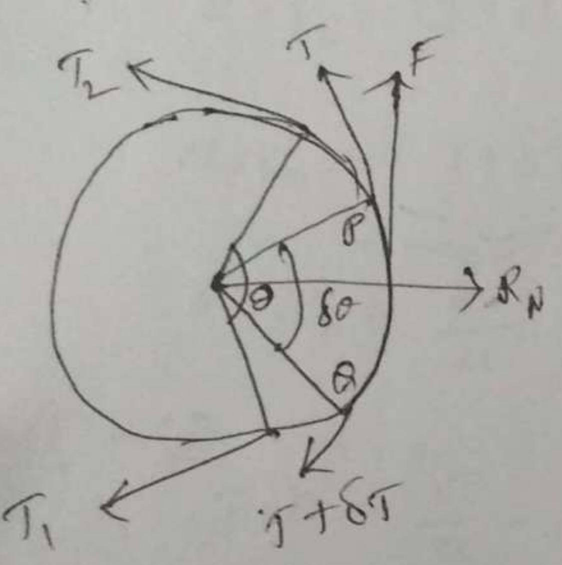
$$L = \frac{\pi}{2}(d_1+d_2) + 2x + \frac{(d_1+d_2)^2}{4x}.$$

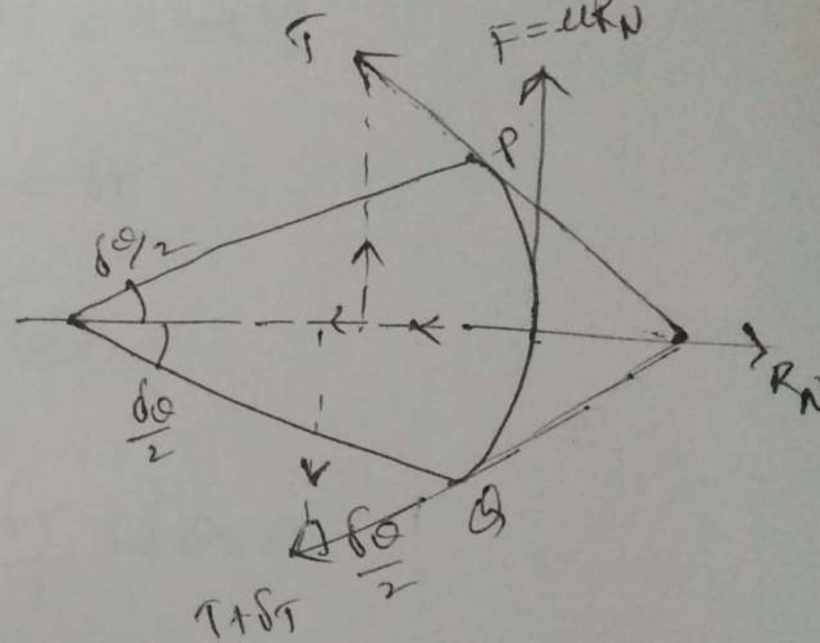
$\therefore$

Power transmitted by Belt:

$P = (T_1 - T_2)v \quad W$

Ratio of driving tensions for flat Belt drive:





θ = Angle of contact in radians.

Resolving all the forces horizontally, we have.

$$R_N = (T+\delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

$$R_N = (T+\delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2}$$

$$= T\frac{\delta\theta}{2} + \delta T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2}$$

$R_N = T\delta\theta$ (i) neglecting

Age : ______ Weight : ______ Gender : ______

Resolving vertical forcess.

$$\mu \times R_N = (T+\delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

$$\mu \times R_N = T+\delta T - T = \delta T$$

$$R_N = \frac{\delta T}{\mu} \quad \text{— ②}$$

① in ②

$$T\delta\theta = \frac{\delta T}{\mu} \Rightarrow \frac{\delta T}{T} = \delta\theta \times \mu$$

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta$$

$$\log_e\left(\frac{T_1}{T_2}\right) = \mu\theta \Rightarrow 2.3\log\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\Rightarrow \frac{T_1}{T_2} = e^{\mu\theta}$$

Centrifugal tension:

$$\Rightarrow T_c = mv^2$$

Total tension is taken into Account

$T_{t_1} = T_1 + T_c \rightarrow$ right side.

$T_{t_2} = T_2 + T_c \rightarrow$ slack side.

Maximum tension in the Belt:

$$T = \sigma \times b \times t$$

σ = Maximum safe stress

b = width of the belt &

t = Thickness of belt.

$T = T_1 + T_c$ (centrifugal force Consider)

Condition for the transmission of maximum power.

$$P = (T_1 - T_2)\,v$$

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad (\text{or}) \quad T_2 = \frac{T_1}{e^{\mu\theta}}$$

$$P = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right)v = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)v = T_1 C v$$

$$C = 1 - \frac{1}{e^{\mu\theta}}$$

$$\Rightarrow T = T_1 + T_c$$

$$T_1 = T - T_c$$

$$P = (T - T_c)\,v \times c$$

$$P = (T - mv^2)\,v \times c = (Tv - mv^3)\,c$$

For Max. power diff

$$\frac{dP}{dv} = 0 \quad (\text{or}) \quad \frac{d}{dv}(Tv - mv^3)\,c = 0$$

$$T - 3mv^2 = 0 \Rightarrow T - 3T_c = 0$$

(or)

$$T = 3T_c$$

$$v = \sqrt{\frac{T}{3m}} \qquad T = 3mv^2$$

Initial tension in the belt:

When a belt is wound round the two pulleys, its two ends are joined together, so that the belt may continuosly move over the pulleys. Since the motion of the belt from driver & follower is governed by a firm grip due to friction b/w the belt & pulleys. In order to increase this grip, the belt is tightend up. At this stage, even when the belt pulleys are stationary, the belt is subjected to some tension, called Initial tension.

Let. T_0 = Initial tension in the belt.

T_1 = tight side

T_2 = slack side

α = Coefficient of increase of the belt length per unit force.

Increase of tension in the tight side $= T_1 - T_0$

" " Length of belt " $= \alpha(T_1 - T_0)$

Decrease of tension slack side $= T_0 - T_2$

Length " " " $= \alpha(T_0 - T_2)$

Assuming that the belt material is perfectly elastic such that the length remains const.

$$\alpha(T_1 - T_0) = \alpha(T_0 - T_2)$$

$$T_0 = \frac{T_1 + T_2}{2}$$

$$= \frac{T_1 + T_2 + 2T_c}{2}$$ [Considering centrifugal tension]

In Actually the belt material is not perfectly elastic.

∴ sum of tensions T_1 & T_2

$$\sqrt{T_1} + \sqrt{T_2} = 2\sqrt{T_0}$$

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Design of cast Iron pulleys:

(i) Dimensions of pulley:

The diameter of the pulley (D) may be obtained either from velocity ratio consideration or centrifugal consideration.

$$\sigma_t = \rho . v^2$$

ρ = Density of the rim material

= 7200 kg/m³ for cast iron.

V = velocity of the rim $= \frac{\pi D N}{60}$

(ii) The width of the belt is known, then width of the pulley (or) face of the pulley (B) is taken 25% greater than the width of belt

$$B = 1.25b$$

$\Rightarrow b$ = width of belt.

(iii) The thickness of the pulley rim (t) varies from $\frac{D}{300} + 2mm$ to $\frac{D}{200} + 3mm$ for single belt & $\frac{D}{200} + 6mm$ for double belt.

2. Dimensions of arm:

(i) The number of arms may be taken as 4 for pulley diameter from 200mm to 600mm & 6 for diam. from 600mm to 1500mm.

(ii) $a_1 = 2b_1$

$$W_T = \frac{T}{R \times n/2} = \frac{2T}{Rn}$$

$$M = \frac{2T}{R \times n} \times R = \frac{2T}{n}$$

$$Z = \frac{\pi}{32} \times b_1 (a_1)^2$$

$$\sigma_b \text{ (or) } \sigma_t = \frac{M}{Z}.$$

(iii) The Arms are tapered from hub to rim. The taper is usually $\frac{1}{48}$ to $\frac{1}{32}$

3. Dimensions of hub (d_1) in terms of shaft dia (d) may be fixed by

$d_1 = 1.5d + 25$ mm.

Oratil

UNIT-V

GEARS

The slipping of a Belt or rope is common phenomenon. it the transmission of Motion or power b/w two shafts. The Effect of slipping is to reduce velocity ratio of the system. In precision machines, in which a definite velocity ratio is importance, the only positive drive is by Gears (or) toothed wheels. A gear drive is also provided, when the distance b/w the driver and the follower is very small.

ADVANTAGES:

1) It transmits exact velocity ratio
2) It may be used to transmit large power
3) It may be used for small centre distance of shafts.
4) It has high effeciency
5) It has reliable service
6) It has Compact layout

Disadvantages:-

1) Since the manufacture of gears require special tools and equipment, therfore it is costlier than other drives
2) The Error in cutting teeth may cause vibrations and noise during operation.
3) It requires suitable lubricant and reliable method of applying it, for the proper operation of gear drives.

1) According to the position of axes of the shafts:
The axes of the two shafts b/w which the motion is to
be transmitted.

a) Parallel

The two parallel and co-planar shafts connected by the
gears are called spur gears. and the arrangment is known
as spur gearing. These gears have teeth parallel to the
axes of the wheel.

Helical Gear in which the teeth are inclined to the axis

(i) Single Helical Gear

(ii) Double Helical Gear (or) herringbone gears.

b) Intersecting.

The two non-parallel (or) intersecting, but coplaner shafts
connected by gears is known as bevel gears and the
arrangment is known as Bevel gearing.

In which also the teeth inclined to the face of the bevel
in which case they are known as helical bevel gears

The two non- intersecting and non - parallel i.e non - coplanar shafts connected by gears are called skew bevel gears (or) spiral gears and arrangment is known as skew bevel gearing (or) spiral gearing. This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surface known as hypeboloids.

Note:

(i) When equal bevel gears (having equal teeth) connect two shaft whose axes are mutually ⊥er then, the bevel gears are known as miters.

2) According to the peripheral velocity of Gears:

a) Low velocity, b) Medium velocity c) High velocity

The gears having velocity less than 3 m/s are termed as low velocity gears.

The velocity b/w 3 to 15 m/s are known as medium velocity.

The velocity more than 15 m/s are known as High velocity gears.

3) According to the type of gearing:-

(a) External Gearing

(b) Internal Gearing

(c) Rack and pinion.

a) External Gearing:- The gears of the two shafts mesh Externally The larger of these two wheels is called spur wheel (or) gear and smaller wheel is called pinion. In an external gearing, the motion of the two wheels is always is unlike.

b) Internal Gearing:- The gears of the two shafts mesh internally with each other. The larger of these two wheels is called annular wheel and the smaller wheel is called pinion. The motion of the wheels is always like.

c) Rack and pinion:- The gear of a shaft meshes externally and internally with the gear in a straight line. The straight line gear is called rack & the circular wheel is called pinion.

4) According to the position of teeth on the gear Surface:-

a) straight

b) Inclined &

c) Curved.

The spur gears have straight teeth whereas helical-gears have their teeth inclined to the wheel rim. In case of Spiral gears, the teeth are curved over the rim Surface.

5) Based on profile:

a) Involute profile gears

b) cycloidal profile gears.

6) Based on pressure angle:

a) Gears with 20° pressure angle

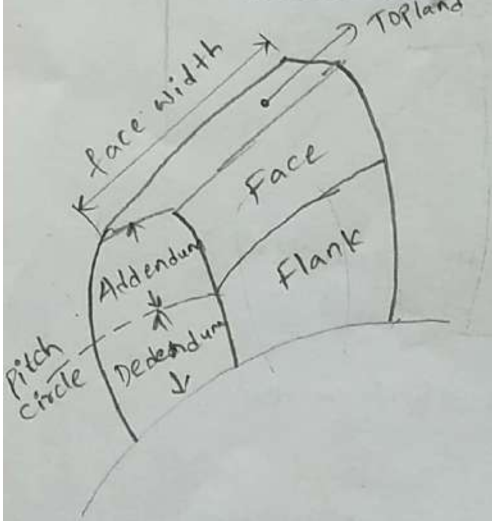
b) Gears with 14½ pressure angle.

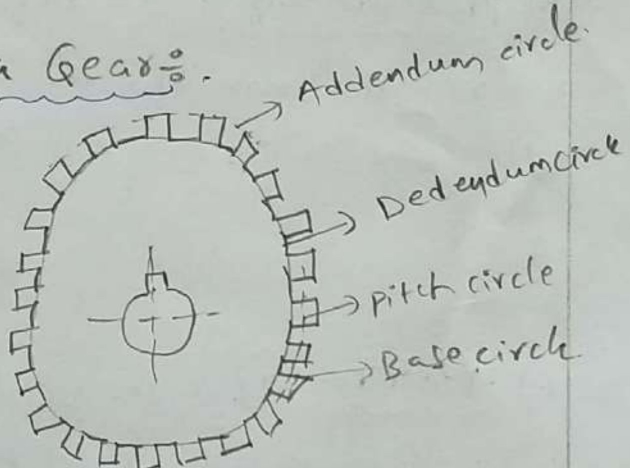
7) Based on tooth height (or) Working depth:

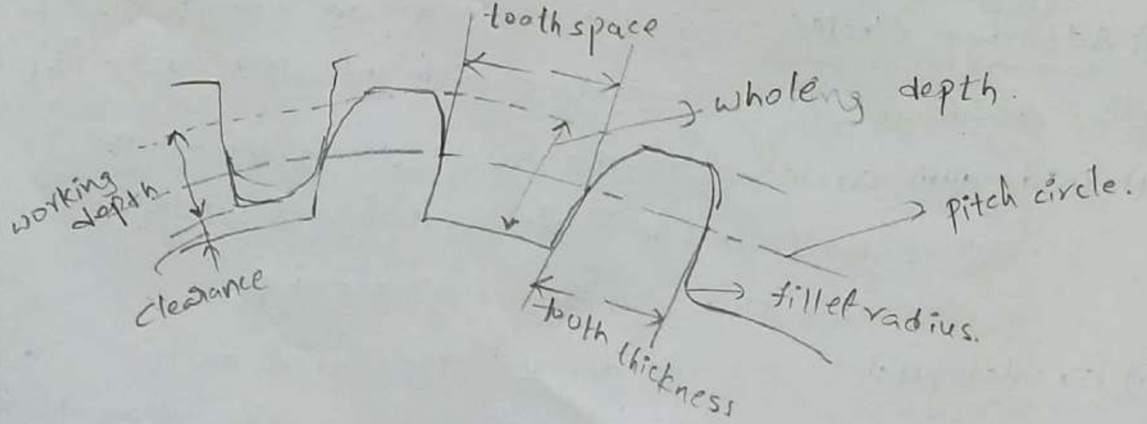
a) Full depth gears (or) standard gears.

b) stub gears.

Design parameters of a Gear:







1) Pitch circle:- It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2) pitch circle diameter:- It is ~~size of~~ diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. (pitch diameter).

3) pitch point:- It is a common point of contanct b/w two pitch circles.

4) pitch surface: It is the surface of the rolling disc which the meshing gears have replaced at the pitch circles.

5) pressure angle: (or) Angle of obliquity:- (ϕ) [$14\frac{1}{2}^\circ$ & 20°]
It is the angle b/w the common normal to two gear teeth at a point of contanct and common tangent at the pitch point.

6) Addendum: It is the radial distance of tooth from the pitch circle to the top of tooth.

7) Dedendum:- It is the radial distance of tooth from the pitch circle to the Bottom of tooth.

8) Addendum Circle:- It is the circle drawn through the top of the teeth and is concentric with the pitch circle. (tip circle)

9) Dedendum circle: It is the circle drawn through the Bottom of the teeth & it is also called root circle.

Root circle dia = pitch circle dia X $\cos\phi$

10) circular pitch: It is the distance measured on the circumference of the pitch circle from a point of one tooth the corresponding point on the next tooth. (P_c)

19) Face of tooth: It is surface of tooth above the pitch surface.

20) Top land: It is surface of the top of the tooth.

21) Flank of tooth: It is surface of tooth below the pitch surface.

22) Face width: It is the width of the gear tooth measured parallel to its axes.

23) profile: It is the curve formed by the face and flank of the tooth.

24) Fillet radius: It is the radius that connects the root circle to the profile of the tooth.

25) Path of contact: It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26) length of the path of contact: It is the length of the common normal cut-off by the addendum circles of the wheel & pinion.

27) Arc of contact: It is the path traced by a point on pitch circle from the begining to the end of engagement of a given pair of teeth.

a) Arc of approch: It is the portion of teeth path of contact from the beginning to the engagement to the pitch point.

b) Arc of recess: It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

circular pitch $P_c = \frac{\pi D}{T}$ (or) $\frac{\pi D}{Z}$

D = Dia of the pitch circle

T (or) Z = Numer of teeth on wheel.

$$P_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \Rightarrow \frac{D_1}{T_1} = \frac{D_2}{T_2}$$

11) Diametral pitch:

It is the ratio of Number of teeth to pitch circle dia. (mm)

$$P_d = \frac{T}{D} = \frac{P_c \pi}{P_c} \qquad P_c = \frac{\pi D}{T}$$

12) Module: It is the ratio the pitch circle dia in mm to the number of teeth. (m)

$$m = \frac{D}{T}$$

13) clearance: It is a radial distance from top of teeth to bottom teeth, in a meshing gear. A circle passing through the top of the meshing gear is known as clearance circle.

14) Total depth: It is the radial distance b/w the addendum and dedenduem circle of a gear.

Total depth = Addendum + Dedendum.

15) Working depth: It is the radial distance from the addendum-circle to clearance circle. It is equal to sum of the addendum of the two meshing gears.

16) Tooth thickness: It is the width of the tooth measured along pitch circle.

17) Tooth space: It is the width of space b/w the two adjacent teeth measured along the pitch circle.

18) Blacklash: It is the differance b/w tooth space and tooth thickness, as measured on the pitch circle.

Condition for constant velocity ratio of gears:-

Law of Gearing:-

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and other on the wheel 2, as shown by thick line curve. Let the two teeth come in contact at a point Q, and the wheels rotate in the directions.

Let TT be the common tangent and MN be the common normal to the curves at point of contact Q. From the centres O_1 and O_2, draw O_1M and O_2N ⊥er to MN. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, & in the direction QD when considered as a point on wheel 2.

Let V_1 and V_2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.

$$V_1 \cos\alpha = V_2 \cos\beta.$$

$$(\omega_1 \times O_1Q) \cos\alpha = (\omega_2 \times O_2Q) \cos\beta.$$

$$\therefore (\omega_1 \times O_1Q)\,\frac{O_1M}{O_1Q} = (\omega_2 \times O_2Q)\,\frac{O_2N}{O_2Q}$$

$$\omega_1 \times O_1M = \omega_2 \times O_2N$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}$$

Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

$\therefore$ In order to have a constant angular velocity ratio for all positions of the wheels, P must be fixed point (pitch point) for the two wheels. In other words, the common normal at the point of contact between pair of teeth must always pass through pitch point. This fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as law of Gearing.

If D_1 and D_2 are diameters of pitch circles of wheel 1 and 2 having teeth $T_1 (Z_1)$ and $T_2 (Z_2)$ respectively, then velocity ratio

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

Gear Materials:

The Material used for the manufacture of gears depend upon the strength and service Conditions. like Wear, noise etc. The gears may be manufactured from metallic or non-metallic Materials. (CI, steel & Bronze) Metallic Materials & non-metallic like wood, rawhide, compressed paper & synthetic resins like nylon are used for gears, especially for reducing noise

The castiron is widely used for the manufacture of gears. due to its good wearing properties, Excellent machinability and ease of producing Complicated shapes by casting Method. The cast iron gears with cut teeth may be employed, where Smooth action is not important.

The steels is used for high strength gears and steel may be plain carbon steel (or) Alloy steel. The steel gears are usually heat treated in order to Combine properly the toughness & tooth Hardness.

Design Considerations for a gear drive:

In the design of gear drive, the following data is given by

1. The power to be transmitted
2. The speed of driving gear
3. The speed of the driven gear or the velocity ratio, and
4. The centre distance.

a) The gear teeth should have sufficient strength so that they will not fail under static loading (or) dynamic loading during normal running conditions.

b) The gear teeth should have wear characteristics so that their life is satisfactory.

c) The use of space and Material should be economical.

d) The alignment of the gears & deflections of the shafts must be considered because they effect on the performance of the gears.

e) The lubrication of gears must be satisfactory.

Beam Strength of Gear teeth – Lewis Equation:

The beam strength of gear teeth is determined from an Equation (known as Lewis Equation) and the load carrying ability of the toothed gears as determined by this equation gives satisfactory results.

consider each tooth as a cantilever beam loaded by normal load (W_N) as shown in diagram. It is resolved into components i.e tangential component (W_T) and radial component (W_R) acting perpendicular and parallel to the centre line of tooth respectively. The tangential component (W_T) induces a bending stress which tends to break the tooth. The radial component (W_R) induces a compressive stress of relatively small magnitude, therefore its effect on the tooth may be neglected. Hence, the bending stress is used as the basis for design calculations. The critical section or the section of Maximum bending stress may be obtained by drawing a parabola through A and tangential to the teeth curves at B and C. This parabola as outlines a beam of uniform strength, i.e if the teeth are shaped like a parabola, it will have the same stress at all sections. But the tooth is larger than the parabola at every section except BC. We therfore, conclude that the section BC is the section of max. stress or the critical section. The max. value of the Bending stress (or the permissible working stress), at the section BC is given by.

$$\sigma_w = \frac{My}{I}$$

M = Maximum Bending Moment at the critical section BC = $W_T \times h$

W_T = Tangential load acting at the tooth

h = Length of the tooth

y = Half the thickness of the teeth (t) at critical section BC = $t/2$

I = Moment of inertia about the centre line of the tooth = $\frac{bt^3}{12}$

b = Width of gear face.

$$\sigma_w = \frac{(W_T \times h)\, t/2}{\frac{bt^3}{12}} = \frac{(W_T \times h) \times 6}{bt^2}$$

$$W_T = \frac{\sigma_w \times b \times t^2}{6h}$$

t and h are variables depending upon the size of the tooth (i.e the circular pitch) & its profile

$t = x \times P_c$ & $h = k \times P_c$ where x & k are constants.

$$W_T = \sigma_w \times b \times \frac{x^2 P_c^2}{6k \cdot P_c}$$

$$= \sigma_w \times b \times P_c \times \frac{x^2}{6k}$$

Substituting $\frac{x^2}{6k} = y$

$$W_T = \sigma_w \cdot b \cdot P_c \cdot y$$

$$W_T = \sigma_w \cdot b \cdot \pi m \cdot y \text{ (or) } F_b = \sigma_b \, b \, P_c \, y \qquad (\because P_c = \pi m)$$

The quantity y is known as Lewis from factor (or) tooth form factor and W_T (Which is the tangential load acting at the tooth) is called the beam strength of the tooth.

$$\text{since } y = \frac{x^2}{6k} = \frac{t^2}{(P_c)^2} \times \frac{P_c}{6h} = \frac{t^2}{6h \cdot P_c}.$$

$\therefore$ in order to find the value of y, the quantities t, h. & P_c may be determined analytically or measured from the drawing similar. It may be noted that if the gear is Enlarged the distances t, h & P_c will each increase proportionately. $\therefore$ The value of y will remain unchanged. A little conside-ration will show that the value of y is independent of the size of the tooth and depends only on the number of teeth on a gear & the system of teeth. The Value of y in terms of the number of teeth may be Expressed as follows.

$$y = 0.124 - \frac{0.684}{T} \quad \text{for } 14\tfrac{1}{2}^\circ \text{ Composite \& full depth involue system}$$

$$y = 0.154 - \frac{0.912}{T}, \quad \text{for } 20^\circ \text{ full depth involute system}$$

$$y = 0.175 - \frac{0.841}{T}, \quad \text{for } 20^\circ \text{ Stub system}$$

$$\therefore F_b = \sigma_b \times b \times \pi m \times y$$
$$= \sigma_b \times b \times \frac{Y}{P_d}$$

$$Y = \pi y \;\&\; \frac{1}{P_d} = m$$

Y = Modified form factor

To find Module:

When the gear transmits the power P. the tangential force produced due to that power is given by.

$$F_t = \frac{P \times K_s}{V} \quad (N)$$

P = Power in Watts.

V = Linear velocity of gears in m/s

K_s = Service factor

Lewis derived the equation for beam strength assuming the load to be static. When the gears are running at high speeds. the gears may be subjected to dynamic effect To account for the dynamic effect, a factor C_v known as velocity factor (or) dynamic factor is considered.

Now, the design tangential force along with dynamic effect is given by.

$$F_D = F_t \times C_v = \frac{P \times K_s \times C_v}{V}$$

C_v developed by Barth depends on the pitch line velocity

$$C_v = \frac{3+V}{3} \quad \text{when } V \leq 10\,m/s \text{ for commerically cut gear.}$$

$$= \frac{6+V}{6} \quad \text{When } V \leq 20\,m/s \text{ for carefully cut gears.}$$

$$= \frac{5.5+\sqrt{V}}{5.5} \quad \text{When } V > 20\,m/s \text{ for precision gears.}$$

$$= \frac{1+V}{1+0.25V} \quad \text{for non-metallic gears.}$$

For safe design, the required beam strength of gear tooth should be greater than the design tangential load.

i.e $F_b \geq F_D$.

$$\therefore \sigma_b \cdot b \cdot \pi m \cdot y \geq \frac{P \cdot K_s \cdot C_v}{V}$$

Buckingham Equation:

When the power is transmitted through gears, apart from static (i.e steady) load produced by the power, some dynamic loads are also applied on the gear tooth due to following reasons like.

1. Inaccuracies of tooth spacing.
2. Irregularities of tooth profiles, and
3. Deflection of tooth under load.

$$F_d = F_t + F_i$$

F_d = Max. dynamic load

F_t = Static load produced by the power $= \frac{P}{V}$

F_i = Incremetal load due to dynamic action

$$F_i = \frac{21V(cb + F_t)}{21V + \sqrt{cb + F_t}}$$

V = pitch line velocity (m/s)

b = face width of Gear tooth (mm)

C = Dynamic factor (or) deformation factor (N/mm)

$$C = \frac{ke}{\frac{1}{E_1} + \frac{1}{E_2}} = \frac{k \cdot E_1 \cdot E_2 \cdot e}{E_1 + E_2}$$

K = A factor depending upon the form & tooth

= 0.107 for $14\frac{1}{2}^\circ$ full depth involute system

= 0.111 for 20° full depth involute system

= 0.115 for 20° stub teeth

E_1 = Young's Modulus for pinion Material, N/mm^2

E_2 = Young's Moduluss for Gear Material, N/mm^2

e = Expected error in tooth profile (or)

Manufacturing error. (mm)

Wear Strength of Gear tooth:

$$F_w = d_1 \cdot b \cdot Q \cdot K_w$$

where F_w = Maximum (or) limiting load for wear (N)

d_1 = Pitch circle diameter of pinion (mm)

b = Face width of pinion (mm)

Q = Ratio factor

$= \frac{2i}{i+1}$ (or) $\frac{2Z_2}{Z_2+Z_1}$ for External Gears.

$= \frac{2i}{i-1}$ (or) $\frac{2Z_2}{Z_2-Z_1}$ for internal Gears.

i = Gear (velocity) Ratio $= \frac{Z_2}{Z_1}$

Z_2 = Number of teeth of gear

Z_1 = Number of teeth of pinion

K_w = Load stress factor (Material Combination factor) N/mm^2.

$$k_w = \frac{\sigma_e^2 \sin\alpha}{1.4}\left(\frac{1}{E_1}+\frac{1}{E_2}\right)$$

α = pressure angle

E_1, E_2 = Young's Modulus of pinion and gear Materials.

σ_e = Surface endurance limit (N/mm^2)

$\sigma_e = 2.8(BHN - 70)\ N/mm^2$

(i) Number of teeth:

The minimum number of teeth on pinion to avoid interference is given by.

$$Z_{1\,min} = \frac{2}{\sin^2\alpha}.$$

$Z_1 \geq 17$ for $20°$ full depth gear.

≥ 32 for $14\frac{1}{2}°$ full depth Gear.

(ii) Face Width:

$b = 8m$ to $12m$

Design a spur Gear drive to transmit 22 kw at 1000 rpm. Speed reduction is 2.5. The centre distance b/w the gear Shaft is approximately 350 mm. The Materials are: Pinion - C45 steel, gear wheel - CI. Grade 30. Design the drive (Use Lewis and Buckingham Equations)

Given

$P = 22\,kw = 22\times10^3\,w$

$n_1 = 1000\,rpm$

$i = 2.5 = \frac{d_2}{d_1} = \frac{n_1}{n_2} \Rightarrow d_2 = 2.5\,d_1$

$a = 350$

Material for pinion = C 45 steel

Material for Gear = C.I Grade 30.

$$T = \frac{60\times P}{2\pi n_1} = \frac{60\times 22\times10^3}{2\pi\times1000} = 210\ N\text{-}m = 210\times10^3\ N\text{-}mm.$$

Centre distance $a = \frac{d_1+d_2}{2} = 350$

$$d_1+d_2 = 700$$

$$d_1 + 2.5\,d_1 = 700$$

$$d_1 = \frac{700}{3.5} = 200\ mm$$

$$d_2 = 2.5\times200 = 500\,mm.$$

$$F_t = \frac{T}{(d/2)} = \frac{210\times10^3}{200/2} = 2100\ N$$

$$F_D = F_t\times K_s\times C_v$$

K_s = service factor = 1.25 for light shock & Medium series.

$$V = \frac{\pi d_1 n_1}{60\times10^3} = \frac{\pi\times200\times1000}{60\times10^3} = 10.5\ m/sec.$$

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$$C_V = \frac{6+V}{6} = \frac{6+10.5}{6} = 2.75$$

$$F_D = 2100 \times 1.25 \times 2.75 = 7220 \text{ N}$$

Module and face width:

$$F_b = \sigma_b . b . \pi m . y$$

$$y = 0.154 - \frac{0.912}{z} \quad \& \quad b = 10m$$

$[\sigma_b]$ for pinion (C45 steel) = 140 N/mm²

$[\sigma_b]$ for Gear (C.I Grade 30) = 70 N/mm²

$Z_1 = 20$ and hence $Z_2 = iZ_1 = 2.5 \times 20 = 50$

$$Y_1 = 0.154 - \frac{0.912}{20} = 0.108$$

$$Y_2 = 0.154 - \frac{0.912}{50} = 0.136$$

Strength factor

for pinion, $[\sigma_b] \times Y_1 = 140 \times 0.108 = 15.12$ N/mm²

for Gear, $[\sigma_b] \times Y_2 = 70 \times 0.136 = 9.52$ N/mm²

Strength factor for Gear is minimum. the design is Based on Gear.

$$F_b \geq F_D$$

$$70 \times 10m \times m \times \pi \times 0.136 \geq 7220 \Rightarrow m = 4.9$$

$$m = 5 \text{ mm}$$

$$b = 10 \times 5 = 50 \text{ mm.}$$

$$Z_1 = \frac{d_1}{m} = \frac{200}{5} = 40 \quad \& \quad Z_2 = \frac{d_2}{m} = \frac{500}{5} = 100$$

corrected Beam Strength.

$$Y = 0.154 - \frac{0.912}{100} = 0.145$$

$$(F_b)_g = 70 \times 50 \times 5 \times \pi \times 0.145 = 7972 \text{ N}$$

$$F_d = F_t + F_i = F_t + \frac{21V(cb + F_t)}{21V + \sqrt{cb + F_t}}$$

$F_t = 2100$ N

$V = 10.5$ m/s

$b = 50$ mm.

$$C = \text{Dynamic factor} = \frac{K \cdot E_1 \cdot E_2 \cdot e}{E_1 + E_2}$$

$E_1 = 2.15 \times 10^5$ N/mm²

$E_2 = 1.1 \times 10^5$ N/mm²

$e = 0.025$ (for carefully cut Gears)

$k = 0.111$ for $20°$ full depth involute system.

$$C = \frac{0.111 \times 2.15 \times 10^5 \times 1.1 \times 10^5 \times 0.25}{2.15 \times 10^5 + 1.1 \times 10^5}$$

$$C = 8077 \times 0.025 = 202 \text{ N/mm}^2$$

$$F_d = 2100 + \frac{21 \times 10.5(202 \times 50 + 2100)}{(21 \times 10.5) + \sqrt{202 \times 50 + 2100}}$$

$F_d = 10228$ N

for safe design $(F_b)_g > F_d$.

But Here, F_d is $> (F_b)_g$. which is not safe.

Hence increase the module & face width.

$m = 6$ mm & $b = 10 \times 6 = 60$ mm

$$Z_1 = \frac{d_1}{m} = \frac{200}{6} = 34$$

$$Z_2 = iZ_1 = 2.5 \times 34 = 85$$

$$Y = 0.154 - \frac{0.912}{85} = 0.1433$$

$$(F_b)_g = 70 \times 60 \times 6 \times \pi \times 0.1433 = \underline{11345}\ N$$

$$F_d = 2100 + \frac{(21 \times 10.5)(202 \times 60 + 2100)}{(21 \times 10.5) + \sqrt{202 \times 60 + 2100}} = \underline{11329\ N}$$

$F_b > F_d$ our design is safe.

Checking for Wear strength:

$$F_w = d_1 \cdot b \cdot Q \cdot K_w$$

$d_1 = 200$ mm

$b = 60$ mm

$$Q = \frac{2i}{i+1} = \frac{2 \times 2.5}{2.5+1} = 1.43$$

$$K_w = \frac{\sigma_e^2 \sin\alpha}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

Assuming BHN for pinion as 250, we get.

$$\sigma_e = (2.8 \times BHN - 70) = 2.8 \times 250 - 70 = 630\ N/mm^2$$

$$K_w = \frac{630 \times \sin 20}{1.4}\left[\frac{1}{2.15 \times 10^5} + \frac{1}{1.1 \times 10^5}\right]$$

$$K_w = 1.33\ N/mm^2$$

$$F_w = 200 \times 60 \times 1.43 \times 1.33$$

$$F_w = 22823\ N$$

Since $F_w > F_d$, our design is $\underline{\text{safe}}$.

Addendum, $h_a = 1m = 1 \times 6 = 6$ mm.

Dedendum, $h_f = 1.25m = 1.25 \times 6 = 7.5$ mm

Tip circle dia. of pinion, $d_{a_1} = d_1 + 2h_a = 200 + (2 \times 6) = 212$ mm

Tip circle dia. of Gear $d_{a_2} = d_2 + 2h_a = 500 + (2 \times 6) = 512$ mm

Root circle dia. of Pinion $d_{f_1} = d_1 - 2h_f = 200 - (2 \times 7.5) = 185$ mm

Root circle dia. of Gear $d_{f_2} = d_2 - 2h_f = 500 - (2 \times 7.5) = 485$ mm

It is required to design a pair of spur gears with 20° full-depth involute teeth consisting of a 20-teeth pinion meshing with a 50 teeth gear. The pinion shaft is connected to a 22.5 kW. 1450 rpm electric motor. The starting torque of the motor can be taken as 150% of the rated torque. The material for pinion is plain carbon steel Fe 410 ($\sigma_{ut} = 410\ N/mm^2$), while the gear is made of Greay cast iron FG200 ($\sigma_{ut} = 200\ N/mm^2$). The factor of safety is 1.5. Design the gears Based on the Lewis Equation & using velocity factor to account for the dynamic load.

Given

$P = 22.5 \times 10^3\ W$

$n_1 = 1450\ rpm,\ Z_1 = 20$

$Z_2 = 50$

Starting torque = 150% Rated torque.

$FS = 1.5$

For pinion $\sigma_{ut} = 410\ N/mm^2$, $\sigma_{b_1} = \frac{410}{3} = 136.67\ N/mm^2$

For Gear $\sigma_{ut} = 200\ N/mm^2$, $\sigma_{b_2} = \frac{200}{3} = 66.67\ N/mm^2$

$$\frac{Z_2}{Z_1} = \frac{n_1}{n_2} \Rightarrow \frac{50}{20} = \frac{1450}{n_2}$$

$$n_2 = \frac{1450 \times 20}{50} = 580\ rpm. \quad \& \quad \underline{b = 10m}$$

$F_b \geq F_D$

$$F_D = \frac{P \times K_s \times C_v}{V} = \frac{22.5 \times 10^3 \times 1.5}{1.517\,m}\left[\frac{3 + 1.517\,m}{3}\right]$$

$k_s = 1.5$

Assuming. $C_v = \frac{3+V}{3}$

$$V = \frac{\pi d n_1}{60} = \frac{\pi \times m \times Z_1 \times \pi}{60}$$

$m = \frac{d_1}{Z_1} \Rightarrow d_1 = mZ_1$

$$V = \frac{\pi \times m \times 20 \times 1450}{60} = 1517.66^{m}\ mm/sec$$

$V = 1.517 \times m \quad m/sec$

$$Y = 0.154 - \frac{0.912}{Z} \qquad [\because \text{ for } 20^\circ \text{ full depth involute profile}]$$

$$Y_1 = 0.154 - \frac{0.912}{20} = 0.1084$$

$$Y_2 = 0.154 - \frac{0.912}{50} = 0.13576$$

Pinion - $\sigma_{b_1} \times Y_1 = 136.67 \times 0.1084 = 14.81$

Gear - $\sigma_{b_2} \times Y_2 = 66.67 \times 0.13576 = 9.0511$

Gear is weaker

So design is Based on Gear

$F_b = \sigma_{b_2} \times b \times \pi m \times Y_2$

$= 66.67 \times 10 \times m \times \pi \times m \times 0.13576 = 284.20\ m^2$

$F_b = F_D$

$$284.20\, m^2 = \frac{22.5 \times 10^3 \times 1.5}{1.517\, m} \left[\frac{3 + 1.517m}{3} \right]$$

$m = 7.112 \cong 7\ mm$

$d_1 = m \times Z_1 = 7 \times 20 = 140\ mm$

$d_2 = m \times Z_2 = 7 \times 50 = 350\ mm$

$b = 10m = 10 \times 7 = 70\ mm$

$\therefore$ check for design.

$$V = \frac{\pi d_1 n_1}{60} = \frac{\pi \times 140 \times 1450}{60} = 10623.66\ mm/sec$$

$= 10.62\ m/sec.$

$V > 10\ m/sec.$ then.

$$C_v = \frac{6 + V}{6}$$

$$C_v = \frac{6+10.62}{6} = 2.77$$

$$F_D = \frac{P \times K_s \times C_v}{V} = \frac{22.5 \times 10^3 \times 1.5 \times 2.77}{10.62} = 8802.9\ N$$

$$F_b = \sigma_{b_2} \times m \times b \times \pi \times Y_2 = 13840.5\ N$$

$$FOS = \frac{F_b}{F_D} = \frac{13840.5}{8802.9} = 1.572$$

The design is satisfactory & the Module should be 7 mm.

It is required to design a pair of spur gears with 20° full-depth involute teeth based on the Lewis Equation. The velocity factor is to be used to accout for dynamic load. The pinion shaft is connected to a 10 kW, 1440 rpm motor. The starting torque of motor is 150% of the rated torque. The speed reduction is 4:1. The pinion as well as the gear is made of plain carbon steel 40C8 ($\sigma_{ut} = 400\ N/mm^2$). The factor of safety can be taken as 1.5. Design the gears. Spesify their dimensions and suggest suitable Hardness for the Gears.

Given

$P = 10\ kW = 10 \times 10^3\ W$

$n_1 = 1440\ rpm$

$i = 4 = \frac{Z_2}{Z_1} = \frac{n_1}{n_2} = \frac{d_2}{d_1} \Rightarrow n_2 = \frac{1440}{4} = 360\ rpm$

Starting torque = 150% Rated torque.

$K_s = 1.5$ & $\sigma_{ut} = 400\ N/mm^2$,

$b = 10 \times m$

The minimum number of teeth for 20° pressure angle is 18

$Z_1 = 18$

$Z_2 = 4 \times 18 = 72$

Both the pinion & Gear Made up Same Material then pinion is Weaker so design is Based on pinion.

$F_b \geq F_D$

$F_D = \frac{P \times K_s \times C_v}{V}$ $\qquad m = \frac{d}{Z} \Rightarrow d = mZ$

$V = \frac{\pi d_1 n_1}{60} = \frac{\pi \times m \times Z_1 \times n_1}{60} = \frac{\pi \times m \times 18 \times 1440}{60} = 1357.16 \times m$

$V = 1.357 \times m$ m/sec

$C_v = \frac{3+V}{3} = \frac{3+1.357m}{3}$

$F_D = \frac{10 \times 10^3 \times 1.5}{1.357 \times m}\left[\frac{3+1.357m}{3}\right]$

$F_b = \sigma_b \times b \times \pi m \times y$

$y = 0.154 - \frac{0.912}{18} = 0.1033$

$F_b = \frac{400}{3} \times 10 \times m \times \pi \times 0.1033 m = \cancel{648.72 \times m^2}\ 432\ m^2$

$F_D = F_b$

$\frac{10 \times 10^3 \times 1.5}{1.357 \times m}\left[\frac{3+1.357m}{3}\right] = 432\ m^2$

$m = 4.2$ mm

$m \cong 5$ mm

$d_1 = mZ_1 = 5 \times 18 = 90$ mm
$d_2 = mZ_2 = 5 \times 72 = 360$ mm
$b = 10 \times 5 = 50$ mm

$V = \frac{\pi d_1 n_1}{60 \times 10^3} = \frac{\pi \times 90 \times 1440}{60 \times 10^3} = 6.78$ m/sec

$C_v = \frac{3+V}{3} = \frac{3+6.78}{3} = 3.26$

$F_D = \frac{P \times K_s \times C_v}{V} = \frac{10 \times 10^3 \times 1.5 \times 3.26}{6.78} = 7212.3$ N

$F_b = \sigma_b \times b \times \pi m \times y = 133.3 \times 50 \times \pi \times 5 \times 0.1033$
$= 10814$

$\frac{F_b}{F_D} = 1.5$ design is safe.

A Reciprocating Compressor is to be connected to an Electric motor with the help of spur gears. The distance between the shafts is to be 500 mm. The speed of the electric motor is 900 rpm and the speed of the compressor shaft is desired to be 200 rpm. The torque to be transmitted is 500 N-m. Taking starting torque as 25% more than the normal torque, determine.

(i) Module and face width of the gears using 20 degrees stub teeth and.

(ii) Number of teeth and pitch circle diameter of each gear. Assume suitable values of velocity factor and Lewis factor.

Given

Torque transmitted $M_t = 500$ N-m
$= 500 \times 10^3$ N-mm

$n_1 = 900$ rpm

$n_2 = 200$ rpm

$a = 500$ mm.

Let the material for pinion is forged steel (Heat-treated) and the material for gear is cast steel (Heat treated)

Design Bending stress for pinion $\sigma_{b_1} = 224$ N/mm²

Design Bending stress for Gear $\sigma_{b_2} = 196$ N/mm²

$$a = \frac{d_1+d_2}{2} \quad \text{(or)} \quad d_1+d_2 = 500 \times 2 = 1000\text{mm}$$

$$\frac{Z_2}{Z_1} = \frac{d_2}{d_1} = \frac{n_1}{n_2} = \frac{900}{200} = 4.5$$

$$d_2 = 4.5 d_1$$

$$d_1 + 4.5 d_1 = 1000 \Rightarrow d_1 = 182\text{mm} \;\&\; d_2 = 818\text{ mm}$$

Scanned by CamScanner

$M_t = 500 \times 10^3$ N-mm

Maximum torque $(M_t) = M_t \times 1.25 = 500 \times 10^3 \times 1.25$

$= 625 \times 10^3$ N-mm.

$$F_t = \frac{M_t}{d_1/2} = \frac{625 \times 10^3}{\frac{182}{2}} = 6.87 \times 10^3 N$$

$F_d = F_t \times C_v$

$$V = \frac{\pi d_1 n_1}{60 \times 10^3} = \frac{\pi \times 182 \times 900}{60 \times 10^3} = 8.6 m/sec$$

$$C_v = \frac{3+V}{3} = \frac{3+8.6}{3} = 3.87 \text{ for } < 10 m/sec.$$

$F_d = 6.87 \times 10^3 \times 3.87$

$= 26.6 \times 10^3$ N

$F_b = \sigma_b \times b \times m \times Y$

Y = Modified form factor

$Z_1 = 20$ & $Z_2 = 20 \times 4.5 = 90$

$\sigma_b \times Y = 224 \times 0.393 = 88 N/mm^2$.

$\sigma_b \times Y = 196 \times 0.503 = 98.6 N/mm^2$

$\sigma_b \times Y$ for pinion is less

$\sigma_b \times b \times m Y \geq F_d$ $\qquad$ $b = 10$ mm

$224 \times 10m \times m \times 0.393 \geq 26.6 \times 10^3$

$$m = \left[\frac{26.6 \times 10^3}{224 \times 10 \times 0.393}\right]^{1/2} \geq 5.5 \text{ mm} = 6 \text{ mm}$$

$b = 10m = 10 \times 6 = 60mm$

$$Z_1 = \frac{d_1}{m} = \frac{182}{6} = 30.3 = 32$$

$$Z_2 = 32 \times 4.5 = 144$$

$$d_1 = m \times Z_1 = 6 \times 32 = 192 \text{ mm}$$

$$d_2 = m \times Z_2 = 6 \times 144 = 864 \text{ mm}$$

$$a = \frac{192 + 864}{2} = \underline{\underline{528 \text{ mm}}}$$

UNIT-V

-: HELICAL-GEAR DRIVES :-

Indroduction:-

Helical Gears are the modified form of spur gears. in which all the teeth are cut at constant angle, known as helix angle, to the axis of the gear, where as in spur gear, teeth are cut parallel to the axis. Helical gears are also employed to transmit power b/w parallel shafts. Because of the inclined structure of teeth, more than one pair of teeth will be in engagement and hence the *inclined structure* the operation may be smooth due to their gradual contact and more power can be transmitted at higher speeds than spur-gear drive.

In Helical gear drive, one of the gears has a right-Hand helix and the mating gear has a left-hand helix.

Types of Helical Gears:-

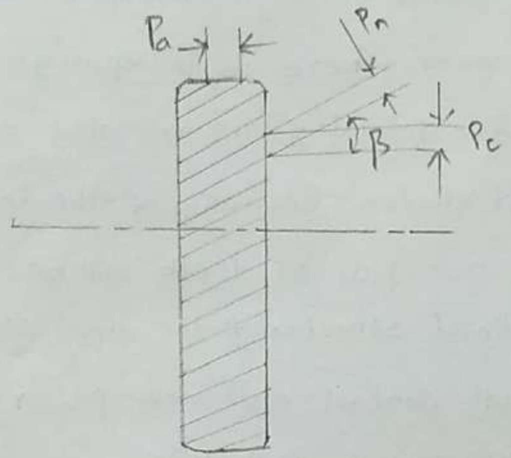
(i) Single-Helical type

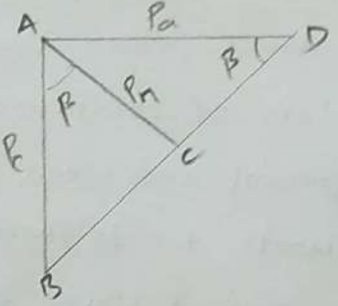
(ii) Double-Helical type. (Herring-Bone Gears)

Double-Helical gears are having both sets of teeth (i.e left hand, and right hand helix) cut on a single Blank. In single helical gears, some axial thrust will be produced b/w the teeth which is a disadvantage. This deficiency may be rectified in double-helical-gears. It is equivalent to two single helical gears in which equal and opposite thrusts are applied on each gear and resulting axial thrust is zero.

Crossed helical gear-drive (or) skew gear-drive which can be operated to transmit power b/w non-parallel and non-intersecting shafts.

Design formulas:





Its designing method is almost same with spur gears with slight modification. Consider a helical gear. Whose teeth are cut at the helix angle(β)

1. Circular pitch: which is the distance b/w the corresponding points of successive teeth, measured along the plane $\perp$er to the axis of gear. (P_c)

2. Norman Pitch (P_n):- Which is the distance b/w two successive teeth measured along the plane $\perp$er to teeth, This plane is at the angle of ($90+\beta$) degress to the axis of gear.

3. Axial pitch (P_a):- Which is distance b/w two succesive teeth measured along the plane parallel to axes of the gear,

Δ^{le} ABC

$$\cos\beta = \frac{P_n}{P_c}$$

$$P_n = P_c \cos\beta$$

$$P_n = \pi m \cos\beta \qquad (\because P_c = \pi m) \quad \left(m = \frac{d}{Z}\right)$$

$$P_n = \pi m_n \qquad \left(\because m_n = m\cos\beta \text{ (or) } m = \frac{m_n}{\cos\beta}\right)$$

Δ^{le} ABD

$$\sin\beta = \frac{P_n}{P_a}$$

$$P_a = \frac{P_n}{\sin\beta} = \frac{P_c\cos\beta}{\sin\beta} = \frac{P_c}{\tan\beta}.$$

$$m = \frac{d_1}{Z_1}$$

$$d_1 = mZ_1 = \frac{m_n Z_1}{\cos\beta}$$

$$d_2 = mZ_2 = \frac{m_n Z_2}{\cos\beta}$$

$$a = \frac{d_1 + d_2}{2}$$

$$a = \frac{\frac{m_n Z_1}{\cos\beta} + \frac{m_n Z_2}{\cos\beta}}{2}$$

$$a = \frac{m_n}{\cos\beta}\left(\frac{Z_1 + Z_2}{2}\right)$$

The usual value of helix angle ranges from 8° to 25° for single helical gears and 25° to 40° for double-Helical gears. For design of herringbone-gears. We can consider that the power transmitted by each portion is half of the total power.

Equivalent spur Gear and virtual Number of teeth:

In order to simply the design of helical gear, the concept of Equivalent spur gear can be adopted. The Equivalent spur gear is an imaginary spur gear which is having Equal strength and power transmitting capacity with actual Existing helical gear. This Equivalent spur gear may be considered for calculating the parameters of helical gear by slightly modifying the spur-gear formula. One of such modification is the number of teeth required for the Equivalent spur-gear called virtual number of teeth

$$\text{Virtual Number of teeth, } Z_v = \frac{Z}{\cos^3\beta}$$

Z = Actual number of teeth of Helical Gear

β = Helix angle

Force Analysis of Helical Gears:

Let F = Resultant (i.e Normal force)

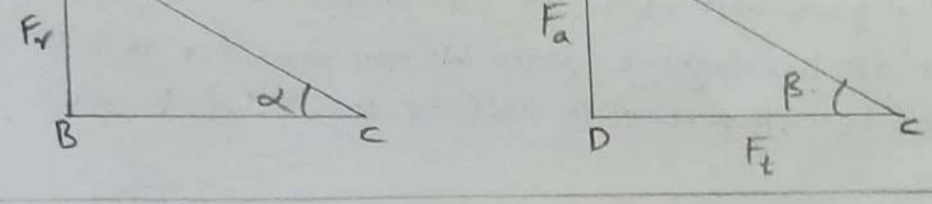
F_t = Tangential Component of normal force

F_r = Radial Component force.

F_a = Axial component of force.

α = pressure Angle

β = Helix Angle.



∴ From Δle ABC

$$\sin\alpha = \frac{F_r}{F}$$

$$\Rightarrow F_r = F\sin\alpha \quad — (1)$$

$$\vec{BC} = F\cos\alpha$$

Δle BDC

$$F_a = \vec{BC}\sin\beta = F\cos\alpha\sin\beta \quad — (2)$$

$$F_t = \vec{BC}\cos\beta = F\cos\alpha\cos\beta \quad — (3)$$

(1) ÷ (3)

$$F_r = F_t\left(\frac{\tan\alpha}{\cos\beta}\right)$$

(2) ÷ (3) $\quad F_a = F_t\tan\beta$

$$F_t = \frac{M_t}{d/2} \Rightarrow \text{where } M_t \text{ (or) } T = \frac{60 \times P}{2\pi N}$$

P = power

N = speed in rpm

d = pitch circle diameter.

Design of Helical Gear By "AGMA" Method:

According to Lewis Equation, the Beam strength of Helical gear tooth is given by.

$$F_b = [\sigma_b].b.\pi m_n.Y_v$$

$[\sigma_b]$ = Allowable contact stress in N/mm^2

b = Face width of gear blank = $10m_n$

m_n = Normal Module which must be Standardized.

Y_v = Lewis form factor, which depends on the Virtual Number of teeth

$$Z_v = \frac{Z}{\cos^3\beta}$$

For safe working, the Beam strength should be greater the design tooth load F_D is.

$$F_D = F_t \times k_s \times C_v = \frac{P \times k_s \times C_v}{V}$$

the values of k_s, C_v, V etc are calculated similar to Spur Gears.

The dynamic load acting on helical gear tooth may be found out using Buckingham Equation as.

$$F_d = F_t + \frac{21V(cb\cos^2\beta + F_t)\cos\beta}{21V + \sqrt{cb\cos^2\beta + F_t}}$$

Wear tooth load is given by.

$$F_w = \frac{d_1 b Q k_w}{\cos^2\beta}$$

The values of Q & k_w etc. are all Common with Spur Gears.

A pair of parallel Helical gears consists of a 20 teeth pinion meshing with a 100 teeth gear. The pinion rotates at 720 rpm. The normal pressure angle is 20°. While the helix angle is 25°. The face width is 40mm and normal Module is 4mm. The pinion as well as gear is made of steel 40C8 ($\sigma_{ut} = 600 N/mm^2$) and heat treated to a surface Hardness of 300BHN. The service factor and factor of safety are 1.5 and 2 respectively. Assume that the velocity factor accounts for the dynamic load and calculate the power transmitting capacity of Gears.

Given.

$Z_1 = 20$ | $m_n = 4\,mm$

$Z_2 = 100$ | $\sigma_{ut} = 600\,N/mm^2$

$n_1 = 720\,rpm$ | $\sigma_{b_1} \& \sigma_{b_2} = \frac{600}{3} = 200\,N/mm^2$

$\alpha = 20°$ | $BHN = 300$

$\beta = 25°$ | $k_s = 1.5$

$b = 40\,mm$ | $FOS = 2.$ & $P = ?$

$$i = \frac{Z_2}{Z_1} = \frac{100}{20} = 5$$

$$V = \frac{\pi d_1 n_1}{60 \times 10^3} = \frac{\pi \times m Z \times n_1}{60 \times 10^3} = \frac{\pi \times 4.41 \times 20 \times 720}{60 \times 10^3} = 3.328\,m/sec.$$

$$m_n = m\cos\beta \Rightarrow m = \frac{m_n}{\cos\beta} = \frac{4}{\cos 25} = 4.41\,mm$$

$$F_b = \sigma_b \times b \times \pi m_n \times Y_v$$

$$Z_v = \frac{Z}{\cos^3\beta} = \frac{20}{\cos^3 25} = 26.87 \simeq 27$$

$$Y_v = 0.154 - \frac{0.912}{Z_v}$$

$$= 0.154 - \frac{0.912}{27}$$

$$Y_v = 0.120$$

$\therefore F_b = 200 \times 40 \times \pi \times 4 \times 0.120$

$F_b = 12063.71\ N$

Wear strength:

$$F_w = \frac{d_1 b Q k_w}{\cos^2\beta}$$

$d_1 = mZ_1 = 4.41 \times 20 = 88.2\ mm$

$b = 40mm$

$$Q = \frac{2Z_2}{Z_2+Z_1} = \frac{2 \times 100}{100+20} = 1.667$$

$$k_w = \frac{\sigma_e^2 \sin\alpha}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

$\sigma_e = 2.8 \times BHN - 70$

$\sigma_e = 2.8 \times 300 - 70 = 770\ N/mm^2$

$$k_w = \frac{(770)^2 \sin 20}{1.4}\left[\frac{1}{200 \times 10^3} + \frac{1}{200 \times 10^3}\right]$$

$k_w = 1.44\ N/mm^2$

$$F_w = \frac{88.2 \times 40 \times 1.667 \times 1.44}{\cos^2 25} = 10318.58\ N$$

Since $F_b > F_w$

$FoS = \frac{F_b}{F_D} \Rightarrow F_b = 2 \times F_D$

$12063.71 = 2 \times \frac{P}{V} \times K_s \times C_V$

$12063.71 = 2 \times \frac{P}{3.328} \times 1.5 \times \left[\frac{3+V}{3}\right]$

$12063.71 = 2 \times \frac{P}{3.328} \times 1.5 \times \left(\frac{3+3.28}{3}\right)$

$P = 6.344\ kW$

(t)

A pair of Helical Gears are to trasmit 15 kw. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 10,000 r.p.m and has 80 mm pitch diameter. The gear has 320 rpm pitch dia. If the gears are made of cast steel having allowable static strength of 100 MPa: Determine a suitable Module & face width from static strength considerations & check the gears for wear ($\sigma_{es} = 618$ MPa).

Given

$P = 15\,kW = 15\times10^3\,W$

$\beta = 45°$

$\alpha = 20°$

$N_1 = 10000$ rpm

$d_1 = 80$ mm

$d_2 = 320$ mm

$\sigma_{b_1} = \sigma_{b_2} = 100\ N/mm^2$

$\sigma_{es} = 618\ N/mm^2$

$$V = \frac{\pi d_1 n_1}{60\times10^3} = \frac{\pi\times80\times10000}{60\times10^3} = 42\ m/sec.$$

$$F_D = \frac{P\times K_s\times C_v}{V}$$

$$C_v = \frac{5.5+\sqrt{V}}{5.5} = 2.178$$

$$F_D = \frac{15\times10^3\times1\times2.178}{42} = 777.85\ N$$

$$F_b = \sigma_b\times b\times\pi m_n\times Y_v$$

$$Z_v = \frac{Z}{\cos^3\beta} = \frac{80}{\cos^3 45} = \frac{226.4}{m}$$

$$Y_v = 0.175 - \frac{0.841}{Z_v}$$

$$Y_v = 0.175 - \frac{0.841}{\frac{226.4}{m}}$$

$Y_v = 0.175 - \frac{0.841m}{226.4}$

$b = 10m_n = 10\,m\cos\beta = 10\cos 45 \times m = 7.071\,m$

$F_b = \sigma_b \times b \times \pi m_n \times Y_v$

$= 100 \times 7.071 \times m \times \pi \times m\cos 45 \times \left(0.175 - \frac{0.841m}{226.4}\right)$

$F_b = 1570.78\,m^2\,(0.175 - 0.0037\,m)$

$F_b > F_D$

$777.85 = 1570.78\,m^2\,(0.175 - 0.0037m)$

$m = 1.71$

$m \cong 2\ mm$

$m_n = m\cos\beta = 2\cos 45 = 1.414\ mm$

$b = 10\,m_n = 10 \times 1.414 = 14.14\ mm$

$k_w = \frac{\sigma_e^2 \sin\alpha}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$

$= \frac{(618)^2 \sin 20}{1.4}\left[\frac{1}{200 \times 10^3} + \frac{1}{200 \times 10^3}\right]$

$k_w = 0.9330\ N/mm^2$

$Q = \frac{2 \times i}{i+1} = \frac{2 \times 4}{4+1} = 1.6$

$i = \frac{d_2}{d_1} = \frac{320}{80} = 4$

$F_w = \frac{d_1 b Q k}{\cos^2\beta} = \frac{80 \times 14.14 \times 1.6 \times 0.933}{\cos^2 45} = 3377.310\,N$

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